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p-n junction

- ❖ The p-n junction is the basic element of all bipolar devices. Its main electrical property is that it rectifies (allow current to flow easily in one direction only). The p-n junction is often just called a DIODE. Applications;
 - >photodiode, light sensitive diode,
 - >LED- ligth emitting diode,
 - >varactor diode-variable capacitance diode

The formation of p-n junction:

- The p-n junction can be formed by pushing a piece of p-type silicon into close contact with a piece of n-type silicon. But forming a p-n junction is not so simply. Because;
- 1) There will only be very few points of contact and any current flow would be restricted to these few points instead of the whole surface area of the junction.
- 2) Silicon that has been exposed to the air always has a thin oxide coating on its surface called the "native oxide". This oxide is a very good insulator and will prevent current flow.
- Bonding arrangement is interrupted at the surface; dangling bonds.

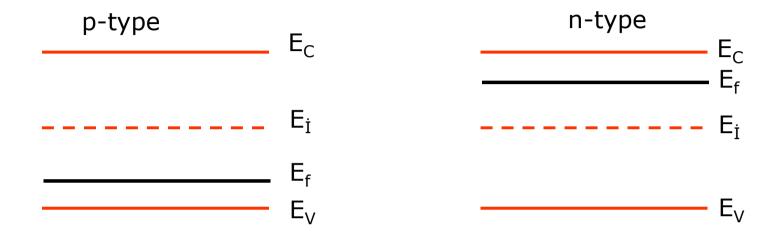
Surface states

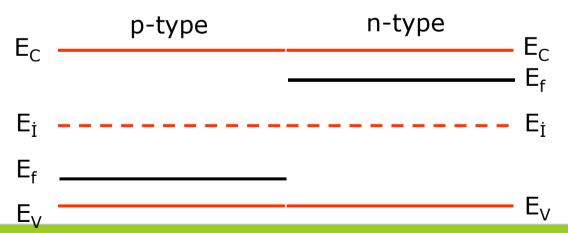
To overcome these surface states problems



p-n junction can be formed in the bulk of the semiconductor, away from the surface as much as possible.

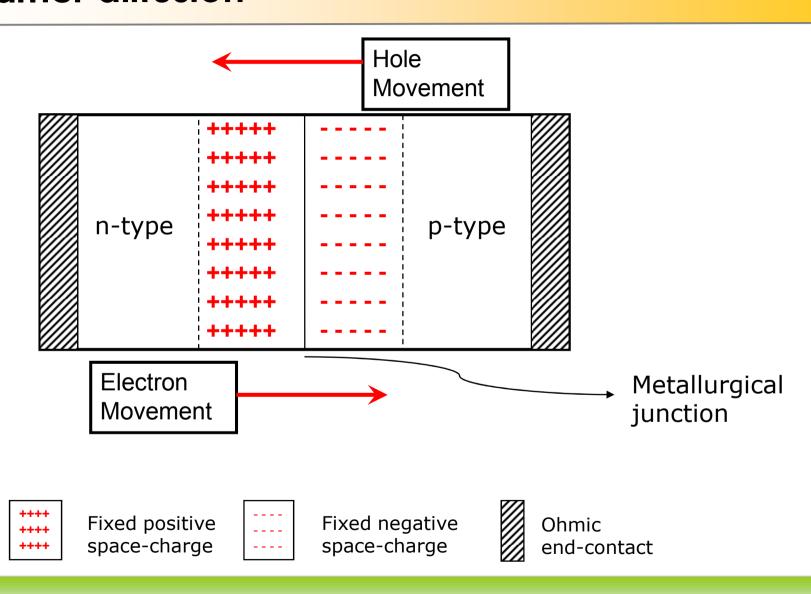
p-n junction





There is a big discontinuity in the fermi level accross the p-n junction.

Idealized p-n junction; recombination of the carrier and carrier diffusion



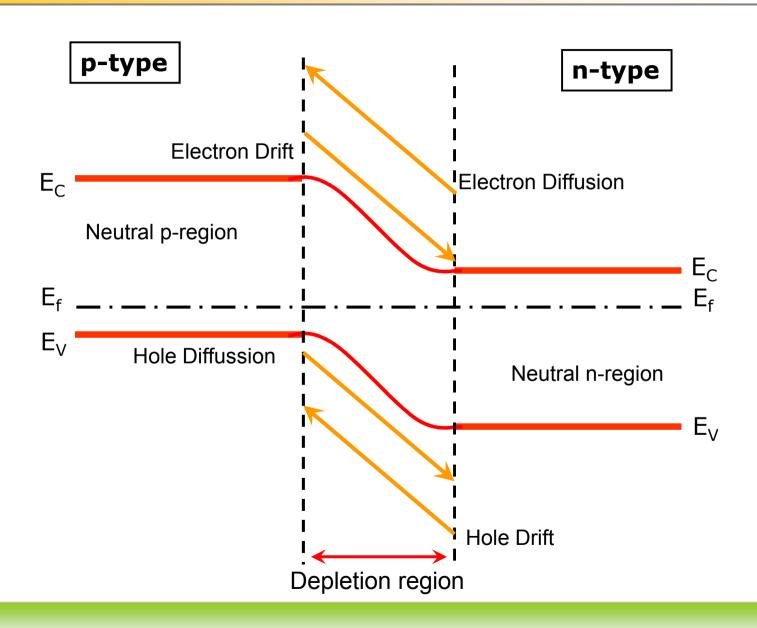
p-n junction

- ❖ Lots of electrons on the left hand side of the junction want to diffuse to the right and lots of holes on the right hand side of the junction want to move to the left.
- ❖ The donors and acceptors fixed,don't move (unless you heat up semiconductors, so they can diffuse) because they are elements (such as arsenic and boron) which are incorporated to lattice.
- However, the electrons and holes that come from them are free to move.

Idealized p-n junction

- Holes diffuse to the left of the metalurgical junction and combine with the electrons on that side. They leave behind negatively charged acceptor centres.
- Similarly, electrons diffusing to the right will leave behind positively charged donor centres. This diffusion process can not go on forever. Because, the increasing amount of fixed charge wants to electrostatically attract the carriers that are trying to diffuse away(donor centres want to keep the electrons and acceptor centres want to keep the holes). Equlibrium is reached.
- This fixed charges produce an electric field which slows down the diffusion process.
- This fixed charge region is known as depletion region or space charge region which is the region the free carriers have left.
- It is called as depletion region since it is depleted of free carriers.

