CHAPTER 3 MIRRORS

3.1 Introduction

In this chapter we will concern with the image formation by flat and ~~curved~~ spherical mirrors.

Image

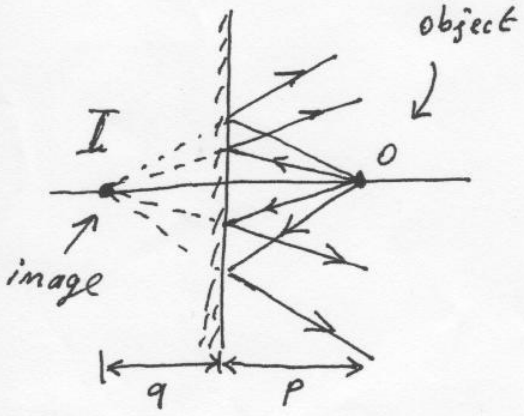
Real

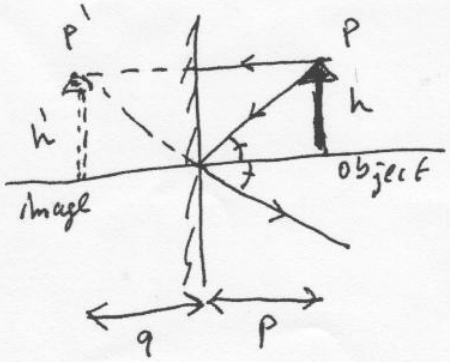
a real image is formed when light rays pass through (intersect) and diverge from the image point.

Virtual

a virtual image is formed when the light rays <u>do not</u> pass through the image point but only appear to diverge from that point.

3.2 Flat Mirror





- p: object distance
  q: image distance
- The image is virtual since extention of the rays intersect at point I.

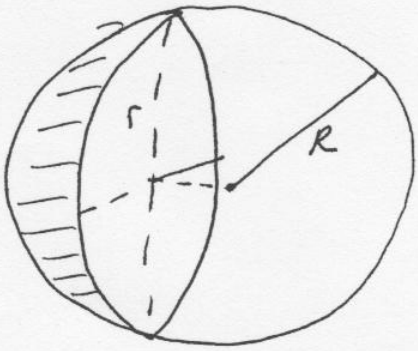
$$p = -q$$

- Lateral magnification

$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} = \underline{1}$$

3.2 Spherical mirrors

A spherical mirror has the shape of a section of sphere.

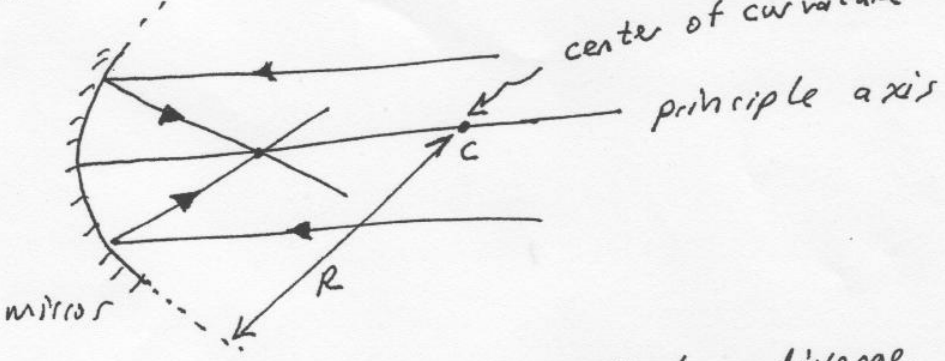


R: radius of curvature
r: mirror radius

- if the internal side is a reflector then the mirror is said to be <u>concave</u> mirror.
- if external side is a reflector then the mirror is <u>convex</u> mirror.

