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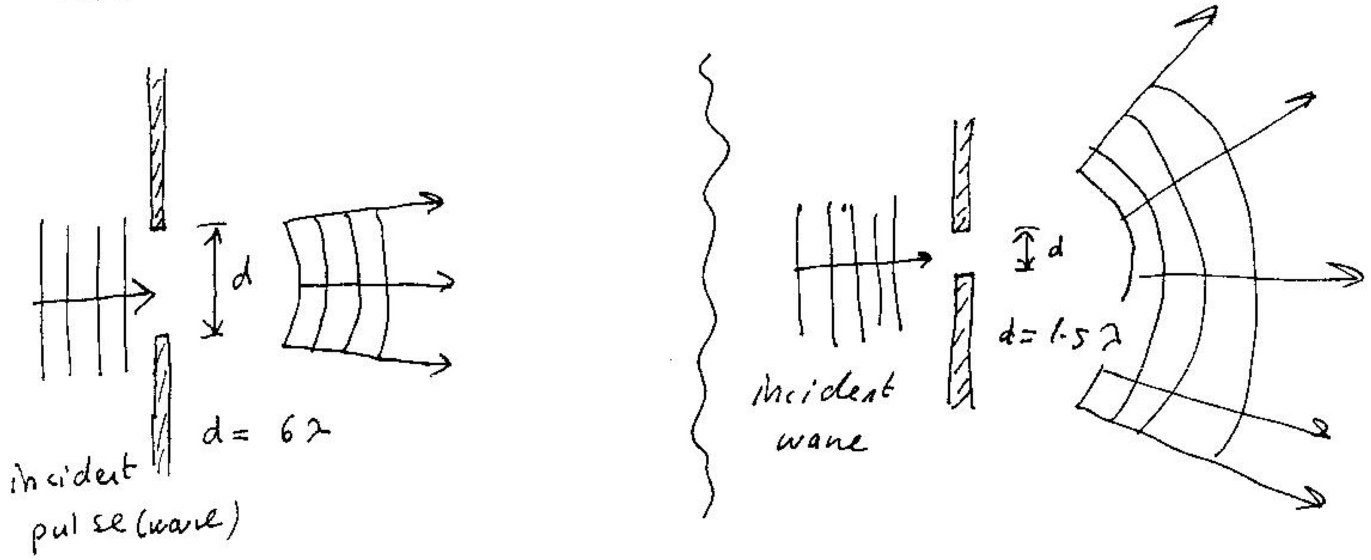
CHAPTER 5 INTERFERENCE

5.1 Introduction

In the previous chapters we assume that light travels in straight lines (ray). In this chapter we remove this restriction.



we will see:

- interference
- diffraction
- polarization

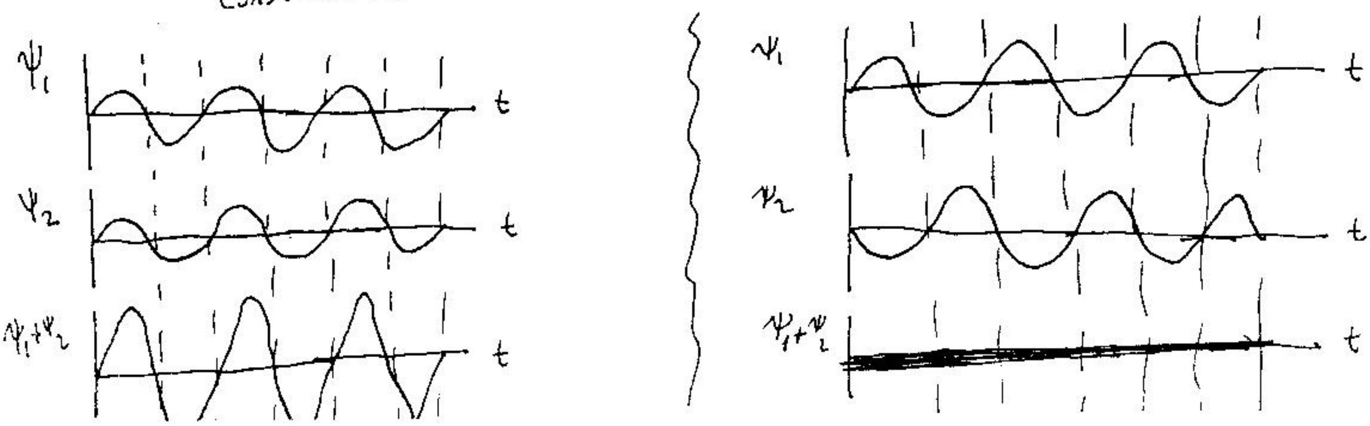
all of them are explained by the wave nature of light.

