EP 122 Final exam 26/05/2014
Answer all questions. Duration 90 minutes.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | | TOT |
|---|---|---|---|---|---|---|---|---|
| | | | | | xxx | xxx | | |

*To be completed only by the lecturer*

| **Student Name, Surname** | **Student Id No** |
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**EDUCATION TYPE:** ☐ **First Education** ☐ **Second Education**

### Question 1 [20%]

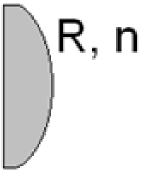


Figure shows a Plano-Convex lens whose radius of curvature *R* and refractive is index *n*. Focal length of such a lens is given by:

$$f = \frac{R}{n-1}$$

Calculate the radius of curvature of the lens and its uncertainty for
f = 0.50 ± 0.01 m and *n* = 1.33 ± 0.01.

### Question 2 [10%]

In a Hospital maximum (systolic) blood pressure of 1500 people are measured. This measurement results in a normal distribution with mean 120 mmHg and standard deviation 15 mmHg. Using normal distribution function, find how many people have blood pressure between 100 mmHg and 140 mmHg.

**Question 3 [20%]**

A temperature measurement element has an input range of 0 to 100 Celsius. The output of the element (milli-volts) is measured under standard conditions and a second-order polynomial fit to the data yields the following calibration function:

$$O(T) = 3 + 2T + T^2$$

(a) Write down the ideal linear response equation

(b) Write down the non-linearity function

(c) Write down the maximum non-linearity

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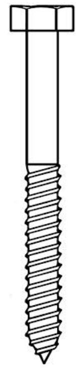
**Question 4 [30%]**

Explain how to perform mass calibration of an electronic balance.

**Question 5 [20%]**



Suppose a machine produces screws whose diameters, *D*, are normally distributed. The mean diameter and standard deviation of the population are given by $\mu = 2$ cm and $\sigma = 0.2$ cm respectively. To determine if the machine is adequately calibrated, a sample of $n = 10$ screws are chosen at random and diameters are measured and the following sample data is obtained:

$$D = \{2.238, 1.759, 1.996, 1.969, 1.679, 2.052, 1.789, 2.283, 1.839, 2.106\}$$

(a) Calculate the sample mean

(b) Calculate the sample standard deviation

(c) Write down the interval that includes the population mean with 95% confidence level.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2} \qquad \sigma_E = \frac{\sigma}{\sqrt{N}}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i/\sigma_i^2}{\sum_{i=1}^{n} 1/\sigma_i^2} \qquad \bar{\sigma} = \frac{1}{\sqrt{\sum_{i=1}^{n} 1/\sigma_i^2}}$$

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_2^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma_n^2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B \mid A) = P(A \cap B)/P(A)$$

$$E[X] = \sum_i x_i\, f(x_i) \qquad E[X] = \int_{-\infty}^{+\infty} x\, f(x)dx$$

$$E[X^2] = \sum_i x_i^2\, f(x_i) \qquad E[X^2] = \int_{-\infty}^{+\infty} x^2\, f(x)dx$$

$$\sigma^2 = E[X^2] - (E[X])^2 = \langle X^2 \rangle - \langle X \rangle^2 \qquad RMS = \sqrt{E[X^2]}$$

$$P_{binom} = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$mean: np \qquad std.dev: \sigma = \sqrt{np(1-p)}$

$$P_{poisson} = \frac{e^{-\lambda}\lambda^k}{k!}$$

mean : $\lambda = np$

$std.dev.\sigma = \sqrt{\lambda}$

$$p_{gauss} = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$mean: \mu \qquad std.dev: \sigma$

$$\rho = \frac{\overline{xy} - \bar{x}.\bar{y}}{\sigma_x \sigma_y} \qquad \overline{xy} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sigma_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\sigma_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2}$$

**Cumulative Distribution Function for Standard Normal Distribution**