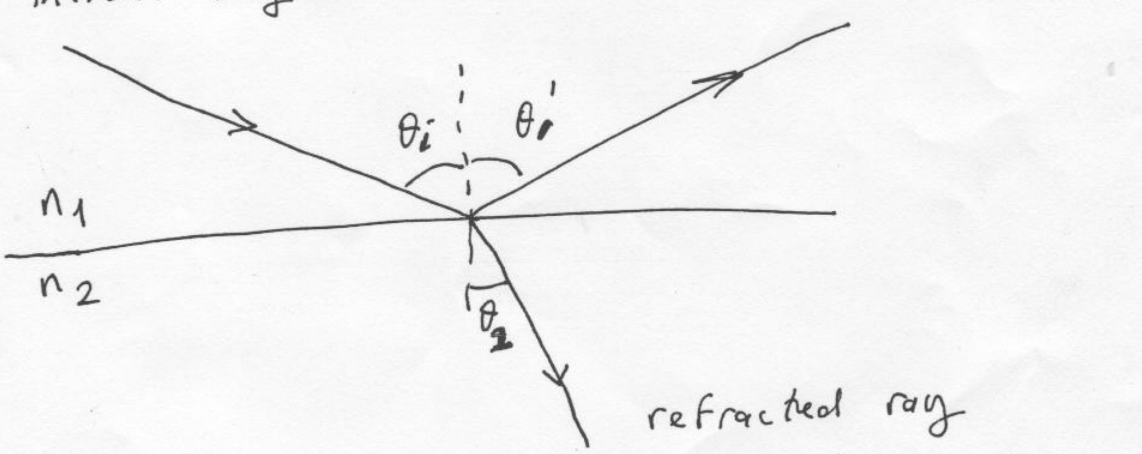
CHAPTER 2 REFLECTION AND REFRACTION

2.1 Introduction

The field of "geometric optics" involves the study of propagation of light with the assumption that light travels in a fixed direction in a straight line.

In this and ~~chapter 3~~ next chapter we will use ray approximation of light.

2.2 Reflection



A part of incident ray is reflected a a part is refracted (transmitted).

Law of Reflection : $\boxed{\theta_1 = \theta_1'}$

2.3 ~~Re~~Refraction

θ_2 : refraction angle w.r.t. normal

Law of refraction : $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$ Snell's Law

refractive index : $n = \frac{c}{v}$ ← speed of light in vacuum / ← speed of light in medium

$$\Rightarrow \quad \frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

or
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \equiv \text{constant} = \frac{n_2}{n_1}$$

EXAMPLE 1 A mono-chromatic beam of light travelling in air is incident on a slab of transparent material. The incident beam makes an angle of 40° with the normal and the refracted beam makes an angle of 26° with the normal. Find the index of refraction of the material.

SOLUTION

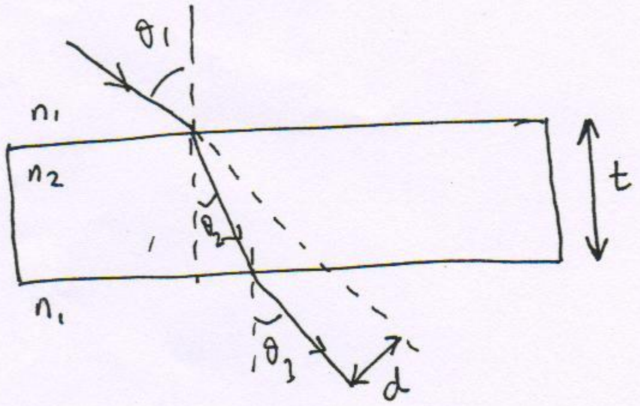
Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1)\sin 40^\circ}{\sin 26^\circ}$$

$$\boxed{n_2 = 1.47}$$

EXAMPLE 2

A light beam passes from medium 1 to 2, with the latter medium being a thick slab of material.



(a) show that the emerging beam is parallel to the incident beam.

