## EP145 Introduction to Engineering

Topic 3
Elements of Engineering Analysis

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## Introduction

In this chapter, we will explain key elements of the engineering analysis.

- Dimensions
- Units
- Unit conversions
- Significant figures (digits)
- Dimensional homogenity of equations


## Dimension

- All physical and engineering quantizes can be described by a combination of basic quantities such as Length, Time, Mass, etc.
- These primary dimensions can be written as

$$
[L],[T],[M] \text {, etc. }
$$

e.g: the speed has the dimension:

$$
\text { speed } \equiv \frac{\text { length }}{\text { time }}=\frac{[L]}{[T]}=\left[L T^{-1}\right]
$$

e.g: the mass density has the dimension:

$$
\text { density } \equiv \frac{\text { mass }}{\text { volume }}=\frac{[M]}{[L]^{3}}=\left[M L^{-3}\right]
$$

## System of Units

- Each primary dimension has a unit.
- There are several systems of units in use today. The most common systems of units are:

1. International System (SI) or MKS units
2. CGS units
3. British Gravitational (BG)
4. U.S. Customary units
e.g: Length can be measured in
meters, centimeters, yards, inches, etc.

## 1. SI Base Units

- The International System of Units (SI) defines seven units of measure as a basic set.
- These SI base units and their physical quantities are:

Meter the fundamental unit of length
Kilogram the fundamental unit of for mass
Second the fundamental unit of time
Ampere the fundamental unit of electric current
Kelvin the fundamental unit of temperature
Mole the fundamental unit of amount of substance
Candela the fundamental unit of luminous intensity
A candle has luminous intensity of approx. 1 candela

| Name | Symbol | Definition |
| :---: | :---: | :---: |
| Meter | m | The length of the path travelled by light in vacuum during a time interval of 1/299 792458 of a second. |
| Kilogram | kg | The mass of the international prototype of the kilogram |
| Second | s | The duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom |
| Ampere | A | The constant electric current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to $2 \quad 10^{-7}$ newton per metre of length |
| Kelvin | K | The fraction $1 / 273.16$ of the thermodynamic temperature of the triple point of water |
| Mole | mol | The amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12 atom |
| Candela | cd | The luminous intensity in a given direction, of a light source that emits monochromatic radiation of frequency $540 \quad 10^{12}$ Hz and that has a radiant intensity in that direction of $1 / 683$ watt per steradian |
|  |  | Sayta 6 |

## Derived SI Units

- Relying on the base units, all other units of measurement can be formed.
- For example,
$>$ the SI derived unit of area is square metre $\left(\mathrm{m}^{2}\right)$
$>$ density is kilograms per cubic metre $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$

Some named derived units:

| Quantity | Symbol | Dimen. | SI | Deriv.SI |
| :---: | :---: | :---: | :---: | :---: |
| Force | F | [MLT ${ }^{-2}$ ] | $\mathrm{kg} . \mathrm{m} / \mathrm{s}^{2}$ | Newton, N |
| Energy | $E$ | [ $\mathrm{ML}^{2} \mathrm{~T}^{-2}$ ] | $\mathrm{kg} . \mathrm{m}^{2} / \mathrm{s}^{2}$ | Joule, J |
| Pressure | $P$ | $\left[\mathrm{ML}^{-1} \mathrm{~T}^{-2}\right]$ | $\mathrm{kg} / \mathrm{m} . \mathrm{s}^{2}$ | Pascal, Pa |