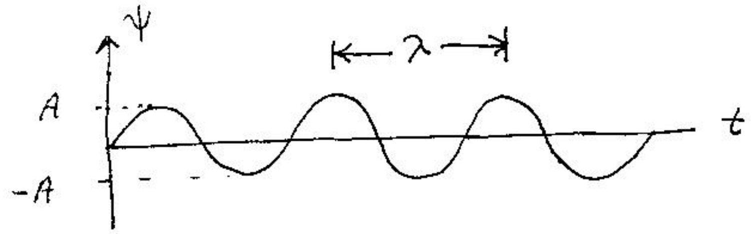
5.2 Interference

Plane wave
$\psi = A \sin(\omega t + \phi)$

Superposition of waves

Constructive interference | Destructive interference



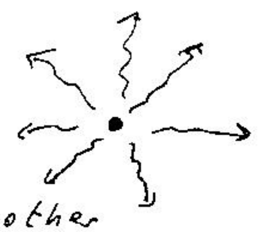


$$\psi = A \sin(\omega t + \phi)$$

amplitude — A; time — t; angular frequency — ω; phase constant — ϕ

$$\omega = 2\pi f = \frac{2\pi}{T}$$

To observe interference in light waves, the following conditions must be satisfied:



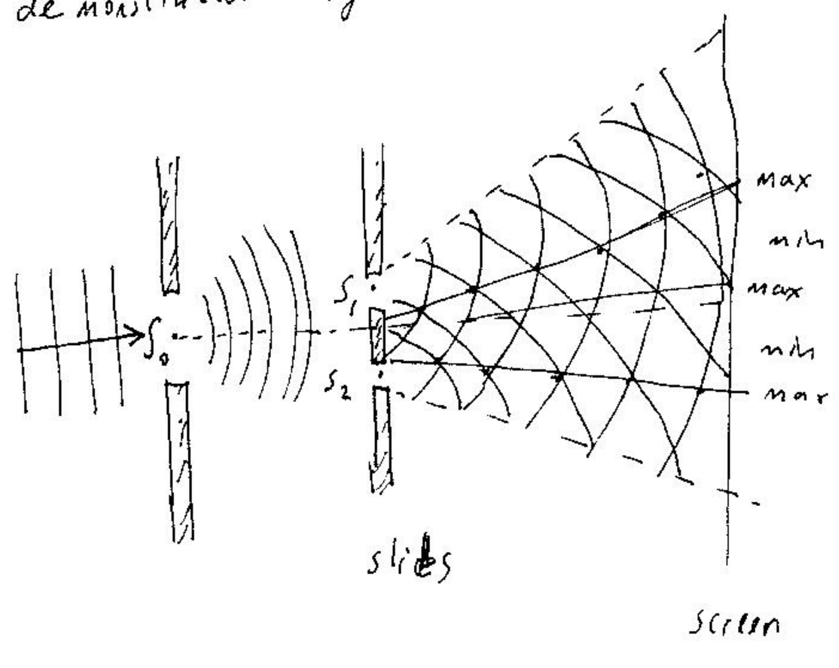
1. The light source must be "coherent"
   $\Rightarrow$ they must maintain a constant phase w.r.t each other