Energy level diagram for the p-n junction in thermal equilibrium



Thermal equilibrium; no applied field; no net current flow

$$J_{p} = J_{p}(drift) + J_{p}(diffusion) = 0$$
 (1)

 J_p is the hole current density $(\frac{A}{2})$

Drift current is due to electric field at the junction; minority carriers.

$$J_p = q\mu_p p E_x - q D_p \frac{dp}{dx} = 0 \quad (2)$$

where

$$E_{x} = \frac{1}{q} \frac{dE_{i}}{dx} \qquad D_{p} = \frac{\mu_{p} kT}{q} \quad (Einstein \ relation)$$

Diffusion current due the to concentration majority gradient; carriers.

Proof

$$J_{p} = \mu_{p} \left(p \frac{dE_{i}}{dx} - kT \frac{dp}{dx} \right) = 0$$
 (3)

$$p = n_i \exp\left(\frac{E_i - E_f}{kT}\right) \Rightarrow \frac{dp}{dx} = \frac{p}{kT}\left(\frac{dE_i}{dx} - \frac{dE_f}{dx}\right)$$

$$J_{p} = \mu_{p} p \frac{dE_{f}}{dx} = 0$$
 (4)

we conclude that $\frac{dE_f}{dx} = 0$ which states that the Fermi Level is a CONSTANT at equilibrium.

$$J_n = \mu_n n \frac{dE_f}{dx} = 0 \qquad (5)$$

Proof

- ❖ The drift and diffusion currents are flowing all the time. But, in thermal equilibrium, the net current flow is zero since the currents oppose each other.
- Under non-equilibrium condition, one of the current flow mechanism is going to dominate over the other, resulting a net current flow.
- The electrons that want to diffuse from the ntype layer to the p-layer have potential barier.

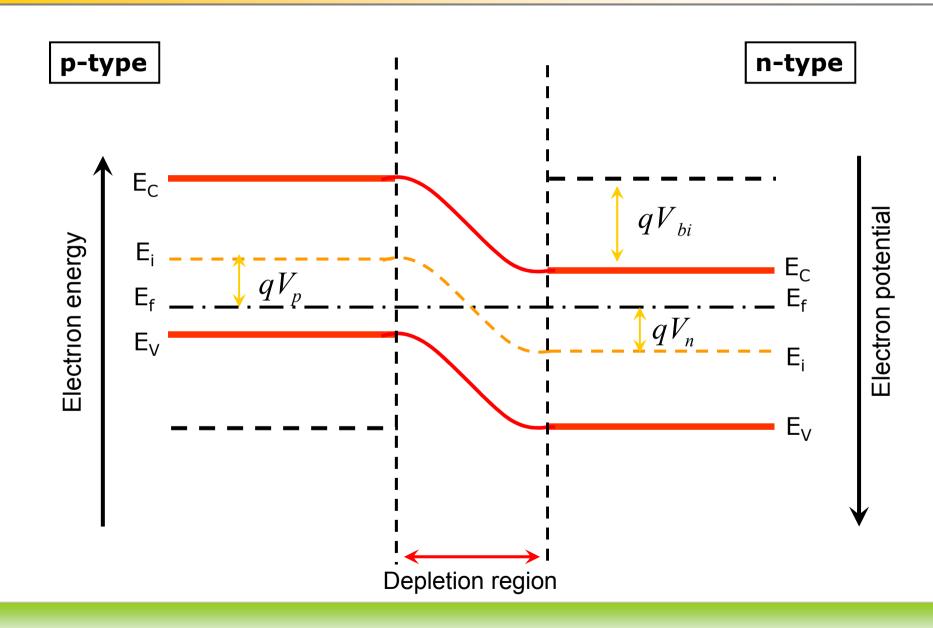
p – **n** junction barrier height, V_{bi}

- * The potential barrier height V_{bi} accross a p-n junction is known as the built in potential and also as the junction potential.
- The potential energy that this potential barrier correspond is

$$qV_{bi}$$

- Electron energy is positive upwards in the energy level diagrams, so electron potentials are going to be measured positive downwards.
- The hole energies and potentials are of course positive in the opposite directions to the electrons

p – n junction barrier height



The p – n junction barrier height

The intrinsic Fermi Level is a very useful reference level in a semiconductor.

$$qV_p = (E_i - E_f) \quad (1) \qquad p = n_i \exp\left(\frac{E_i - E_f}{kT}\right)$$

$$\left|V_{p}\right| = \frac{kT}{q} \ln \frac{N_{A}}{n_{i}} \quad (2)$$

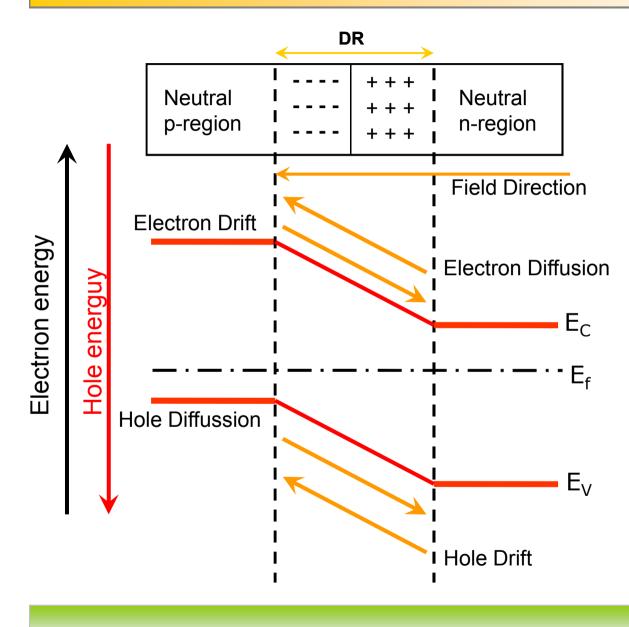
Similarly for $|V_n|$

$$|V_n| = \frac{kT}{q} \ln \frac{N_D}{n_i}$$
 (3)

