A COMPARISON WITH ABİYEV BALANCED SQUARE AND OTHER MAGIC SQUARE

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Abstract: A magic square is an $n \times n$ matrix of the integers from the 1 to n^2 such that the sum of every row, column, and diagonals is constant depending upon the the size of the matrix. The sum equals to $S = \frac{n^2+1}{2}n$ and called the magic number. The magic square, which is known as the recreation game for a long time, can be applicable to different branch of science and technology. In this study, we compared the abiyev balanced square with other perfect magic squares in the literature. Because, abiyev balanced square is written by a perfect algorithm in which static mass moment vectors according to the concentric frames are invariant with respect to the symmetric interchanges of numbers. This invariant property of abiyev balanced square allows to facilitate applications in cryptology, physics, mathematics, and genetics.

Key Words: Magic square, balanced square, cryptology, genetic, invariant.

1. Introduction

In this work, assigned numbers in the cells according to the algorithm described below and elsewhere [1] assume as the masses and concentric frames represent the mass system. The nature of the distribution of masses in the concentric frames is investigated and presented here. Certainly, different vectorial values can be assigned in the cells to obtain measurable relationships, such as electric field, magnetic field, etc.

Let us take the origin of the radius-vector $\vec{r_i} = m_i \vec{r_i}$ at the center of the magic square, and assume that the members of the magic square are mass point and located at the center of their respective cells. The calculation shows that for all magic square (except Franklin's magic squares) is equal to zero. That is:

$$\sum_{i=1}^{n^2} m_i \vec{r_i} = 0 \, , \quad [1],$$

Here, we present that the center of mass system of the magic square and its geometric center coincide. Because of this reason, the magic square named as the balanced square.

In this study, we are not going to focus on how different published magic squares were constructed. However, we present the comparisons of the other perfect squares with the abiyev balanced squares in order to delineate the basic properties of abiyev balanced squares. The importance of these properties which allow different applications will also be discussed [2].

2 Balanced Abiyev's Square

The abiyev balanced square algorithm that was founded in 1996 [3] enables to create balanced square in any desired numbers which could be rational, irrational, and complex numbers, etc [4].

The main idea of the algorithm is as: four arithmetical sequences are named as $\alpha, \beta, \gamma,$ and δ with arithmetical constants $+1, +n, -1,$ and $-n,$ respectively. The cells of constituent of each sequences were painted orange (set α), red (set β), blue (set γ), and violet (set δ). The numbers in the cells of concentric frames were assignedby means of closed graphs [5,6] The balanced abiyev's square can be created in a given order by visiting the http://www1.gantep.edu.tr/~abiyev web site [7]. The balanced square created by this algorithm according to transcendent numbers π and e shown in Table 1. Here the magic number equals to $\sqrt{19}$. In Table 1, the perfect distribution of irrational numbers is shown. Thus, the relationship between the transcendent numbers and irrational numbers can be established.

The properties of balanced squares were investigated previously [8]. These properties confirm that abiyev balanced squares are distinct comperatively from the other published perfect magic squares [8].

One of the properties which is absent in other magic squares presently is the distribution of static mass vectors of concentric frames. The mass vectors are invariant in respect to the symmetric interchanges of numbers. In balanced squares, the coloring of cells of numbers according to a determined algorithm allow to facilitate the comparison of different squares created by others as shown in Tables 3-6.

3. Comparison of Squares

In this study, we compared the Franklin's square [9], the Tien Tao Kuo's square [10], the Kwon Yong Shin's square [11], the Tamori's square [12], and the magic square created by MATLAB [13]. The comparison of the static mass moments on each concentric frames of magic squares [9-12] with balanced abiyev's square of 16th order is shown in Table 2. In Table 2 the perfect abiyev balanced square is distinct from other published magic squares. Except the Franklin's square all others including abiyev balanced square have the sum of *A(x)* and *A(y)* values are equal to zero (Table 2). That is, the mass centers and the geometric centers are congruent to each other in these magic squares (Table 2). As shown in Table 2, abiyev balanced square *A(y)* component will be 18 in 18th order, 20 in the 20th order etc [1], however, *A(x)* and *A(y)* components of vector $\vec{A}$ are needed to be calculated for each order for other published magic squares (Table 2).