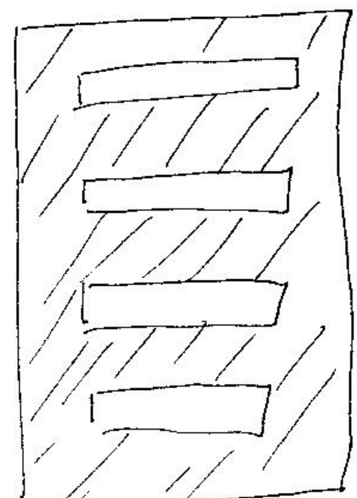
2. The light sourse must be "monochromatic"
   $\Rightarrow$ they must have a single wavelength

3. The superposition principle must apply

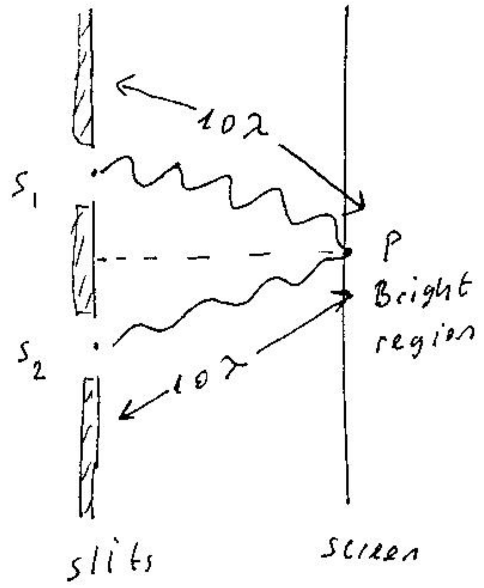
5.3 Young's Double Slit Experiment

Interference in light waves from two sources was first demonstrated by Thomas Young in 1801.

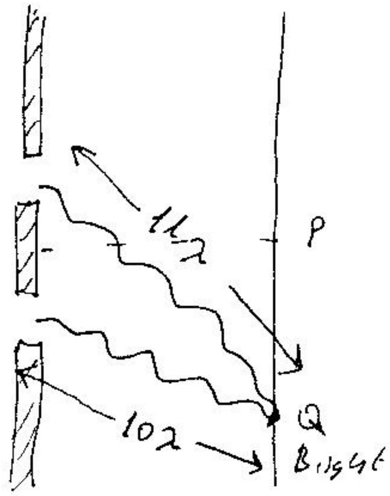




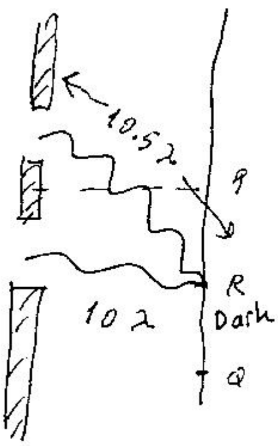
fringe pattern form on the screen



Constructive interference occur at point P.

