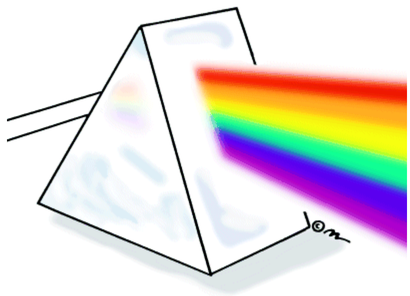
EP118 Optics

TOPIC 10
PHOTOMETRY



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Content

10.1 Introduction

- In optics, the electromagnetic radiation measurement is studied in two groups:

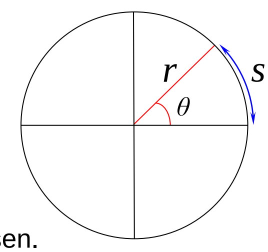
1. *radiometry* is the measurement of optical radiation including visible light
2. *photometry* is the measurement of visible light only.

- In this chapter, we will discuss the details of these two concepts and their units in SI.

10.2 Angle

- Angle in two-dimension (2D) defined as

$$\theta = k\frac{s}{r}$$

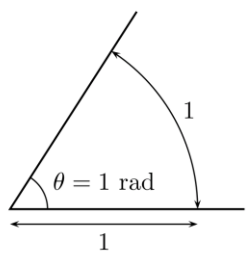


where *k* is a proportionality constant and depends on the unit of measurement that is chosen.

for radian measure *k* = 1
for degree measure *k* = 180/π ≈ 57.3

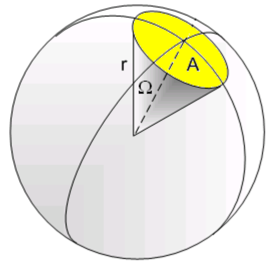
- Full circle is 2π radians:

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \ \text{rad}$$



- 1 radian defines an arc of a circle that has the same length as the circle's radius.
- 1 rad = 57.3°

10.3 Solid Angle



A : Surface area subtended from the center

r : Radius of the sphere

- The solid angle, Ω, is the 2D angle in 3D space that an object subtends at a point.
- Definition

$$\Omega = \frac{A}{r^2}$$

- It is a measure of how large that object appears to an observer looking from that point.
- SI unit is steradian (sr)
- The solid angle of a sphere measured from a point in its interior is 4π sr.

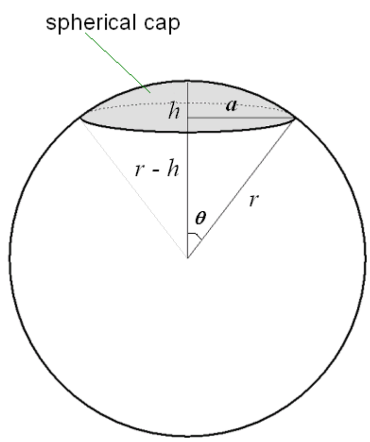
$$\Omega = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \text{ sr}$$

- Area of a spherical cap:

$$A = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$$

- Solid angle subtended:

$$\Omega = \frac{A}{r^2} = 2\pi(1 - \cos\theta)$$



Try yourself to prove these two relations!

EXAMPLE 1

What is the solid angle of the Moon subtended from the Earth?

Distance of the Moon to the Earth is 384,400 km and the radius of the Moon of 1738 km.

SOLUTION

We can assume that the area of the moon is approximately equal to the spherical cap since the Moon-Earth distance (d) is much more grater than the radius (R) of the moon (d>>R).

10.4 SI Base Units

The International System of Units (SI) defines seven units of measure as a basic set from which all other SI units are derived.