(b) Determine the offset distance d.

SOLUTION

a) Snell's law at the upper surface

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 \quad (1)$$

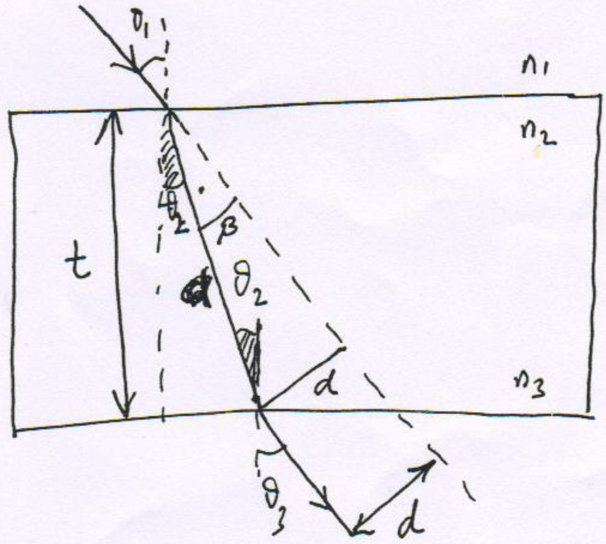
at the bottom surface

$$\sin\theta_3 = \frac{n_2}{n_1}\sin\theta_2 \quad (2)$$

Subsituting (1) into (2) yields

$$\sin\theta_3 = \frac{n_2}{n_1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin\theta_1$$

$\therefore \boxed{\theta_3 = \theta_1}$ the ray does not alter its direction



(b) From figure:

$$a\cos\theta_2 = t \rightarrow a = \frac{t}{\cos\theta_2}$$

$$\theta_2 + \beta = \theta_1 \rightarrow \beta = \theta_1 - \theta_2$$

$$a\sin\beta = d$$

$$\therefore\ d = a\sin\beta = \left(\frac{t}{\cos\theta_2}\right)\sin(\theta_1 - \theta_2)$$

Note that if θ_1 is small $\Rightarrow \theta_2$ is small $\rightarrow$ $\sin(\theta_1-\theta_2) \approx \theta_1 - \theta_2$, $\cos\theta_2 \approx 1$

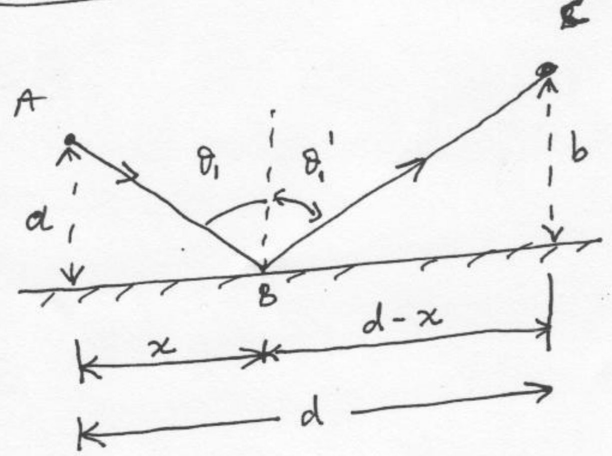
$$d \approx t(\theta_1 - \theta_2)$$

2.4 Fermat's Principle

"The path between two points taken by a light ray is traversed in least time"

EXAMPLE 3 Using Fermat's principle derive law of reflection

SOLUTION



time to travel for a ray along the path ABC is

$$t = \frac{|AB|}{v} + \frac{|BC|}{v}$$
$$= \frac{\sqrt{a^2+x^2}}{v} + \frac{\sqrt{b^2+(d-x)^2}}{v}$$