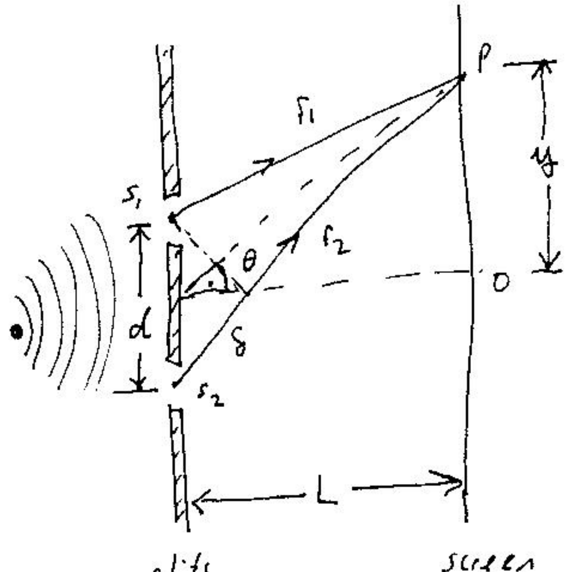


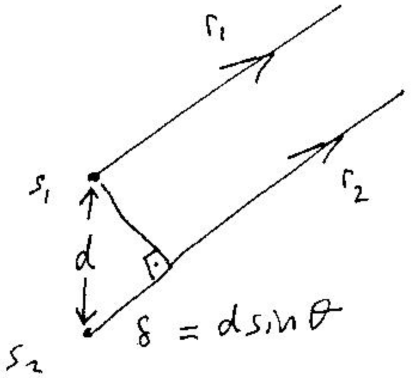
Constructive interference at point Q



Destructive interference at point R

Qualitative analysis of the experiment





$L \sim$ a few meters

$d \sim$ a few 100 micrometers.

- If $L \gg d \Rightarrow$ we can assume that $r_1 \parallel r_2$
- the path difference (δ) between two rays:

$$\delta = d\sin\theta \quad \text{and} \quad \sin\theta \approx \tan\theta = \frac{y}{L}$$

If δ is zero or integral multiple of λ, two waves at point P are in phase and constructive interference results.

condition for bright fringes

$$\boxed{\delta = m\lambda} \qquad m = 0, \mp 1, \mp 2, \dots$$

$$d \sin\theta = m\lambda$$

$$d \frac{y}{L} = m\lambda$$

If δ is half integral of λ, the waves at P are $180°$ out of phase and will result in destructive interference

condition for dark fringes

$$\boxed{\delta = \left(m + \frac{1}{2}\right)\lambda} \qquad m = 0, \mp 1, \mp 3 \dots$$

The integer number m is called "order number".
For bright fringes (maxima)
$m = 0 \rightarrow$ zeroth order maximum
$m = \mp 1 \rightarrow$ first order maximum.

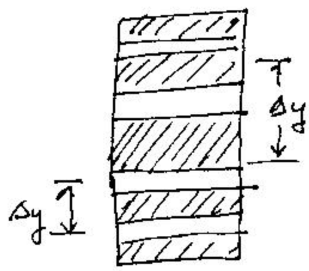
* The position of

bright fringes : $y_b = \frac{\lambda L}{d} m$

dark fringes : $y_d = \frac{\lambda L}{d} (m + 1/2)$

* The distance between adjacent fringes:

bright : $\Delta y_b = y_{m+1} - y_m$
$$= \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d}(m)$$
$$= \frac{\lambda L}{d}$$



dark : $\Delta y_d = \frac{\lambda L}{d}(m + \frac{1}{2} + 1) - \frac{\lambda L}{d}(m + \frac{1}{2})$
$$= \frac{\lambda L}{d}$$

$\Delta y_b = \Delta y_d \Leftarrow$ same for both case!

Example 1

A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.03 mm. The second-order bright fringe ($m=2$) is 4.5 cm from the center line.

(a) Determine the wavelength of the light.

$$y_b = \frac{\lambda L}{d} m \rightarrow \lambda = \frac{dy}{mL} = \frac{(3\times10^{-5})(4.5\times10^{-2}\,m)}{(2)(1.2)} = 5.6\times10^{-7}\,m$$
$$= 560\ nm.$$

(b) Calculate the distance between two adjacent bright fringes.

$$\Delta y = \frac{\lambda L}{d} = \frac{(5.6\times10^{-7})(1.2)}{3\times10^{-5}} = 2.2\times10^{-2}\,m = 2.2\ cm$$

Example 2

In a double-slit arrangement, $d = 0.15$ mm, $L = 140$ cm, $\lambda = 643$ nm and $y = 1.8$ cm.