For full ionization, the built - in voltage is a sum of

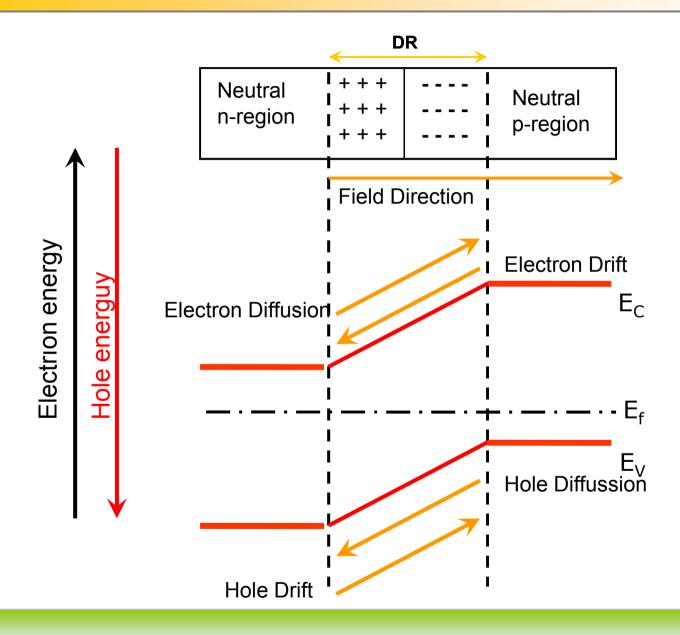
$$\left|V_{bi}\right| = \left|V_{n}\right| + \left|V_{p}\right| = \frac{kT}{q} \ln \frac{N_{A}N_{D}}{n_{i}^{2}} \qquad (4)$$

p – n junction in thermal equilibrium



- Current Mechanisms,
- Diffusion of the carriers cause an electric in DR.
- Drift current is due to the presence of electric field in DR.
- Diffusion current is due to the majority carriers.
- Drift current is due to the minority carriers.

n – p junction at equilibrium



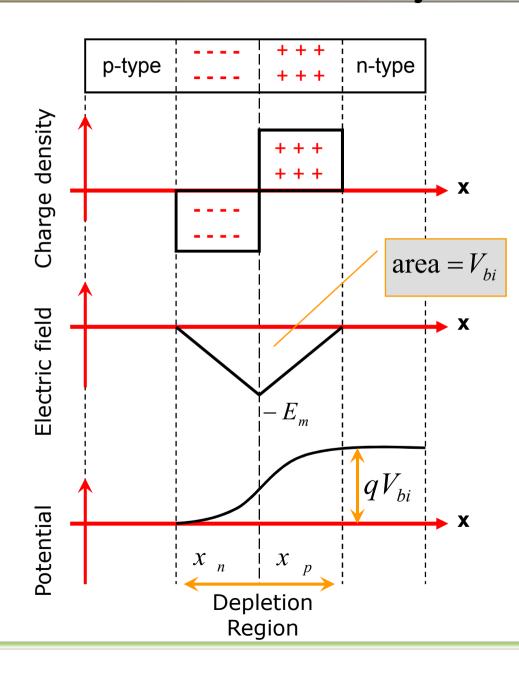
Diffusion:

- ❖ When electrons and holes are diffusing from high concentration region to the low concentration region they both have a potential barrier. However, in drift case of minority carriers there is no potential barrier.
- Built in potential;

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

At fixed T, V_{bi} is determined by the number of N_A and N_D atoms.

Depletion Approximation, Electric Field and Potential for pn junction



- At equilibrium, there is no bias, i.e. no applied voltage.
- The field takes the same sign as the charge
- The sign of the electric field is opposite to that of the potential;

$$E_{v} = -\frac{dV_{n}}{dx}$$

Depletion Approximation, Electric Field and Potential for pn junction

- Charge density is negative on p-side and positive on n-side.
- As seen from the previous diagram, the charge distribution is very nice and abrupt changes occur at the depletion region (DR) edges. Such a junction is called as an <u>abrupt</u> <u>junction</u> since the doping abruptly changes from p- to n-type at the metallurgical junction (ideal case).

$$x_n \rightarrow$$
 the width of the DR on n-side

 $x_p \rightarrow$ the width of the DR on p-side

Depletion Approximation, Electric Field and Potential for pn junction

❖ In reality, the charge distribution tails-off into the neutral regions, i.e. the charge distrubition is not abrupt if one goes from depletion region into the neutral region. This region is called as a transition region and since the transition region is very thin, one can ignore the tail-off region and consider the change being abrupt. So this approximation is called as DEPLETION APPROXIMATION.

Depletion Approximation, Electric Field and Potential for pn junction

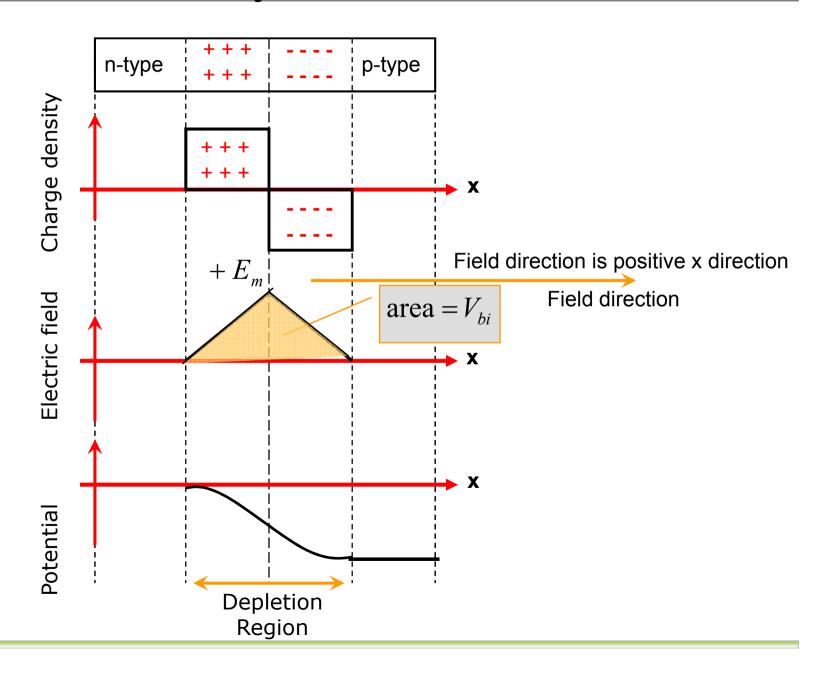
Electric Field Diagram:

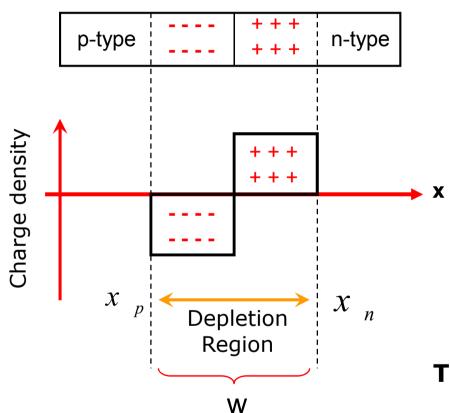
❖ The electric field is zero at the edge of the DR and increases in negative direction. At junction charge changes its sign so do electric field and the magnitude of the field decreases (it increases positively).