## Scaling Prefixes of SI Units

| Multiplication Factor | Prefix |  | ysmbol |
| :---: | :---: | :---: | :---: |
| $1,000,000,000,000,000,000,000,000=10^{24}$ | yotta | Y |  |
| $1,000,000,000,000,000,000,000=10^{21}$ | zetta | Z |  |
| $1,000,000,000,000,000,000=10^{18}$ | exa | E |  |
| $1,000,000,000,000,000=10^{15}$ | peta | P |  |
| $1,000,000,000,000=10^{12}$ | tera | T |  |
| $1,000,000,000=10^{9}$ | giga | G |  |
| $1,000,000=10^{6}$ | mega | M |  |
| $1000=10^{3}$ | kilo | k |  |
| $100=10^{2}$ | hecto | h |  |
| $10=10^{1}$ | deka | da |  |
| $0.1=10^{-1}$ | deci | d | Examples: |
| $0.01=10^{-2}$ | centi | c | $1 \mathrm{GHz}=10^{9} \mathrm{~Hz}$ |
| $0.001=10^{-3}$ | milli | m |  |
| $0.000,001=10^{-6}$ | micro | $\mu$ | $1 \mathrm{MW}=10^{6} \mathrm{~W}$ |
| $0.000,000,001=10^{-9}$ | nano | n |  |
| $0.000,000,000,001=10^{-12}$ | pico | p | $1 \mathrm{kPa}=10^{3} \mathrm{~Pa}$ |
| $0.000,000,000,000,001=10^{-15}$ | femto | f | $1 \mathrm{~mm}=10^{-3} \mathrm{~m}$ |
| $0.000,000,000,000,000,001=10^{-18}$ | atto | a | $m m=10^{-3} \mathrm{~m}$ |
| $0.000,000,000,000,000,000,001=10^{-21}$ | zepto | z | $1 \mu F=10^{-6} \mathrm{~F}$ |
| $0.000,000,000,000,000,000,000,001=10^{-24}$ | yocto | y |  |

## EXAMPLE 1 [Engineering Fundamentals, 4th Ed, Cencage Learning]

Acceleration is sometimes measured in g's, where $1 \mathrm{~g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$.
How many g's correspond to the steady acceleration of a car doing zero to fifty km/h in 9.0 seconds?
(Ans: 0.2 g )

## SOLUTION

## 2. CGS Units

The centimetre-gram-second system (abbreviated CGS or cgs)

- The unit of time is seconds (s).
- The unit of length is centimeters (cm)
- The unit of mass is grams (g)
- The derived unit of force is dyne (dyn) 1 dyn $=10^{-5} \mathrm{~N}$
- The derived unit of energy is erg (erg) $1 \mathrm{erg}=10^{-7} \mathrm{~J}$
- The derived unit of pressure is barye $(\mathrm{Ba}) 1 \mathrm{Ba}=10^{-1} \mathrm{~Pa}$

See also:

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http://en.wikipedia.org/wiki/Cgs_unit
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## 3. British Gravitational Units

- The unit of time is seconds (s).
- The unit of length is a foot ( ft ). $1 \mathrm{ft}=0.3048 \mathrm{~m}$.
- The unit of force is a pound (lb). $1 \mathrm{lb}=4.448 \mathrm{~N}$.
- The unit of mass is a slug. $1 \mathrm{lb}=(1 \mathrm{slug})\left(1 \mathrm{ft} / \mathrm{s}^{2}\right)$
- The unit of temperature is degree Fahrenheit ( ${ }^{\circ} \mathrm{F}$ ) or degree Rankine ( ${ }^{\circ} \mathrm{R}$ )

$$
\begin{aligned}
& T\left({ }^{\circ} \mathrm{R}\right)=T\left({ }^{\circ} \mathrm{F}\right)+459.67 \\
& T\left({ }^{\circ} \mathrm{F}\right)=\frac{9}{5} T\left({ }^{\circ} \mathrm{C}\right)+32 \\
& T\left({ }^{\circ} \mathrm{R}\right)=\frac{9}{5} T(K) \\
& T\left({ }^{\circ} \mathrm{C}\right)=T(K)-273.15
\end{aligned}
$$

## 4. U.S. Customary Units

Some engineers still use the U.S. Customary system of units.

- The unit of time is seconds (s).
- The unit of length is a foot ( ft ). $\quad 1 \mathrm{ft}=0.3048 \mathrm{~m}$
- The unit of mass is a pound mass (lbm). $1 \mathrm{lbm}=0.453592 \mathrm{~kg}$
- The unit of force is pound force (lbf) $1 \mathrm{lbf}=4.448 \mathrm{~N}$
- The unit of temperatures are identical to BG.
- Note that 1 slug $\approx 32.2 \mathrm{lbm}$.

