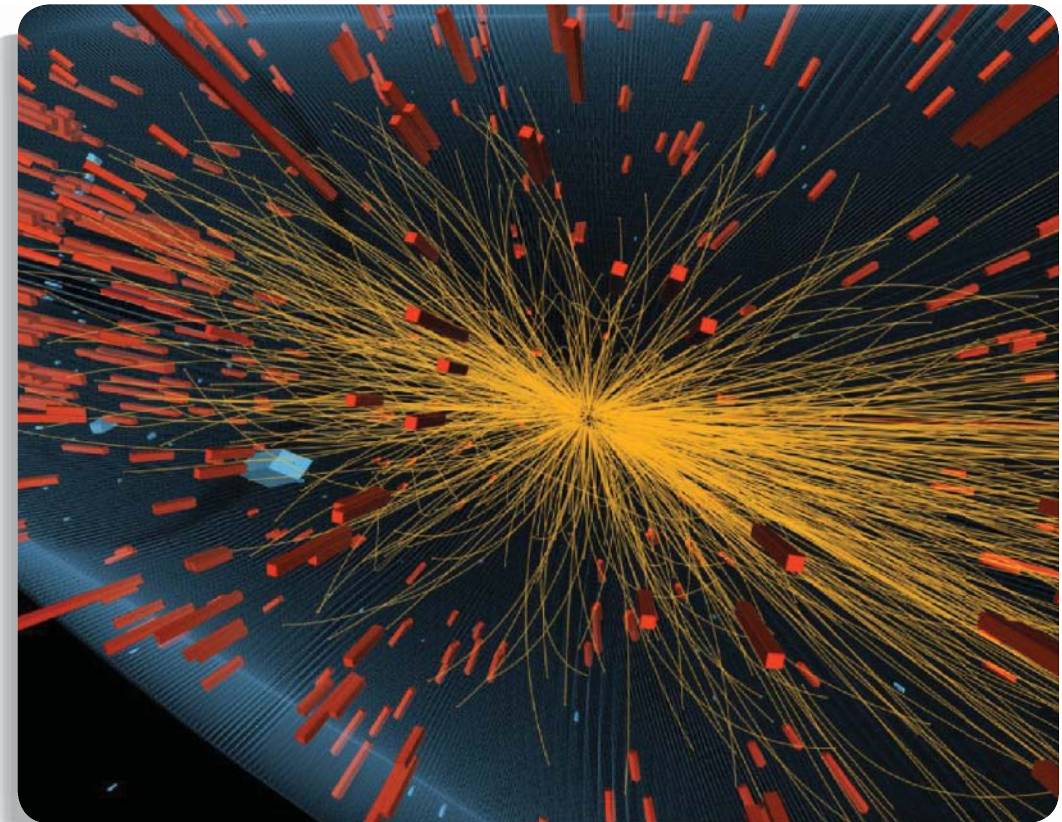
Chapter 14

ELEMENTARY PARTICLES



Particle tracks from a head-on collision of a high-energy proton and a lead ion at the Large Hadron Collider at CERN. Thousands of product particles are produced in each collision. As they travel outward from the collision site at the center, their energy loss and the curvature of their path in a magnetic field help to identify the particles. The goal of this experiment is to produce a "soup" of quarks and gluons, which is believed to characterize the universe just microseconds after the Big Bang. Courtesy of CERN

The search for the basic building blocks of nature has occupied the thoughts of scientific investigators since the Greeks introduced the idea of atomism 2500 years ago. As we look carefully at complex structures, we find underlying symmetries and regularities, which help us to understand the laws that determine how they are put together. The regularities of crystal structure, for example, suggest to us that the atoms of which the crystal is composed must follow certain rules for arranging themselves and joining together. As we look more deeply, we find that although nature has constructed all material objects out of roughly 100 different kinds of atoms, we can understand these atoms in terms of only three particles: the electron, proton, and neutron. Our attempts to look further within the electron have been unsuccessful—the electron seems to be a fundamental particle, with no internal structure. However, when nucleons collide at high energy, the result is more complexity rather than simplicity; hundreds of new particles can emerge as products of these reactions. If there are hundreds of basic building blocks, it seems unlikely that we could ever uncover any fundamental dynamic laws of their behavior. However, experiments show a new, underlying regularity that can be explained in terms of a small number of truly fundamental particles called *quarks*.

In this chapter, we examine the properties of many of the particles of physics, the laws that govern their behavior, and the classifications of these particles. We also show how the quark model helps us to understand some properties of the particles.

14.1 THE FOUR BASIC FORCES

All of the known forces in the universe can be grouped into four basic types. In order of increasing strength, these are: *gravitation*, the *weak interaction, electromagnetism*, and the *strong interaction*.

1. The Gravitational Interaction Gravity is of course exceedingly important in our daily lives, but on the scale of fundamental interactions between particles in the subatomic realm, it is of no importance at all. To give a relative figure, the gravitational force between two protons just touching at their surfaces is about 10^{-38} of the strong force between them. The principal difference between gravitation and the other interactions is that, on the practical scale, gravity is cumulative and infinite in range. Tiny gravitational interactions, such as the force exerted by one atom of the Earth on one atom of your body, combine to produce observable effects. The other forces, while much stronger than gravity at the microscopic level, do not affect objects on the large scale, either because they have a short range (the strong and weak forces) or their effect is negated by shielding (electromagnetism).

2. The Weak Interaction The weak interaction is responsible for nuclear beta decay (see Section 12.8) and other similar decay processes involving fundamental particles. It does not play a major role in the binding of nuclei. The weak force between two neighboring protons is about 10^{-7} of the strong force between them, and the range of the weak force is on the scale of 0.001 fm. Nevertheless, the weak force is important in understanding the behavior of fundamental particles, and it is critical in understanding the evolution of the universe.

3. The Electromagnetic Interaction Electromagnetism is important in the structure and the interactions of the fundamental particles. For example, some particles interact or decay primarily through this mechanism. Electromagnetic forces are of infinite range, but the shielding effect generally diminishes their effect for ordinary objects. Many common macroscopic forces (such as friction, air resistance, drag, and tension) are ultimately due to electromagnetic forces at the atomic level. Within the atom, electromagnetic forces dominate. The electromagnetic force between neighboring protons in a nucleus is about 10^{-2} of the strong force, but within the nucleus the electromagnetic forces can act cumulatively because there is no shielding. As a result, the electromagnetic force can compete with the strong force in determining the stability and the structure of nuclei.

4. The Strong Force The strong force, which is responsible for the binding of nuclei, is the dominant one in the reactions and decays of most of the fundamental particles. However, as we shall see, some particles (such as the electron) do not feel this force at all. It has a relatively short range, on the order of 1 fm.

The relative strength of a force determines the time scale over which it acts. If we bring two particles close enough together for any of these forces to act, then a longer time is required for the weak force to cause a decay or reaction than for the strong force. As we shall see, the mean lifetime of a decay process is often a signal of the type of interaction responsible for the process, with strong forces being at the shortest end of the time scale (often down to 10^{-23} s). Table 14.1 summarizes the four forces and some of their properties.

Particles can interact with one another in decays and reactions through any of the basic forces. Table 14.1 indicates which particles can interact through each of the four forces. All particles can interact through the gravitational and weak forces. A subset of those can interact through the electromagnetic force (for example, the neutrinos are excluded from this category), and a still smaller subset can interact through the strong force. When two strongly interacting particles are within the range of each other's strong force, we can often neglect the effects of the weak and electromagnetic forces in decay and reaction processes; because their relative strengths are so much smaller than that of the strong force, their effects are much smaller than those of the strong force. (However, these forces are not always negligible—the weak interaction between protons is responsible for a critical step in one of the fusion processes that occurs in stars.)

Even though the proton is a strongly interacting particle, a proton and an electron will *never* interact through the strong force. The electron is able to

TABLE 14.1 The Four Basic Forces

Type	Range	Relative Strength	Characteristic Time	Typical Particles
Strong	1 fm	1	$< 10^{-22}$ s	π, K, n, p
Electromagnetic	∞	10^{-2}	$10^{-14} - 10^{-20}$ s	e, μ, π, K, n, p
Weak	10^{-3} fm	10^{-7}	$10^{-8} - 10^{-13}$ s	All
Gravitational	∞	10^{-38}	Years	All

TABLE 14.2 The Field Particles

Force	Field Particle	Symbol	Charge (e)	Spin ($\hbar$)	Rest Energy (GeV)
Strong	Gluon	g	0	1	0
Electromagnetic	Photon	γ	0	1	0
Weak	Weak boson	W^+, W^- Z^0	±1 0	1 1	80.4 91.2
Gravitational	Graviton		0	2	0

ignore the strong force of the proton and respond only to its weak or electromagnetic force.

Each of the four forces can be represented in terms of the emission or absorption of particles that carry the interaction, just as we represent the force between nucleons in the nucleus in terms of the exchange of pions (see Section 12.4). Associated with each type of force is a field that is carried by its characteristic particle, as shown in Table 14.2.

- **The strong force** between quarks is carried by particles called *gluons*, which have been observed through indirect techniques.
- **The electromagnetic force** between particles can be represented in terms of the emission and absorption of *photons*.
- **The weak force** is carried by the *weak bosons* $W^{\pm}$ and Z^0, which are responsible for processes such as nuclear beta decay. For example, the beta decay of the neutron (a weak interaction) can be represented as

$$n \rightarrow p + W^- \qquad \text{followed by} \qquad W^- \rightarrow e^- + \overline{\nu}_e$$

  Because the decay $n \rightarrow p + W^-$ would violate energy conservation, the existence of the W^- is restricted by the uncertainty principle, and its range can be determined in a manner similar to that of the pion (see Eq. 12.8).
- **The gravitational force** is carried by the *graviton*, which is expected to exist based on theories of gravitation but has not yet been observed.

14.2 CLASSIFYING PARTICLES

One way of studying the elementary particles is to classify them into different categories based on certain behaviors or properties and then to look for similarities or common characteristics among the classifications. We have already classified some particles in Table 14.1 according to the types of forces through which they interact. Another way of classifying them might be according to their masses. In the early days of particle physics, it was observed that the lightest particles (including electrons, muons, and neutrinos) showed one type of behavior, the heaviest group (including protons and neutrons) showed a different behavior, and a middle group (such as pions and kaons) showed a still different behavior. The names originally given to these groups are based on the Greek words for light, middle, and heavy: *leptons* for the light particles, *mesons* for the middle group, and *baryons* for the heavier particles. Even though the classification by mass is now obsolete (leptons and mesons have been discovered that are more massive than protons or neutrons), we keep the original names, which now describe instead a group or *family* of particles with similar properties. When we compare our first two ways of classifying particles, we find an interesting result: The leptons do not interact through the strong force, but the mesons and baryons do.