Fermat's Principle: $\frac{dt}{dx} = 0$

$$\frac{dt}{dx} = \frac{d}{dx}\left\{ \frac{(a^2+x^2)^{1/2}}{v} + \frac{(b^2+(d-x)^2)^{1/2}}{v} \right\} = 0$$

$$= \frac{1}{v}\frac{1}{2}(a^2+x^2)^{-1/2}(2x) + \frac{1}{v}\frac{1}{2}(b^2+(d-x)^2)^{-1/2}(2x-2d$$

$$= \frac{x}{v\sqrt{a^2+x^2}} + \frac{x-d}{v\sqrt{b^2+(d-x)^2}} = 0$$

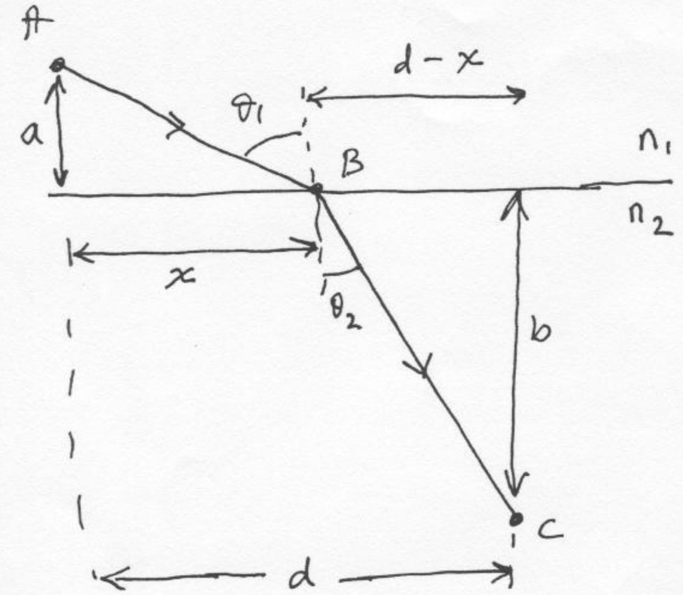
$$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$\sin\theta_1 = \sin\theta_1'$$

$\therefore$ $\boxed{\theta_1 = \theta_1'}$ law of reflection

EXAMPLE 4 Using Fermat's principle derive Snell's law of refraction.

SOLUTION



time to travel along the path ABC

$$t = \frac{|AB|}{v_1} + \frac{|BC|}{v_2}$$

$$= \frac{\sqrt{a^2+x^2}}{v_1} + \frac{\sqrt{b^2+(d-x)^2}}{v_2}$$

$$v_1 = \frac{c}{n_1} \quad \text{and} \quad v_2 = \frac{c}{n_2}$$

Fermat's Principle: $\dfrac{dt}{dx} = 0$

$$\frac{dt}{dx} = \frac{d}{dx}\left\{ \frac{\left[a^2+x^2\right]^{1/2}}{c/n_1} + \frac{\left[b^2+(d-x)^2\right]^{1/2}}{c/n_2} \right\} = 0$$

$$= \frac{n_1}{c}\,\frac{1}{2}\left[a^2+x^2\right]^{-1/2}(2x) + \frac{n_2}{c}\,\frac{1}{2}\left[b^2+(d-x)^2\right]^{-1/2}(2x-2d)$$