More details can be found at:
http://en.wikipedia.org/wiki/Unit_converter
You can download a simple converter executable program from: http://www1.gantep.edu.tr/~bingul/ep145/converter.exe

| Dimension | System of Units |  |  | Conversion Factors |
| :---: | :---: | :---: | :---: | :---: |
|  | SI | BG | U.S. Customary |  |
| Length | Meter (m) | Foot (ft) | Foot (ft) | $\begin{aligned} & 1 \mathrm{ft}=0.3048 \mathrm{~m} \\ & 1 \mathrm{~m}=3.2808 \mathrm{ft} \end{aligned}$ |
| Time | Second (s) | Second (s) | Second (s) |  |
| Mass | Kilogram (kg) | Slugs* | Pound mass ( $\mathrm{lb}_{\mathrm{m}}$ ) | $\begin{aligned} & 1 \mathrm{lb}_{\mathrm{m}}=0.4536 \mathrm{~kg} \\ & 1 \mathrm{~kg}=2.2046 \mathrm{lb}_{\mathrm{m}} \\ & 1 \text { slug }=32.2 \mathrm{lb}_{\mathrm{m}} \end{aligned}$ |
| Force | Newton (N) $1 \mathrm{~N}=(1 \mathrm{~kg})\left(1 \frac{m}{s^{2}}\right)$ | $1 \mathrm{lb}_{\mathrm{f}}=(1 \text { slug })\left(1 \frac{\mathrm{ft}}{\mathrm{~s}^{2}}\right)$ | One pound mass <br> weighs one pound force at sea level | $\begin{aligned} & 1 \mathrm{~N}=224.809 \mathrm{E}-3 \mathrm{lb}_{\mathrm{f}} \\ & 1 \mathrm{lb}_{\mathrm{f}}=4.448 \mathrm{~N} \end{aligned}$ |
| Temperature | Degree Celsius $\left({ }^{\circ} \mathrm{C}\right)$ <br> or Kelvin (K) $\mathrm{K}={ }^{\circ} \mathrm{C}+273.15$ | Degree Fahrenheit ( ${ }^{\circ} \mathrm{F}$ ) or degree Rankine ( ${ }^{\circ} \mathrm{R}$ ) ${ }^{\circ} \mathrm{R}={ }^{\circ} \mathrm{F}+459.67$ | Degree Fahrenheit ( ${ }^{\circ} \mathrm{F}$ ) or degree Rankine ( ${ }^{\circ} \mathrm{R}$ ) ${ }^{\circ} \mathrm{R}={ }^{\circ} \mathrm{F}+459.67$ | $\begin{aligned} & { }^{\circ} \mathrm{C}=\frac{5}{9}\left[{ }^{\circ} \mathrm{F}-32\right] \\ & { }^{\circ} \mathrm{F}=\frac{2}{5}{ }^{\circ} \mathrm{C}+32 \\ & \mathrm{~K}=\frac{5}{9}{ }^{\circ} \mathrm{R} \\ & { }^{\circ} \mathrm{R}=\frac{2}{5} \mathrm{~K} \end{aligned}$ |
| Work, Energy | Joule $(\mathrm{J})=(1 \mathrm{~N})(1 \mathrm{~m})$ | $\mathrm{lb}_{\mathrm{f}} \cdot \mathrm{ft}=\left(1 \mathrm{lb}_{\mathrm{f}}\right)(1 \mathrm{ft})$ <br> Commonly written as $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}}$ | $\mathrm{lb}_{\mathrm{f}} \cdot \mathrm{ft}=\left(1 \mathrm{lb}_{\mathrm{f}}\right)(1 \mathrm{ft})$ | $\begin{aligned} & 1 \mathrm{~J}=0.7375 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} \\ & 1 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}}=1.3558 \mathrm{~J} \\ & 1 \mathrm{Btu}=778.17 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} \end{aligned}$ |
| Power | $\begin{aligned} & \text { Watt }(\mathrm{W})=\frac{1 \text { loule }}{1 \text { second }} \\ & \mathrm{kW}=1000 \mathrm{~W} \end{aligned}$ | $\frac{\mathrm{lbfff}}{\text { second }}=\frac{(1 \mathrm{lbf}(1 \mathrm{ff})}{1 \text { second }}$ | $\frac{\mathrm{lbf} \cdot \mathrm{ff}}{\mathrm{~second}}=\frac{(1 \mathrm{lbf}(1 \mathrm{ft})}{1 \text { second }}$ | $\begin{aligned} & 1 \mathrm{~W}=0.7375 \frac{\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}}}{\mathrm{~s}} \\ & 1 \mathrm{hp}=550 \frac{\mathrm{ft} \cdot \mathrm{~b}_{\mathrm{f}}}{\mathrm{~s}} \\ & 1 \mathrm{hp}=0.7457 \mathrm{~kW} \end{aligned}$ |
|  |  |  |  | Sayfa 14 |

