Introduction

In this chapter we will consider some concepts related to Force in Engineering.

Details can be found in [1] and [2].
**Force**

- **What is Force?**
  Force represents the interaction of two objects and it is a **push** or a **pull**.

- **Units of Force** \((F = ma)\)
  **SI:**
  \[1 \text{ Newton} = (1 \text{ kg})(1 \text{ m/s}^2)\]
  \[1 \text{ N} = 1 \text{ kg.m/s}^2\]

  **BG:**
  \[1 \text{ pound force} = 1 \text{ lbf} = (1 \text{ slug})(1 \text{ ft/s}^2)\]
  \[1 \text{ lbf} = 4.448 \text{ N}\]

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**Newton’s Laws of Motion**

In 1687, Newton published his famous book: *Mathematical Principles of Natural Philosophy*

You can download the book at: [http://archive.org/details/philosophiaenatu00newt](http://archive.org/details/philosophiaenatu00newt)
In Principia, Newton mentioned three laws of motion:

1. If net force acting on a body is zero \( F = 0 \) then, the body keeps on doing what it is doing.

2. If net force acting on the body is non-zero then, the body will accelerate according to: \( F = ma \).

3. Forces are generated in pairs. Any action results in a reaction.

**Contact Force**

- \( F \): applied force
- \( N \): normal force
- \( W \): weight
- \( F_f \): frictional force
- \( g \): gravitational acceleration

\[ g = 9.8 \, \text{m/s}^2 \]
\[ g = 32.174 \, \text{ft/s}^2 \]
Spring Force

Hooke’s Law:
over the elastic range the deformation of a spring is directly proportional to the applied force, according to

\[ F = kx \]

- \( F \) = applied force (N)
- \( k \) = spring constant (N/m)
- \( x \) = deformation of the spring (m)

Example 1
For a given spring, in order to determine the value of the spring constant, we have attached dead weights to one end of the spring, as shown in Figure. We have measured and recorded the deflection caused by the corresponding weights as given in Table. What is the value of the spring constant?

<table>
<thead>
<tr>
<th>Weight (N)</th>
<th>The Deflection of the Spring (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>9</td>
</tr>
<tr>
<td>10.0</td>
<td>17</td>
</tr>
<tr>
<td>15.0</td>
<td>29</td>
</tr>
<tr>
<td>20.0</td>
<td>35</td>
</tr>
</tbody>
</table>

Solution will be given in the lecture.
Surface Frictional Force

\[ F_{\text{fmax}} = \mu_S N = \mu_S mg \]

\[ F_{\text{fkin}} = \mu_K N = \mu_K mg \]

\( \mu_S \) = coefficient of static friction

\( \mu_K \) = coefficient of kinetic friction

In general

\( \mu_K < \mu_S \)

Coefficient of static and kinetic frictions for some materials:

<table>
<thead>
<tr>
<th>Material 1</th>
<th>Material 2</th>
<th>( \mu_S )</th>
<th>( \mu_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>Aluminum</td>
<td>1.05 - 1.35</td>
<td>1.40</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>Cast Iron</td>
<td>1.10</td>
<td>0.15</td>
</tr>
<tr>
<td>Nickel</td>
<td>Nickel</td>
<td>0.70 - 1.10</td>
<td>0.53</td>
</tr>
<tr>
<td>Copper</td>
<td>Cast Iron</td>
<td>1.05</td>
<td>0.29</td>
</tr>
<tr>
<td>Glass (dry)</td>
<td>Glass (dry)</td>
<td>0.90 - 1.00</td>
<td>0.40</td>
</tr>
<tr>
<td>Glass (wet)</td>
<td>Glass (wet)</td>
<td>0.10 - 0.60</td>
<td>0.10</td>
</tr>
<tr>
<td>Teflon</td>
<td>Teflon</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Steel (Mild)</td>
<td>Lead</td>
<td>0.95</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Example 2
An object is placed on an inclined plane
As shown in Fig. The mass of the object
is \( m = 2 \text{ kg} \) and it is in equilibrium.
(a) What is the magnitude of the static
friction force on the object?
(b) What is the value of the coefficient of static friction (\( \mu_s \))?

Solution will be given in the lecture.

Example 3
A variable horizontal force acts on a block of
mass \( m = 2 \text{ kg} \) which is initially at
rest on a table as shown in Fig a.
The acceleration of the block as a
function of applied force is drawn in the Fig b.
(a) What are the coefficients of
static friction (\( \mu_s \)) and kinetic friction (\( \mu_k \))
between the table and the block respectively?
(b) Plot the frictional force (\( F_f \)) versus applied
force (\( F \)) graph for the block.

Solution will be given in the lecture.
Drag Force

In fluid Mechanics, drag force (air resistance or fluid resistance) is a force that resists the motion of a body moving through a fluid.
At low speeds, drag force is proportional to the speed of the solid in the fluid:

\[ F_d = bv \]

where
- \( b \) is constant that depends on the properties of the fluid and the dimensions of the object.
- \( v \) is the speed of the object.