3.3 Concave Mirrors

This type of mirror focuses incoming parallel rays to a point



we shall consider only rays that diverge from the object and make a small angle with the principle axis. Such rays are called "paraaxial rays".

special rays

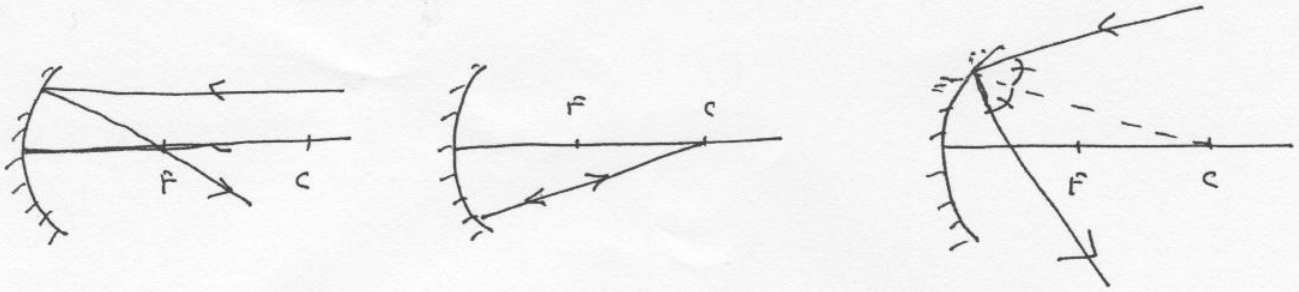
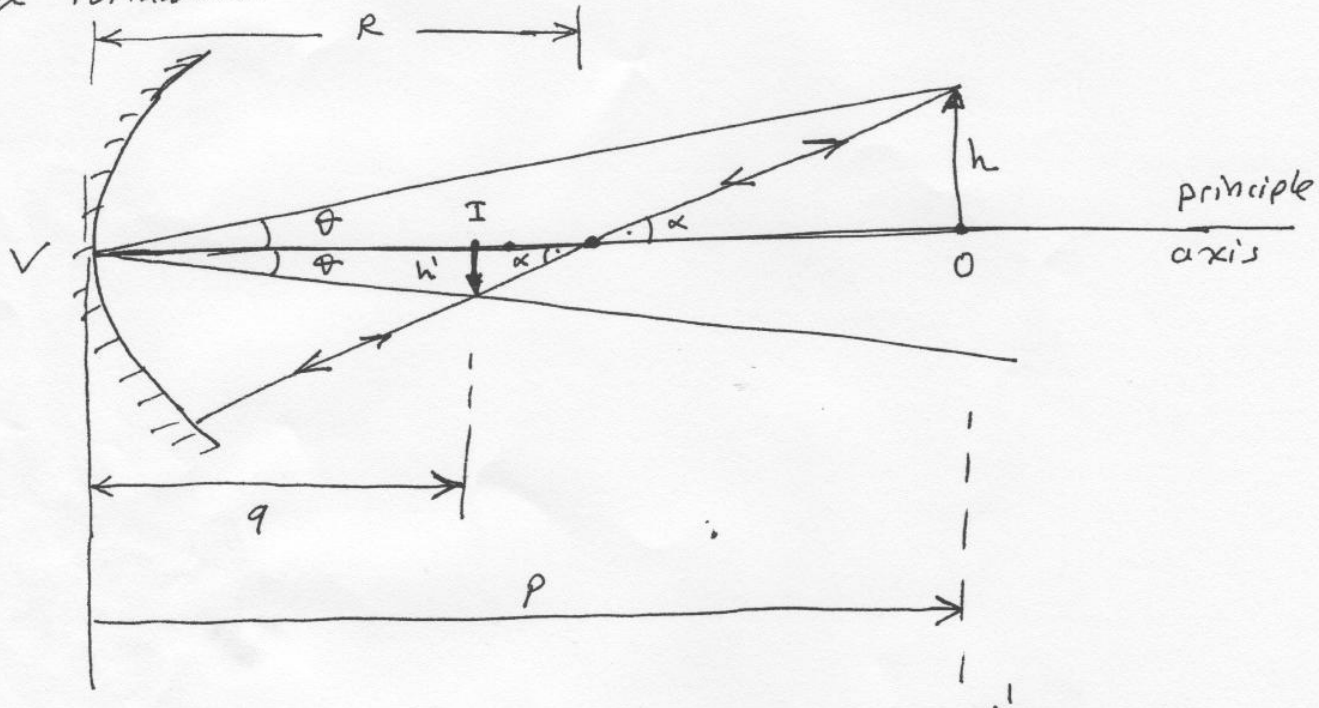


image formation



from figure $\tan\theta = h/p$ and $\tan\theta = -h'/q$

↑ image is inverted

magnification: $m = \frac{h'}{h} = -\frac{q}{p}$

$$\tan\alpha = \frac{h}{p-R} \quad \text{and} \quad \tan\alpha = -\frac{h'}{R-q}$$

or $$\frac{h'}{h} = -\frac{R-q}{p-R} = -\frac{q}{p}$$

After simple algebra we obtain

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{2}{R}}$$

mirror equation.

