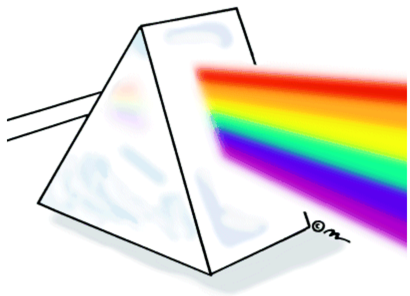
EP118 Optics

TOPIC 9
ABERRATIONS



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Content

9.1 Introduction

- In an ideal optical system, all rays of light from a point in the object plane would converge to the same point in the image plane, forming a clear image.

- The influences which cause different rays to converge to different points are called **aberrations**.

- Aberration leads to blurring of the image produced by an image-forming optical system. It occurs when light from one point of an object after transmission through the system does not converge into (or does not diverge from) a single point.

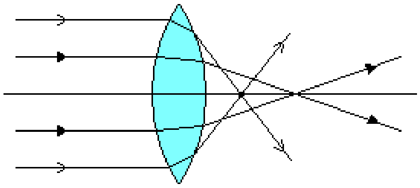
- Optical Instrument-makers need to correct optical systems to compensate for aberration.

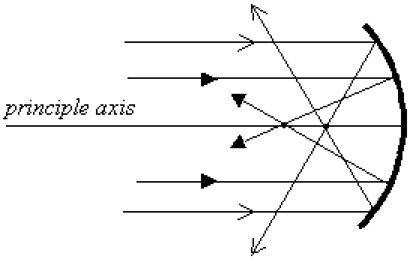
- In this chapter, we will consider spherical and chromatic aberrations and their corrections.

9.2 Spherical Aberrations (SA)

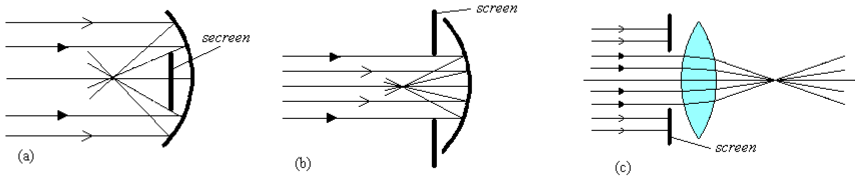
- Spherical aberrations occur because the focal points of rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis.

- Figures given below illustrate *spherical aberrations* for parallel rays passing through a converging lens, and reflecting from a concave mirror. Rays are converged to different points instead of a single focal point.





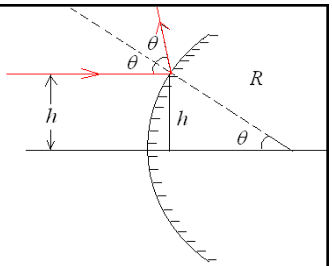
- Spherical aberration can be reduced by a screening method.
- To do this, a screen (pupil) can be placed in front of the mirror or lens allowing parallel rays to focus on a single point.



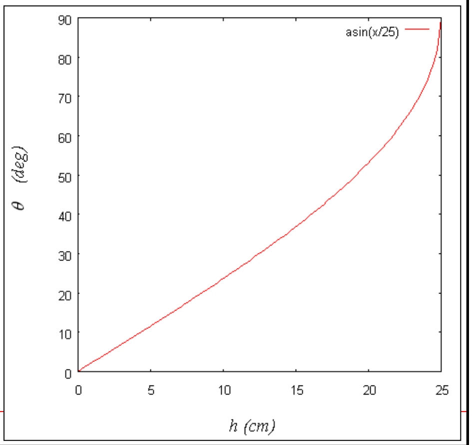
- NOTE THAT
  - Screening reduces the brightness (light intensity) of the image while it reduces the spherical aberrations.
  - Many cameras have an adjustable aperture to control light intensity and reduce spherical aberration.

EXAMPLE 1

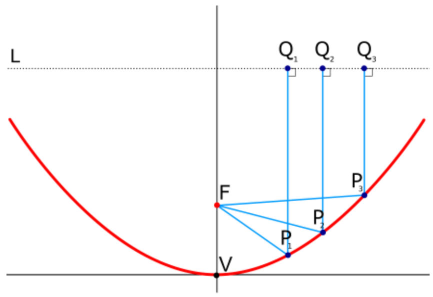
A ray traveling parallel to the principal axis strike a convex mirror whose radius of curvature is R = 25 cm as shown in Figure. Assume that the ray is at distances h = 5 cm. Find the angle θ.