Potential Diagram:

❖ Since the electric field is negative through the whole depletion region ,DR, the potential will be positive through the DR. The potential increases slowly at left hand side but it increases rapidly on the right hand side. So the rate of increase of the potential is different an both sides of the metallurgical junction. This is due to the change of sign of charge at the junction.

Depletion Approximation, Electric Field and Potential for np junction





 The amount of uncovered negative charge on the left hand side of the junction must be equal to the amount of positive charge on the right hand side of the metalurgical junction. Overall space-charge neutrality condition;

$$N_A x_p = N_D x_n$$



The higher doped side of the junction has the narrower depletion width

when
$$N_A = N_D \Rightarrow x_n = x_p$$

* x_n and x_p is the width of the depletion layer on the n-side and p-side of the junction, respectively.

When
$$N_D >> N_A$$
 (unequal impurity concentrations) and $x_p >> x_n$, $W \cong x_p$

Unequal impurity concentration results an unequal depletion layer widths by means of the charge neutrality condition;

$$N_A.x_p = N_D.x_n$$

W = total depletion region

When
$$N_A \gg N_D \implies x_n \gg x_p \implies W \cong x_n$$

Depletion layer widths for n-side and p-side

$$x_n = \frac{1}{N_D} \sqrt{\frac{2\varepsilon_{Si} V_{bi} N_A N_D}{q(N_A + N_D)}}$$

$$x_{p} = \frac{1}{N_{A}} \sqrt{\frac{2\varepsilon_{Si} V_{bi} N_{A} N_{D}}{q(N_{A} + N_{D})}}$$

For equal doping densities $W = x_n + x_p$

Total depletion layer width, W

$$W = \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \sqrt{\frac{2 \varepsilon_{Si} V_{bi} N_A N_D}{q \left(N_A + N_D\right)}}$$

$$W = \sqrt{\frac{2 \varepsilon_{Si} V_{bi} (N_A + N_D)}{q N_A N_D}}$$

$$\mathcal{E}_{Si} = \mathcal{E}_o \mathcal{E}_r$$

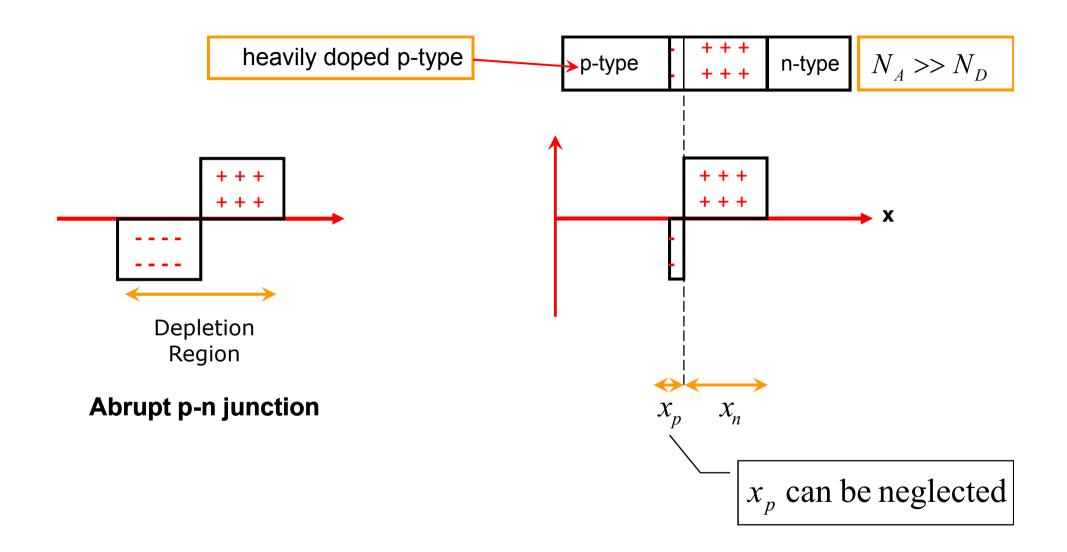
$$\varepsilon_o = \text{permittivity of vacuum} = 8.85 \times 10^{-12} \, F/m$$

$$\varepsilon_r$$
 = relative permittivity of Silicon = 11.9

 x_n, x_p and W depends on N_A, N_D and V_{bi}

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

One-Sided abrupt p-n junction



One-Sided abrupt p-n junction

$$W \cong x_n = \frac{1}{N_D} \sqrt{\frac{2\varepsilon_{Si} V_{bi} N_A N_D}{q \left(N_A + N_D\right)}} = \frac{1}{N_D} \sqrt{\frac{2\varepsilon_{Si} V_{bi} N_A N_D}{q N_A}}$$