We can also classify particles by their intrinsic spins. Every particle has an intrinsic spin; you will recall that the electron has a spin of $\frac{1}{2}$, as do the proton and neutron. We find that the leptons all have spins of $\frac{1}{2}$, the mesons all have integral spins $(0, 1, 2, \ldots)$, and the baryons all have half-integral spins $(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots)$.

Antiparticles

One additional property that is used to classify a particle is the nature of its *antiparticle*.* Every particle has an antiparticle, which is identical to the particle in such properties as mass and lifetime, but differs from the particle in the sign of its electric charge (and in the sign of certain other properties, as we discuss later). The antiparticle of the electron is the positron e^+, which was discovered in the 1930s through reactions initiated by cosmic rays. The positron has a charge of $+e$ (opposite to that of the electron) and a rest energy of 0.511 MeV (identical to that of the electron). The antiproton $\overline{\mathrm{p}}$ was discovered in 1956 (see Example 2.21); it has a charge of $-e$ and a rest energy of 938 MeV. A stable atom of antihydrogen could be constructed from a positron and an antiproton; the properties of this atom would be identical to those of ordinary hydrogen.

Antiparticles of stable particles (such as the positron and the antiproton) are themselves stable. However, when a particle and its antiparticle meet, the *annihilation reaction* can occur: the particle and antiparticle both vanish, and instead two or more photons can be produced. Conservation of energy and momentum requires that, neglecting the kinetic energies of the particles, when two photons are emitted each must have an energy equal to the rest energy of the particle. Examples of annihilation reactions are:

$$\mathrm{e}^- + \mathrm{e}^+ \rightarrow \gamma_1 + \gamma_2 \qquad (E_{\gamma_1} = E_{\gamma_2} = 0.511 \text{ MeV})$$
$$\mathrm{p} + \overline{\mathrm{p}} \rightarrow \gamma_1 + \gamma_2 \qquad (E_{\gamma_1} = E_{\gamma_2} = 938 \text{ MeV})$$

We call the kind of stuff of which we are made *matter* and the other kind of stuff *antimatter*. There may indeed be galaxies composed of antimatter, but we cannot tell by the ordinary techniques of astronomy, because *light and antilight are identical*! To put it another way, the photon and antiphoton are the same particle, so matter and antimatter emit the same photons. The only way to tell the difference is by sending a chunk of our matter to the distant galaxy and seeing whether or not it is annihilated with the corresponding emission of

*We use two systems to indicate antiparticles. Sometimes the symbol for the particle will be written along with the electric charge to indicate particle or antiparticle, as, for example, e^+ and e^-, or μ^+ and μ^-. Other times the antiparticle will be written with a bar over the symbol—for example, ν and $\overline{\nu}$ or p and $\overline{\mathrm{p}}$.

a burst of photons. (It is indeed possible, but *highly unlikely*, that the first astronaut to travel to another galaxy may suffer such a fate! The first intergalactic handshake would indeed be quite an event!)

In our classification scheme it is usually easy to distinguish particles from antiparticles. We begin by defining *particles* to be the stuff of which ordinary matter is made—electrons, protons, and neutrons. Ordinary matter is not composed of neutrinos, so we have no basis for distinguishing a neutrino from an antineutrino, but the conservation laws in the beta decay process can be understood most easily if we define the *antineutrino* to be the particle that accompanies negative beta decay and the *neutrino* to be the particle that accompanies positron decay and electron capture. For a heavy baryon, such as the Λ (lambda), we take advantage of its radioactive decay, which leads eventually to ordinary protons and neutrons; that is, the Λ is the particle that decays to n, and the $\overline{\Lambda}$ ("anti-lambda") therefore decays to $\overline{\mathrm{n}}$. Similarly, in the case of the leptons, the μ^- and the μ^+ are antiparticles of one another; because μ^- decays to ordinary e^- (and has many properties in common with the electron) it is the *particle*, while μ^+ is the antiparticle.

Three Families of Particles

Table 14.3 summarizes the three families of material particles.

Leptons The leptons interact only through the weak or electromagnetic interactions. No experiment has yet been able to reveal any internal structure for the leptons; they appear to be truly fundamental particles that cannot be split into still smaller particles. All known leptons have spin $\frac{1}{2}$.

Table 14.4 shows the six known leptons, grouped as three pairs of particles. Each pair includes a charged particle (e^-, μ^-, τ^-) and an uncharged

TABLE 14.3 Families of Particles

Family	Structure	Interactions	Spin	Examples
Leptons	Fundamental	Weak, electromagnetic	Half integral	e, ν
Mesons	Composite	Weak, electromagnetic, strong	Integral	π, K
Baryons	Composite	Weak, electromagnetic, strong	Half integral	p, n

TABLE 14.4 The Lepton Family

Particle	Antiparticle	Particle Charge (e)	Spin ($\hbar$)	Rest Energy (MeV)	Mean Life (s)	Typical Decay Products
e^-	e^+	−1	$\frac{1}{2}$	0.511	∞	
ν_e	$\overline{\nu}_e$	0	$\frac{1}{2}$	< 2 eV	∞	
μ^-	μ^+	−1	$\frac{1}{2}$	105.7	2.2×10^{-6}	$e^- + \overline{\nu}_e + \nu_\mu$
ν_μ	$\overline{\nu}_\mu$	0	$\frac{1}{2}$	< 0.19	∞	
τ^-	τ^+	−1	$\frac{1}{2}$	1776.9	2.9×10^{-13}	$\mu^- + \overline{\nu}_\mu + \nu_\tau$
ν_τ	$\overline{\nu}_\tau$	0	$\frac{1}{2}$	< 18	∞	

neutrino (ν_e, ν_μ, ν_τ). Each lepton has a corresponding antiparticle. We have already discussed the electron neutrino and antineutrino in connection with beta decay (Section 12.8), and the decay of cosmic-ray muons was discussed as confirming the time dilation effect in special relativity (Section 2.4). The neutrino masses are very small but nonzero. The rest-energy limits shown in Table 14.4 come from attempts at direct measurement, but indirect evidence from astrophysics and cosmology suggests that the rest energies of all three neutrinos are less than 1 eV.

Mesons Mesons are strongly interacting particles having integral spin. A partial list of some mesons is given in Table 14.5. Mesons can be produced in reactions through the strong interaction; they decay to other mesons or leptons through the strong, electromagnetic, or weak interactions. For example, pions can be produced in reaction of nucleons, such as

$$\mathrm{p} + \mathrm{n} \to \mathrm{p} + \mathrm{p} + \pi^- \qquad \text{or} \qquad \mathrm{p} + \mathrm{n} \to \mathrm{p} + \mathrm{n} + \pi^0$$

and the pions can decay according to

$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad (\text{mean life} = 2.6 \times 10^{-8}\ \text{s})$$
$$\pi^0 \to \gamma + \gamma \qquad (\text{mean life} = 8.5 \times 10^{-17}\ \text{s})$$

The first decay is caused by the weak interaction (indicated by the lifetime and by the presence of a neutrino among the decay products) and the second is caused by the electromagnetic interaction (indicated by the lifetime and the photons).

Because mesons are not observed in ordinary matter, the classification into particles and antiparticles is somewhat arbitrary. For the charged mesons such

TABLE 14.5 Some Selected Mesons

Particle	Antiparticle	Charge* (e)	Spin ($\hbar$)	Strangeness*	Rest Energy (MeV)	Mean Life (s)	Typical Decay Products
π^+	π^-	+1	0	0	140	2.6×10^{-8}	$\mu^+ + \nu_\mu$
π^0	π^0	0	0	0	135	8.5×10^{-17}	$\gamma + \gamma$
K^+	K^-	+1	0	+1	494	1.2×10^{-8}	$\mu^+ + \nu_\mu$
K^0	$\overline{K}^0$	0	0	+1	498	0.9×10^{-10}	$\pi^+ + \pi^-$
η	η	0	0	0	548	5.1×10^{-19}	$\gamma + \gamma$
ρ^+	ρ^-	+1	1	0	775	4.4×10^{-24}	$\pi^+ + \pi^0$
η'	η'	0	0	0	958	3.4×10^{-21}	$\eta + \pi^+ + \pi^-$
D^+	D^-	+1	0	0	1870	1.0×10^{-12}	$K^- + \pi^+ + \pi^+$
J/ψ	J/ψ	0	1	0	3097	7.1×10^{-21}	$e^+ + e^-$
B^+	B^-	+1	0	0	5279	1.6×10^{-12}	$D^- + \pi^+ + \pi^-$
Υ	Υ	0	1	0	9460	1.2×10^{-20}	$e^+ + e^-$

*The charge and strangeness are those of the particle. Values for the antiparticle have the opposite sign. The spin, rest energy, and mean life are the same for a particle and its antiparticle.

as π^+ and π^- or K^+ and K^-, which are not part of ordinary matter, the positive and negative particles are antiparticles of one another but there is no way to choose which is matter and which is antimatter. For some uncharged mesons (such as π^0 and η), the particle and antiparticle are identical, while for others (such as K^0 and $\overline{K}^0$) they may be distinct.

Baryons The baryons are strongly interacting particles with half-integral spins ($\frac{1}{2}, \frac{3}{2}, \ldots$). A partial listing of some baryons is given in Table 14.6. Like the leptons, the baryons have distinct antiparticles. Like the mesons, the baryons can be produced in reactions with nucleons through the strong interaction; for example, the Λ^0 baryon can be produced in the following reaction:

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \Lambda^0 + \mathrm{K}^+$$

The Λ^0 then decays through the weak interaction according to

$$\Lambda^0 \rightarrow \mathrm{p} + \pi^- \qquad (\text{mean life} = 2.6 \times 10^{-10}\ \mathrm{s})$$

Even though neutrinos are not produced in this decay process, the lifetime indicates that the decay proceeds through the weak interaction. Other baryons can be identified in Table 14.6 that decay through the strong, electromagnetic, or weak interactions.