The striking difference between abiyev balanced squares and the other published magic squares can be realized by the visual and numerical comparisons, respectively (Tables 3-6,7and 8). The symmetrical perfect color distribution of integers in 18th order of natural square is given in Table 6 and the symmetrical color distribution of abiyev balanced odd and even order and MATLAB perfect magic squares are given comperatively in Tables 3-5. The symmetry of color distribution of 16th, 17th and 18th order show the perfect order of abiyev's balanced square compared to MATLAB (Tables 3-6). The numerical values of vector $\vec{A}$ components of MATLAB and abiyev balanced squares are given in tables 7 and 8. Even and odd order number distribution of MATLAB magic squares does not have the order displayed in the abiyev balanced squares (Tables 7 and 8). The components of the static mass moment of the odd orders as shown in table 8, the abiyev's balanced square versus to MATLAB has the multipliers which sum up to the given odd orders. The multipliers of $A(x)$ and $A(y)$ are calculated according to formula which is given below for abiyev's balanced odd order square does not hold true for MATLAB (Table 8). For example, the multipliers for $A(x)$ and $A(y)$ are 8 and 9 for 17th order and 9 and 10 for 19th order, respectively in abiyev balanced odd order square (Table 8). Such relationship can not be ascertained for MATLAB (Table 8).

$$A(x) = \frac{n-1}{2} \cdot B; \quad where\ B = \frac{k-1}{2}\left[-\frac{(k-3)(k-2)}{3} + (n-k)(k-1)\right]$$

$$A(y) = \frac{n+1}{2} \cdot B;$$

where n-the order of square, k-the order concentric frames. The odd order balanced square concentric frames studies have been reported previously [14]. Also, the formation of vector $\sum m_i \vec{r_i} = \vec{A}$ in arbitrary odd or even order balanced squares were elaborated in details earlier [3].

This perfect algorithm of balanced squares is also named as Abiyev's theorem. All balanced squares written by this algorithm have same properties for a given order. The invariant property of vector $\vec{A}$ in a balanced square offers many applications in diverse fields.

4. Conclusion

We have presented the invariant properties of abiyev balanced square and the existence of these properties were investigated in other published perfect magic squares. Finally, the magic squares brought into the applications in science by the abiyev balanced squares.

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$\frac{2\sqrt{19}-15\pi+15e}{12}$	$\frac{10\sqrt{19}-21\pi-27e}{12}$	$\frac{2\sqrt{19}-3\pi-6e}{3}$	$\frac{-\sqrt{19}+3\pi+6e}{3}$	$\frac{2\sqrt{19}+3\pi-9e}{6}$	$\frac{-4\sqrt{19}+15\pi+15e}{6}$
$\frac{3\pi-e}{2}$	$\frac{2\sqrt{19}-9\pi+9e}{12}$	$\frac{2\sqrt{19}-3\pi-5e}{4}$	$\frac{\sqrt{19}-3e}{3}$	$\frac{-2\sqrt{19}+9\pi+9e}{6}$	$\frac{2\sqrt{19}-9\pi+3e}{6}$
$\frac{2\sqrt{19}-7\pi-e}{4}$	π	$\frac{2\sqrt{19}+3\pi-3e}{12}$	$\frac{2\sqrt{19}-3\pi-3e}{6}$	$\frac{\sqrt{19}-3\pi}{3}$	$\frac{-\sqrt{19}+6\pi+3e}{3}$
$\frac{-2\sqrt{19}+21\pi+3e}{12}$	$\frac{-2\sqrt{19}+15\pi+9e}{12}$	$\frac{\pi+e}{2}$	$\frac{2\sqrt{19}-3\pi+3e}{12}$	$\frac{2\sqrt{19}-5\pi-3e}{4}$	$\frac{2\sqrt{19}-6\pi-3e}{3}$
$\frac{-2\sqrt{19}+9\pi+7e}{4}$	$\frac{4\sqrt{19}-9\pi-9e}{6}$	$\frac{-2\sqrt{19}+9\pi+15e}{12}$	e	$\frac{2\sqrt{19}+9\pi-9e}{12}$	$\frac{10\sqrt{19}-27\pi-21e}{12}$
$\frac{2\sqrt{19}-5\pi-5e}{2}$	$\frac{-2\sqrt{19}+7\pi+9e}{4}$	$\frac{-2\sqrt{19}+3\pi+21e}{12}$	$\frac{2\sqrt{19}-\pi-7e}{4}$	$\frac{-\pi+3e}{2}$	$\frac{2\sqrt{19}+15\pi-15e}{12}$

$$a_0=\frac{2\sqrt{19}-15\pi+15e}{12};\quad b=\frac{-2\sqrt{19}+9\pi+3e}{12};\quad c=\frac{2\sqrt{19}-3\pi-9e}{12};\quad S=\frac{2a_0+5(b+c)}{2}6=\sqrt{19}$$

Table 1. Abiyev's balanced square of 6th order of irrational and transcendental numbers. The first term associated with the transcendental numbers is named as a_0, the constants of the arithmetical sequences are termed as b and c, and the magic number is shown as *S*.