$$\underset{\text{since N}_A >> N_D}{\text{neglegted}}$$

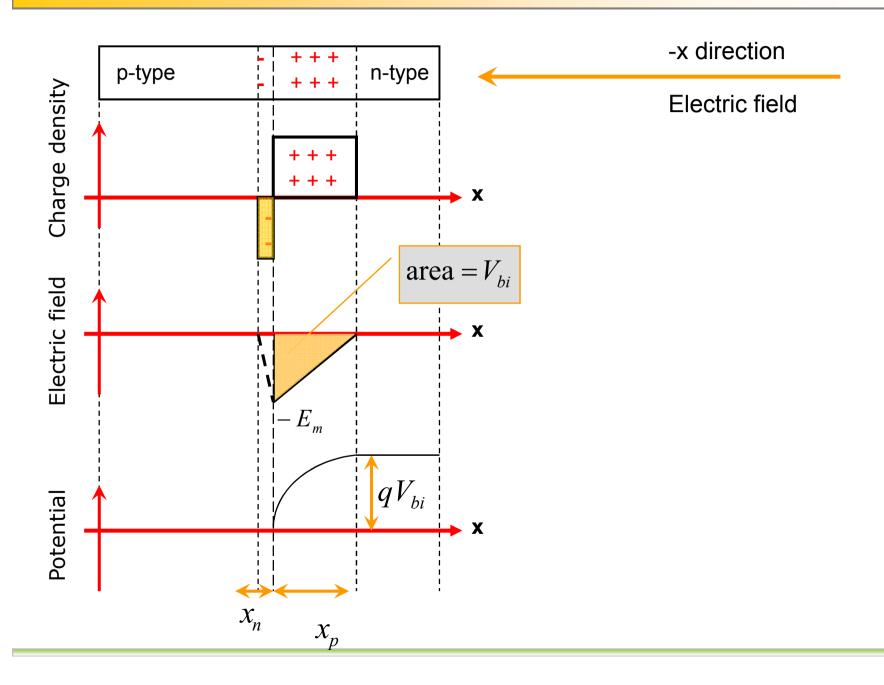
$$W \cong \sqrt{\frac{2\varepsilon_{Si}V_{bi}}{qN_D}}$$

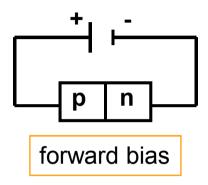
obtain a similar equation for $W \cong x_p$

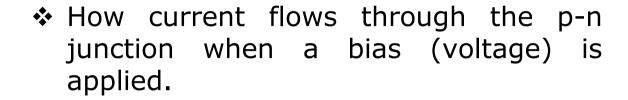
in the case of $N_D >> N_A$

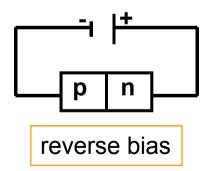
One-sided abrupt junction

One-Sided abrupt p-n junction

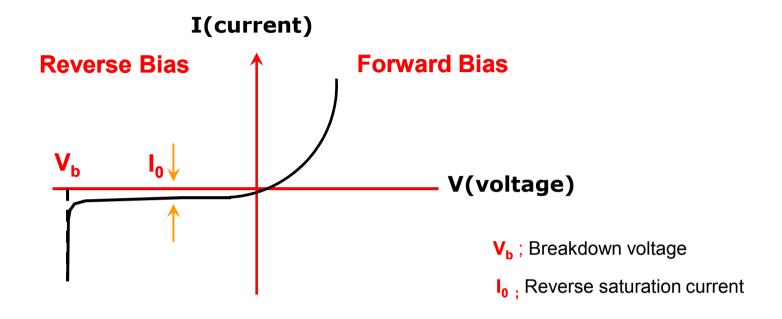




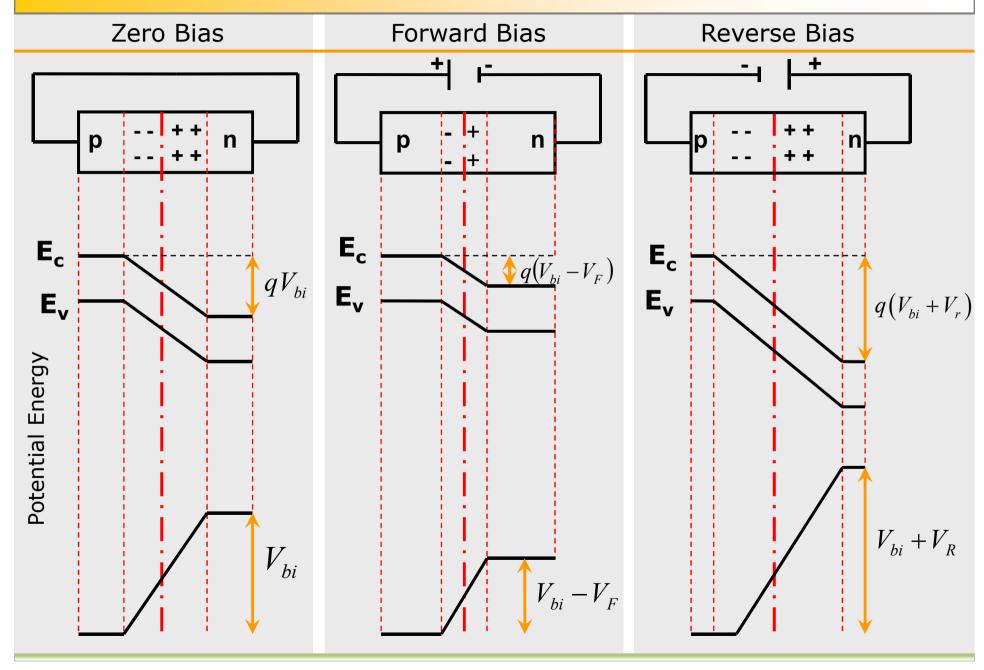




❖ The current flows all the time whenever a voltage source is connected to the diode. But the current flows rapidly in forward bias, however a very small constant current flows in reverse bias case.



- ❖ There is no turn-on voltage because current flows in any case. However, the turn-on voltage can be defined as the forward bias required to produce a given amount of forward current.
- ❖ If 1 m A is required for the circuit to work, 0.7 volt can be called as turn-on voltage.



$$V_F \rightarrow$$
 forward voltage $V_R \rightarrow$ reverse voltage

When a voltage is applied to a diode, bands move and the behaviour of the bands with applied forward and reverse fields are shown in previous diagram.

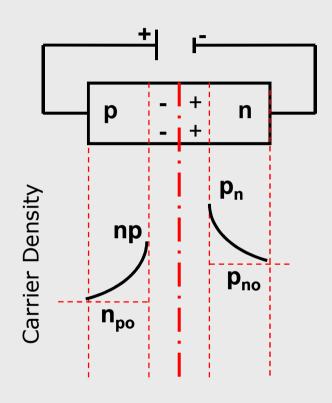
Forward Bias

- Junction potential reduced
- Enhanced hole diffusion from p-side to n-side compared with the equilibrium case.
- Enhanced electron diffusion from n-side to p-side compared with the equilibrium case.
- Drift current flow is similar to the equilibrium case.
- Overall, a large diffusion current is able to flow.
- Mnemonic. Connect positive terminal to p-side for forward bias.
- ❖ Drift current is very similar to that of the equilibrium case. This current is due to the minority carriers on each side of the junction and the movement minority carriers is due to the built in field accross the depletion region.