TABLE 14.6 Some Selected Baryons

Particle	Antiparticle	Charge* (e)	Spin ($\hbar$)	Strangeness*	Rest Energy (MeV)	Mean Life (s)	Typical Decay Products
p	$\overline{\mathrm{p}}$	+1	$\frac{1}{2}$	0	938	∞	
n	$\overline{\mathrm{n}}$	0	$\frac{1}{2}$	0	940	880	$\mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}$
Λ^0	$\overline{\Lambda}^0$	0	$\frac{1}{2}$	−1	1116	2.6×10^{-10}	$\mathrm{p} + \pi^-$
Σ^+	$\overline{\Sigma}^+$	+1	$\frac{1}{2}$	−1	1189	8.0×10^{-11}	$\mathrm{p} + \pi^0$
Σ^0	$\overline{\Sigma}^0$	0	$\frac{1}{2}$	−1	1193	7.4×10^{-20}	$\Lambda^0 + \gamma$
Σ^-	$\overline{\Sigma}^-$	−1	$\frac{1}{2}$	−1	1197	1.5×10^{-10}	$\mathrm{n} + \pi^-$
Ξ^0	$\overline{\Xi}^0$	0	$\frac{1}{2}$	−2	1315	2.9×10^{-10}	$\Lambda^0 + \pi^0$
Ξ^-	$\overline{\Xi}^-$	−1	$\frac{1}{2}$	−2	1322	1.6×10^{-10}	$\Lambda^0 + \pi^-$
Δ^*	$\overline{\Delta}^*$	+2, +1, 0, −1	$\frac{3}{2}$	0	1232	5.6×10^{-24}	$\mathrm{p} + \pi$
Σ^*	$\overline{\Sigma}^*$	+1, 0, −1	$\frac{3}{2}$	−1	1385	1.8×10^{-23}	$\Lambda^0 + \pi$
Ξ^*	$\overline{\Xi}^*$	−1, 0	$\frac{3}{2}$	−2	1533	7.2×10^{-23}	$\Xi + \pi$
Ω^-	$\overline{\Omega}^-$	−1	$\frac{3}{2}$	−3	1672	8.2×10^{-11}	$\Lambda^0 + \mathrm{K}^-$

*The charge and strangeness are those of the particle. Values for the antiparticle have the opposite sign. The spin, rest energy, and mean life are the same for a particle and its antiparticle.

14.3 CONSERVATION LAWS

In the decays and reactions of elementary particles, conservation laws provide a way to understand why some processes occur and others are not observed, even though they are expected on the basis of other considerations. We frequently use the conservation of energy, linear momentum, and angular momentum in our analysis of physical phenomena. These conservation laws are closely connected with the fundamental properties of space and time; we believe those laws to be absolute and inviolable.

We also use other kinds of conservation laws in analyzing various processes. For example, when we combine two elements in a chemical reaction, such as hydrogen + oxygen → water, we must balance the reaction in the following way:

$$2H_2 + O_2 \rightarrow 2H_2O$$

The process of balancing a reaction can also be regarded as a way of accounting for the electrons that participate in the process: a molecule of water contains 10 electrons, and so the atoms that combine to make up the molecule must likewise include 10 electrons.

In nuclear processes, we are concerned not with electrons but with protons and neutrons. In the alpha decay of a nucleus, such as

$$^{235}_{92}U_{143} \rightarrow {}^{231}_{90}Th_{141} + {}^{4}_{2}He_{2}$$

or in a reaction such as

$$p + {}^{63}_{29}Cu_{34} \rightarrow {}^{63}_{30}Zn_{33} + n$$

we balance the number of protons and also the number of neutrons. We might be tempted to conclude that nuclear processes conserve both proton number and neutron number, but the separate conservation laws are not satisfied in beta decays; for example,

$$n \rightarrow p + e^- + \overline{\nu}_e$$

which does not conserve either neutron number or proton number. However, it does conserve the total neutron number plus proton number, which is equal to 1 both before and after the decay. (This conservation law of total nucleon number includes the separate laws of conservation of proton number and neutron number as a special case.)

Pictorial Parade / Getty Images

Emmy Noether (1882–1935, Germany-United States). Known both as a mathematician and a theoretical physicist, she explored the role of conservation laws in physics. In an important result now known as Noether's theorem, she discovered that each symmetry of the mathematical equations describing a phenomenon gives a conserved quantity. For example, the symmetry of equations to translations in time leads to conservation of energy, and the invariance to translations in space leads to conservation of linear momentum.

Lepton Number Conservation

In negative beta decay, we always find an antineutrino emitted, never a neutrino. Conversely, in positron beta decay, it is the neutrino that is always emitted. We account for these processes by assigning each particle a *lepton number L*. The electron and neutrino are assigned lepton numbers of +1, and the positron and antineutrino are assigned lepton numbers of −1; all mesons and baryons are assigned lepton numbers of zero. Lepton number conservation in positive and negative beta decay then works as follows:

$$\begin{array}{rccccccc} & \mathrm{n} & \to & \mathrm{p} & + & \mathrm{e}^- & + & \overline{\nu}_\mathrm{e} \\ L: & 0 & \to & 0 & + & 1 & + & (-1) \end{array}$$

$$\begin{array}{rccccccc} & \mathrm{p} & \to & \mathrm{n} & + & \mathrm{e}^+ & + & \nu_\mathrm{e} \\ L: & 0 & \to & 0 & + & (-1) & + & 1 \end{array}$$

You can see that the total lepton number is 0 both before and after these decays, which accounts for the appearance of the antineutrino in negative beta decay and the neutrino in positron decay.

According to the lepton conservation law, these processes are forbidden:

$$\begin{array}{rccccccc} & \mathrm{e}^- & + & \mathrm{p} & \to & \mathrm{n} & + & \overline{\nu}_\mathrm{e} \\ L: & 1 & + & 0 & \to & 0 & + & (-1) \end{array}$$

$$\begin{array}{rccccc} & \mathrm{p} & \to & \mathrm{e}^+ & + & \gamma \\ L: & 0 & \to & -1 & + & 0 \end{array}$$

In keeping track of leptons, we must count each type of lepton (e, μ, τ) separately. Evidence for this comes from a variety of experiments. For example, the distinction between electron-type and muon-type leptons is clear from an experiment in which a beam of muon-type antineutrinos is incident on a target of protons:

$$\overline{\nu}_\mu + \mathrm{p} \to \mathrm{n} + \mu^+$$

If there were no difference between electron-type and muon-type leptons, the following reaction would be possible: $\overline{\nu}_\mu + \mathrm{p} \to \mathrm{n} + \mathrm{e}^+$. However, this outcome is never observed, which indicates the distinction between the two types of leptons and the need to account separately for each type.

Another example of the difference between the types of leptons comes from the failure to observe the decay $\mu^- \to \mathrm{e}^- + \gamma$. If there were only one common type of lepton number, this decay would be possible. The failure to observe this decay (in comparison with the commonly observed decay $\mu^- \to \mathrm{e}^- + \overline{\nu}_\mathrm{e} + \nu_\mu$, which conserves both muon-type and electron-type lepton number) suggests the need for the different kinds of lepton numbers. We call these lepton numbers L_e, L_μ, and L_τ, and we have the following conservation law for leptons:

> *In any process, the lepton numbers for electron-type leptons, muon-type leptons, and tau-type leptons must each remain constant.*

The following examples illustrate the conservation of these lepton numbers.

$$\begin{array}{rccccccc} & \overline{\nu}_\mathrm{e} & + & \mathrm{p} & \to & \mathrm{e}^+ & + & \mathrm{n} \\ L_\mathrm{e}: & -1 & + & 0 & \to & -1 & + & 0 \end{array}$$

$$\begin{array}{rccccccc} & \nu_\mu & + & \mathrm{n} & \to & \mu^- & + & \mathrm{p} \\ L_\mu: & 1 & + & 0 & \to & 1 & + & 0 \end{array}$$

$$\begin{array}{rccccccc} & \mu^- & \to & \mathrm{e}^- & + & \overline{\nu}_\mathrm{e} & + & \nu_\mu \\ L_\mathrm{e}: & 0 & \to & 1 & + & (-1) & + & 0 \\ L_\mu: & 1 & \to & 0 & + & 0 & + & 1 \end{array}$$

$$\begin{array}{rccccc} & \pi^- & \to & \mu^- & + & \overline{\nu}_\mu \\ L_\mu: & 0 & \to & 1 & + & (-1) \end{array}$$

Studying these examples, we can understand why sometimes neutrinos appear and sometimes antineutrinos appear.

Baryon Number Conservation

Baryons are subject to a similar conservation law. All baryons are assigned a baryon number $B = +1$, and all antibaryons are assigned $B = -1$. All non-baryons (mesons and leptons) have $B = 0$. We then have the law of conservation of baryon number:

In any process, the total baryon number must remain constant.

(The conservation of nucleon number A is a special case of conservation of baryon number, in which all the baryons are nucleons. In particle physics, it is customary to use B instead of A to represent all baryons, including the nucleons.) No violation of the law of baryon conservation has ever been observed, although the Grand Unified Theories (see Section 14.8) suggest that the proton can decay in a way that would violate conservation of baryon number.

As an example of conservation of baryon number, consider the reaction that was responsible for the discovery of the antiproton:

$$\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \mathrm{p} + \overline{\mathrm{p}}$$

On the left side, the total baryon number is $B = +2$. On the right side, we have three baryons with $B = +1$ and one antibaryon with $B = -1$, so the total baryon number is $B = +2$ on the right side also. On the other hand, the reaction $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \overline{\mathrm{n}}$ violates baryon number conservation and is therefore forbidden.

Strangeness Conservation

The number of mesons that can be created or destroyed in decays or reactions is not subject to a conservation law like the number of leptons or baryons. For example, the following reactions can be used to produce pions:

$$\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{n} + \pi^+ \qquad \mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{n} + \pi^+ + \pi^0$$
$$\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \pi^0 \qquad \mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \pi^+ + \pi^-$$

As long as enough energy is available, any number of pions can be produced in these reactions.

If we try the same type of reaction to produce K mesons, a different type of behavior is observed. The reactions $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{n} + \mathrm{K}^+$ and $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \mathrm{K}^0$ never occur, even though the incident proton is given enough energy to produce this particle. We do, however, observe reactions such as $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{n} + \mathrm{K}^+ + \overline{\mathrm{K}}^0$ and $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \mathrm{K}^+ + \mathrm{K}^-$, which are very similar to the reactions that produce two pions. Why do reactions producing π mesons give any number (odd or even) but reactions producing K mesons give them only in pairs?

Here's another example of this unusual behavior. The reaction $\pi^- + \mathrm{p} \to \pi^+ + \Sigma^-$ conserves electric charge and baryon number and so would be expected to occur, but it does not. Instead, the following reaction is easily observed: $\pi^- + \mathrm{p} \to \mathrm{K}^+ + \Sigma^-$. Usually when we fail to observe a reaction or decay process that is expected to occur, we look for the violation of some conservation law such as electric charge or baryon number. Is there a new conserved quantity whose violation prohibits the reaction from occurring?

There are also decay processes that suggest that our labeling of the particles is incomplete. The uncharged η and π^0 mesons decay very rapidly (10^{-16} – 10^{-18} s) into two photons; on the basis of the systematic behavior of mesons, we would expect the K^0 to decay similarly to two photons in a comparable time. The observed decay of the K^0 takes place much more slowly (10^{-10} s); moreover, the decay products are not photons, but π mesons and leptons. Is a new conservation law responsible for restricting the decay of the K^0?

As a final example of the need for a new conservation law, the heavy charged mesons are all strongly interacting particles, and we expect them to decay into the lighter mesons through the strong interaction with very short lifetimes. For example, the decay $\rho^+ \to \pi^+ + \pi^0$ occurs in a lifetime of about 10^{-23} s. But the decay $K^+ \to \pi^+ + \pi^0$ occurs very slowly, in a time of the order of 10^{-8} s, and in fact the different decay mode $K^+ \to \mu^+ + \nu_\mu$ is more probable. What is responsible for slowing the decay of the K meson by 15 orders of magnitude?