16th Order										
	Franklin		Tien Tao Kuo		Kwon Yong Shin		Tamori		Abiyev	
K	*A(x)*	*A(y)*	*A(x)*	*A(y)*	*A(x)*	*A(y)*	*A(x)*	*A(y)*	*A(x)*	*A(y)*
2	1	8x16	15	0	1	- 1x16	- 1	16	- 1	16
4	3	- 40x16	- 19	0	19	- 19x16	1	- 16	0	0
6	5	104x16	- 71	0	85	- 85x16	- 35	35x16	1	- 16
8	7	- 200x16	75	0	231	- 231x16	35	- 35x16	0	0
10	- 41	248x16	195	0	- 327	327x16	- 165	165x16	1	- 16
12	37	- 216x16	- 207	0	- 245	245x16	165	- 165x16	- 1	16
14	- 61	152x16	- 331	0	- 51	51x16	- 455	455x16	1	- 16
16	49	- 88x16	343	0	287	- 287x16	455	- 455x16	- 1	16
	0	- 32x16	0	0	0	0	0	0	0	0

Table2. The comparison of the static mass moments of the concentric frames of Franklin square [9], the Tien Tao Kuo's square [10], the Kwon Yong Shin's square [11], the Tamori's square [12] with the abiyev balanced square of 16th order. The *A(x)* and *A(y)* are the components of the vector $\vec{A}$. The last row shows the sum of the *A(x)* and *A(y). K* represents the order of frame.

1	242	14	244	12	246	10	249	248	7	251	5	253	3	255	16
240	18	227	29	229	27	231	232	25	234	22	236	20	238	31	17
33	223	35	212	44	214	42	217	216	39	219	37	221	46	34	224
208	50	206	52	197	59	199	200	57	202	54	204	61	51	207	49
65	191	67	189	69	182	74	185	184	71	187	76	68	190	66	192
176	82	174	84	172	86	167	168	89	170	91	85	173	83	175	81
97	159	99	157	101	155	103	152	153	106	102	156	100	158	98	160
144	114	142	116	124	118	138	137	136	119	123	133	125	131	127	129
128	143	126	141	140	139	122	121	120	135	134	117	132	115	130	113
145	111	147	109	149	107	151	105	104	154	150	108	148	110	146	112
96	162	94	164	92	166	90	88	169	87	171	165	93	163	95	161
177	79	179	77	181	75	183	72	73	186	70	188	180	78	178	80
64	194	62	196	60	198	58	56	201	55	203	53	205	195	63	193
209	47	211	45	213	43	215	41	40	218	38	220	36	222	210	48
32	226	30	228	28	230	26	24	233	23	235	21	237	19	239	225
241	15	243	13	245	11	247	9	8	250	6	252	4	254	2	256

256	2	3	253	252	6	7	249	248	10	11	245	244	14	15	241
17	239	238	20	21	235	234	24	25	231	230	28	29	227	226	32
33	223	222	36	37	219	218	40	41	215	214	44	45	211	210	48
208	50	51	205	204	54	55	201	200	58	59	197	196	62	63	193
192	66	67	189	188	70	71	185	184	74	75	181	180	78	79	177
81	175	174	84	85	171	170	88	89	167	166	92	93	163	162	96
97	159	158	100	101	155	154	104	105	151	150	108	109	147	146	112
144	114	115	141	140	118	119	137	136	122	123	133	132	126	127	129
128	130	131	125	124	134	135	121	120	138	139	117	116	142	143	113
145	111	110	148	149	107	106	152	153	103	102	156	157	99	98	160
161	95	94	164	165	91	90	168	169	87	86	172	173	83	82	176
80	178	179	77	76	182	183	73	72	186	187	69	68	190	191	65
64	194	195	61	60	198	199	57	56	202	203	53	52	206	207	49
209	47	46	212	213	43	42	216	217	39	38	220	221	35	34	224
225	31	30	228	229	27	26	232	233	23	22	236	237	19	18	240
16	242	243	13	12	246	247	9	8	250	251	5	4	254	255	1

Table 3. The comparison of the visual distribution of the numbers of abiyev's balanced square (left) and MATLAB's magic square (right) of 16th order. The color distribution of the integers in the cells are the same for both abiyev balanced square and the MATLAB magic square. The perfect symmetry of color distribution of integers in abiyev balanced square is noticable.