Reverse Bias

- Junction potential increased
- ❖ Reduced hole diffusion from p-side to n-side compared with the equilibrium case.
- Reduced electron diffusion from n-side to p-side compared with the equilibrium case
- Drift current flow is similar to the equilibrium case.
- Overall a very small reverse saturation current flows.
- Mnemonic. Connect positive terminal to n-side for reverse bias.

Qualitative explanation of forward bias



p-n junction in forward bias

- ❖ Junction potential is reduced from V_{bi} to V_{bi}-V_F
- ❖ By forward biasing a large number of electrons are injected from n-side to p-side accross the depletion region and these electrons become minority carriers on p-side, and the minority recombine with majority holes so that the number of injected minority electrons decreases (decays) exponentially with distance into the p-side.

Qualitative explanation of forward bias

- ❖ Similarly, by forward biasing a large number of holes are injected from p-side to n-side across the DR. These holes become minority carriers at the depletion region edge at the n-side so that their number (number of injected excess holes) decreases with distance into the neutral n-side.
- In summary, by forward biasing in fact one injects minority carriers to the opposite sides. These injected minorites recombine with majorities.

Qualitative explanation of forward bias

- How does current flow occur if all the injected minorities recombine with majorities?
- If there is no carrier; no current flow occurs.
- Consider the role of ohmic contacts at both ends of p-n junction.
- The lost majority carriers are replaced by the majority carriers coming in from ohmic contacts to maintain the charge neutrality.
- ❖ The sum of the hole and electron currents flowing through the ohmic contacts makes up the total current flowing through the external circuit.

"o" subscript denotes the equilibrium carrier concentration.

 $n_{no} \rightarrow$ equilibrium electron concentration in n - type material.

 $n_{po} \rightarrow$ equilibrium electron concentration in p - type material.

 $p_{po} \rightarrow$ equilibrium hole concentration in p - type material.

 $p_{no} \rightarrow$ equilibrium hole concentration in n - type material.

$$n.p = n_i^2$$

$$n.p = n_i^2$$

At equilibrium case (no bias)

$$n_{no}.p_{no} = n_i^2 \rightarrow n - type \ material$$
 (1)

$$n_{po}.p_{po} = n_i^2 \rightarrow p - type \ material$$
 (2)

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$
 assuming full ionization

$$N_A \cong p_{po}$$
; $N_D \cong n_{no}$ majority carriers

$$V_{bi} = \frac{kT}{q} \ln \frac{p_{po}.n_{no}}{n_i^2} \qquad n_{no}.p_{no} = n_i^2 \text{ for n-type}$$

$$V_{bi} = \frac{kT}{q} \ln \frac{p_{po}}{p_{no}} \Longrightarrow p_{po} = p_{no} \exp\left(\frac{qV_{bi}}{kT}\right) \quad (3)$$

Similarly, from equation (2)

$$V_{bi} = \frac{kT}{q} \ln \frac{p_{po}.n_{no}}{n_i^2}$$
 $n_{po}.p_{po} = n_i^2$

$$V_{bi} = \frac{kT}{q} \ln \frac{n_{no}}{n_{po}} \Rightarrow n_{no} = n_{po} \exp\left(\frac{qV_{bi}}{kT}\right) \quad (4)$$

This equation gives us the equilibrium majority carrier concentration.

What happens when a voltage appears across the p-n junction?

- Equations (3) and (4) still valid but you should drop (0) subscript and change V_{bi} with
- i. $V_{bi} V_{F}$ if a forward bias is applied.
- ii. $V_{bi} + V_{R}$ if a reverse bias is applied.

V_F: forward voltage

V_R : reverse voltage

With these biases, the carrier densities change from equilibrium carrier densities to non- equilibrium carrier densities.

Non-equilibrium majority carrier concentration in forward bias;

$$p_{p} = p_{n} e x p \left(\frac{q (V_{bi} - V_{F})}{k T} \right)$$

For example; n_n for reverse bias

$$n_n = n_p e x p \left(\frac{q (V_{bi} + V_R)}{k T} \right)$$

• When a voltage is applied; the equilibrium n_{no} changes to the non equilibrium n_{no}

Assumption; low level injection

• For low level injection; the number of injected minorities is much less than the number of the majorities. That is the injected minority carriers do not upset the majority carrier equilibrium densities.

$$n_n \approx n_{no}$$
 $p_p \approx p_{po}$

 Non equilibrium electron concentration in n-type when a forward bias is applied ,

$$n_n = n_p \exp\left(\frac{q(V_{bi} - V_F)}{kT}\right)$$
 non-equilibrium.

$$n_n \approx n_{no}$$
 $n_{no} = n_p \left(\frac{q(V_{bi} - V_F)}{kT} \right)$ (5)

$$n_{no} = n_{po} \exp\left(\frac{qV_{bi}}{kT}\right) \quad (6)$$

combining (5) and (6)

$$n_p \exp\left[\frac{q(V_{bi} - V_F)}{kT}\right] = n_{po} \exp\left(\frac{qV_{bi}}{kT}\right)$$

❖ Solving for non-equilibrium electron concentration in p-type material, i.e. n_p

$$n_p = n_{po} \exp\left(\frac{qV_F}{kT}\right)$$
 and subtracting n_{po} from both sides

$$n_p - n_{po} = n_{po} \left[\exp\left(\frac{qV}{kT}\right) - 1 \right] = \delta_n$$

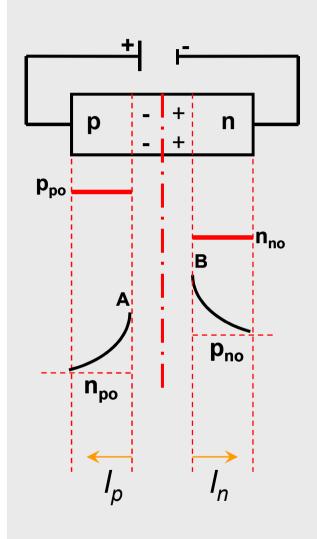
 δ_n = the excess concentration of minority electrons over the equilibrium concentration at the edge of the DR

Similarly,

$$p_n - p_{no} = p_{no} \left[\exp\left(\frac{qV}{kT}\right) - 1 \right] = \delta_p$$
 the non-equilibrium

 δ_p = the excess concentration of minority holes over the equilibrium concentration at the edge of the DR

Forward-bias diode; injection of minority carriers across DR



point
$$A \rightarrow n_p - n_{po}$$

point $B \rightarrow p_p - p_{po}$

- • l_p is the distance from DR edge into p-side
- • l_n is the distance from DR edge into n-side

When a forward bias is applied; majority carriers are injected across DR and appear as a minority carrier at the edge of DR on opposite side. These minorities will diffuse in field free opposite-region towards ohmic contact. Since ohmic contact is a long way away, minority carriers decay exponentially with distance in this region until it reaches to its equilibrium value.