These SI base units and their physical quantities are:

* meter	for length
* kilogram	for mass
* second	for time
* ampere	for electric current
* kelvin	for temperature
* candela	for luminous intensity
* mole	for the amount of substance

Name	Symbol	Definition
Meter	m	*The length* of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.
Kilogram	kg	*The mass* of the international prototype of the kilogram
Second	s	*The duration* of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom
Ampere	A	The constant *electric current* which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 10^{-7} newton per metre of length
Kelvin	K	The fraction 1/273.16 of the *thermodynamic temperature* of the triple point of water
Mole	mol	*The amount of substance* of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12 atom
Candela	cd	*The luminous intensity* in a given direction, of a light source that emits monochromatic radiation of frequency 540 10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian

10.5 Radiometry

- Radiometry is the field that studies the measurement of electromagnetic radiation, including *visible light*.
- Some SI radiometric units

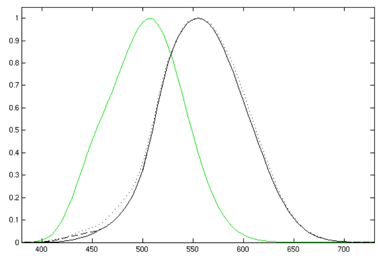
Quantity	Symbol	SI unit	Abbr.
Radiant energy	***Q***	**Joule**	**J**
Radiant flux or Radiant power	***Φ***	**Watt**	**W**
Radiant intensity	***I***	**Watt per steradian**	**W/sr**
Irradiance	***E***	**Watt per square-meter**	**W/m²**
Radiance	***L***	**Watt per steradian per meter-square**	**W/sr.m²**

10.6 Photometry

- Photometry is the science of the measurement of light, in terms of its perceived brightness to the human eye.

- The human eye is not equally sensitive to all wavelengths of visible light.

- Photometry attempts to account for this by weighing the measured power at each wavelength with a factor that represents how sensitive the eye is at that wavelength (see later).



Photopic (black) and scotopic (green) luminosity functions.

- *For everyday light levels*, the **photopic** curve (black) best approximates the response of the human eye.
- *For low light levels*, the response of the human eye changes, and the **scotopic** curve (green) applies.

- Some SI photometric units

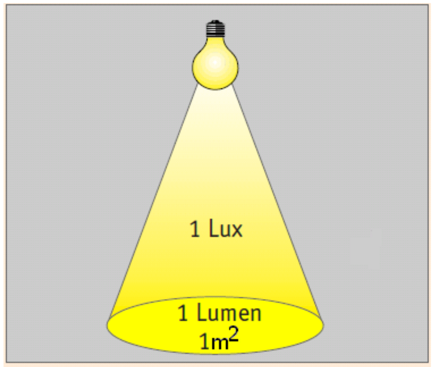
Quantity	Symbol	SI unit	Abbr.
Luminous energy	Q_v	**lumen.second**	**lm.s**
Luminous flux or Luminous power	Φ_v	**lumen**	**lm**
Luminous intensity	I_v	**candela**	**cd = lm/sr**
illuminance	E_v	**lumen per meter-square**	**lux = lm/m^2**
Luminance	L_v	**lumen per steradian per meter-square**	**lm/sr.m^2 = cd/m^2**

Turkish names:

Flux = Akı
Intensity = Şiddet
illuminance = Aydınlanma
Luminance = Işıldama

- **Typical illuminaces:**

*** Direct sun light**	**100,000 lux**
*** Working desk**	**500 lux**
*** Hospital corridors**	**20-50 lux**

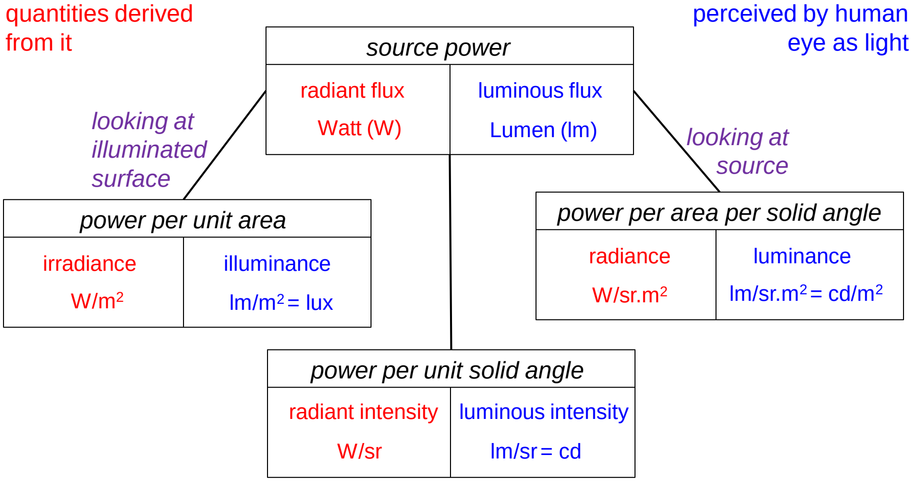


$1\,lx = 1\,lm/m^2$

10.7 Unit Comparison

Radiometry measures the entire radiant power and quantities derived from it

Photometry measures that part of radiant power perceived by human eye as light



10.8 Photometer

- Photometer is an instrument for measuring **light intensity**.
- Most of the, photometers are used to measure iluminance (E_v) or irradiance (E).
- Measuring E_v is important in *illumination Engineering.*
- Most photometers detect the light with photoresistors, photodiodes or photomultipliers (*we will see later*).

10.9 Radiometric & Photometric Equations

Radiative flux of point source:

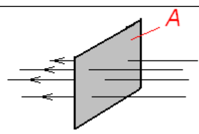
$$\Phi = 4\pi I$$

Luminous flux of point source:

$$\Phi_v = 4\pi I_v$$

Irradiance on area *A*:

$$E = \frac{\Phi}{A}$$

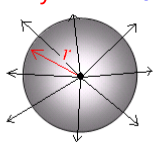


illuminance on area *A*:

$$E_v = \frac{\Phi_v}{A}$$

Irradiance of a point source of intensity *I*

$$E = \frac{\Phi}{A} = \frac{4\pi I}{4\pi r^2} = \frac{I}{r^2}$$

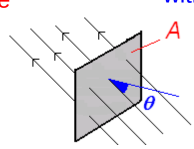


illuminance of a point source of intensity I_v

$$E_v = \frac{\Phi_v}{A} = \frac{4\pi I_v}{4\pi r^2} = \frac{I_v}{r^2}$$

If radiation direction makes an angle θ with the normal of irradiated surface

$$E = \frac{I}{r^2}\cos\theta$$



If radiation direction makes an angle θ with the normal of illuminated surface

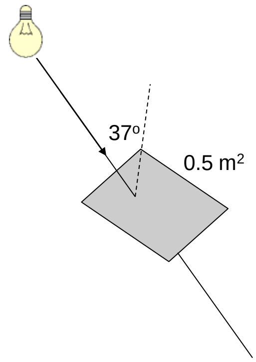
$$E_v = \frac{I_v}{r^2}\cos\theta$$

EXAMPLE 2

The light rays emerging from a point source of intensity 100 cd fall on a planar surface whose area is 0.5 m^2 at distance 1 m from the source. The rays make an angle of 37^o with the normal of a planar surface.

(a) Find the total flux of the source.
(b) Find the illuminance on the surface.
(c) Find the flux on the surface.

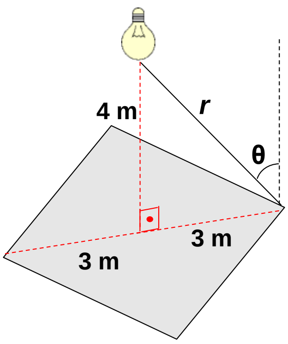
SOLUTION



EXAMPLE 3

A 1000 cd-bulb is hang at a height of 4 m from the center of the floor of a room having square shape with diagonal length of 6 m as shown in figure. Calculate the illuminance of the bulb at any corner of the floor.