(a) What is the path difference for the two rays from the slits arriving at P?

$$\delta = d\sin\theta \approx d\tan\theta = d\frac{y}{L} = (0.15\times10^{-3})\left[\frac{1.8}{140}\right] = 1.93\times10^{-6}\,m$$
$$= 1.93\,\mu m$$

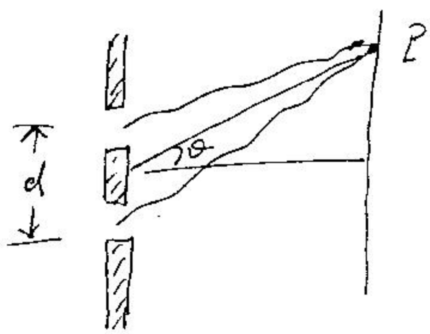
(b) Express this path difference in terms of λ.

$$\frac{\delta}{\lambda} = \frac{1.93\times10^{-6}\,m}{643\times10^{-9}\,m} = 3 \Rightarrow \delta = 3\lambda$$

(c) Does point P correspond to a maximum or minimum?

point P is a <u>maximum</u> since δ is <u>integral multiple</u> of λ.

Note that the intensity (energy transfered across a unit area per unit time) of waves on the screen is given by

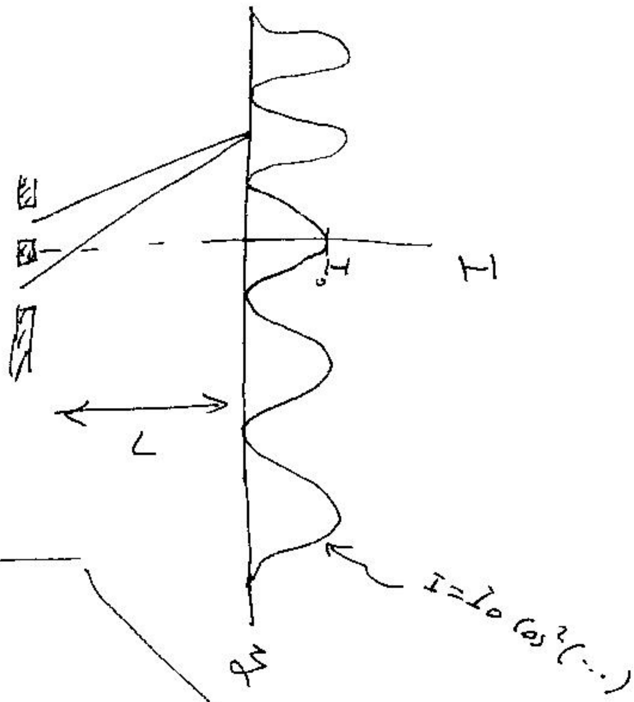


$$I = I_0 \cos^2\left[\frac{\pi d \sin\theta}{\lambda}\right] \; *$$

again for small angles $\sin\theta \simeq y/L = \tan\theta$

$$I = I_0 \cos^2\left[\frac{\pi d}{\lambda L} y\right]$$

I_0: maximum intensity



(*) Proof is omitted.

EXAMPLE 3 Let $L = 120$ cm and $d = 0.25$ cm. The slits are illuminated with $\lambda = 600$ nm light. Calculate the distance y above the central maximum for which the intensity on the screen is 75% of the maximum.

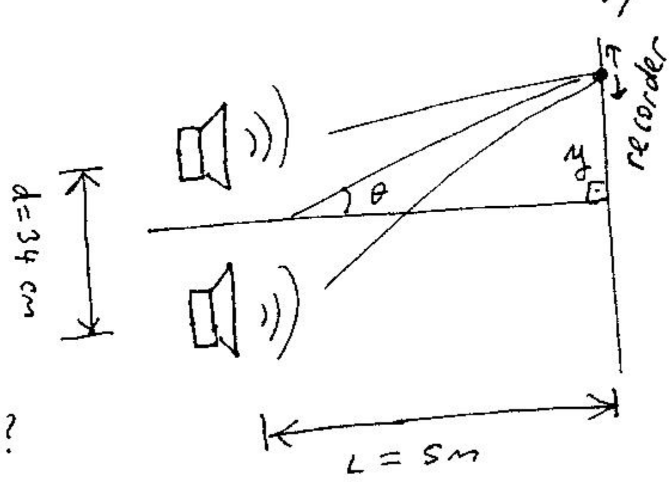
SOLUTION

$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda L} y\right) \longrightarrow y = \frac{\cos^{-1}\left(\sqrt{I/I_0}\right)}{\pi d/\lambda L}$$