At high speeds, drag force is proportional to the square of the speed:

\[ F_d = \frac{1}{2} \rho v^2 C_d A \]

where
- \( \rho \) is density of the fluid
- \( A \) is the cross-sectional area of object
- \( C_d \) is the drag coefficient.
The speed of falling objects reaches a terminal (final) value due to drag force. (Terminal speed is a constant).

At low speeds: \( F_d = bv_T = mg \)

Terminal speed is: \( v_T = \frac{mg}{b} \)
Example 4
Calculate the drag force acting on a wooden sphere (density 0.83 g/cm$^3$ and radius 8 cm) falling through air (density 1.23 kg/m$^3$)
(a) for low speeds with $b = 0.5$ kg/s and $v = 1$ m/s.
(b) for high speeds with $C_d = 0.5$ and $v = 25$ m/s.

Solution will be given in the lecture.

Example 5
Estimate the terminal speed of a wooden sphere (density 0.83 g/cm$^3$ and radius 8 cm) falling through air (density 1.23 kg/m$^3$)
(a) for low speeds with $b = 0.5$ kg/s
(b) for high speeds with $C_d = 0.5$

Solution will be given in the lecture.
**Gravitational Force**

Newton's law of universal gravitation states that:

> every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

\[
F = G \frac{m_1 m_2}{r^2}
\]

where \( G \) is called the universal gravitational constant and has the value:

\[ G = 6.673 \times 10^{-11} \text{ N.m}^2/\text{kg}^2 \]

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**Example 6**

What is the magnitude of the gravitational force acting on an object whose mass is 1 kg due to Earth?

Mass of Earth is \( M_E = 6 \times 10^{24} \text{ kg} \).

Radius of Earth \( R_E = 6.4 \times 10^6 \text{ m} \).

**Solution:**

\[
F = G \frac{M_E m}{R_E^2} = \left(6.673 \times 10^{-11} \text{ N.m}^2/\text{kg}^2\right) \frac{(6 \times 10^{24} \text{ kg})(1 \text{ kg})}{(6.4 \times 10^6 \text{ m})^2} = 9.8 \text{ kg.m/s}^2
\]

Note that gravitational acceleration is given by:

\[
g = \frac{F}{m} = \frac{9.8 \text{ kg.m/s}^2}{1 \text{ kg}} = 9.8 \text{ m/s}^2
\]
Example 7

Two people have the same mass of \( m_1 = m_2 = 75 \, \text{kg} \).

(a) What is the magnitude of the gravitational force between the people if the separation between them is 0.5 m.

(b) What is the value of the force exerted by Earth on each of them?

Solution:

(a) \[
F = (6.673 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2) \left( \frac{75 \, \text{kg} \times 75 \, \text{kg}}{(0.5 \, \text{m})^2} \right) = 1.5 \times 10^{-6} \, \text{N}
\]

(b) \[
F = mg = (75 \, \text{kg})(9.8 \, \text{m/s}^2) = 735 \, \text{N}
\]

The ratio:
\[
\frac{735 \, \text{N}}{1.5 \times 10^{-6} \, \text{N}} \approx 10^8
\]

Free-Fall Acceleration \( g \) as a function of altitude \( h \):

\[
g = \frac{F}{m} = \frac{GM_E}{(R_E + h)^2}
\]

<table>
<thead>
<tr>
<th>Altitude ( h ) (km)</th>
<th>( g ) (m/s(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.80</td>
</tr>
<tr>
<td>1000</td>
<td>7.33</td>
</tr>
<tr>
<td>2000</td>
<td>5.68</td>
</tr>
<tr>
<td>3000</td>
<td>4.53</td>
</tr>
<tr>
<td>4000</td>
<td>3.70</td>
</tr>
<tr>
<td>5000</td>
<td>3.08</td>
</tr>
<tr>
<td>6000</td>
<td>2.60</td>
</tr>
<tr>
<td>7000</td>
<td>2.23</td>
</tr>
<tr>
<td>8000</td>
<td>1.93</td>
</tr>
<tr>
<td>9000</td>
<td>1.69</td>
</tr>
<tr>
<td>10000</td>
<td>1.49</td>
</tr>
<tr>
<td>50000</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Example 8
The International Space Station operates at an altitude of 350 km. It has a weight (measured at the Earth's surface) of $4.22 \times 10^6$ N. What is its weight when in orbit?

Solution will be given in the lecture.

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**Elastic Properties of Solids**

Except springs, we have assumed that objects remain rigid when external forces act on them.

In reality, all objects are deformable. Forces can change the shape or the size of a solid. Internal forces in the object resist the deformation.

We will discuss the deformation of solids by using the concepts of **stress** and **strain**.
Tensile stress is the external force acting on an object per unit cross-sectional area.

\[ \text{stress} = \sigma = \frac{F}{A} \]

The result of a stress is (tensile) strain defined by

\[ \text{strain} = \varepsilon = \frac{\Delta L}{L} \]

For sufficiently small stresses, strain is proportional to stress;

\[ \sigma = E \varepsilon \]

Which piece of material will stretch more, when subjected to the same force?
There are three types of deformation and define an elastic modulus for each:

1. **Young's modulus**
   measures the resistance of a solid to a change in its length

2. **Shear modulus**
   measures the resistance to motion of the planes within a solid parallel to each other

3. **Bulk modulus**
   measures the resistance of solids or liquids to changes in their volume

We will consider only Young's modulus.