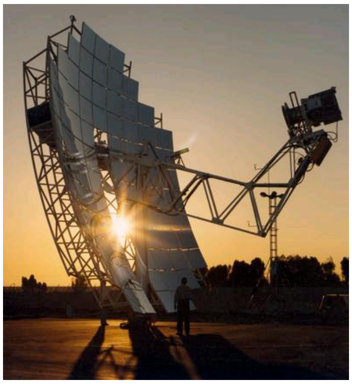
SOLUTION



- To overcome the spherical aberration in mirrors, parabolic reflecting surface rather than a spherical surface is used.
- Parabolic surfaces are not used often --> *very expensive to make*.
- Parallel light rays incident on a parabolic surface focus at a common point, regardless of their distance from the principal axis.
- Parabolic reflecting surfaces are used in many astronomical telescopes to enhance image quality.
- One can prove why we need parabolic reflecting surface (See Appendix at the end)



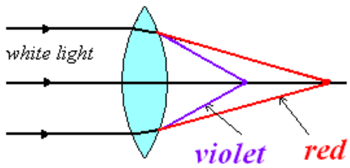
A parabolic solar dish

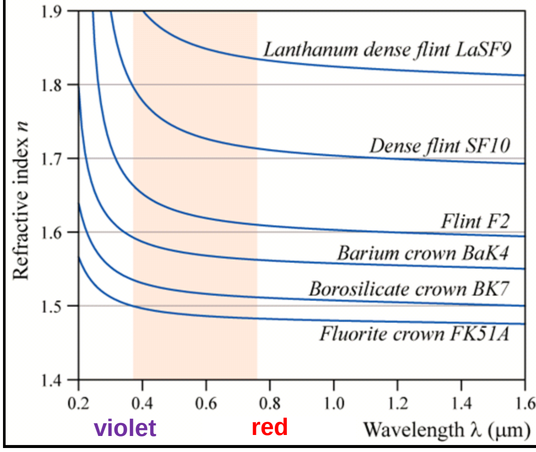


The mirror of the Hubble Space Telescope

9.3 Chromatic Aberrations (CA)

- A lens will not focus different colors in exactly the same place.
- CA occurs because lenses have a different refractive index for different wavelengths of light (the dispersion of the lens).



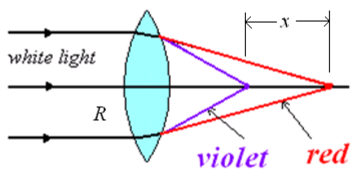


$$v = \frac{c}{n(\lambda)}$$

$$1 < n(\lambda_{\text{red}}) < n(\lambda_{\text{yellow}}) < n(\lambda_{\text{blue}})$$

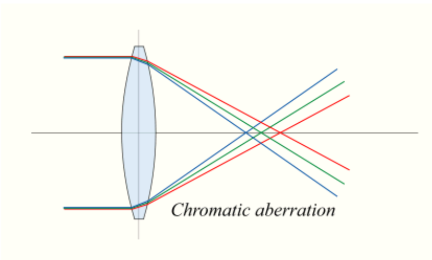
EXAMPLE 2

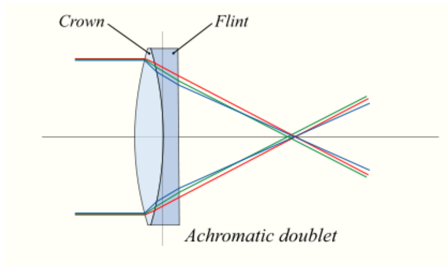
Figure shows a bi-convex lens of same radius (*R = 10 cm*) made up of a <u>dense flint SF10</u>. Determine the distance (*x*) between focal length of the violet and red lights if the lens is illuminated by a white light.