146	130	114	98	82	66	50	34	1	274	258	242	226	210	194	178	162
164	148	132	116	100	84	68	35	19	3	276	260	244	228	212	196	180
182	166	150	134	118	102	69	53	37	21	5	278	262	246	230	214	198
200	184	168	152	136	103	87	71	55	39	23	7	280	264	248	232	216
218	202	186	170	137	121	105	89	73	57	41	25	9	282	266	250	234
236	220	204	171	155	139	123	107	91	75	59	43	27	11	284	268	252
254	238	205	189	173	157	141	125	109	93	77	61	45	29	13	286	270
272	239	223	207	191	175	159	143	127	111	95	79	63	47	31	15	288
273	257	241	225	209	193	177	161	145	129	113	97	81	65	49	33	17
2	275	259	243	227	211	195	179	163	147	131	115	99	83	67	51	18
20	4	277	261	245	229	213	197	181	165	149	133	117	101	85	52	36
38	22	6	279	263	247	231	215	199	183	167	151	135	119	86	70	54
56	40	24	8	281	265	249	233	217	201	185	169	153	120	104	88	72
74	58	42	26	10	283	267	251	235	219	203	187	154	138	122	106	90
92	76	60	44	28	12	285	269	253	237	221	188	172	156	140	124	108
110	94	78	62	46	30	14	287	271	255	222	206	190	174	158	142	126
128	112	96	80	64	48	32	16	289	256	240	224	208	192	176	160	144

155	174	193	212	231	250	269	288	1	20	39	58	77	96	115	134	153
173	192	211	230	249	268	287	17	19	38	57	76	95	114	133	152	154
191	210	229	248	267	286	16	18	37	56	75	94	113	132	151	170	172
209	228	247	266	285	15	34	36	55	74	93	112	131	150	169	171	190
227	246	265	284	14	33	35	54	73	92	111	130	149	168	187	189	208
245	264	283	13	32	51	53	72	91	110	129	148	167	186	188	207	226
263	282	12	31	50	52	71	90	109	128	147	166	185	204	206	225	244
281	11	30	49	68	70	89	108	127	146	165	184	203	205	224	243	262
10	29	48	67	69	88	107	126	145	164	183	202	221	223	242	261	280
28	47	66	85	87	106	125	144	163	182	201	220	222	241	260	279	9
46	65	84	86	105	124	143	162	181	200	219	238	240	259	278	8	27
64	83	102	104	123	142	161	180	199	218	237	239	258	277	7	26	45
82	101	103	122	141	160	179	198	217	236	255	257	276	6	25	44	63
100	119	121	140	159	178	197	216	235	254	256	275	5	24	43	62	81
118	120	139	158	177	196	215	234	253	272	274	4	23	42	61	80	99
136	138	157	176	195	214	233	252	271	273	3	22	41	60	79	98	117
137	156	175	194	213	232	251	270	289	2	21	40	59	78	97	116	135

Table 4. The comparison of the visual distribution of the integers of abiyev's balanced square (left) and MATLAB's magic square (right) of 17th order. The color distribution of the integers in the cells are the same for both abiyev balanced square and the MATLAB magic square. The perfect symmetry of color distribution of integers in odd order of abiyev balanced square is distinct from the MATLAB magic square.

1	308	16	310	14	312	12	314	315	10	317	7	319	5	321	3	323	18
306	20	291	33	293	31	295	29	297	298	26	300	24	302	22	304	35	19
37	287	39	274	50	276	48	278	46	279	281	43	283	41	285	52	38	288
270	56	268	58	257	67	259	65	261	262	62	264	60	266	69	57	269	55
73	251	75	249	77	240	84	242	82	243	245	79	247	86	76	250	74	252
234	92	232	94	230	96	223	101	225	226	98	228	103	95	231	93	233	91
109	215	111	213	113	211	115	206	118	207	209	120	114	212	112	214	110	216
198	128	196	130	194	132	192	134	189	190	137	133	193	131	195	129	197	127
145	161	178	159	176	157	174	173	172	171	152	169	150	167	148	165	146	162
180	179	147	177	149	175	151	155	154	153	170	156	168	158	166	160	164	163
144	182	142	184	140	186	138	188	136	135	191	187	139	185	141	183	143	181
199	125	201	123	203	121	205	119	208	117	116	210	204	122	202	124	200	126
108	218	106	220	104	222	102	224	99	100	227	97	229	221	105	219	107	217
235	89	237	87	239	85	241	83	244	81	80	246	78	248	238	88	236	90
72	254	70	256	68	258	66	260	63	64	263	61	265	59	267	255	71	253
271	53	273	51	275	49	277	43	280	45	44	282	42	284	40	286	272	54
36	290	34	292	32	294	30	296	27	28	299	25	301	23	303	21	305	289
307	17	309	15	311	13	313	11	9	316	8	318	6	320	4	322	2	324