These unusual behaviors are explained by the introduction of a new conserved quantity. This quantity is called the *strangeness* S, and we can use it to explain the properties of the K-meson decays. The K^0 and K^+ are assigned strangeness of $S = +1$; the π mesons and leptons are nonstrange particles ($S = 0$). The decay $K^0 \to \gamma + \gamma$, which is an electromagnetic decay (as indicated by the photons), is forbidden because the electromagnetic interaction conserves strangeness ($S = +1$ on the left, $S = 0$ on the right). The decay $K^+ \to \pi^+ + \pi^0$ does not occur in the typical strong interaction time of 10^{-23} s because the strong interaction cannot change strangeness. It occurs in the typical weak interaction time of 10^{-8} s (and the corresponding weak interaction decay $K^+ \to \mu^+ + \nu_\mu$ occurs as often) because the weak interaction *does not* conserve strangeness; decays that are caused by the weak interaction can change the strangeness by one unit.

We can summarize these results in the *law of conservation of strangeness*:

> *In processes governed by the strong or electromagnetic interactions, the total strangeness must remain constant. In processes governed by the weak interaction, the strangeness either remains constant or changes by one unit.*

The strangeness quantum numbers of the mesons and baryons are given in Tables 14.5 and 14.6. The strangeness of an antiparticle has the opposite sign to that of the corresponding particle.

Conservation of strangeness in the strong interaction explains why the K mesons are always produced in pairs in proton–proton collisions. The protons and neutrons are non-strange particles ($S = 0$), so the only way to conserve strangeness in the collisions that produce K mesons is to produce them in pairs, always one with $S = +1$ and the other with $S = -1$.

The baryons also come in strange and nonstrange varieties. Looking at the lifetimes in Table 14.6, we see that the Λ^0 decays into $p + \pi^-$ with a lifetime of about 10^{-10} s, while we would expect a strongly interacting particle to decay to other strongly interacting particles with a lifetime of about 10^{-23} s. If the strangeness of the Λ^0 is assigned as -1, these decays change S and are forbidden to go by the strong interaction, and so must be due to the weak interaction, with the characteristic 10^{-10} s lifetime. The strangeness violation also tells us why the electromagnetic decay $\Lambda^0 \to n + \gamma$ does not occur (while the decay $\Sigma^0 \to \Lambda^0 + \gamma$ does occur, with a typical electromagnetic lifetime of 10^{-19} s). It also explains why the reaction $\pi^- + p \to \pi^+ + \Sigma^-$, which is permitted by all other conservation laws, is never observed—the initial state has $S = 0$ and the final state has $S = -1$, so it violates strangeness conservation.

The weak interaction can change the strangeness by at most *one* unit. As a result, processes such as $\Xi^0 \rightarrow n + \pi^0$ ($S = -2 \rightarrow S = 0$) are absolutely forbidden, even by the weak interaction.

Example 14.1

The Ω^- baryon has $S = -3$. (*a*) It is desired to produce the Ω^- using a beam of K^- incident on protons. What other particles are produced in this reaction? (*b*) How might the Ω^- decay?

Solution

(*a*) Reactions usually proceed only through the strong interaction, which conserves strangeness. We consider the reaction

$$K^- + p \rightarrow \Omega^- + ?$$

On the left side, we have $S = -1, B = +1$, and electric charge $Q = 0$. On the right side, we have $S = -3$, $B = +1$, and $Q = -1$. We must therefore add to the right side particles with $S = +2$, $B = 0$, and $Q = +1$. Scanning through the tables of mesons and baryons, we find that we can satisfy these criteria with K^+ and K^0, so one possible reaction is

$$K^- + p \rightarrow \Omega^- + K^+ + K^0$$

(*b*) The Ω^- cannot decay by the strong interaction, because no $S = -3$ final states are available. It must therefore decay to particles having $S = -2$ through the weak interaction, which can change S by one unit. One of the product particles must be a baryon in order to conserve baryon number. Two possibilities are

$$\Omega^- \rightarrow \Lambda^0 + K^- \quad \text{and} \quad \Omega^- \rightarrow \Xi^0 + \pi^-$$

14.4 PARTICLE INTERACTIONS AND DECAYS

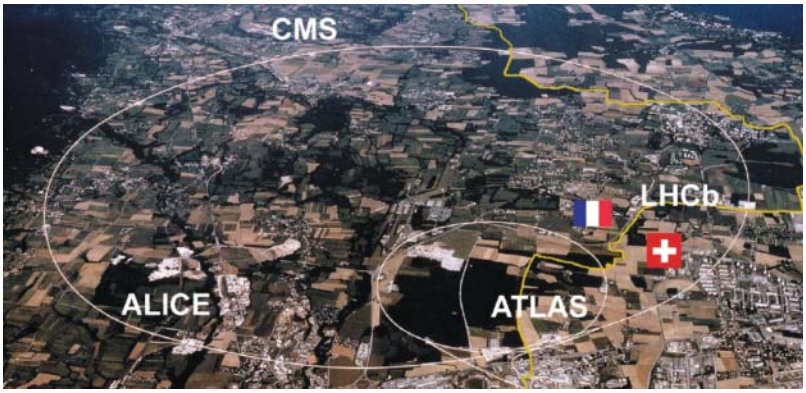
In this section, we briefly summarize the properties of the elementary particles and how they are measured.

Atoms and molecules can be taken apart relatively easily and nonviolently, enabling us to study their structure. However, the elementary particles, most of which are unstable and do not exist in nature, must be created in violent collisions. (The particle theorist Richard Feynman once compared this process with studying fine Swiss watches by smashing them together and looking at the pieces that emerge from the collision.) For this purpose, we need a high-energy beam of particles and a suitable target of elementary particles.

For a beam of particles, protons generally are the choice. As stable particles, they can be accelerated and stored for long periods. Electric and magnetic fields can be used to accelerate and shape beams of protons to very high energies. Some high-energy accelerators smash beams of protons against a fixed target of protons, often in the form of liquid hydrogen to provide greater density than a gas target. In other accelerators, such as the Large Hadron Collider (Figure 14.1), beams of protons traveling in opposite directions are steered into head-on collisions. Some accelerators seek to create new types of particles from collisions involving high-energy electrons, including electron–positron head-on collisions.

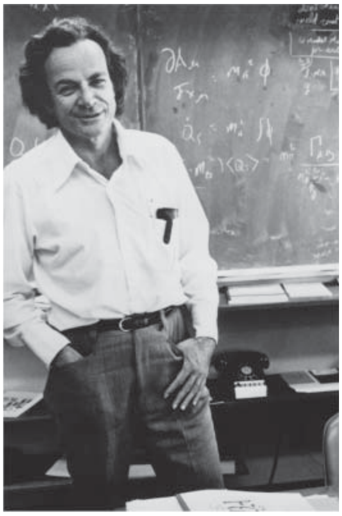
One type of particle physics reaction can thus be represented as

$$p + p \rightarrow \text{product particles}$$



Courtesy of CERN

FIGURE 14.1 The Large Hadron Collider accelerator at the European Organization for Nuclear Research (CERN) on the border between France and Switzerland. The large ring, 27 km in circumference, shows the location of the accelerator tubes which are more than 100 m underground. Beams of protons accelerated to 7 TeV circulate in opposite directions and are made to collide at the locations of the detectors.



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Richard P. Feynman (1918–1988, United States). Seldom is one person known for both exceptional insights into theoretical physics and exceptional methods of teaching first-year physics. He received the Nobel Prize for his work on the theory that couples quantum mechanics to electromagnetism, and his text and film *Lectures on Physics* give unusual perspectives to many areas of basic physics for undergraduates.

Among the product particles may be a variety of mesons or even heavier particles of the baryon family, of which the nucleons are the lightest members. The study of the nature and properties of these particles is the goal of particle physics.

In many cases, conservation laws restrict the nature of the product particles, and it would be desirable to have other types of beams available. One possibility is indicated in Figure 14.2. A proton beam is incident on a target—the nature of the target is not important. Like Feynman's Swiss watch parts, many different particles emerge. By suitable focusing and selection of the momentum, we can extract a beam of the *secondary* particles created in the reactions. The particle must live long enough to be delivered to a second target, which might be tens of meters away; even if the particle were traveling at the speed of light, it would need about 10^{-7} s to make its journey. Although this is a very short time interval by ordinary standards, on the time scale of elementary particles, it is a very long time—in fact, none of the unstable mesons or baryons (except the neutron) lives that long.

Although our efforts to make a secondary beam would seem to be in vain, we have forgotten one very important detail. The lifetime of the particle is measured in its rest frame, while we are observing its flight in the laboratory frame, in which the particle is moving at speeds extremely close to the speed of light. The *time dilation* factor results in a lifetime, observed in our frame of reference, which might be hundreds of times longer than the *proper lifetime*. This factor extends the range of available secondary beams to those particles with lifetimes as short as 10^{-10} s and makes it possible to obtain secondary beams to study such reactions as

$$\pi + \mathrm{p} \rightarrow \text{particles}$$

and

$$\mathrm{K} + \mathrm{p} \rightarrow \text{particles}$$

even though the proper lifetimes of the π and K are in the range of 10^{-10} to 10^{-8} s.

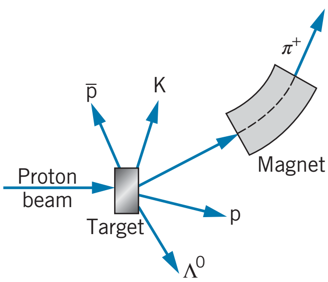


FIGURE 14.2 The production of secondary particle beams. The magnet helps to select the mass and momentum of the desired particle.

Detecting Particles

Observing the products of these reactions, which may involve dozens of high-energy charged and uncharged particles, poses a great technological problem for the experimenter. The detector must completely surround the reaction area, so that particles are recorded no matter what direction they travel after the reaction. The particles must produce visible tracks in the detector, so that their identity and direction of travel can be determined. The detector must provide sufficient mass to stop the particles and measure their energy. A magnetic field must be present, so that the resulting curved trajectory of a charged particle can be used to determine its momentum and the sign of its charge. Figure 14.3 shows tracks left in a *bubble chamber*, a large tank filled with liquid hydrogen in which the passage of a charged particle causes microscopic bubbles resulting from the ionization of the hydrogen atoms. The bubbles can be illuminated and photographed to reveal the tracks. Figure 14.4 shows a large detector system that is used both to display the tracks of particles and to measure their energies; Figure 14.5 shows a sample of the results that are obtained with this type of detector.