SOLUTION

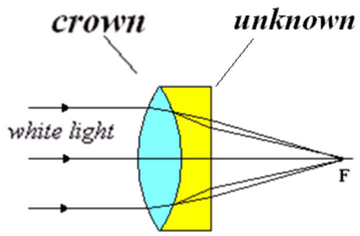
- One way to minimize this aberration is to use glasses of different dispersion in a doublet or other combination.
- The use of a strong positive lens made from a low dispersion glass like **crown glass** coupled with a weaker high dispersion glass like **flint glass** can correct the *chromatic aberration* for two colors, e.g., red and blue.
- Such doublets are often cemented together and called **achromat**.
- An achromatic lens (or achromat) is designed to limit the effects of chromatic and spherical aberration.

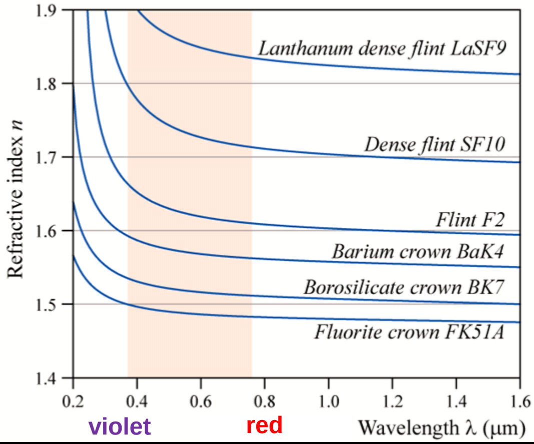




EXAMPLE 3

Figure shows an achromatic lens consisting of a crown glass BK7 (bi-convex converging lens of same radius), and an unknown glass (plano-concave diverging lens). Lenses are cemented together. The refractive indices of some materials as a function of wavelength are given figure. Both lenses have the same radius of curvature R.





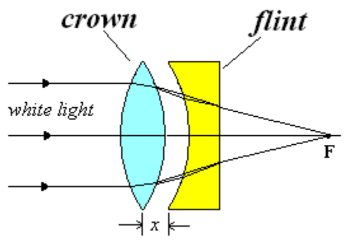
Find the best unknown material such that focal points of the red and violet are the same for achromat lens.

SOLUTION

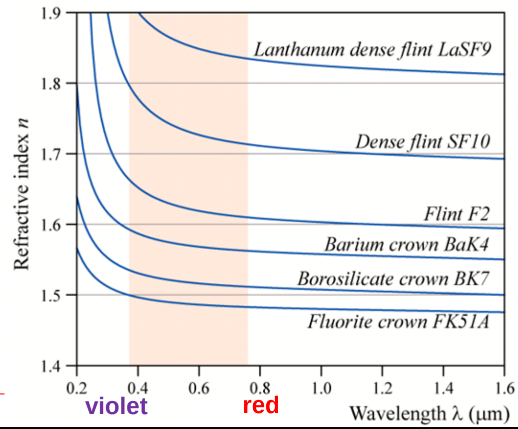
From the figure we can roughly find the refractive index for red and violet for the crown glass (BK7) as $n_C^{red} = 1.51$ *and* $n_C^{violet} = 1.54$ *respectively.*

EXAMPLE 4

Figure shows an achromatic lens consisting of a crown glass BK7 (bi-convex converging lens of same radius), and flint glass F2 (plano-concave diverging lens). Their refractive indices as a function of wavelength are given below. Lenses are separated by a distance *x*. Both lenses have the same radius of curvature *R* = 10 cm.



Find a value of *x* such that the focal points of the red and violet are the same.



SOLUTION

From the figure we can roughly find the refractive index as follows:

Lens	Refractive index (n)	
	Red(700 nm)	Violet(380 nm)
Crown	1.51	1.54
Flint	1.61	1.67

Applying Lens Maker's Equation for both lenses yields:

For crown glass $f_C = \dfrac{R/2}{n_C - 1}$ *and for flint glass* $f_F = \dfrac{-R}{n_F - 1}$

$$f_C^r = \frac{R/2}{n_C^r - 1} = \frac{10/2}{1.51 - 1} = 9.8\,\text{cm} \qquad f_F^r = \frac{-R}{n_F^r - 1} = \frac{-10}{1.61 - 1} = -16.4\,\text{cm}$$

$$f_C^v = \frac{R/2}{n_C^v - 1} = \frac{10/2}{1.54 - 1} = 9.3\,\text{cm} \qquad f_F^v = \frac{-R}{n_F^v - 1} = \frac{-10}{1.67 - 1} = -14.9\,\text{cm}$$