SOLUTION



10.10 Radiometry and Photometry Conversion

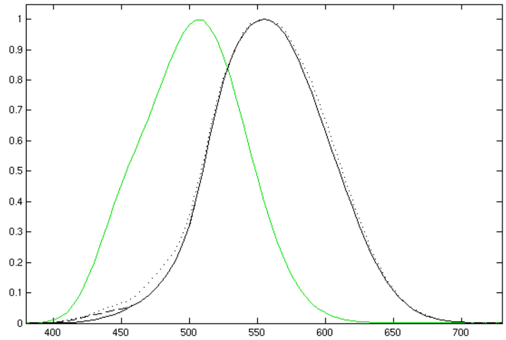
The radiant power at each wavelength is weighted by a luminosity function V(λ) that models human brightness sensitivity.

For photopic curve (black):

$$V(\lambda) = 1.019 e^{-285.4(\lambda - 0.559)^2}$$

For scotopic curve (green)

$$V'(\lambda) = 0.992 e^{-312.9(\lambda - 0.503)^2}$$



- We know from the definition of the candela that there are 683 lumens per watt at a wavelength 555 nm (in vacuum or air). This is the wavelength that corresponds to the maximum spectral responsivity of the *human eye*.

- The conversion from watts to lumens at any other wavelength involves the product of the power (watts) and the V(λ) value at the wavelength of interest. For *mono-chromatic wave* we can use

$$\Phi_v = (683 \text{lm/W})\, \Phi V(\lambda)$$

- In order to convert a source with *non-monochromatic* spectral distribution to a luminous quantity, the situation is decidedly more complex. We must know the spectral nature of the source, because it is used in an equation of the form:

$$\Phi_v = (683 \text{lm/W}) \int_0^\infty \Phi(\lambda) V(\lambda) d\lambda$$

EXAMPLE 4

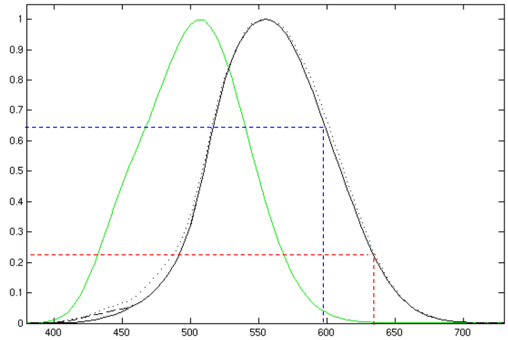
Compare brightness' of two 5 mW laser pointers at 635 nm and 600 nm.

SOLUTION

* at λ =600 nm, V(λ) = 0.650 --> $\Phi_v = (683\frac{\text{lm}}{\text{W}})(0.005\,\text{W})(0.65) = 2.22\ \text{lm}$

* at λ = 635 nm, V(λ) = 0.217 --> $\Phi_v = (683\frac{\text{lm}}{\text{W}})(0.005\,\text{W})(0.217) = 0.74\ \text{lm}$

The shorter wavelength (600 nm) laser pointer will create a spot that is almost 3 times as bright as the longer wavelength (635 nm) laser assuming the same beam diameter.



10.11 Efficiency and Efficacy of a Light Source

- Efficiency and efficacy may be defined as follows*

$$\text{Efficiency} = \frac{\text{Visible Radiant Flux}}{\text{Power Consumed}} = \frac{\Phi}{P} \longrightarrow \frac{\text{Watts}}{\text{Watts}} \equiv \text{unitless}$$

$$\text{Efficacy} = \frac{\text{Luminous Flux}}{\text{Radiant Flux}} = \frac{\Phi_v}{\Phi} \longrightarrow \frac{\text{lumens}}{\text{Watts}} \equiv \frac{\text{lm}}{\text{W}}$$

**** Depending on context, the "power" can be either***
the radiant flux of the source's output, or
the total electric power consumed by the source.