EXAMPLE 2 [Engineering Fundamentals, 4th Ed, Cencage Learning] A person who is 6 feet and 1 inch tall and weighs 185 pound force (lbf) is driving a car at a speed of 65 miles per hour over a distance of 25 miles. The outside air temperature is $80^{\circ} \mathrm{F}$ and has a density of 0.0735 pound mass per cubic foot (lbm/ft ${ }^{3}$ ). Convert all of the values given in this example from U.S. Customary Units to SI units.

## SOLUTION

## Other Units of Length

## Physics:

- fermi (fm) $\quad 1 \mathrm{fm}=1$ femtometre $=10^{-15} \mathrm{~m} \quad$ (Nuclear Physics)
- angstrom ( $\AA$ ) $1 \AA=10^{-10} \mathrm{~m}$ (Atomic Physics)
- micron 1 micron $=10^{-6} \mathrm{~m}$
- Bohr radius $\left(\mathrm{a}_{0}\right) \quad 1 \mathrm{a}_{0}=5.29 \times 10^{-11} \mathrm{~m} \quad$ (Atomic Physics)
- Planck length $\left(\ell_{\mathrm{P}}\right) \quad 1 \ell_{\mathrm{P}}=1.6 \times 10^{-35} \mathrm{~m} \quad$ (Particle Physics)


## Astronomy:

- Earth radius (RE) $1 \mathrm{RE}=6370 \mathrm{~km}$
- Astronomical unit (AU) $1 \mathrm{AU}=1.5 \times 10^{9} \mathrm{~m}$ (dis. between Sun and Earth)
- Light year (ly) $\quad 1 \mathrm{ly}=9.46 \times 10^{12} \mathrm{~m} \quad$ (dis. travelled by light in a year)
- parsec (pc) $1 \mathrm{pc}=30.810^{12} \mathrm{~m}$

Imperial/US units:

- Inches (in) $1 \mathrm{in}=2.54 \mathrm{~cm}$ and $1 \mathrm{ft}=12 \mathrm{in}$
- yard (yd) $1 \mathrm{yd}=3 \mathrm{ft}=36 \mathrm{in}=0.9144 \mathrm{~m}$
- mile (mi) $1 \mathrm{mi}=5280 \mathrm{ft}=1609.344 \mathrm{~m}$


## Other Units of Volume

- Liter (L) $1 \mathrm{~L}=1 \mathrm{dm}^{3}=0.001 \mathrm{~m}^{3}$.
- Gallon (gal)
> The imperial (UK) gallon* $1 \mathrm{gal}=4.54609 \mathrm{~L}$
$>$ The US liquid gallon $\quad 1 \mathrm{gal}=3.785411784 \mathrm{~L}$
$>$ The US dry gallon $\quad 1$ gal $=4.40488377086 \mathrm{~L}$

See also:
http://www.nist.gov/pml/wmd/h44-12.cfm

* The imperial gallon is used in everyday life in UK, Ireland and Canada


## Other Units of Force

Force $(\boldsymbol{F})$ is any influence that causes an $\quad \mathbf{F}=m \mathbf{a}$ object to undergo a certain change, either concerning its movement, direction, or geometrical construction.