$$y = \frac{\cos^{-1}\left(\sqrt{0.75\, I_0/I_0}\right)}{(\pi)(0.25\times 10^{-2})/(600\times 10^{-9})(1.2)} = 48\times 10^{-6}\text{ m} = 48\,\mu\text{m}$$

EXAMPLE 4

A 2 kHz sound wave is produced from two speakers as shown in figure.



(a) At what angles does the recorder records the best quality?

(b) If the sound intensity for $y = 0$ is 50 dB [deci bell] what is the expected intensity at a point corresponding to $y = 0.2$ m? (Hint: $v_{sound} = 340 \frac{m}{s}$)

SOLUTION

(a) path difference:

$$\delta = d\sin\theta = m\lambda \quad \leftarrow \quad \lambda = \frac{v_s}{f} = \frac{340}{2000} = 0.17 \text{ m}$$

$$\sin\theta = m\frac{\lambda}{d} = m\frac{v_s}{fd}$$

$$= m\frac{340}{(2000)(0.34)}$$

$$= 0.5 \text{ m}$$

$m = 0 \rightarrow \sin\theta = 0 \rightarrow \theta = 0°$ ✓ (ok accepted)

$m = \mp 1 \rightarrow \sin\theta = \mp 0.5 \rightarrow \theta = \mp 30°$ ✓

$m = \mp 2 \rightarrow \sin\theta = \mp 1 \rightarrow \theta = \mp 90°$ X

(b) $I_0 = 50$ dB

$$I = (50\text{ dB})\cos^2\left\{\frac{\pi d}{\lambda L}y\right\}$$

$$= (50\text{ dB})\cos^2\left\{\underbrace{\frac{(\pi)\,0.34}{(0.17)(5)}(0.2)}_{14.4°}\right\} = 46.9 \text{ dB}$$

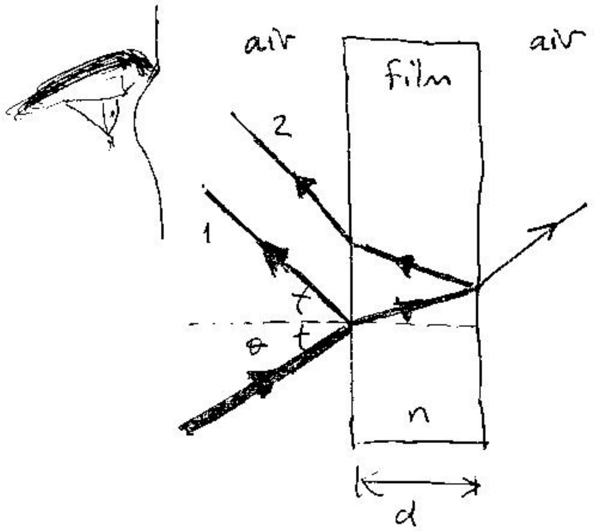
5.4 Interference in Thin Films (STRING ANALOGY)

Interference effects are commonly observed in thin films, such as:

* thin layers of oil on water and
* soap bubbles

The varied colors observed when white light is incident on such films results from interference of waves reflected from the two surfaces of the film.

Figure shows a transparent film of uniform thickness d. Assume that the light rays in air are nearly normal to the two surfaces of the film



- ~~A wave travelling from a medium of index of refrac~~

(1) 180° phase change
(2) no-phase change
$\theta \approx 0°$

- A wave travelling from a medium of index of refraction n_1 (air) toward a medium of index of refraction n_2 (film) undergoes a 180° phase change if $n_2 > n_1$. There is no phase change if $n_2 < n_1$

- The wavelength of a light
  is λ_n in the film
  is λ in the air

$$v_{air} = f\lambda \qquad v_{film} = f\lambda_n$$

$$\frac{v_{air}}{v} = \frac{\lambda_n}{\lambda} = \frac{c/n_{air}}{c/n}$$

$$\Rightarrow \boxed{\lambda_n = \frac{\lambda}{n}}$$

Ray ① is 180° out of phase with ray ②.