290	301	312	323	1	12	23	34	45	209	220	231	242	163	174	104	115	126
300	311	322	252	11	22	33	44	46	219	230	241	171	173	184	114	125	127
310	321	251	253	21	32	43	54	56	229	240	170	172	183	194	124	135	137
320	250	261	263	31	42	53	55	66	239	169	180	182	193	204	134	136	147
6	260	262	273	284	52	63	65	76	168	179	181	192	203	214	144	146	157
259	270	272	283	51	62	64	75	5	178	189	191	202	213	224	145	156	86
269	271	282	293	61	72	74	4	15	188	190	201	212	223	234	155	85	96
279	281	292	303	71	73	3	14	25	198	200	211	222	233	235	84	95	106
280	291	302	313	81	2	13	24	35	199	210	221	232	243	164	94	105	116
47	58	69	80	244	255	266	277	288	128	139	150	161	82	93	185	196	207
57	68	79	9	254	265	276	287	289	138	149	160	90	92	103	195	206	208
67	78	8	10	264	275	286	297	299	148	159	89	91	102	113	205	216	218
77	7	18	20	274	285	296	298	309	158	88	99	101	112	123	215	217	228
249	17	19	30	41	295	306	308	319	87	98	100	111	122	133	225	227	238
16	27	29	40	294	305	307	318	248	97	108	110	121	132	143	226	237	167
26	28	39	50	304	315	317	247	258	107	109	120	131	142	153	236	166	177
36	38	49	60	314	316	246	257	268	117	119	130	141	152	154	165	176	187
37	48	59	70	324	245	256	267	278	118	129	140	151	162	83	175	186	197

Table 5. The comparison of the visual distribution of the integers of abiyev's balanced square (left) and MATLAB's magic square (right) of 18th order. The color distribution of the integers in the cells are the same for both abiyev balanced square and the MATLAB magic square. The perfect symmetry of color distribution of integers in even and odd order of abiyev balanced square is conspicuous from the MATLAB magic square.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144
145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162
163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198
199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216
217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234
235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252
253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288
289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306
307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324

Table 6. 18th order natural square. The symmetrical distribution of colors is based on the color assignment rule explained above. The color harmony in this natural square supports the perfectness of the abiyev balanced square algorithm.

	MATLAB		ABİYEV			MATLAB		ABİYEV	
K	A(x)	A(y)	A(x)	A(y)	K	A(x)	A(y)	A(x)	A(y)
2	- 1	16	- 1	16	2	2	- 91	- 1	18
4	1	- 16	0	0	4	155	- 730	0	0
6	- 33	528	1	- 16	6	368	- 1939	- 1	18
8	33	- 528	0	0	8	- 231	- 2724	1	- 18
10	- 97	1552	1	- 16	10	- 588	- 2154	- 1	18
12	97	- 1552	- 1	16	12	- 99	1530	1	- 18
14	- 193	3088	1	- 16	14	1192	- 362	- 1	18
16	193	- 3088	- 1	16	16	127	2659	1	- 18
	0	0	0	0	18	- 926	3811	1	- 18
						0	0	0	0

Table 7. The comparison of 16th and 18th order static mass components of vector $\vec{A}$ according to concentric frames in MATLAB and abiyev's balanced squares. The last row is the sum of the mass components. The irregularities of MATLAB are evident compared to abiyev balanced square.

	MATLAB		ABİYEV	
K	A(x)	A(y)	A(x)	A(y)
3	6	252	8x28= 224	9x28= 252
5	- 636	590	8x92= 736	9x92= 828
7	- 1078	1236	8x160= 1280	9x160= 1440
9	- 1288	1630	8x200= 1600	9x200= 1800
11	- 1506	1348	8x180= 1440	9x180= 1620
13	- 816	544	8x68= 544	9x68= 612
15	1154	- 1206	8x(-168)= - 1344	9x(-168)= - 1512
17	4164	- 4394	8x(-560)= - 4480	9x(-560)= - 5040
	0	0	0	0

Table 8. The 17th odd order mass components of vector $\vec{A}$ in MATLAB's and abiyev's balanced square. The irregulation is also exist for odd order magic squares of MATLAB. The last row shows the sum of the mass components.