- MKS (SI) unit is the newton ( N ) $\quad \mathrm{N} \equiv \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$.
- CGS unit is dyne (dyn) dyn $\equiv \mathrm{g} . \mathrm{cm} . \mathrm{s}^{-2} .1$ dyn $=10^{-5} \mathrm{~N}$
- $1 \mathrm{lbF}=4.448222 \mathrm{~N}$
- $1 \mathrm{pdl}=0.138255 \mathrm{~N}$

See also:
http://en.wikipedia.org/wiki/Force

## Other Units of Pressure

Pressure ( $p$ ) is the ratio of force to the area over which that force is distributed.

$$
p=\frac{F}{A}
$$

- SI unit is the pascal (Pa) $\mathrm{Pa} \equiv \mathrm{N} / \mathrm{m}^{2} \equiv \mathrm{~kg} \cdot \mathrm{~m}^{-1} \cdot \mathrm{~s}^{-2}$.
- $1 \mathrm{bar}=10^{5} \mathrm{~Pa}$
- $1 \mathrm{~atm}=1.01325 \times 10^{5} \mathrm{~Pa} \approx 1 \mathrm{bar}$
- 1 Torr $=133.3224 \mathrm{~Pa} \approx 1 \mathrm{mmHg}$
- $1 \mathrm{psi}=6.8948 \times 10^{3} \mathrm{~Pa}=\mathrm{lbf} / \mathrm{in}^{2}$ (pound per square inch)

See also:
http://en.wikipedia.org/wiki/Pressure

## Other Units of Power

Power $(P)$ is the rate at which energy is transferred, used, or transformed.

$$
P=\frac{\text { Energy }}{\text { time }}
$$

- SI unit is the watt $(W) \quad W \equiv \mathrm{~J} / \mathrm{s} \equiv \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{~s}^{-3}$.
- CGS unit is erg/s
- horsepower (hp) $1 \mathrm{hp}=746 \mathrm{~W}=550 \mathrm{ft} . \mathrm{lbf} / \mathrm{s}$
- BG (Btu/h) 1 Btu/h = $1.055 \mathrm{~kJ} / \mathrm{h}$

See also:
http://en.wikipedia.org/wiki/Power_(physics)

## Angular SI Units: Angle

- Angle in two-dimension (2D) defined as

$$
\theta=k \frac{s}{r}
$$

where $k$ is a proportionality constant and depends on the unit of measurement that is chosen.

for radian measure $k=1$
for degree measure $k=180 / \pi \approx 57.3$
for grad measure $k=200 / \pi \approx 63.7$

- Full circle is $2 \pi$ radians:

$$
\theta=\frac{s}{r}=\frac{2 \pi r}{r}=2 \pi \mathrm{rad}
$$

- 1 radian defines an arc of a circle that has the same length as the circle's radius.


Conversion of some common angles:

| Units | Values |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Turns | 0 | $1 / 12$ | $1 / 8$ | $1 / 6$ | $1 / 4$ | $1 / 2$ | $3 / 4$ | 1 |  |
| Degrees | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ | $180^{\circ}$ | $270^{\circ}$ | $360^{\circ}$ |  |
| Radians | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3 \pi}{2}$ | $2 \pi$ |  |
| Grads | $0^{\mathrm{g}}$ | $3313^{9}$ | $50^{\mathrm{g}}$ | $66^{2 / 3^{\mathrm{g}}}$ | $100^{\mathrm{g}}$ | $200^{\mathrm{g}}$ | $300^{\mathrm{g}}$ | $400^{\mathrm{g}}$ |  |

## EXAMPLE 3

In NATO countries including Turkish Army, an angular mil ( Wh $^{\prime}$ ) unit is used. This angular measurement is generally employed by Artillery (Topçu). If 1 Wh $=1 / 6400$ of a circle, evaluate 542 财 in degrees, radians and grads.
(Ans: $15.24^{\circ}, 0.27$ rad, $16.94{ }^{9}$ )

## SOLUTION

## Angular SI Units: Solid Angle

- The solid angle, $\Omega$, is the 2 D angle in 3D space that an object subtends at a point.
- Definition

$$
\Omega=\frac{A}{r^{2}}
$$

- It is a measure of how large that object appears to an observer looking from that point.
- SI unit is steradian (sr)