This is equivalent to a path difference: $\lambda_n/2$.

Ray ② travels an extra distance $2d$ before the waves recombine. For example,

if $2d = \lambda_n/2 \Rightarrow$ ray ① and ② recombine in phase and result is constructive interference

In general:

$$2d = \left(m+\frac{1}{2}\right)\lambda_n \qquad (m = 0, 1, 2, \dots)$$

substituting $\lambda_n = \lambda/n$ yields:

$$\boxed{2nd = \left(m+\frac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots)}$$ maxima condition

If the extra distance $(2d)$ travelled by ray ② corresponds to a multiply of λ_n, two waves combine out of phase and the result is destructive interference.

$$\boxed{2nd = m\lambda \qquad (m = 0, 1, 2, \dots)}$$ minima

EXAMPLE 5 A water film ($n = 1.33$) in air is 320 nm thick. If it is illuminated by a white light at normal incidence, what color will it appear to be in reflected light?

SOLUTION

- maxima (reflected): $\lambda = \frac{2nd}{m + 1/2} = \frac{(2)(1.33)(320\text{ nm})}{m + 1/2} = \frac{850\text{ nm}}{m+1/2}$

$$\left.\begin{array}{llll}
m = 0 \rightarrow & \lambda = \frac{850}{0+1/2} & = 1700\text{ nm} ; & \text{infrared} \\
m = 1 \rightarrow & \lambda = \frac{850}{1+1/2} & = 567\text{ nm} ; & \text{visible} \\
m = 2 \rightarrow & \lambda = \frac{850}{2+1/2} & = 340\text{ nm} ; & \text{visible} \\
m = 3 \rightarrow & \lambda = \frac{850}{3+1/2} & = 243\text{ nm} ; & \text{ultra-violett}
\end{array}\right\}\text{reflected}$$

- minima (transmitted): $\lambda = \frac{2nd}{m} = \frac{850\text{ nm}}{m}$

$$\left.\begin{array}{llll}
m = 1 \rightarrow & \lambda = 850\text{ nm} & & ; \text{infrared} \\
m = 2 \rightarrow & \lambda = \frac{850}{2} & = 425\text{ nm} ; & \text{visible} \\
m = 3 \rightarrow & \lambda = \frac{850}{3} & = 283\text{ nm} ; & \text{ultra-violett}
\end{array}\right\}\text{transmitted}$$

EXAMPLE 6 Calculate the minimum thickness of a soap bubble film ($n = 1.33$) that results in constructive interference in the reflected light if the film is illuminated by light of $\lambda = 600\,nm$

SOLUTION

$$2nd = \left(m + \frac{1}{2}\right)\lambda$$

$$d = \frac{(m + 1/2)\,\lambda}{2n} = \frac{(m+1/2)(600\,nm)}{(2)(1.33)} = 225\,nm\,(m + 1/2)$$