From a careful analysis of the paths of particles, such as those of Figures 14.3 or 14.5, we can deduce the desired quantities of mass, linear momentum, and energy. The other important property we would like to know is the lifetime of the decay of the product particles, because many of the products are often unstable. If we know the speed of a particle, we can find its lifetime by simply observing the length of its track in a bubble chamber photograph. (Even for uncharged particles, which leave no tracks, we can

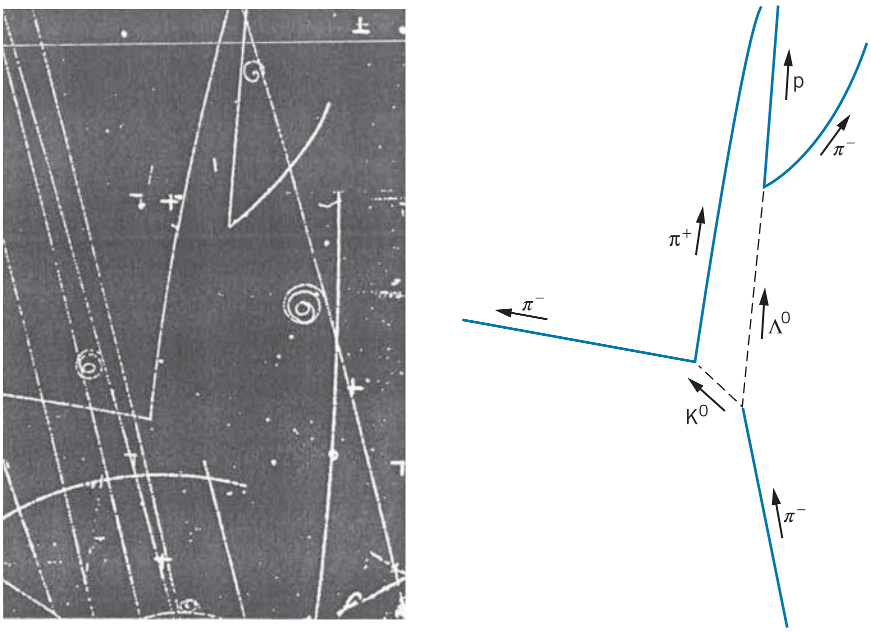


FIGURE 14.3 A bubble chamber photograph of a reaction between particles. At right is shown a diagram indicating the particles that participate in the reaction. An incident pion collides with a proton in the liquid hydrogen, producing a K^0 and a Λ^0, both of which subsequently decay.

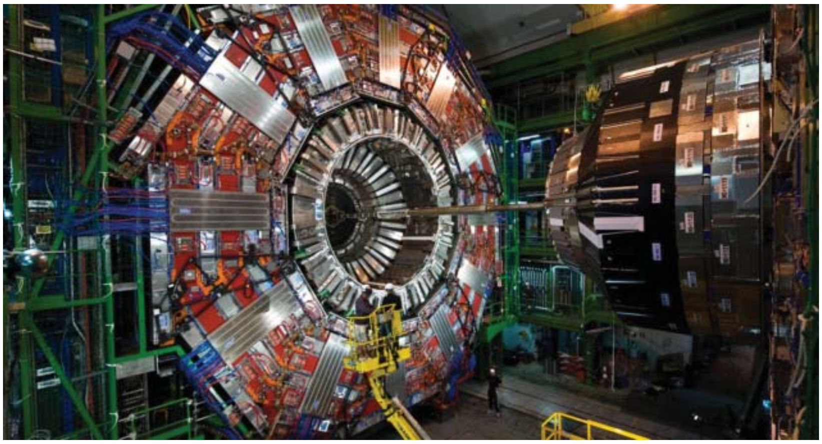
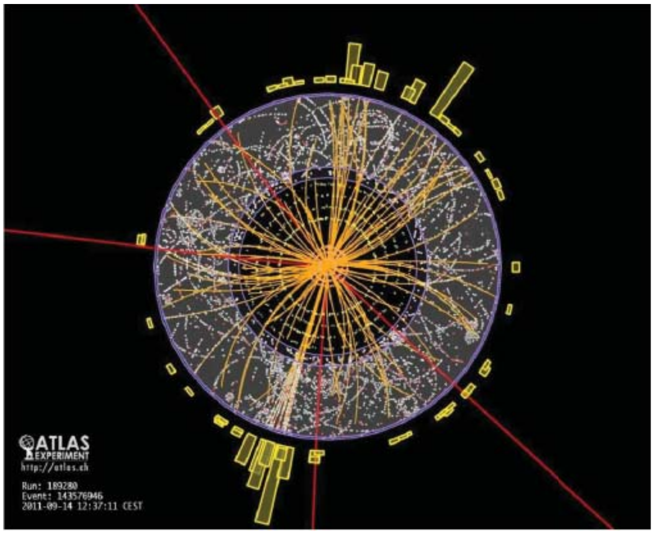

Courtesy of CERN

FIGURE 14.4 One of the detector systems at the Large Hadron Collider. The proton beams travel along the axis of the detector and collide in its interior. A series of concentric detectors surrounds the collision area and records the directions and energies of the particles that emerge from the collisions.



Courtesy of CERN

FIGURE 14.5 A sample of the particle tracks leaving the collision area of one of the detectors at the Large Hadron Collider. A magnetic field causes the tracks to curve, which allows measurement of the momentum of the particles. Positive and negative charges curve in opposite directions.

use this method to deduce the lifetime, because the subsequent decay of the uncharged particle into two charged particles defines the length of its path rather clearly, as shown in Figure 14.3.)

This method works well if the lifetime is of the order of 10^{-10} s or so, such that the particle leaves a track long enough to be measured (millimeters to centimeters). With careful experimental technique and clever data analysis, this can be extended to track lengths of the order of 10^{-6} m, and so lifetimes down to about 10^{-16} s can be measured in this way (with a little help from the time dilation factor). But many of our particles have lifetimes of only 10^{-23} s, and a particle moving at even the speed of light travels only the diameter of a nucleus in that time! How can we measure such a lifetime? Furthermore, how do we even know such a particle exists at all? Consider the reaction

$$\pi + \mathrm{p} \rightarrow \pi + \mathrm{p} + \mathrm{x}$$

where x is an unknown particle with a lifetime of about 10^{-23} s, which decays into two π mesons according to $\mathrm{x} \rightarrow \pi + \pi$. How do we distinguish the above reaction from the reaction

$$\pi + \mathrm{p} \rightarrow \pi + \mathrm{p} + \pi + \pi$$

which leads to the same particles as actually observed in the laboratory?

Experimental evidence suggests that the two π mesons in this type of reaction may combine for an instant (10^{-23} s) to form an entity with all of the usual properties of a particle—a definite mass, charge, spin, lifetime, etc. These states are known as *resonance particles*, and we now look at the indirect evidence from which we infer their existence.

Suppose that you receive a package in the mail from a friend. When you open it, you find it contains many small, irregular pieces of broken glass. How do you learn whether your friend sent you a beautiful glass vase that was broken in shipment or a package of broken glass as a practical joke? You try to put the pieces together! If the pieces fit together, it is a good assumption that the vase was once whole, although the mere fact that they fit together doesn't *prove* that it was once whole. It's just the simplest possible assumption *consistent with our experience*. (An alternative assumption that the pieces were manufactured separately and just happen by chance to fit together is highly improbable.)

How then do we detect a "particle" that lives for only 10^{-23} s? We look at its decay products (which live long enough to be seen in the laboratory), and putting the pieces back together, we infer that they once may have been a whole particle.

For example, suppose in the laboratory we observe two π mesons emitted as shown in Figure 14.6. We measure the direction of travel and the linear momentum of the π mesons as shown. A second and a third event each produces two π mesons as also shown in the figure. Are these three events consistent with the existence of the same resonance particle?

Let us assume that in each case, a particle moving at an unknown speed decayed into the two particles as shown, one with energy E_1 and momentum $\vec{\mathbf{p}}_1$ and the other with energy E_2 and momentum $\vec{\mathbf{p}}_2$. Each decay must conserve energy and momentum, so we can use the decay information to find the energy $E = E_1 + E_2$ and momentum $\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2$ of the decaying particle, and then we can find its rest energy according to $mc^2 = \sqrt{E^2 - c^2\vec{\mathbf{p}}^2} = \sqrt{(E_1 + E_2)^2 - c^2(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)^2}$. Carrying out the calculation, we find that, for the decay shown in part (*a*) of Figure 14.6, $mc^2 = 764$ MeV, while for part (*b*), $mc^2 = 775$ MeV. It is therefore possible that these two events result from the decays of identical particles. Part (*c*) of the figure gives $mc^2 = 498$ MeV, which differs considerably from parts (*a*) and (*b*).

Of course, these two events are not sufficient to identify conclusively the existence of a resonance particle with a rest energy in the range of 770 MeV.

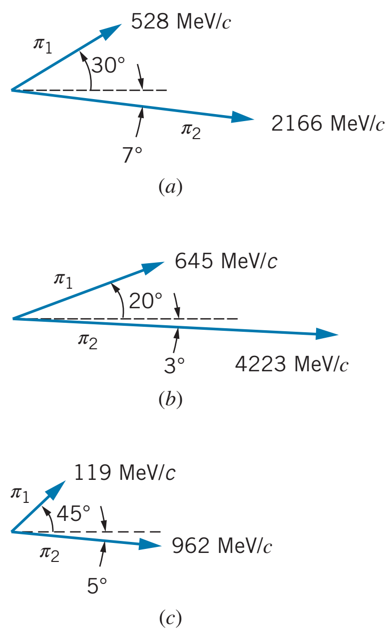


FIGURE 14.6 Three possible decays of an unknown particle into two π mesons. The direction and momentum of each π meson are indicated.

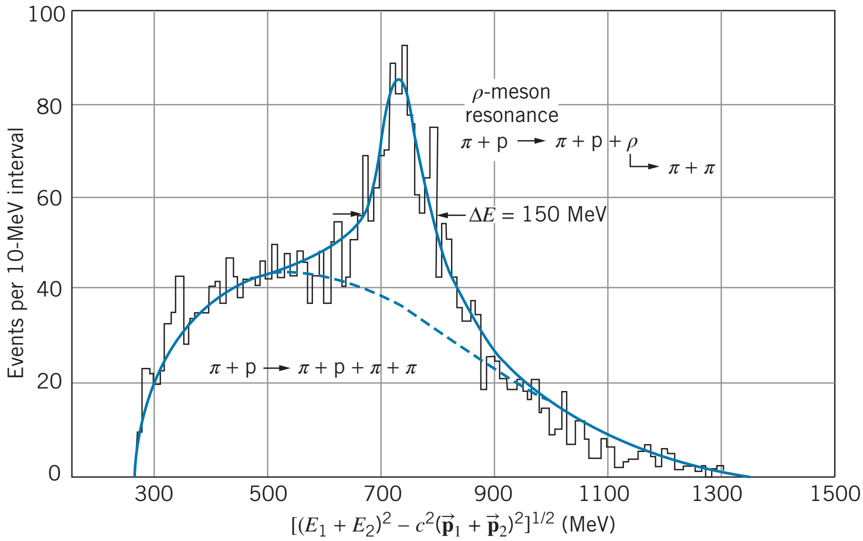


FIGURE 14.7 The resonance identified as the ρ meson. The horizontal axis shows the energy and momentum of the two decay π mesons, combined to be equivalent to the mass of the resonance particle.