A : Surface area subtended from the center
$r$ : Radius of the sphere

- The solid angle of a sphere measured from a point in its interior is $4 \pi \mathrm{sr}$.

$$
\Omega=\frac{A}{r^{2}}=\frac{4 \pi r^{2}}{r^{2}}=4 \pi \mathrm{sr}
$$

- Area of a spherical cap:

$$
A=\pi\left(a^{2}+h^{2}\right)=2 \pi r^{2}(1-\cos \theta)
$$

- Solid angle subtended:

$$
\Omega=\frac{A}{r^{2}}=2 \pi(1-\cos \theta)
$$



Try yourself to prove these two relations!

## EXAMPLE 4

What is the solid angle of the Moon subtended from the Earth? (The distance of the Moon to Earth is $384,400 \mathrm{~km}$ and the radius of the Moon is 1738 km)

## SOLUTION

We can assume that the area of the moon is approximately equal to the spherical cap since the Moon-Earth distance (d) is much more grater than the radius $(R)$ of the moon ( $d \gg R$ ).

$$
\Omega=\frac{A}{r^{2}} \approx \frac{\pi R^{2}}{d^{2}}=\frac{\pi(1738)^{2}}{(384000)^{2}}=6.4 \times 10^{-5} \mathrm{sr}
$$

Note that Moon covers about $5 \times 10^{-4} \%$ of the sky since

$$
\frac{6.4 \times 10^{-5} \mathrm{sr}}{4 \pi \mathrm{sr}}=5.1 \times 10^{-6}
$$

## Angular SI Units: Hz and rpm

- Rotational or angular speed $(\omega)$ is rate of change of angular displacement

$$
\omega \equiv \frac{\text { angular displaceme nts }}{\text { time }}=\frac{\Delta \theta}{\Delta \mathrm{t}} \equiv \frac{\mathrm{rad}}{\mathrm{~s}}
$$

- rpm stand for revolutions per minute (rpm is not a unit).
$1 \mathrm{rpm}=\frac{1 \text { revolution } \mathrm{s}}{1 \text { minute }}=\frac{2 \pi \mathrm{rad}}{60 \mathrm{~s}}=\frac{\pi}{30} \mathrm{rad} / \mathrm{s} \approx 0.105 \mathrm{rad} / \mathrm{s}$
- Rotational frequency ( $f$ ) and angular velocity is related by:

$$
f=\frac{\omega}{2 \pi} \equiv \frac{\mathrm{rad} / \mathrm{s}}{\mathrm{rad}} \equiv \frac{1}{s} \equiv \mathrm{~Hz}
$$

## EXAMPLE 5

Modern ultrasonic dental drills can rotate at up to $800,000 \mathrm{rpm}$. Calculate the maximum rotational speed in rad/s and Hz units.
(Ans: 84,000 rad/s and 13.3 kHz )


## SOLUTION

## More Examples:

> The "second" hand of a conventional analogue clock rotates at 1 rpm .
> DVD players rotates at $1530 \mathrm{rpm}(25.5 \mathrm{~Hz})$
> A washing machine's drum may rotate at 500 to $2000 \mathrm{rpm}(8-33 \mathrm{~Hz})$.
> Modern Automobile engines are typically operated around 2000-3000 rpm $(33-50 \mathrm{~Hz}$ ).

## Dimensional Homogenity of Equations

- All formulas used in engineering analysis must be dimensionally homogeneous.
- You can not add someone's height who is 175 cm tall to his weight of 68 kg and his body temperature of $37^{\circ} \mathrm{C}$.
- Consider the equation $\mathrm{X}=\mathrm{a}+\mathrm{b}-\mathrm{c}$. If variable $X$ has dimension of length, then the variables $\mathrm{a}, \mathrm{b}$, and c should also have dimensions of length.
- Therefore equations must be dimensionally homogenous.