Now we can calculate the focal length of the compound lens system. The distance from flint lens to the new focus as function of x for both wavelength can be as follows:

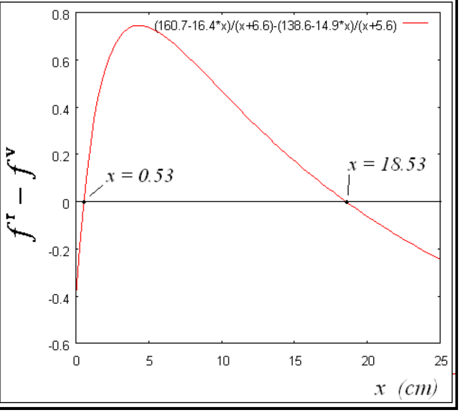
$$f^{\mathrm{r}}(x) = \frac{f_{\mathrm{F}}^{\mathrm{r}}(x - f_{\mathrm{C}}^{\mathrm{r}})}{x - (f_{\mathrm{C}}^{\mathrm{r}} + f_{\mathrm{F}}^{\mathrm{r}})} = \frac{(-16.4)(x - 9.8)}{x - (9.8 - 16.4)} = \frac{160.7 - 16.4x}{x + 6.6}$$

$$f^{\mathrm{v}}(x) = \frac{f_{\mathrm{F}}^{\mathrm{v}}(x - f_{\mathrm{C}}^{\mathrm{v}})}{x - (f_{\mathrm{C}}^{\mathrm{v}} + f_{\mathrm{F}}^{\mathrm{v}})} = \frac{(-14.9)(x - 9.3)}{x - (9.3 - 14.9)} = \frac{138.6 - 14.9x}{x + 5.6}$$

The aim is to find a value of x minimizing the distance between two focal points: That is $f^{\mathrm{r}} - f^{\mathrm{v}} = 0$

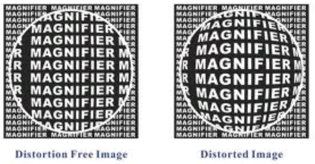
$$\frac{160.7 - 16.4x}{x + 6.6} = \frac{138.6 - 14.9x}{x + 5.6}$$

$\boxed{x = 0.53 \text{ cm}}$ or $\boxed{x = 18.53 \text{ cm}}$

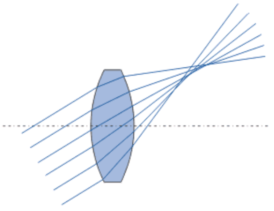


9.4 Other Types of Aberrations

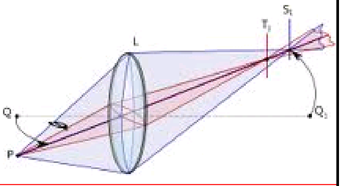
- Distortion is a deviation from rectilinear projection.



- Coma refers to aberration due to imperfection in the lens



- Astigmatism is one where rays that propagate in two perpendicular planes have different focal lengths.

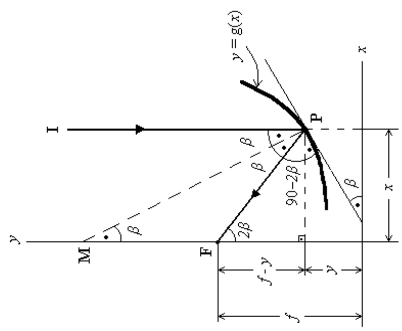


Appendix: Derivation of parabolic surface

1.1 Eğrisel Bir Yüzeyin Odak Uzaklığı

Bu kısımda herhangi bir eğrisel yüzeye paralel gelen ışınların yansıdıktan sonra asal ekseni kestiği noktanın (odak noktası) bulunuşu gösterilecektir. Basit ve daha anlaşılır olması açısından problem yüzey yerine bir eğri için çözülecektir.

Şekil 4'de denklemi $y = g(x)$ ile belli olan bir eğriye y eksenine paralel bir I ışını geldiğni düşünelim. Bu ışın P noktasında yansıdıktan sonra y eksenini F noktasında kessin. Bu durumda ışının y eksenini kestiği nokta, f, odak uzaklığıdır.