$m = 0 \rightarrow$ $\boxed{d = 112.5\ nm}$ ← minimum thickness

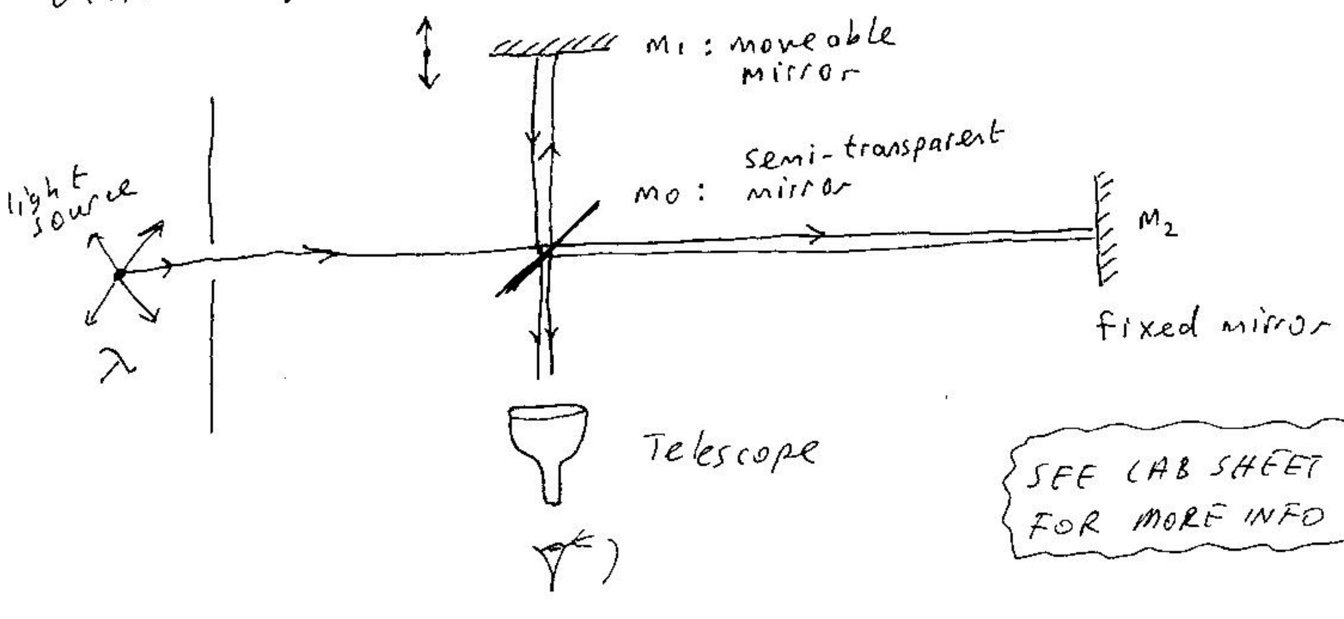
$m = 1 \rightarrow$ $d = 337.5\ nm$

$m = 2 \rightarrow$ $d = 562.5\ nm$

$\vdots$ $\vdots$

5.5 Michelson Interferometer

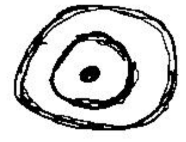
The interferometer splits a light beam into two parts and then recombines them to form an interference pattern. The device can be used to measure wavelengths or other lengths accurately.



SEE LAB SHEET FOR MORE INFO

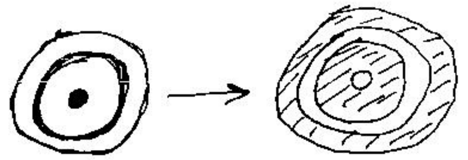
The interference condition for two rays is determined by their path difference (δ).

* Interference pattern is a dark or bright circular fringes in the telescope



* As m_1 is moved (up or down) the fringe pattern collapses or expands!

* Consider we have dark circle at the center.



If m_1 is moved a distance $\frac{\lambda}{4}$ toward m_0 the path diff. changes to $\frac{\lambda}{2}$. As a result dark circle at the center becomes bright circle

$\therefore$ λ can be measured by counting the number of fringe shifts for a given displacement of m_1

<u>EXAMPLE 7</u> Monochromatic light is beamed into a Michelson interferometer. The moveable mirror is displaced 0.382 mm causing the interference pattern to produce itself 1700 times. Determine wavelength of the light.

<u>SOLUTION</u>

path diff. $\delta = (3.82 \times 10^{-4} m) * 2 = 1700\,\lambda$

or $\lambda = 4.49 \times 10^{-7} m = 449$ nm $\leftarrow$ blue!