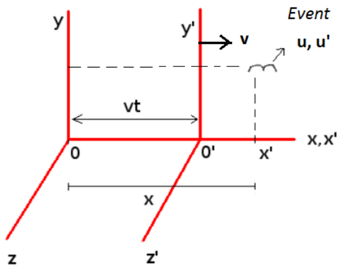
EP 228 Particle Physics, Lecture Notes on Special Theory of Relativity, Oct 2020



Galilean Transformations (Classical Relativity)

Event is a four dimensional coordinate in space-time. Consider two inertial (non-accelerating) frames whose origins are O and O' respectively. O is at rest and O' is moving at a constant speed v in the direction of +x-axis. An event is represented by four-vector (x, y, z, t) in O and by (x', y', z', t') in O The connection between these two coordinates are given by Galilean Transformations as follows:

Galilean coordinate transformations in 1D.	*Galilean velocity transformations in 1D.*	*Galilean transformations in 3D.*
$x' = x - vt$	$u_x' = u_x - v$	$\mathbf{r}' = \mathbf{r} - \mathbf{v}t$
$y' = y$	$u_y' = u_y$	$\mathbf{u}' = \mathbf{u} - \mathbf{v}$
$z' = z$	$u_z' = u_z$	$t' = t$
$t' = t$		

where for the 3D transformations

the position vectors are: $\mathbf{r} = (x, y, z)$ and $\mathbf{r'} = (x', y', z')$.

the velocity of the object (bird): $\mathbf{u} = (u_x, u_y, u_z)$ and $\mathbf{u'} = (u'_x, u'_y, u'_z)$

the velocity of frame O' with respect to O: $\mathbf{v} = (v_x, v_y, v_z)$.

Lorentz Transformations (Extracted from Einstein's "Theory of Special Relativity")

Postulate 1: The laws of physics are the same for all inertial observers.

Postualte 2: Speed of light as measured by all observers is $c = 3\text{x}10^8$ m/s independent of motion of the source.

Lorentz coordinate transformations in 1D.	*Lorentz velocity transformations in 1D.*	*Lorentz coordinate transformations in 3D.*
$x' = \gamma(x - vt)$	$u_x' = \dfrac{u_x - v}{1 - (v/c^2)u_x}$	$\mathbf{r}' = \mathbf{r} + \mathbf{v}\left(\dfrac{(\gamma-1)\mathbf{v}\cdot\mathbf{p}}{v^2} - \gamma t\right)$
$y' = y$	$u_y' = \dfrac{u_y/\gamma}{1 - (v/c^2)u_x}$	$t' = \gamma(t - \mathbf{v}\cdot\mathbf{r}/c^2)$
$z' = z$	$u_z' = \dfrac{u_z/\gamma}{1 - (v/c^2)u_z}$	
$t' = \gamma(t - \dfrac{vx}{c^2})$		

where

$$\beta = v/c, \quad \boldsymbol{\beta} = \mathbf{v}/c \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

Time Dilation : $t = \gamma t_0$

Length Contraction: $L = L_0 \,/\, \gamma$

Relativistic Kinematics

Let rest mass be m_0 and particle speed be u. Then,

Gamma factor $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$

Relativistic mass: $m = \gamma m_0$

Relativistic Momentum: $p = \gamma m_0 u = mu$

Force: $F = dp/dt$

Relativistic Total energy: $E = \gamma m_0 c^2 = mc^2$

Relativistic Kinetic energy: $T = (\gamma - 1)m_0 c^2 = mc^2 - m_0 c^2$

Energy-momentum relation: $E^2 = p^2c^2 + m_0^2 c^4$

Particle speed: $u = pc^2 \,/\, E$

For massless particles ($m_0 = 0$, like photon) → $K = E = pc$ and $u = c$

Lorentz transformations for energy and momentum

in 1D.	*in 3D.*
$p_x' = \gamma(p_x - (v/c)E)$	$\mathbf{p}' = \mathbf{p} + \dfrac{\gamma\mathbf{v}}{c^2}\left(\dfrac{\gamma\mathbf{v}.\mathbf{p}}{\gamma+1} - E\right)$
$p_y' = p_y$	$E' = \gamma(E - \mathbf{v}.\mathbf{p}c)$
$p_z' = p_z$	
$E' = \gamma(E - vp_x)$	

Doppler shift for light:

Substitute $E' = hf'$ and $E = hf = p_x c$ → $E' = \gamma(E - vE/c)$ → $f' = \gamma f(1 - \beta)$

Hints for Decay: $X \to Y + Z$

For all frames:

* $m_X > m_Y + m_Z$ (mass is not conserved)
* $E_X = E_Y + E_Z$
* $\mathbf{p}_X = \mathbf{p}_Y + \mathbf{p}_Z$
* Kinetic energy may or may not be conserved

In the rest frame of mother, X, (that is $\mathbf{p}_X = 0$) center of mass energies of products are:

$E_Y^* = (m_X^2 - m_Y^2 + m_Z^2)c^2 \,/\, 2m_X$

$E_Z^* = (m_X^2 + m_Y^2 - m_Z^2)c^2 \,/\, 2m_X$

Hints for Collision: $1 + 2 \to 3 + 4$

For all frames:

* $m_1 + m_2 \neq m_3 + m_4$ (mass is not conserved)
* $E_1 + E_2 = E_3 + E_4$
* $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_3 + \mathbf{p}_4$
* Kinetic energy may or may not be conserved

Invariant mass (W) has the same value in any inertial frame

Invariant mass for single particle:

$$W^2c^2 = E^2 - p^2c^2$$

Invariant mass for n particles:

$$W^2c^2 = \left(\sum_{i=1}^{n} E_i\right)^2 - c^2\left(\sum_{i=1}^{n} \mathbf{p}_i\right)^2$$

$$W^2c^2 = (E_1 + E_2 + \cdots + E_n)^2 - (\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n)^2c^2$$

EP 228 Particle Physics, Lecture Notes on Special Theory of Relativity, Oct 2020

Solved Problems

[1]. As measured by O, an event occurs at x = 100 km, y = 10 km, z = 1 km at $t = 5 \times 10^{-4}$ s. What are the coordinates x',y',z' and t' of this event as determined by by a second observer, O', moving relative to O at −0.8c along the common x-x' axis? (Ans: x'=367 km, y'=10 km, z'=1km, $t'=12.8 \times 10^{-4}$ s)

[2]. An airplane is moving with respect to earth at constant speed of 600 m/s. Its proper length is 50 m. By how much will it appear to be shortened to an observer on earth? (Ans: 10^{-10} m)

[3]. Atmospheric muons (μ) are created at an altitude around 25-30 km. Muons are unstable particles since they decay (are converted) to electrons. In the rest frame of muons, mean lifetime of muons are about 2×10^{-6} s. The speed of a muon is 0.99c on average. How far can a muon travel in the atmosphere before it decays

(a) if relativistic effects are ignored?(Ans: about 600 m)

(b) if relativistic effects are not ignored? (Ans: about 30 km)

[4]. It takes 10^5 years for light to reach us from the most distant part of our galaxy. Could a human travel there, at a constant speed, in 50 years? (Yes. the speed is v = 0.999 999 875c)

[5]. Rocket A travels to right and rocket B travels to left, with velocities 0.8c and 0.6c respectively, relative to the earth. What is the velocity of rocket A measured from rocket B? (Ans: 0.946c)

[6]. Rocket A travels to right and a laser beam travels to left, with velocities 0.8c and c respectively, relative to the earth. What is the velocity of laser beam measured by rocket A? (Ans: c)

EP 228 Particle Physics, Lecture Notes on Special Theory of Relativity, Oct 2020

[7]. Mass of the proton is 1.67x10 $^{-27}$ kg. Calculate the rest mass of a proton in GeV/c^2.
(Ans: 0.938 GeV/c^2)

[8]. Kinetic energy of an electron is 10 MeV. Calculate
(a) total energy of the electron in MeV

(b) the momentum of the electron in MeV/c

(c) the speed of the electron in m/s

(d) the relativistic mass of the electron in MeV/c^2

(e) the relativistic mass of the electron in kg

[9]. *Doppler shift.* For which value of velocity v you can you see a red light as green in a traffic?

[10]. Find the energies of the decay products in the decay $X \rightarrow Y + Z$ if the mother particle is at rest.

EP 228 Particle Physics, Lecture Notes on Special Theory of Relativity, Oct 2020

[11]. Consider a neutrino originating from the decay $\pi \rightarrow \mu + \nu$.
(a) What is the speed of the muon if pion is at rest?

(b) What are the minimum and maximum speeds of the muon if pion has momentum of 10 GeV/c?

[12]. What is the wavelength of each photon (gamma ray) emitted from $\pi^0 \rightarrow \gamma + \gamma$ decay where neutral pion is at rest?

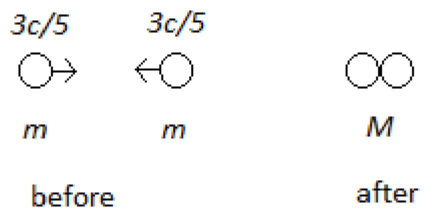
[13]. Consider π^0 is flying in *x*-direction in the decay $\pi^0 \rightarrow \gamma + \gamma$ decay. What is the angle between photons if photon energies are measured as E_1 = 2 GeV and E_2 = 6 GeV?

[14]. The Bevatron at Berkeley was built with the idea of producing antiprotons by the reaction

$$p + p \rightarrow p + p + p + \bar{p}$$

That is, a high-energy proton strikes a proton at rest, creating (in addition to the original particles) a proton-antiproton pair. What is the threshold energy for this reaction (i.e., the minimum energy of the incident proton)? Let mass of a proton is m. (Ans: $E = 7mc^2 \approx 7$ GeV)

[15].
Two lumps of clay, each of mass m, collide head-on at $3c/5$. They stick together. What is the mass M of the final composite lump? (Ans: $M = 2.5m$)



[16].
A particle of mass M, initially at rest, decays into two pieces, each of mass m.
What is the speed of each piece as it flies off?
(Ans: $v = c\sqrt{1-(2m/M)^2}$)