if the object is very far from the mirror

$$p \to \infty$$

then $\frac{1}{p} \to 0$ and $q \approx \frac{R}{2}$

we call the image point in this case the "focal point" and image distance "focal length", f, where

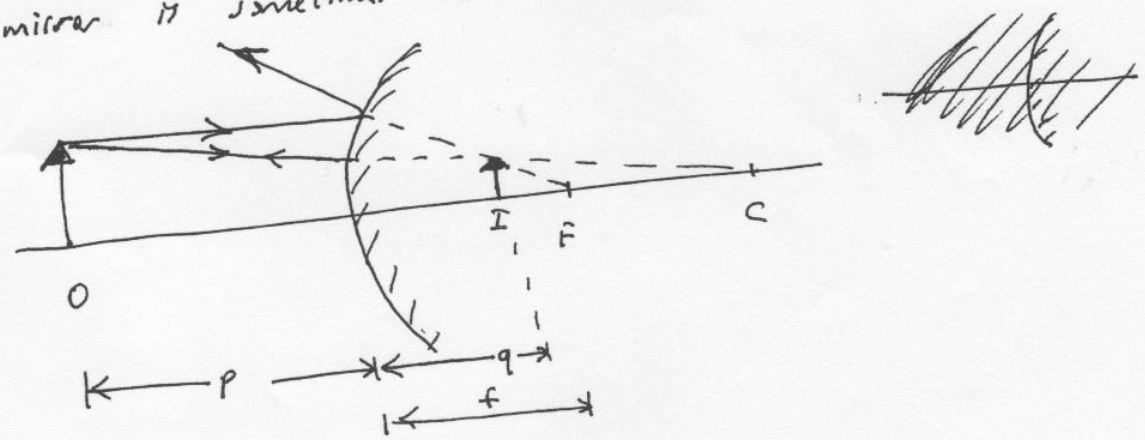
$$f = \frac{R}{2}$$

Hence:

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}}$$

3.4 Convex Mirrors

This mirror is sometimes called as diverging mirror.



We do not derive any equations for convex mirrors since we can use the the ~~convex~~ concave mirror equation

Sign conversions for mirrors

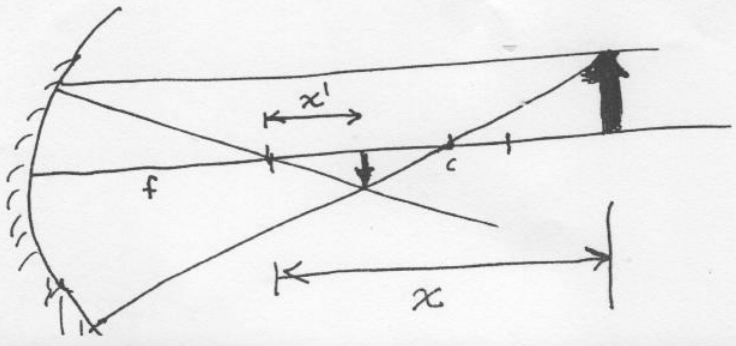
Quantity	Positive when	Negative when
Object location (p)	object is in front of mirror (real object)	object is in back of mirror (virtual obj.)
Image location (q)	Image is in front of mirror (real image)	Image is in back of mirror (virtual im
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	mirror is concave	mirror is convex
magnification (m)	Image is upright	Image is inverted

Note that for both mirrors:

$$\frac{h'}{h} = \frac{f}{x} \quad \text{and} \quad \frac{h'}{h} = \frac{x'}{f}$$

Combining these two equations:

$$\boxed{f^2 = x x'}$$ Newton's equation



EXAMPLE 1

Assume that a certain sperical mirror has a focal length of +10 cm. Locate, describe and draw the image for object distance of (a) 25 cm (b) 10 cm (c) 5 cm.