It could be a mere accident, just like the chance fitting together of two pieces of broken glass. What is needed is a large (statistically significant) number of events, in which we can combine the energy and momenta of the two emitted π mesons in such a way that the deduced mass of the resonance particle is always the same. Figure 14.7 is an example of such a result. There is a background of events with a continuous distribution of energies, like beta decay electrons; these come from events like part (c) of Figure 14.6. There is also present a very prominent peak at 775 MeV. We identify this energy as the rest energy of the resonance particle, which is known as the ρ (rho) meson. (How do we know it is a meson? It must be a strongly interacting particle, because it decays so rapidly. Therefore, the only possibilities are mesons, with integral spin, or baryons, with half-integral spin. Pi mesons have integral spin, and two integral spins can combine to give only another integral spin, so it must be a meson.)

We can also infer the lifetime of the particle from Figure 14.7. The particle lives only for about 10^{-23} s, and so if we are to measure its rest energy we have only 10^{-23} s in which to do it. But the uncertainty principle requires that an energy measurement made in a time interval Δt be uncertain by an amount roughly $\Delta E \cong \hbar/\Delta t$. This energy uncertainty ΔE is observed as the *width* of the peak in Figure 14.7. We don't always deduce the same value 775 MeV for the rest energy of the ρ meson; sometimes our value is a bit larger and sometimes a bit smaller. *The width of the resonance peak tells us the lifetime of the particle.* (The width is not really precisely defined, but physicists usually take as the width the interval between the two points where the height of the resonance is one-half its maximum value above the background, as shown in Figure 14.7.) The width of $\Delta E = 150$ MeV leads to a value of $\Delta t = \hbar/\Delta E = 4.4 \times 10^{-24}$ s for the lifetime of the ρ meson.

Example 14.2

When K^- mesons are incident on protons, the reaction probability shows several peaks like that of Figure 14.7, but when K^+ mesons are incident on protons, the reaction probability is smooth with no peaks. Explain.

Solution

The peaks in the $K^- + p$ spectrum are due to baryon resonances such as those corresponding to Λ and Ξ baryons. Like the K^-, these resonances have strangeness -1. The K^+ meson has strangeness $+1$, and there are no baryons with strangeness $+1$ so no baryon resonances appear in the $K^+ + p$ spectrum.

14.5 ENERGY AND MOMENTUM IN PARTICLE DECAYS

In analyzing the decays and reactions of elementary particles, we apply many of the same laws that we used for nuclear decays and reactions: energy, linear momentum, and total angular momentum must be conserved, and the total value of the quantum numbers associated with electric charge, lepton number, and baryon number (which we previously called nucleon number) must be the same before and after the decay or reaction. In reactions of elementary particles, we are often concerned with the production of new varieties of particles. The energy necessary to manufacture these particles comes from the kinetic energy of the reaction constituents (usually the incident particle); this energy is usually quite large (hence the name *high-energy physics* for this type of research), so *relativistic equations* must be used for energy and momentum.

The decays of elementary particles can be analyzed in a way similar to the decays of nuclei, following the same two basic rules:

1. The energy available for the decay is the difference in rest energy between the initial decaying particle and the particles that are produced in the decay. By analogy with our study of nuclear decays, we call this the Q value:
$$Q = (m_\mathrm{i} - m_\mathrm{f})c^2 \tag{14.1}$$
where $m_\mathrm{i}c^2$ is the rest energy of the initial particle and $m_\mathrm{f}c^2$ is the total rest energy of all the final product particles. (Of course, the decay will occur only if Q is positive.)
2. The available energy Q is shared as kinetic energy of the decay products in such a way as to conserve linear momentum. As in the case of nuclear decays, for a decay of a particle at rest into two final particles, the particles have equal and opposite momenta, and we can find unique values for the energies of the two final particles. For decays into three or more particles, each particle has a spectrum or distribution of energies from zero up to some maximum value (as was the case with nuclear beta decay).

Example 14.3

Compute the energies of the proton and π meson that result from the decay of a Λ^0 at rest.

Solution

The decay process is $\Lambda^0 \rightarrow p + \pi^-$. Using the rest energies from Tables 14.5 and 14.6, we have:

$$\begin{aligned} Q &= (m_{\Lambda^0} - m_p - m_{\pi^-})c^2 \\ &= 1116 \text{ MeV} - 938 \text{ MeV} - 140 \text{ MeV} \\ &= 38 \text{ MeV} \end{aligned}$$

and so the total kinetic energy of the decay products must be:

$$K_p + K_\pi = 38 \text{ MeV}$$

Using the relativistic formula for kinetic energy, we can write this as

$$K_p + K_\pi = \left(\sqrt{c^2p_p^2 + m_p^2c^4} - m_pc^2\right) + \left(\sqrt{c^2p_\pi^2 + m_\pi^2c^4} - m_\pi c^2\right) = 38 \text{ MeV}$$

Conservation of momentum requires $p_p = p_\pi$. Substituting for one of the unknown momenta in the above equation and solving algebraically for the other, we obtain

$$p_\pi = p_p = 101 \text{ MeV}/c$$

The kinetic energies can be found by substituting these momenta into the relativistic formula:

$$K_\pi = 33 \text{ MeV} \qquad \text{and} \qquad K_p = 5 \text{ MeV}$$

Example 14.4

What is the maximum kinetic energy of the electron emitted in the decay $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$?

Solution

The Q value for this decay is $Q = m_\mu c^2 - m_e c^2 = 105.2$ MeV, because the neutrinos have negligible rest energy. If the μ^- is at rest, this energy is shared by the electron and the neutrinos: $Q = K_e + E_{\bar{\nu}_e} + E_{\nu_\mu}$. When the electron has its maximum kinetic energy, the two neutrinos carry away the minimum energy. This minimum cannot be zero, because that would violate momentum conservation: the electron would be carrying momentum that would not be balanced by the neutrino momenta to give a net of zero (the μ^- is at rest, so $\sum \vec{\mathbf{p}}_i = \sum \vec{\mathbf{p}}_f = 0$). We assume that the electron has its maximum energy when the neutrinos are emitted in exactly the opposite direction to the electron; otherwise some of the decay energy is "wasted" by providing transverse momentum components for the neutrinos, and not as much energy will be available for the electron. It does not matter which of the neutrinos carry the energy and momentum (they may even share it in any proportion), so we let E_ν and p_ν be the total neutrino energy and momentum; these are of course related by $E_\nu \cong cp_\nu$, because neutrinos are presumed to be of negligible mass and thus to travel at nearly the speed of light. If we let E_e and p_e represent the energy and momentum of the electron, then linear momentum conservation gives

$$p_e - p_\nu = 0$$

For the electron, $E_e = \sqrt{c^2p_e^2 + m_e^2c^4}$. Together, these equations give:

$$\begin{aligned} Q &= K_e + E_\nu = E_e - m_ec^2 + cp_\nu = E_e - m_ec^2 + cp_e \\ &= E_e - m_ec^2 + \sqrt{E_e^2 - m_e^2c^4} \end{aligned}$$

Solving, we find $E_e = Q/2m_\mu c^2 + m_ec^2$ and so

$$K_e = E_e - m_ec^2 = Q^2/2m_\mu c^2 = 52.3 \text{ MeV}$$

The original rest energy of the μ^- is shared essentially equally by the electron and the two neutrinos in this case: $(K_e)_{max} = (E_\nu)_{max} \cong Q/2$. Note how different this is from the case of the beta decay of the neutron, where the heavy proton resulting from the decay could absorb considerable recoil momentum at a cost of very little energy, so nearly all of the available energy could be given to the electron, and in that case $(K_e)_{max} \cong Q$.

Example 14.5

Find the maximum energy of the positrons and of the π mesons produced in the decay $K^+ \rightarrow \pi^0 + e^+ + \nu_e$.

Solution

The Q value for this decay is

$$Q = (m_K - m_\pi - m_e)c^2 = 494\ \text{MeV} - 135\ \text{MeV} - 0.5\ \text{MeV}$$
$$= 358.5\ \text{MeV}$$

This energy must be shared among the three products:

$$Q = K_\pi + K_e + E_\nu$$

The electron and π meson have their maximum energies when the neutrino has negligible energy: $Q = K_\pi + K_e$, and conservation of momentum in this case (if the neutrino has negligible momentum) requires $p_\pi = p_e$. Using relativistic kinetic energy, we have

$$Q = K_\pi + K_e = \sqrt{(pc)^2 + (m_\pi c^2)^2} - m_\pi c^2 + \sqrt{(pc)^2 + (m_e c^2)^2} - m_e c^2$$

where $p = p_e = p_\nu$. Inserting the numbers, we obtain

$$494\ \text{MeV} = \sqrt{(pc)^2 + (135\ \text{MeV})^2} + \sqrt{(pc)^2 + (0.5\ \text{MeV})^2}$$

Clearing the two radicals involves quite a bit of algebra, but we can simplify the problem if we inspect this expression and notice that the solution must have a large value of pc, certainly greater than 100 MeV. (Otherwise the two terms could not sum to nearly 500 MeV.) Thus, $(pc)^2 \gg (0.5\ \text{MeV})^2$, and we can neglect the electron rest energy term in the second radical, which simplifies the equation somewhat:

$$494\ \text{MeV} = \sqrt{(pc)^2 + (135\ \text{MeV})^2} + pc$$

Solving, we find $pc = 229$ MeV, which gives $(E_e)_{\max} = 229$ MeV and $(E_\pi)_{\max} = 266$ MeV. Figure 14.8 shows the observed energy spectra of e^+ and π^0 from the K^+ decay, and the energy maxima are in agreement with the calculated values. (The shapes of the energy distributions are determined by statistical factors, as in the case of nuclear beta decay. The statistical factors are different for e^+ and π^0, because the π^0 also has its maximum energy when the e^+ appears at rest and the ν carries the recoil momentum.)

You should repeat this calculation and convince yourself that (1) the π^0 has its maximum energy also when $K_e = 0$ ($E_e = m_e c^2$) and (2) the e^+ does *not* have its maximum energy when $K_\pi = 0$.

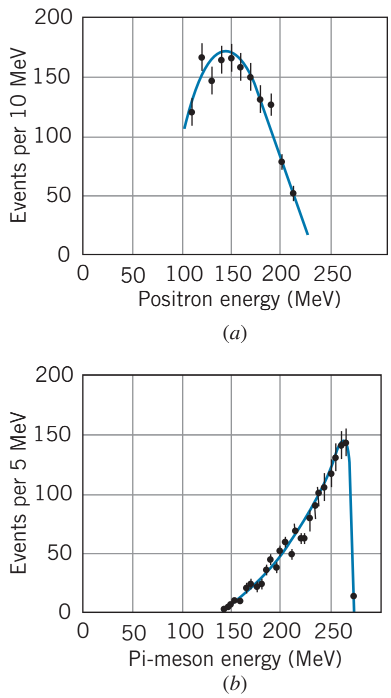
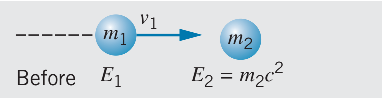


FIGURE 14.8 The spectrum of positrons and π mesons from the decay of the K^+ meson.