Exponential decay of injected minority carriers on opposition sides

The excess injected minorities decay exponentially as

$$\delta p(l_p) = \delta n(0) \exp\left(-\frac{l_p}{L_n}\right)$$

$$\delta n(l_n) = \delta p(0) \exp \left(-\frac{l_n}{L_p}\right)$$

 L_n and L_p are diffusion lengths for electrons and holes

Number of Injected Minority Holes Across The Depletion Region

- By means of forward-biasing a p-n junction diode, the holes diffuse from left to right accross the DR and they become minority carriers.
- These holes recombine with majority electrons when they are moving towards ohmic constants.
- So, the number of minority holes on the n-region decreases exponentially towards the ohmic contact. The number of injected minority holes; excess holes;

$$\delta p(l_n) = \delta p(0) \exp(-\frac{l_n}{L_p})$$

Distance into then region from the Depletion Region

Diffusion Length for holes

Number of Injected Minority Holes Across The Depletion Region

point
$$A \rightarrow n_p - n_{po}$$

$$\delta n(l_p) = \delta n(0) \exp(-\frac{l_p}{L_n})$$

$$\delta n(0) = n_{po} \left[\exp(\frac{qV}{kT}) - 1 \right]$$

$$\delta p(0) = p_{no} \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]$$

 Similarly by means of forward biasing a p-n junction, the majority electrons are injected from right to left across the Depletion Region. These injected electrons become minorities at the Depletion Region edge on the p-side, and they recombine with the majority holes. When they move into the neutral pside, the number injected excess electrons decreases exponentially.

$$\delta_n(l_p) = \delta_n(0) \exp(-\frac{l_p}{L_n})$$

$$\delta_n(l_p) = n_{po} \left[\exp(\frac{qV}{kT}) - 1 \right] \cdot \exp(-\frac{l_p}{L_n})$$

Diffusion current density for electrons;

$$J_{n} = qD_{n}\frac{dn}{dx} = qD_{n}\frac{d}{dx}\delta n(l_{p}) = -q\frac{D_{n}n_{po}}{L_{n}}\left[\exp\left(\frac{qV}{kT}\right) - 1\right]\exp\left(-\frac{l_{p}}{L_{n}}\right)$$

$$J_n(l_p = 0) = \frac{-qD_n n_{po}}{L_n} \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]$$
 Minus sign shows that electron current

density is in opposite direction to increasing l_p . That is in the positive x direction. Similarly for holes;

$$J_{p}(l_{n}=0) = -qD_{p}\frac{dp}{dx} = \frac{qD_{p}p_{no}}{L_{p}}\left[\exp\left(\frac{qV}{kT}\right) - 1\right]$$

The total current density;

$$J_{Total} = J_n + J_p = q \left[\frac{D_n n_{po}}{L_n} + \frac{D_p p_{no}}{L_p} \right] \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]$$

$$J_{Total} = q \left[\frac{D_n n_{po}}{L_n} + \frac{D_p p_{no}}{L_p} \right] \left[\exp\left(\frac{qV}{kT}\right) - 1 \right] = J_o \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]$$

multiplying by area;

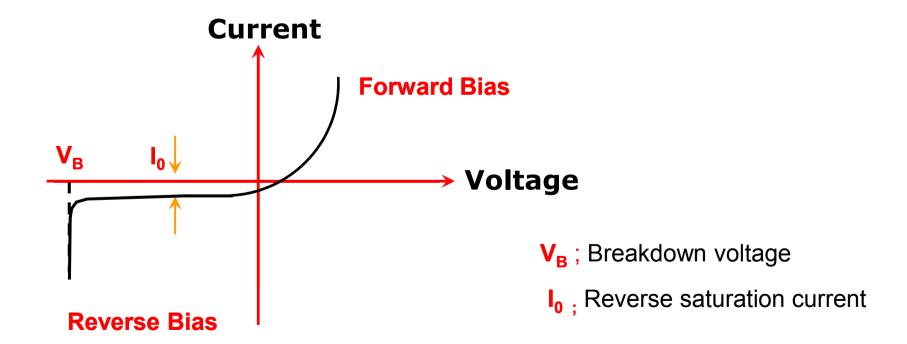
$$I = I_o \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]$$

Ideal diode equation

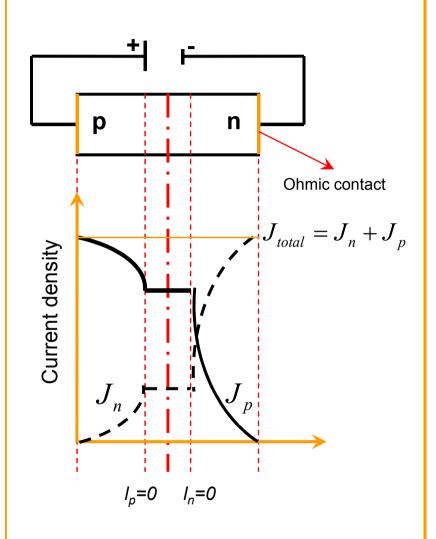
This equation is valid for both forward and reverse biases; just change the sign of V.