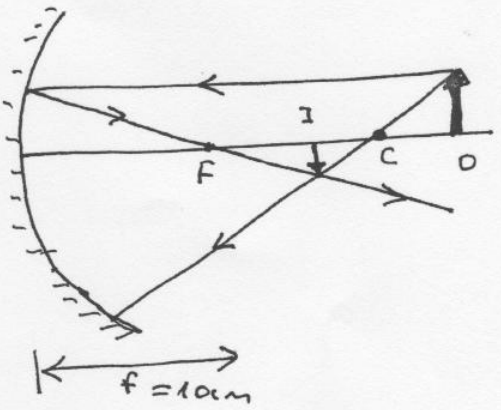
SOLUTION The mirror is concave since $f = +10\,cm > 0$.

(a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{1}{25} + \frac{1}{q} = \frac{1}{10}$

$q = 16.7\,cm$

$m = -\frac{q}{p} = -\frac{16.7}{25} = -0.668.$

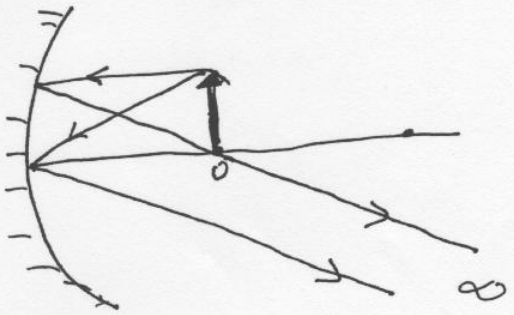


The image is inverted smaller than object and real ($q > 0$).

(b) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{1}{10} + \frac{1}{q} = \frac{1}{10}$

$q = \infty$



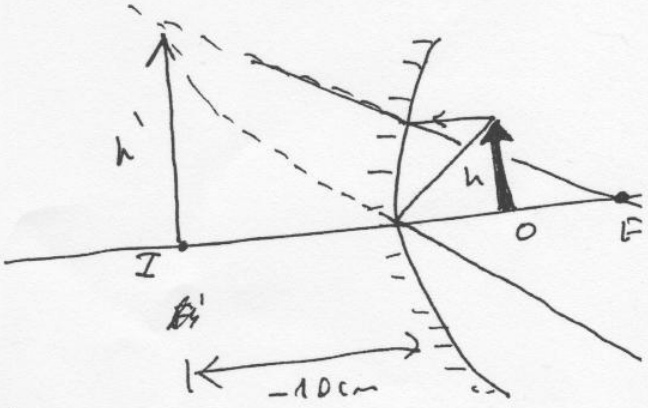
The image is formed at infinity

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{1}{5} + \frac{1}{q} = \frac{1}{10}$

$q = -10\,cm$

$m = 2 = h'/h$



The image is virtual ($q < 0$) larger than object and upright ($m > 0$)

EXAMPLE 2 A convex mirror has a radius of curvature of $R = -0.5$ m. (a) Determine the position of the image of an object at a distance $p = 3$ m from the mirror. (b) Find the magnification of the image.

SOLUTION

$$f = \frac{R}{2} = -\frac{0.5}{2} = -0.25 \text{ m}.$$

(a) $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{3} + \frac{1}{q} = \frac{1}{-0.25}$$