Şekil 4: $y = g(x)$ eğrisine gelen bir ışının yansıması

Şekilden:

$$\tan 2\beta = \frac{x}{f-y} \qquad (2)$$

olduğu açıktır. Fonksiyonun P noktasındaki teğetinin eğimi o noktadaki türevine eşittir. Bundan dolayı:

$$\text{eğim} = \tan\beta = \frac{dy}{dx} = y' \qquad (3)$$

Diğer taraftan

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} \qquad (4)$$

dır. (2) ve (3) numaralı denklemler Denklem (4)' yerleştirildiğinde odak uzaklığı için:

$$f = \frac{x(1-y'^2)}{2y'} + y \qquad (5)$$

bağıntısı elde edilir.

Denklem (5) f odak uzaklığının ile x, y ve y' değişkenlerine bağlı olarak hesaplanmasına imkan sağlar. Gerçekte bu denklem doğrusal olmayan bir diferansiyel denklemdir ve şöyle de yazılabilir:

$$\left(\frac{dy}{dx}\right)^2 + \frac{2(f-y)}{x}\left(\frac{dy}{dx}\right) = 1 \qquad (6)$$

Bu denklemin çözümü, $y = f(x)$, sonsuz uzaklıktan bir yüzeye paralel gelen ışınları tek bir sabit noktada, f, toplayan yüzeyin denklemini verecektir. $u = y/x$ dönüşümü yapıldıktan sonra denklemin çözümü:

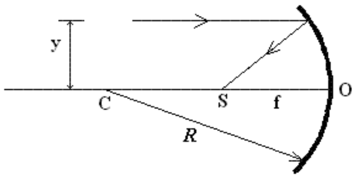
$$y = \frac{x^2}{4f} \qquad (7)$$

dir. Bu bir p a r a b o l denklemidir. Öyle ise, sonsuzdan gelen ışınları ancak pabolik yüzeyler kusursuz olarak odaklayabilir.

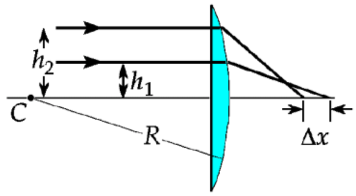
9.5 Exercises

1. Does the image formed by a plane mirror suffer from any aberration?
2. What are spherical and chromatic of aberrations?
3. Explain how can the spherical and chromatic aberrations be reduced?
4. The magnitudes of the radii of curvature are 30 cm and 40 cm for the two faces of a biconcave lens. The glass has index of refraction 1.54 for violet light and 1.50 for red light. For a very distant object, locate and describe (a) the image formed by violet light, and (b) the image formed by red light.
5. In EXAMPLE 1, find an expression for the focal length (extension of the ray reflected on principle axis) of the mirror as a function of *h*.
6. Repeat EXAMPLE 4 for LaSF9 and FK51A glasses.

7. A light ray traveling parallel to the principal axis at a distance y from the principal axis strike a concave mirror having a radius of curvature R as shown. The ray is focused on point S such that |OS| = f.
(a) Find an expression for f in terms of y and R.
(b) What is the value of f when $R >> y$?
Conclude the result. *Hint*: $\tan 2\theta = 2\tan\theta/(1-\tan 2\theta)$

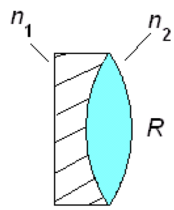


8. (Serway 6th Edition ref[1])
Two rays traveling parallel to the principal axis strike a large plano-convex lens having a refractive index of 1.6. If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs). Assume this face has a radius of curvature of 20.0 cm and the two rays are at distances h1 = 0.5 cm and h2= 12.0 cm from the principal axis. Find the difference Δx in the positions where each crosses the principal axis.



9. Two lenses made of kinds of glass having different refractive indices n_1 and n_2 are cemented together to form what is called an optical doublet. Optical doublets are often used to correct chromatic aberrations in optical devices. The first lens of a doublet has one flat side and one concave side of radius of curvature R. The second lens has two convex sides of radius of curvature R. Show that the doublet can be modeled as a single thin lens with a focal length described by:

$$f = \frac{R}{2n_2 - n_1 - 1}$$



9.6 References

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