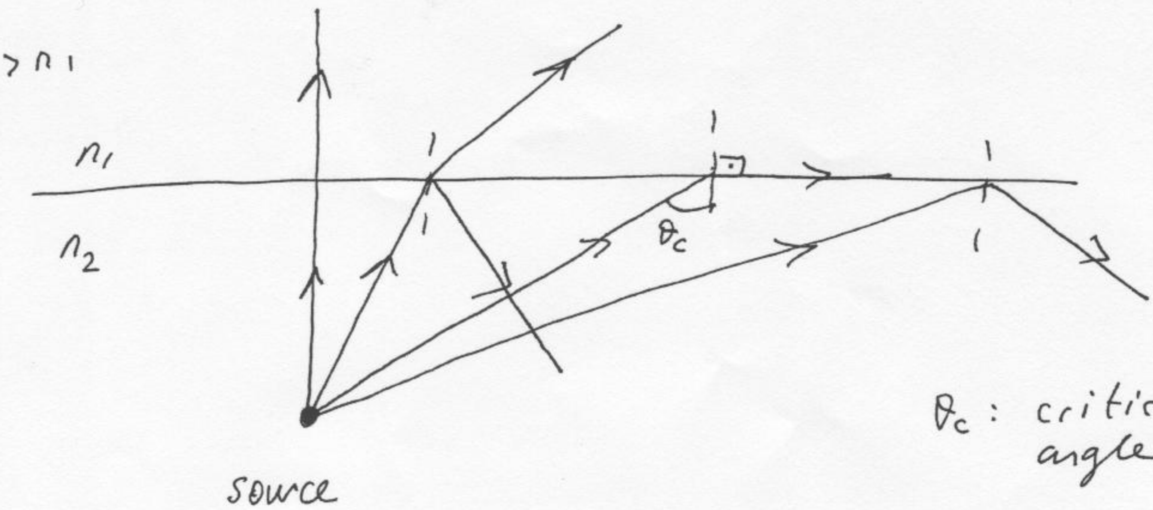
$$= \frac{n_1}{c}\,\frac{x}{\sqrt{a^2+x^2}} + \frac{n_2}{c}\,\frac{x-d}{\sqrt{b^2+(d-x)^2}} = 0$$

$$\Rightarrow \quad \frac{n_1}{\cancel{c}}\,\frac{x}{\sqrt{a^2+x^2}} = \frac{n_2}{\cancel{c}}\,\frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$ law of refraction

2.5 Totally Internal Reflection

$n_2 > n_1$



θ_c : critical angle

Snell's law: $\quad n_1 \sin 90^\circ = n_2 \sin\theta_c$

or $\quad \sin\theta_c = \dfrac{n_1}{n_2}$

for $n_1 = \underline{1}$ (air) and $n_2 = 1.333$ (water)

$$\sin\theta_c = \frac{1}{1.333} = 0.75$$

$$\theta_c = \sin^{-1}(\theta_c) = 48.6^\circ$$

$\boxed{\text{HW}}$ Determine the critical angle

(a) a diamond-air

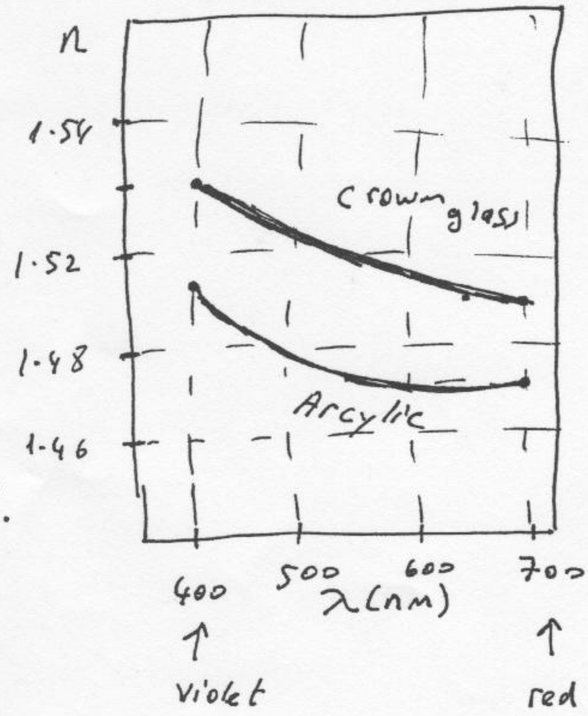
(b) a water-glass

interfaces.

2.6 Dispersion

An important property of refraction index (n) is that, for a given material index varies with the wavelength of the light passing through the material.