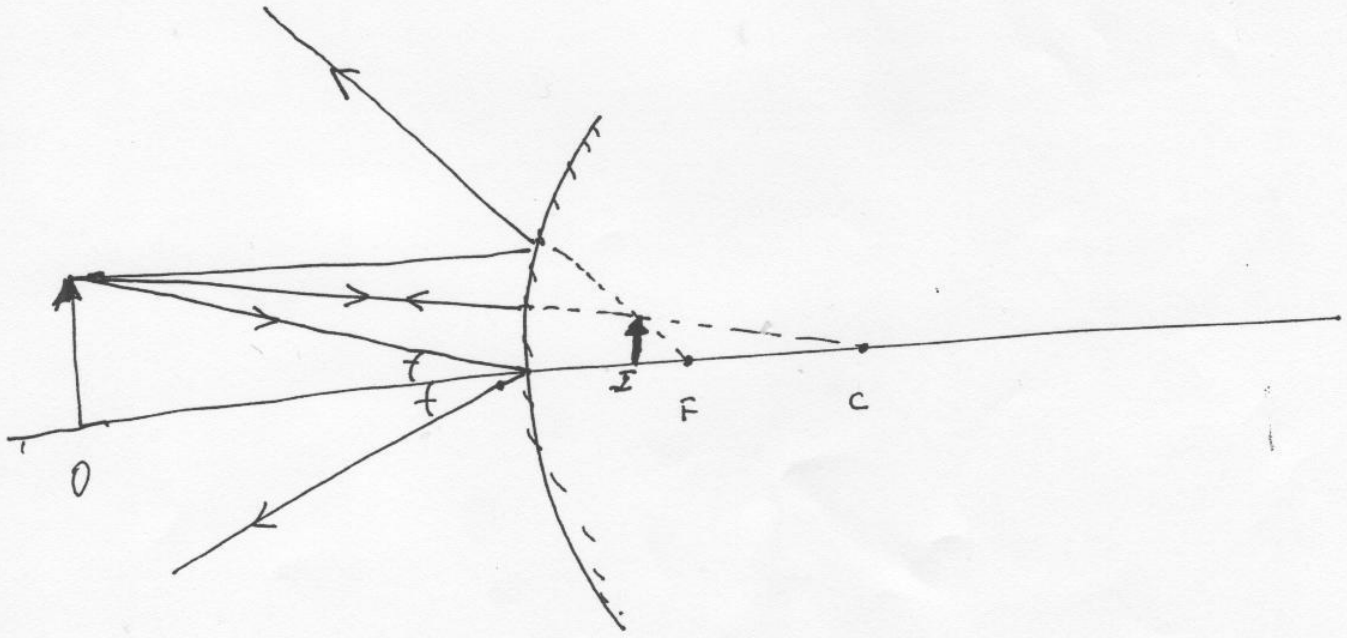
$$q = -0.23 \text{ m}$$

(b) $$m = -\frac{q}{p}$$

$$= -\left(-\frac{0.23}{3}\right)$$

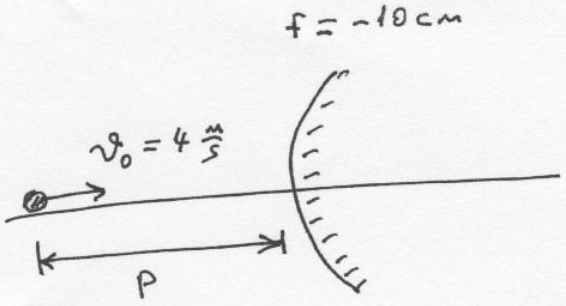
$$= +0.08$$

The image is smaller than the object and it is upright because m is positive and image is virtual since $q < 0$.



EXAMPLE 3

A ball approaches a convex mirror at a constant speed of $v_o = 6$ cm/s. Find the <u>speed</u> of its image when $p = 5$ cm.



HW

Calculate the average velocity of the image between $p = 6$ cm and $p = 4$ cm

SOLUTION

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$q = \frac{pf}{p-f}$$

image speed

$$v_q = \frac{dq}{dt} = \frac{d}{dt}\left(\frac{pf}{p-f}\right)$$

$$= \frac{dq}{dp}\underbrace{\frac{dp}{dt}}_{v_o}$$

$$= v_o \frac{dq}{dp}$$

$$= v_o \frac{d}{dp}\left[\frac{pf}{p-f}\right]$$

$$= v_o \left\{-\frac{f^2}{(p-f)^2}\right\}$$

$$= -\frac{v_o f^2}{(p-f)^2}$$

$$= -\frac{(6)(-10)^2}{[5-(-10)]^2}$$

$$= -2.67 \text{ cm/s}$$

↑ negative direction ←

Note that when $p \to 0$, $v_q = -v_o$

as object moves very close to mirro the mirror looks lik a plane mirror!