## EXAMPLE 6 [Engineering Fundamentals, 4th Ed, Cencage Learning]

The heat transfer rate through a solid material is governed by Fourier's law:

$$
q=k A \frac{T_{1}-T_{2}}{L}
$$

where

$q=$ heat transfer rate
$k=$ thermal conductivity in $\mathrm{W} / \mathrm{m} .{ }^{\circ} \mathrm{C}$
SOLUTION
$T_{1}-T_{2}=$ temperature difference in ${ }^{\circ} \mathrm{C}$
$A=$ area in $\mathrm{m}^{2}$
$L=$ thickness of the material in $m$ What is the appropriate unit for $q$ ?
(Ans: W)

## Measurement Errors for Some Devices

- The values of experimental measurements have uncertainties due to measurement limitations.
- Here we will show the uncertainty for two mostly used devices in the labs.


## Ruler

In Fig, the pointer indicates a value between 23 and 24 mm . With this millimeter scale one strategy is to take the center of the bin as the estimate of the value, the maximum error is then half the width of the bin. So in this case our measurement is 23.50 .5 mm .

The value of 0.5 mm is the estimate of the random error.


## Digital Measuring Devices

All digital measuring devices has a maximum uncertainty of the order of half its last digit. For example, in Fig, for the reading from a digital voltmeter, the uncertainty is $0.01 / 2$ Volts. Thus, assuming the voltmeter is calibrated accurately, the value is

$$
19.1600 .005 \mathrm{~V}
$$



- The measurement readings fall between the smallest scale division of each instrument:

$12.880 \quad 0.005 \mathrm{~V}$

$$
35.5 \quad 0.5 \mathrm{~mm}
$$



## Significant Figures (Digits)

- Engineers record the results of measurements and calculations using numbers.
- Calculator will result many digits as its display will
 hold. But, How many of those digits actually contribute toward achieving the purpose of engineering!
- The significant figures of a number are those digits that carry meaning contributing to its precision.
- The accuracy of a measurement system is the degree of closeness of measurements of a quantity to its true (actual) value.
- $L=10$ meters
means your accuracy is 10 m
- $L=10$. meters
means your accuracy is 1 m
- $L=10.0$ meters
means your accuracy is 0.1 m
- $L=10.00$ meters means your accuracy is 0.01 m
- $L=10.000$ meters means your accuracy is 0.001 m
- Significant digits are numbers $0,1,2,3,4,5,6,7,8,9$
- When zeros are used to show the position of a decimal point, they are not considered significant digits.
- Each of the following numbers has three significant digits: 251, 404, 35.3, 1.67, and 0.00144 .
- 1500 has two significant digits since $1500=1.5 \times 10^{3}=15 \times 10^{2}=0.015 \times 10^{5}$.
- 1500.0 has four significant digits which means the accuracy is $1 / 10000$.