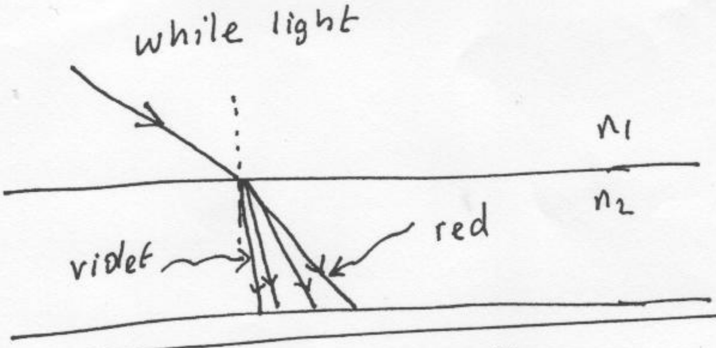
This behaviour is called "dispersion".



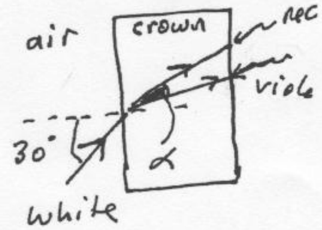
Because n is function of λ

$$n(\lambda)$$

light of different wavelegths is bent at different angles



EXAMPLE 5: A white light falls on a crown glass as shown in figure. Find the angle between violet (400 nm) and red (700) nm rays inside the glass. (USE FIGURE ABOVE)



SOLUTION

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1/2 = 1 \cdot \sin 30 = n_2 \sin\theta_2$$

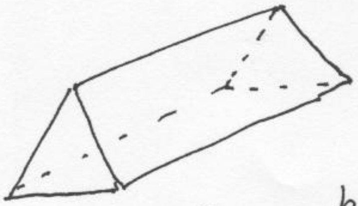
$$\theta_2 = \sin^{-1}\left(\frac{1}{2n_2}\right) \rightarrow$$

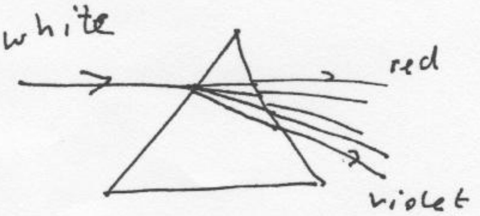
$$\theta_2^{\text{violet}} = \sin^{-1}\left(\frac{1}{2 * 1.53}\right) = 19.1°$$

$$\theta_2^{\text{red}} = \sin^{-1}\left(\frac{1}{2 * 1.49}\right) = 19.6°$$

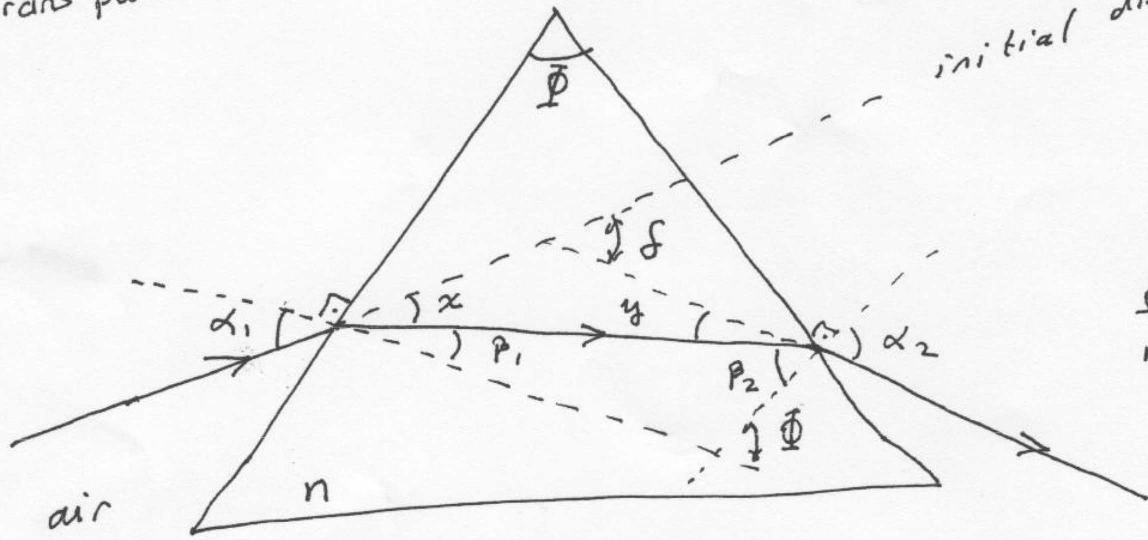
$$\alpha = \Delta\theta = 19.6° - 19.1° = \underline{0.5°}$$