Type	Luminous Efficiency	Luminous Efficacy (lm/W)
Sun	**12 %**	**80**
100–200 W tungsten lamb	**2 %**	**13 – 15**
10–30 W Fluorescent lamb	**8 % – 11%**	**46 – 75**
White LED	**1 % – 22 %**	**5 – 150**
Ideal monochromatic 555 nm source	**100 %**	**683**

EXAMPLE 5

On a table, one needs a 60 lux illuminance. A 40 W-lamp whose luminous efficacy is 100 lm/W and efficiency is 20% will be used for illumination. Calculate the height of the lamb that must be hang from the table.

SOLUTION

Total flux of the lamb: $\Phi_v = (100\frac{\text{lm}}{\text{W}})(40\,\text{W})(0.2) = 800.0\ \text{lm}$

The luminous intensity of the bulb: $I_v = \frac{\Phi_v}{4\pi} = \frac{800.0\ \text{lm}}{4\pi} = 63.7\,\text{cd}$

From: $I_v = \frac{E_v}{d^2} \longrightarrow d = \sqrt{\frac{E_v}{I_v}} = \sqrt{\frac{60\ \text{lux}}{63.7\,\text{cd}}} = 0.97\,\text{m} \approx 1.0\,\text{m}$

10.12 Exercises

1. What is the difference between the radiometry and photometry?
Answer: The only real difference between radiometry and photometry is that radiometry includes the entire optical radiation spectrum, while photometry is limited to the visible spectrum as defined by the response of the eye.

2. The solid angle of a half-sphere measured from a point in its center is
(a) 4π sr (b) 2π sr (c) π sr (d) π/2 sr

3. The solid angle subtended at the center of a cube by one of its faces is
(a) 2π sr (a) π/2 sr (c) π/3 sr (d) 2π/3 sr

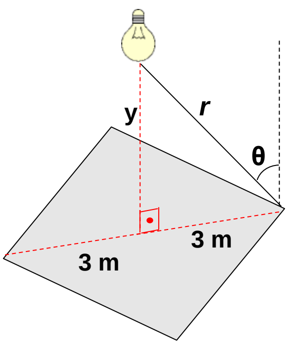
4. What is the SI unit of
(a) radiant intensity (b) radiant flux (c) radiance (d) irradiance
(e) luminous intensity (f) luminous flux (g) luminance (h) illuminance?

5. What is the luminous intensity of a light bulb having the luminous flux of 270 lm?

6. For reading a book one needs a 50 lux illuminance. A bulb of 100 cd is used for illumination. Calculate the required distance of the lamb to the book.

7. On a table, one needs a 500 lux illuminance. A 200 W-lamp whose luminous efficacy is 100 lm/W and efficiency is 16% will be used for illumination. Calculate the height of the lamb that must be hang from the table.

8.
A 1000 cd-bulb is hang at a height *y* from the center of the floor of a room having square shape with diagonal length of 6 m as shown in figure.



For which value of the y the illuminace at the bottom corner of the floor is maximum?

Answer: y = 2.12 m

9. What is the irradiance of a 100 W lamp, radiating light in all directions, on a surface of area 0.1 m^2?

10. A lens with a diameter of 3 cm and a focal length of 5 cm projects the image of a lamp capable of producing 3000 cd/cm^2. Find the illuminance in lm/m^2 on a screen at a distance 0.6 m from the lens.

11. Calculate irradiance and illuminace of a 10 mW laser pointer at 620 nm on an area of 4 mm^2.

12. Table gives data on measured radiant power spectrum of a light source at various wavelengths measured in nm. Convert each radiant power given in watts to lumens and compute total luminous flux of the source.

λ (nm)	Φ (mW)	Φ_v (lm)
400	2.52	
450	6.11	
500	0.20	
550	6.25	
575	8.49	
700	0.15	

10.13 References

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