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Addition and Subtraction rules:
The sum or difference of two values should contain no significant figures farther to the right of the decimal place than occurs in the least precise number in the operation.
\(113.2+1.43=114.63 \quad\) (your calculator will display)
\(113.2+1.43=114.6 \quad\) (But the result should be recorded in this way)
\(113.2-1.43=111.77 \quad\) (your calculator will display)
113.2-1.43 = \(111.8 \quad\) (But the result should be recorded in this way)
\(113.212-113.0=0.212 \quad\) (your calculator will display)
\(113.212-113.0=0.2\) (result should be recorded in this way)
```


## Multiplication and Division rules:

The product or quotient should contain no more significant figures than are contained by the term with the least number of significant figures used in the operation.
(113.2) $\times(1.43)=161.876$ (your calculator will display)
(113.2) $\times(1.43)=162 \quad$ (result should be recorded in this way)
(113.2) / (1.43) $=79.16 \quad$ (your calculator will display)
(113.2) $/(1.43)=79.2 \quad$ (result should be recorded in this way)

## Questions

1. How many seconds are there in a year?
2. Convert the value of area $A=100 \mathrm{~cm}^{2}$ to $\mathrm{m}^{2}$.
3. Convert the value of volume $V=100 \mathrm{~mm}^{3}$ to $\mathrm{in}^{3}$.
4. Convert the value of atmospheric pressure, $P=10^{4} \mathrm{~N} / \mathrm{m}^{2}$ to $\mathrm{lbf} / \mathrm{in}^{2}$.
5. Convert the value of the density of water, $\rho=1 \mathrm{~g} / \mathrm{cm}^{3}$ to $\mathrm{lbm} / \mathrm{ft}^{3}$.
6. If the pressure in the tire on your car is $34.0 \mathrm{lbf} / \mathrm{in}^{2}$ (or psi), what is its pressure in SI units?
7. What is the area in SI system of the skin of a spherical apple that is 3.8 inches in diameter?
8. The gravitational acceleration on Earth surface is $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$. Express $g$ in BG units. Show the conversion steps.
9. How many lbf does it take for a $4.0 \times 10^{3} \mathrm{lbm}$ car to achieve 0 to 50 mph in 10.0 seconds?
10. What would the 8.33 slug mass weigh on the Moon where the acceleration of gravity is only $1 / 6$ of that on Earth?
11. (a) What is the weight on Earth in SI units of a 10 kg mass?
(b) What is the weight on Earth in Engineering English units of a 10.0 lbm mass?
(c) What is the mass of a 10.0 lbm object on the Moon?
(d) What is the weight of that 10.0 lbm object on the Moon?
12. Calculate with the correct significant figures:
(a) 10/6, (b) 10.0/6, (c) 10/6.0, (d) 10./6.0, (e) 10.0/6.00
13. Calculate with the correct significant figures:
(a) $11 \times 73$,
(b) $11.0 \times 73.0$, (c) $11.00 / 73.3$
(d) $11.0 \times 73.33$, (e) $11.00 \times 73.44$
14. Gravitational force between two objects of masses $m_{1}$ and $m_{2}$ is given by:

$$
F=G \frac{m_{1} m_{2}}{r^{2}}
$$

where $r$ is the distance between the masses and $G$ is the universal gravitational constant. What is the SI unit of the $G$ if the above equation is to be dimensionally homogenous?
15. The "second" hand of a conventional analogue clock rotates at 1 rpm . Express the rotational speed in Hz unit.
16. Convert the following SI units into BG
$140 \mathrm{~km} / \mathrm{h} \quad$--> miles $/ \mathrm{h}$ and $\mathrm{ft} / \mathrm{s}$
1240 W --> Btu/h and hp
$10 \mathrm{~m}^{3} \quad$--> $\mathrm{ft}^{3}$
$16 \mathrm{~kg} \quad-->\mathrm{lbm}$
$1 \mathrm{~g} / \mathrm{cm}^{3} \quad$--> $\quad \mathrm{lbm} / \mathrm{ft}^{3}$
$150 \mathrm{~N} \quad$--> lbf
$10 \mathrm{kPa} \quad-->\quad \mathrm{lbf} / \mathrm{in}^{2}$
$9.8 \mathrm{~m} / \mathrm{s}^{2} \quad-->\quad \mathrm{ft} / \mathrm{s}^{2}$
17. The solid angle of a half-sphere measured from a point in its center is
(a) $4 \pi \mathrm{sr}$
(b) $2 \pi \mathrm{sr}$
(c) m sr
(d) $\pi / 2 \mathrm{sr}$
18. The solid angle subtended at the center of a cube by one of its faces is
(a) $2 \pi \mathrm{sr}$
(b) $\pi / 2 \mathrm{sr}$
(c) $\pi / 3 \mathrm{sr}$
(d) $2 \pi / 3 \mathrm{sr}$
19. What is the value of reading given below?
(a) 28.50 .5 mm
(b) $28.0 \quad 0.5 \mathrm{~mm}$
(c) $29.0 \quad 0.5 \mathrm{~mm}$
(d) 27.50 .5 mm

20. The last digit of the following electronic ballance can be $0,2,4,6,8$. What is the value of reading on display?
(a) 12.580 .10 g
(b) $12.58 \quad 0.20 \mathrm{~g}$
(c) $12.58 \quad 0.01 \mathrm{~g}$
(d) $12.58 \quad 0.02 \mathrm{~g}$

## References

1. P. Kosky et al., Exploring Engineering, 2nd Ed.

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2. S. Moaveni, Engineering Fundamentals, 4th Ed. Cengage Learning (2011)