2.7 Prism





Transparent medium between two planes is called a "prism"



δ: angle of deviation
Φ: apex angle
n: index of refraction

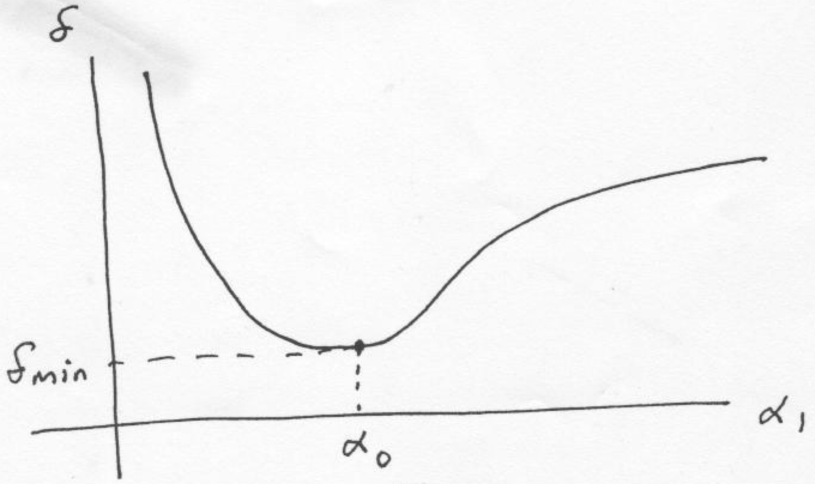
from figure: $x = \alpha_1 - \beta_1$ and $y = \alpha_2 - \beta_2$

$\delta = x + y$ and $\Phi = \beta_1 + \beta_2$

$$\therefore \quad \delta = x + y = \alpha_1 - \beta_1 + \alpha_2 - \beta_2 = \underbrace{\alpha_1 + \alpha_2 - \Phi}_{\text{measureable angles.}}$$

using snell's law:

$$\frac{\sin\alpha_1}{\sin\beta_1} = n \quad \text{and} \quad \frac{\sin\alpha_2}{\sin\beta_2} = n$$



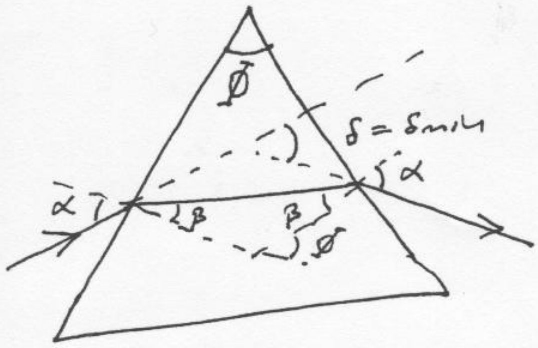
For a specific value of α_1 (say α_0), the deviation δ will be minimum.

In this case

$$\left\{\begin{aligned} \alpha_1 &= \alpha_2 = \alpha \\ \beta_1 &= \beta_2 = \beta \end{aligned}\right\} *$$

Hence we have symmetric line with respect to apex point

* Prove these relations (Hw



$$\delta_{min} = 2\alpha - \Phi$$

$$\Phi = 2\beta$$

$$n = \frac{\sin\alpha}{\sin\beta}$$

$$\boxed{n = \frac{\sin\left(\frac{\Phi + \delta_{min}}{2}\right)}{\sin\left(\frac{\Phi}{2}\right)}}$$

Hence, knowing Φ and measuring δ_{min}, we can calculate index of refraction of the prisma