14.6 ENERGY AND MOMENTUM IN PARTICLE REACTIONS

The basic experimental technique of particle physics consists of studying the product particles that result from a collision between an incident particle (accelerated to high energies) and a target particle (often at rest).



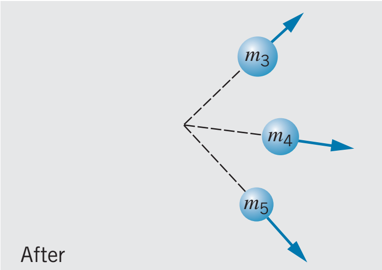


FIGURE 14.9 A reaction between particles in the laboratory reference frame.

The kinematics of the reaction process must be analyzed using relativistic formulas, because the kinetic energies of the particles are usually comparable to or greater than their rest energies. In this section, we derive some of the relationships that are needed to analyze these reactions, using the formulas for relativistic kinematics we obtained in Chapter 2. An important purpose of these reactions is the production of new varieties of particles, so we concentrate on calculating the threshold energy needed to produce these particles. (You might find it helpful to review the discussion in Chapter 13 on *nonrelativistic* reaction thresholds.)

Consider the following reaction:

$$m_1 + m_2 \rightarrow m_3 + m_4 + m_5 + \cdots$$

where the m's represent both the particles and their masses. Any number of particles can be produced in the final state. Here m_1 is the incident particle, which has total energy E_1, kinetic energy $K_1 = E_1 - m_1c^2$, and momentum $cp_1 = \sqrt{E_1^2 - m_1^2c^4}$ in the *laboratory* frame of reference. The target particle m_2 is at rest in the laboratory. Figure 14.9 illustrates this reaction in the laboratory frame of reference. As in the case of decays, if there are only two product particles, they have unique values of energy; if there are three or more product particles, each has a continuous distribution of energy from 0 up to a maximum value.

Just as we did for nuclear reactions, we define the Q value to be the difference between the initial and final rest energies:

$$Q = (m_i - m_f)c^2 = [m_1 + m_2 - (m_3 + m_4 + m_5 + \cdots)]c^2 \tag{14.2}$$

If Q is positive, rest energy is turned into kinetic energy, so that the product particles $m_3, m_4, m_5, \ldots$ have more combined kinetic energy than the initial particles m_1 and m_2. If Q is negative, some of the initial kinetic energy of m_1 is turned into rest energy.

Example 14.6

Qualitatively compare the pion energy spectrum from the following two reactions:

$$\text{p} + \text{p} \rightarrow \pi + \text{p} + \text{n} \quad \text{and} \quad \text{p} + \text{p} \rightarrow \pi + \text{d}.$$

Solution

The first reaction has a 3-body final state like beta decay, and so the pions can emerge with a continuous range of energies from zero up to some maximum determined by the energy of the incident proton. The second reaction has a 2-body final state so the pions emerge with a single energy. The energy of the pions in the second reaction will be a bit larger than the maximum energy of the pions in the first reaction because of the extra energy made available by the deuteron binding energy.

Example 14.7

Compute the Q values for the reactions (*a*) $\pi^- + \text{p} \rightarrow \text{K}^0 + \Lambda^0$; (*b*) $\text{K}^- + \text{p} \rightarrow \Lambda^0 + \pi^0$.

Solution

(*a*) Using rest energies from Tables 14.5 and 14.6,

$$\begin{aligned} Q &= [m_{\pi^-} + m_\text{p} - (m_{\text{K}^0} + m_{\Lambda^0})]c^2 \\ &= 140 \text{ MeV} + 938 \text{ MeV} - 498 \text{ MeV} - 1116 \text{ MeV} \\ &= -536 \text{ MeV} \end{aligned}$$

This reaction has a negative Q value, and energy must be supplied in the form of initial kinetic energy to produce the additional rest energy of the products.

(*b*)

$$Q = [m_{K^-} + m_p - (m_{\Lambda^0} + m_{\pi^0})]c^2$$
$$= 494 \text{ MeV} + 938 \text{ MeV} - 1116 \text{ MeV} - 135 \text{ MeV}$$
$$= 181 \text{ MeV}$$

A positive Q value indicates that there is enough rest energy in the initial particles to produce the final particles; in fact, there is 181 MeV of energy (plus the kinetic energy of the incident particle) left over for kinetic energy of the Λ^0 and π^0.

Threshold Energy

When the Q value is negative, there is a minimum kinetic energy that m_1 must have in order to initiate the reaction. As in the nonrelativistic nuclear physics case, this *threshold kinetic energy* K_{th} is larger than the magnitude of Q. The Q value is the energy necessary to create the additional mass of the product particles, but to conserve momentum the product particles cannot be formed at rest, so the threshold energy must not only create the additional particles but must also give them sufficient kinetic energy so that linear momentum is conserved in the reaction.

If Figure 14.9 represents a reaction with a negative Q value, clearly the reaction is not being done at the threshold kinetic energy. In the reaction as it is drawn, not only have the new particles been created, they have been given both forward momentum (to the right in the figure), which is necessary to conserve the initial momentum of m_1, as well as transverse momentum. This transverse momentum, which must sum to zero in order to conserve momentum, is not necessary either to create the particles or to satisfy conservation of momentum. At the minimum or threshold condition, this transverse momentum is zero.

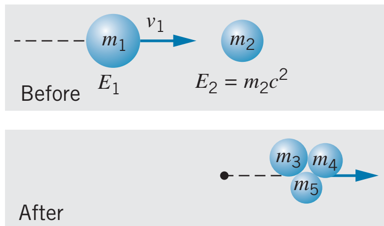


FIGURE 14.10 The reaction of Figure 14.9 when m_1 has the threshold kinetic energy. The product particles move together as a unit in the direction of the original momentum.

Also at threshold, the most efficient way to provide momentum to the final particles is to have them all moving together with the same speed, as in Figure 14.10. (This is equivalent to creating the particles at rest if we examine the collision from a reference frame in which the total initial momentum is zero, such as in a head-on collision of two particles.) Let's represent the bundle of final particles, all moving as a unit, as a total mass M. Then conservation of momentum ($p_{\text{initial}} = p_{\text{final}}$) gives $p_1 = p_M$ and conservation of total relativistic energy ($E_{\text{initial}} = E_{\text{final}}$) gives $E_1 + E_2 = E_M$, where p_M and E_M represent the momentum and total relativistic energy of the final bundle of particles. Then

$$\sqrt{(p_1c)^2 + (m_1c^2)^2} + m_2c^2 = \sqrt{(p_Mc)^2 + (m_Mc^2)^2} = \sqrt{(p_1c)^2 + (m_Mc^2)^2} \qquad (14.3)$$

Squaring both sides and solving, we obtain

$$\sqrt{(p_1c)^2 + (m_1c^2)^2} = \frac{(Mc^2)^2 - (m_1c^2)^2 - (m_2c^2)^2}{2m_2c^2} \qquad (14.4)$$

The threshold kinetic energy of m_1 is then

$$K_{\text{th}} = E_1 - m_1c^2 = \sqrt{(p_1c)^2 + (m_1c^2)^2} - m_1c^2$$
$$= \frac{(Mc^2)^2 - (m_1c^2)^2 - (m_2c^2)^2}{2m_2c^2} - m_1c^2 \tag{14.5}$$
$$= \frac{(Mc^2 - m_1c^2 - m_2c^2)\,(Mc^2 + m_1c^2 + m_2c^2)}{2m_2c^2}$$

With $Q = m_1c^2 + m_2c^2 - Mc^2$ and $M = m_3 + m_4 + m_5 + \cdots$, this becomes

$$K_{\text{th}} = (-Q)\frac{m_1 + m_2 + m_3 + m_4 + m_5 + \cdots}{2m_2} \tag{14.6}$$

This can also be written as

$$K_{\text{th}} = (-Q)\frac{\text{total mass of all particles involved in reaction}}{2 \times \text{mass of target particle}} \tag{14.7}$$

In the limit of low speeds, the relativistic threshold formula reduces to the nonrelativistic formula for nuclear reactions derived in Chapter 13 (see Problem 22).

Example 14.8

Calculate the threshold kinetic energy to produce π mesons from the reaction $p + p \rightarrow p + p + \pi^0$.

Solution

The Q value is

$$Q = m_pc^2 + m_pc^2 - (m_pc^2 + m_pc^2 + m_\pi c^2)$$
$$= -m_\pi c^2 = -135 \text{ MeV}$$

Using Eq. 14.7 we can find the threshold kinetic energy:

$$K_{\text{th}} = (-Q)\frac{4m_p + m_\pi}{2m_p}$$
$$= (135 \text{ MeV})\frac{4(938 \text{ MeV}) + 135 \text{ MeV}}{2(938 \text{ MeV})} = 280 \text{ MeV}$$

Such energetic protons are produced at many accelerators throughout the world, and as a result the properties of the π mesons can be carefully investigated.

Example 14.9

In 1956, an experiment was performed at Berkeley to search for the antiproton, which could be produced in the reaction $p + p \rightarrow p + p + p + \overline{p}$. What is the threshold energy for this reaction?

Solution

The rest energy of the antiproton is identical to the rest energy of the proton (938 MeV), so the Q value is

$$Q = m_pc^2 + m_pc^2 - 4(m_pc^2) = -2m_pc^2$$

Thus,

$$K_{\text{th}} = (2m_pc^2)\frac{6m_pc^2}{2m_pc^2} = 6m_pc^2 = 5628 \text{ MeV}$$
$$= 5.628 \text{ GeV}$$

For the discovery of the antiproton produced in this reaction, Owen Chamberlain and Emilio Segrè were awarded the Nobel Prize in physics in 1959.

It is interesting to compute the "efficiency" of these reactions—that is, how much of the initial kinetic energy we supply actually goes into producing the final particles and how much is "wasted" in the laboratory kinetic energies of the reaction products. In the first example, we supply 280 MeV of kinetic energy to produce 135 MeV of rest energy, for an efficiency of about 50%. In the second example, $6m_pc^2$ of kinetic energy produces only $2m_pc^2$ of rest energy, for an efficiency of only 33%. As the rest energies of the product particles become larger, the efficiency decreases, and relatively more energy must be supplied. For example, to produce a particle with a rest energy of 50 GeV in a proton–proton collision, we need to supply about 1250 GeV of initial kinetic energy. Only 4% of the energy supplied actually goes into producing the new particles; the remaining 96% must go to kinetic energy of the products in order to balance the large initial momentum of the incident particle. To produce a 100-GeV particle requires not twice as much energy, but four times as much. This is obviously not a pleasant situation for particle physicists, who must build increasingly more powerful accelerators to accomplish their goals of producing more massive particles.