 Change V with -V for reverse bias. When qV>a few kT; exponential term goes to zero as

$$I = I_o \left[\exp \left(\frac{-qV}{kT} \right) - 1 \right] \qquad \qquad I = -I_o \qquad \qquad \text{Reverse saturation current}$$

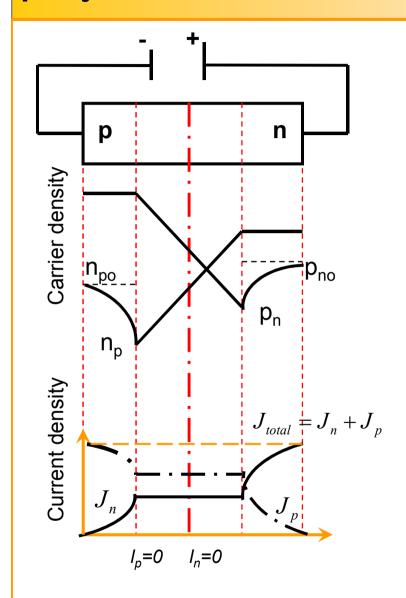


Forward bias current densities



- Minority current densities decreases exponentially into the the neutral sides whereas the current densities due to the majorities increase into the neutral sides.

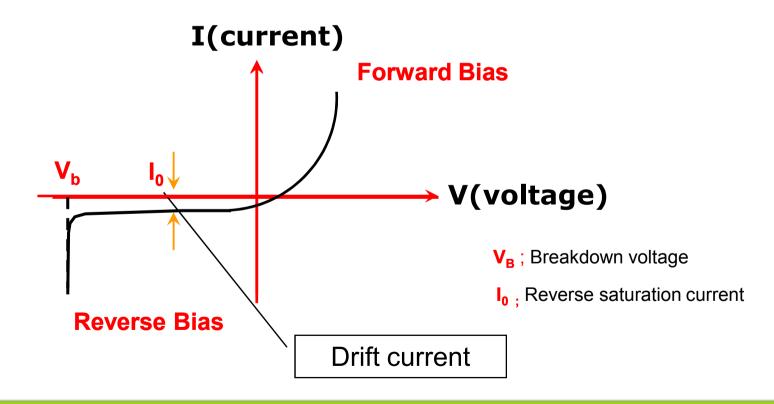
p-n junction in reverse bias



- Depletion region gets bigger with increasing reverse bias.
- Reverse bias prevents large diffusion current to flow through the diode.
- However; reverse bias doesn't prevent the small current flow due to the minority carrier. The presence of large electric field across the DR extracts almost all the minority holes from the nregion and minority electrons from the p-region.
- This flow of minority carriers across the junction constitudes I_{0} , the reverse saturation current.
- These minorities are generated thermally.

p-n junction in reverse bias

 The flow of these minorities produces the reverse saturation current and this current increases exponentially with temperature but it is independent of applied reverse voltage.



Junction breakdown or reverse breakdown

- An applied reverse bias (voltage) will result in a small current to flow through the device.
- At a particular high voltage value, which is called as breakdown voltage V_B , large currents start to flow. If there is no current limiting resistor which is connected in series to the diode, the diode will be destroyed. There are two physical effects which cause this breakdown.
- **1) Zener breakdown** is observed in highly doped p-n junctions and occurs for voltages of about 5 V or less.
- **2) Avalanche breakdown** is observed in less highly doped p-n junctions.

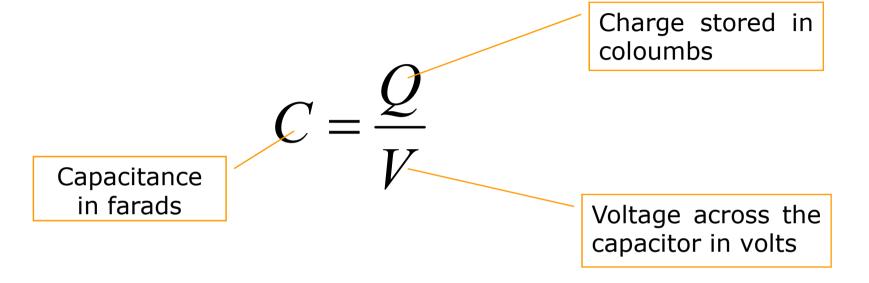
Zener breakdown

- Zener breakdown occurs at highly doped p-n junctions with a tunneling mechanism.
- In a highly doped p-n junction the conduction and valance bands on opposite side of the junction become so close during the reverse-bias that the electrons on the p-side can tunnel from directly VB into the CB on the n-side.

Avalanche Breakdown

- Avalanche breakdown mechanism occurs when electrons and holes moving through the DR and acquire sufficient energy from the electric field to break a bond i.e. create electron-hole pairs by colliding with atomic electrons within the depletion region.
- The newly created electrons and holes move in opposite directions due to the electric field and thereby add to the existing reverse bias current. This is the most important breakdown mechanism in p-n junction.

- When a reverse bias is applied to p-n junction diode, the depletion region width, W, increases. This cause an increase in the number of the uncovered space charge in depletion region.
- Whereas when a forward bias is applied depletion region width of the p-n junction diode decreases so the amount of the uncovered space charge decreases as well.
- So the p-n junction diode behaves as a device in which the amount of charge in depletion region depends on the voltage across the device. So it looks like a capacitor with a capacitance.



- ❖ Capacitance of a diode varies with W (Depletion Region width)
- ❖ W (DR width varies width applied voltage V)

Capacitance per unit area of a diode;

$$C_{DEP} = \frac{\varepsilon_{Si}}{W} \frac{F}{cm^2}$$

For one-sided abrupt junction; e.g. $N_A >> N_D \Rightarrow x_n >> W \cong x_n$ $N_A x_p = N_D x_n$

$$C_{DEP} = \frac{\varepsilon_{Si}}{W} \cong \frac{\varepsilon_{Si}}{x_n} \Rightarrow x_n = \sqrt{\frac{2\varepsilon_{Si}V_{bi}}{qN_D}} \quad \text{for } N_A >> N_D$$

The application of reverse bias;

$$C_{DEP} = \frac{\varepsilon_{Si}}{\sqrt{\frac{2\varepsilon_{Si}(V_{bi} + V_{R})}{qN_{D}}}} = \sqrt{\frac{q\varepsilon_{Si}N_{D}}{2(V_{bi} + V_{R})}}$$

If one makes C - V measurements and draw $1/C^2$ against the voltage V_R ; obtain built-in voltage and doping density of low-doped side of the diode from the intercept and slope.

$$\frac{1}{C^2} = \frac{2(V_{bi} + V_R)}{q\varepsilon_{Si}N_D}$$

$$slope = \frac{2}{q\varepsilon_{Si}N_D}$$

$$V_{bi} = \frac{kT}{q} \ln(\frac{N_A N_D}{n_i^2})$$