NOTE THAT

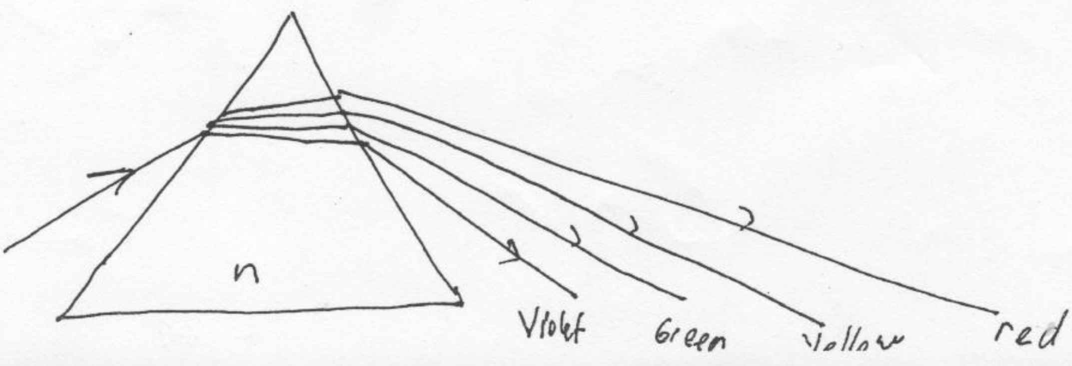
- if Φ is small $\Rightarrow$ δ_{min} will be small so using the approximation $\sin\theta \approx \theta$

$$n \approx \frac{\frac{\delta_{min} + \Phi}{2}}{\frac{\Phi}{2}} = \frac{\delta_{min}}{\Phi} + 1$$

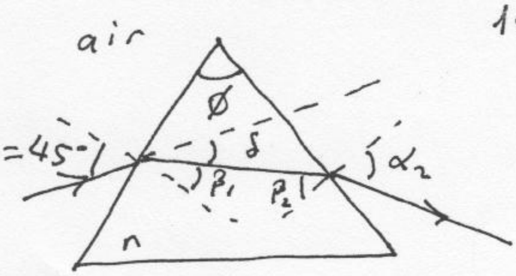
or

$$\delta_{min} = (n-1)\,\Phi$$

- if incident ray is a white-light (a combination of all visible wavelengths), the rays that emerge spread out in a series of colors known as "visible spectrum".



EXAMPLE 6



A light ray enter from one surface of a prism whose apex angle is $\phi = 60°$ and index of refraction $n = 1.5$.
The ray leaves the prism from another surface with an angle α_2.

(a) Find α_2
(b) Is the deviation minimum? ($\delta \overset{?}{=} \delta_{min}$)

SOLUTION

(a) • Snell's law at the first surface

$$\sin\alpha_1 = n \sin\beta_1 \rightarrow \quad \sin\beta_1 = \frac{\sin\alpha_1}{n} = \frac{\sin 45}{1.5} = \frac{\sqrt{2}}{3}$$

$$\beta_1 = \sin^{-1}(\sqrt{2}/3) = 28.2°$$

$$\beta_2 = \phi - \beta_1 = 60° - 28.1° = 31.9°$$

• Snell's law at the second surface

$$\sin\alpha_2 = n \sin\beta_2 = (1.5)\sin(31.9°) = 0.793$$

$$\alpha_2 = \sin^{-1}(0.793) = 52.4°$$

(b) $$\delta = \alpha_1 + \alpha_2 - \phi = 45.0 + 52.4 - 60 = 37.4°$$

$$n = \frac{\sin[(\phi + \delta_{min})/2]}{\sin[\phi/2]} \rightarrow \quad \delta_{min} = 2\sin^{-1}\left[n\sin\left(\frac{\phi}{2}\right)\right] - \phi$$

$$= 2\sin^{-1}\left[(1.5)\sin 30°\right] - 6$$

$$= 37.2°$$

$\therefore\ \delta > \delta_{min} \rightarrow$ this is not a minimum deviation

$\delta - \delta_{min} = 0.2°$ (very close to min. deviation)