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**Typical Values for Elastic Moduli**

<table>
<thead>
<tr>
<th>Substance</th>
<th>Young's Modulus (N/m²)</th>
<th>Shear Modulus (N/m²)</th>
<th>Bulk Modulus (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tungsten</td>
<td>35 x 10¹⁰</td>
<td>14 x 10¹⁰</td>
<td>20 x 10¹⁰</td>
</tr>
<tr>
<td>Steel</td>
<td>20 x 10¹⁰</td>
<td>8.4 x 10¹⁰</td>
<td>6 x 10¹⁰</td>
</tr>
<tr>
<td>Copper</td>
<td>11 x 10¹⁰</td>
<td>4.2 x 10¹⁰</td>
<td>14 x 10¹⁰</td>
</tr>
<tr>
<td>Brass</td>
<td>9.1 x 10¹⁰</td>
<td>3.5 x 10¹⁰</td>
<td>6.1 x 10¹⁰</td>
</tr>
<tr>
<td>Aluminum</td>
<td>7.0 x 10¹⁰</td>
<td>2.5 x 10¹⁰</td>
<td>7.0 x 10¹⁰</td>
</tr>
<tr>
<td>Glass</td>
<td>7.0 x 10¹⁰</td>
<td>3.0 x 10¹⁰</td>
<td>5.2 x 10¹⁰</td>
</tr>
<tr>
<td>Quartz</td>
<td>5.6 x 10¹⁰</td>
<td>2.6 x 10¹⁰</td>
<td>2.7 x 10¹⁰</td>
</tr>
<tr>
<td>Water</td>
<td>-</td>
<td>-</td>
<td>0.2 x 10¹⁰</td>
</tr>
<tr>
<td>Mercury</td>
<td>-</td>
<td>-</td>
<td>2.8 x 10¹⁰</td>
</tr>
</tbody>
</table>
**Young's modulus** (symbol \( E \) or \( Y \)) measures the resistance of a solid to a change in its length.

\[
Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L}
\]

Tensile tests are performed to measure the modulus of elasticity and strength of solid materials.

See also the videos on the course web page:
- Tensile_Test_Stainless_Steel_Specimen.avi
- Stress_Strain_Test.avi
Example 9
A structural member with a rectangular cross section, as shown in Figure is used to support a load of 4000 N distributed uniformly over the cross-sectional area of the member. What type of material should be used to carry the load safely?

Solution:

The average tensile stress is:

\[ \sigma = \frac{F}{A} = \frac{4000 \text{ N}}{(0.05 \text{ m})(0.005 \text{ m})} = 16 \times 10^6 \text{ Pa} = 16 \text{ MPa} \]

So, we can use Steel or Tungsten to carry the load safely.

Example 10
A block mass \( m = 90 \text{ kg} \) is attached to a ceiling by a steel wire of length \( L = 10 \text{ m} \) as shown in Figure. What diameter should the wire have if we don't want it to stretch more than \( \Delta L = 0.5 \text{ cm} \)?

Solution will be given in the lecture.
Example 11
(a) Evaluate Young's modulus for the material whose stress versus strain curve is shown in Figure.
(b) What is the maximum force that can be exerted on a wire made of this material if the wire diameter is 5 mm?

Solution will be given in the lecture.

Example 12
Assume that Young's modulus is $1.50 \times 10^{10}$ N/m$^2$ for bone and that the bone will fracture if stress greater than $1.50 \times 10^8$ N/m$^2$ is imposed on it.
(a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm?
(b) If this much force is applied compressively, by how much does the 25.0-cm-long bone shorten?

Solution will be given in the lecture.
### Four Fundamental Forces (Interactions)

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Particles affected</th>
<th>Range</th>
<th>Relative Strength</th>
<th>Role in Universe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>Quarks</td>
<td>$10^{-15}$ m</td>
<td>1</td>
<td>Holds quarks together to form nucleons</td>
</tr>
<tr>
<td></td>
<td>Hadrons</td>
<td></td>
<td></td>
<td>Holds nucleons together to form atomic nuclei</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>Charged particles</td>
<td>$\infty$</td>
<td>$10^{-2}$</td>
<td>Determines the structure of atoms, molecules, solids and liquids, is important factor in astronomical universe, is responsible for frictional force</td>
</tr>
<tr>
<td>Weak</td>
<td>Quarks and Leptons</td>
<td>$10^{-18}$ m</td>
<td>$10^{-5}$</td>
<td>Mediates transformations of quarks and leptons, helps determine compositions of atomic nuclei</td>
</tr>
<tr>
<td>Gravitational</td>
<td>All</td>
<td>$\infty$</td>
<td>$10^{-38}$</td>
<td>Assembles matter into planets, stars, galaxies.</td>
</tr>
</tbody>
</table>
Questions
Not available.

References