$\boxed{\text{NOTE THAT}}$

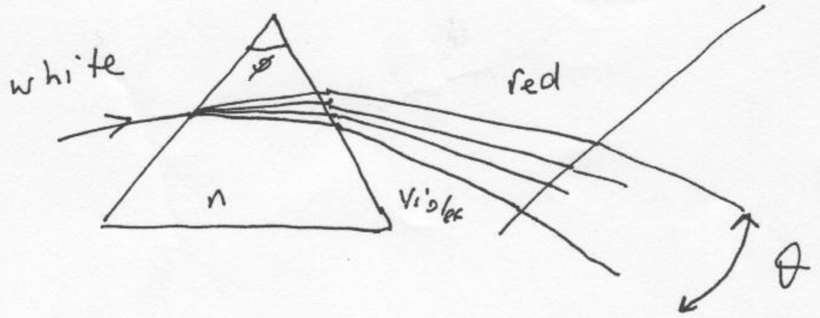
for the minimum condition: $\quad \phi = 2\beta \rightarrow \beta = \frac{\phi}{2}$

$$\sin\alpha = n\sin\beta = n\sin\left(\frac{\phi}{2}\right)$$

$$= (1.5)\sin(30°)$$

$$\Rightarrow \alpha_1 = \alpha = 48.6°$$

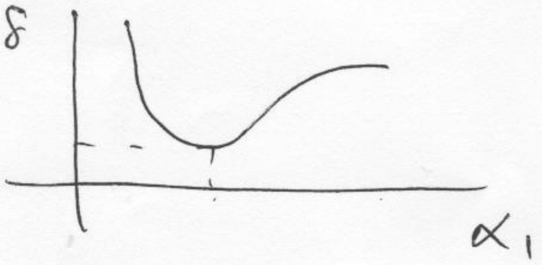
HW



find the angle θ for $\phi = 60°$ if the prism is made up of crown glass.

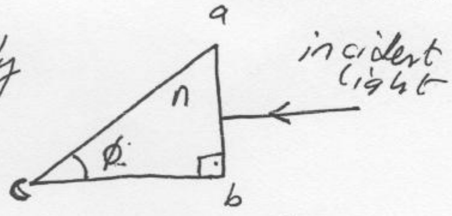
$n(\text{violet}) = 1.53$
$n(\text{red}) = 1.49$

$$\delta(\alpha_1) = \alpha_1 + \sin^{-1}\left[n \sin\left\{\phi - \sin^{-1}\left(\frac{\sin\alpha_1}{n}\right)\right\}\right] - \phi$$



Example

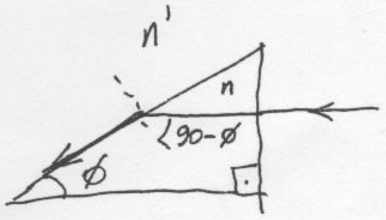
A ray of light is incident normally on the face ab of a glass prism ($n = 1.52$) as shown in figure.



(a) Assuming that the prism is ~~ain~~ immersed in air, Find largest value for the angle ϕ so that the ray is totally reflected at face ac.

(b) Find ϕ if the prism is immersed in water ($n_w = 4/3$)

Snell's law:



$$n \sin(90-\phi) = n' \sin 90^\circ$$

$$n \cos\phi = n'$$

$$\cos\phi = n'/n \rightarrow \phi = \cos^{-1}(n'/n)$$

(a) $n' = 1.00 \text{ (air)} \rightarrow \phi = \cos^{-1}(1/1.52) = 48.9^\circ$ $\boxed{\phi \leq 48.9^\circ}$

(b) $n' = 1.33 \text{ (water)} \rightarrow \phi = \cos^{-1}(1/1.33) = 28.0^\circ$ $\boxed{\phi \leq 29.0^\circ}$