One way out of this difficulty would be to do an experiment in which two particles with equal and opposite momentum collide head-on. In effect, we would be doing this experiment in the center-of-mass (CM) frame, where at threshold the production of new particles is 100% efficient—*none* of the initial kinetic energy goes into kinetic energy of the products, which are produced at rest in the CM frame. Thus, a 50-GeV particle could be produced by a head-on collision between two protons with as little as 25 GeV of kinetic energy. Of course, this great gain in efficiency is at a cost of the technological difficulty of making such collisions occur.

There are now *colliding beam* accelerators in operation, in which beams of particles (such as electrons or protons) can occasionally be made to collide. The Large Hadron Collider (hadron meaning strongly interacting particles), which is on the border between Switzerland and France, became operational in 2009; it collides two beams of protons each at an energy of 7 GeV in order to search for new particles in an even higher range of rest energies. Other colliding beam accelerators bring together electrons and positrons at energies of 50 to 100 GeV. In each case, all of the available energy can go into the production of new particles.

14.7 THE QUARK STRUCTURE OF MESONS AND BARYONS

Without the periodic table, it would be difficult to understand the structure of the more than 100 chemical elements. When they are laid out in the table, the groups with similar chemical and physical properties (such as the 3*d* transition metals, alkalis, halogens, or inert gases) lead us to suspect that there is a simplifying underlying structure, which we now know can be achieved with three fundamental building blocks: protons, neutrons, and electrons.

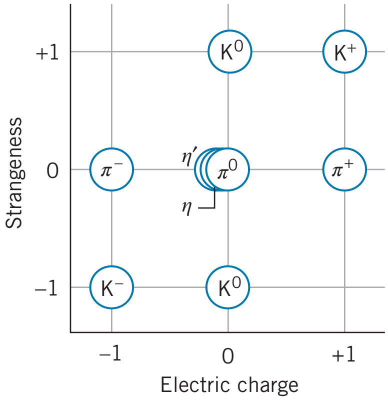


FIGURE 14.11 The relationship between electric charge and strangeness for the spin-0 mesons.

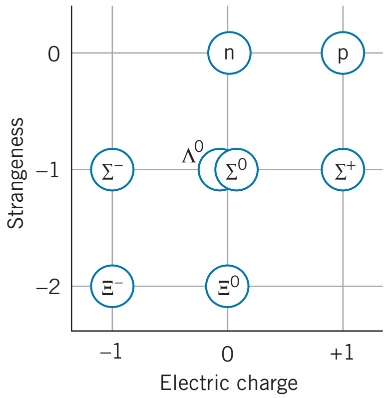


FIGURE 14.12 The relationship between electric charge and strangeness for the spin-$\frac{1}{2}$ baryons.

We can achieve a similar understanding of the underlying structure of the elementary particles by displaying them on a two-dimensional diagram, with strangeness along the y axis and electric charge along the x axis. Figures 14.11 and 14.12 show the resulting diagrams for the spin-0 mesons and the spin-$\frac{1}{2}$ baryons.

In 1964, Murray Gell-Mann and George Zweig independently and simultaneously recognized that such regular patterns are evidence of an underlying structure in the particles. They showed that they could duplicate these patterns if the mesons and baryons were composed of three fundamental particles, which soon became known as *quarks*. These three quarks, known as up (u), down (d), and strange (s), have the properties listed in Table 14.7. We now believe that six quarks are necessary to account for all known mesons and baryons, as we will discuss later.

Let us see how the quark model works in the case of the spin-0 mesons. The quarks have spin $\frac{1}{2}$, so the simplest scheme to form a spin-0 meson would be to combine two quarks, with their spins directed oppositely. However, the mesons have baryon number $B = 0$, while a combination of two quarks would have $B = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$. A combination of a quark and an antiquark, on the other hand, would have $B = 0$, because the antiquark has $B = -\frac{1}{3}$.

For example, suppose we combine a u quark with a $\bar{\mathrm{d}}$ ("antidown") quark, obtaining the combination $\mathrm{u}\bar{\mathrm{d}}$. This combination has spin zero and electric charge $\frac{2}{3}e + \frac{1}{3}e = +e$. (A d quark has charge $-\frac{1}{3}e$, so $\bar{\mathrm{d}}$ has charge $+\frac{1}{3}e$.) The properties of this combination are identical with the π^+ meson, and so we identify the π^+ with the combination $\mathrm{u}\bar{\mathrm{d}}$. Continuing in this way, we find nine possible combinations of one of the three original quarks from Table 14.7 with an antiquark, as listed in Table 14.8, and plotting those nine combinations on a graph of strangeness against electric charge, we obtain Figure 14.13, which looks identical to Figure 14.11.

The baryons have $B = +1$ and spin $\frac{1}{2}$ or $\frac{3}{2}$, which suggests immediately that three quarks make a baryon. The three spins can be coupled either to a value of $\frac{3}{2}$ if they all point in the same direction or to $\frac{1}{2}$ if two are in one direction and one is in the opposite direction. The 10 possible combinations of the three original quarks are listed in Table 14.9, and we can arrange the spin-$\frac{1}{2}$ combinations into the pattern shown in Figure 14.14, which is identical to Figure 14.12.

TABLE 14.7 Properties of the Three Original Quarks

Name	Symbol	Charge (e)	Spin ($\hbar$)	Baryon Number	Strangeness	Antiquark
Up	u	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	0	$\bar{\mathrm{u}}$
Down	d	$-\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	0	$\bar{\mathrm{d}}$
Strange	s	$-\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	-1	$\bar{\mathrm{s}}$

Values shown for the charge, baryon number, and strangeness are those for the quark; values for the antiquark have the opposite sign.

TABLE 14.8 Possible Quark–Antiquark Combinations

Combination	Charge (e)	Spin ($\hbar$)	Baryon Number	Strangeness
$u\bar{u}$	0	0, 1	0	0
$u\bar{d}$	+1	0, 1	0	0
$u\bar{s}$	+1	0, 1	0	+1
$d\bar{u}$	−1	0, 1	0	0
$d\bar{d}$	0	0, 1	0	0
$d\bar{s}$	0	0, 1	0	+1
$s\bar{u}$	−1	0, 1	0	−1
$s\bar{d}$	0	0, 1	0	−1
$s\bar{s}$	0	0, 1	0	0

TABLE 14.9 Possible Three-Quark Combinations

Combination	Charge (e)	Spin ($\hbar$)	Baryon Number	Strangeness
uuu	+2	$\frac{3}{2}$	+1	0
uud	+1	$\frac{1}{2}, \frac{3}{2}$	+1	0
udd	0	$\frac{1}{2}, \frac{3}{2}$	+1	0
uus	+1	$\frac{1}{2}, \frac{3}{2}$	+1	−1
uss	0	$\frac{1}{2}, \frac{3}{2}$	+1	−2
uds	0	$\frac{1}{2}, \frac{3}{2}$	+1	−1
ddd	−1	$\frac{3}{2}$	+1	0
dds	−1	$\frac{1}{2}, \frac{3}{2}$	+1	−1
dss	−1	$\frac{1}{2}, \frac{3}{2}$	+1	−2
sss	−1	$\frac{3}{2}$	+1	−3

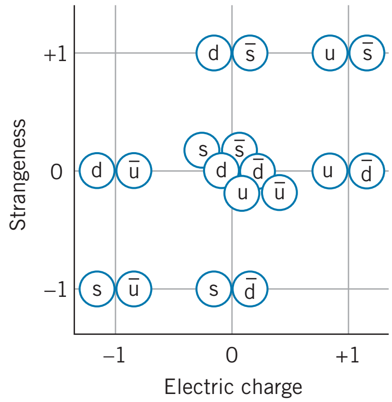


FIGURE 14.13 Spin-0 quark–antiquark combinations; compare with Figure 14.11.

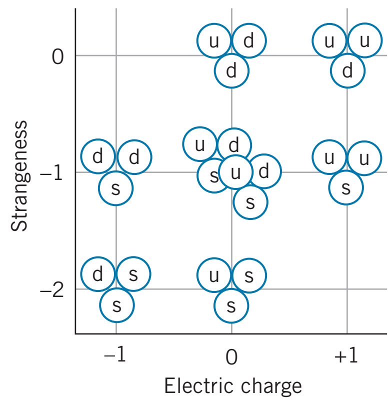


FIGURE 14.14 Spin-$\frac{1}{2}$ three-quark combinations; compare with Figure 14.12.

Using the quark model, we can analyze the decays and reactions of the elementary particles, based on two rules:

1. Quark–antiquark pairs can be created from energy quanta, and conversely can annihilate into energy quanta. For example,

$$\text{energy} \rightarrow u + \bar{u} \quad \text{or} \quad d + \bar{d} \rightarrow \text{energy}$$

This energy can be in the form of gamma rays (as in electron–positron annihilation), or else it can be transferred to or from other particles in the decay or reaction.

2. The weak interaction can change one type of quark into another through emission or absorption of a W^+ or W^-, for example $s \to u + W^-$. The W then decays by the weak interaction, such as $W^- \to \mu^- + \overline{\nu}_\mu$ or $W^- \to d + \overline{u}$. The strong and electromagnetic interactions cannot change one type of quark into another.

Example 14.10

(*a*) For the spin-$\frac{1}{2}$ baryons illustrated in Figures 14.12 and 14.14, explain why each of the 3-quark couplings corresponds to a single particle except for the two particles with the uds combination. (*b*) Would you expect the same behavior for the spin-$\frac{3}{2}$ uds combination?

Solution

(*a*) Other than the uds coupling, each three-quark combination includes a pair of identical quarks. According to the Pauli principle, these two identical quarks must have opposite spins and therefore couple to $S = 0$. There is only one possible way to couple that combination to the spin of the third quark to give a total of $\frac{1}{2}$. With the three non-identical quarks of the uds combination, the spins can be coupled in two different but unique ways: u and d coupled parallel to give $S = 1$ with s opposite to give a total $S = \frac{1}{2}$, and u and d antiparallel giving $S = 0$ coupled to s making $S = \frac{1}{2}$. (*b*) The uds combination with $S = \frac{3}{2}$ has all quark spins parallel, for which there is only one possible configuration. (The Pauli principle doesn't apply because the three quarks are different.)

Example 14.11

Analyze (*a*) the reaction $\pi^- + p \to \Lambda^0 + K^0$ and (*b*) the decay $\pi^+ \to \mu^+ + \nu_\mu$ in terms of the constituent quarks.

Solution

(*a*) The reaction $\pi^- + p \to \Lambda^0 + K^0$ can be rewritten as follows:

$$d\overline{u} + uud \to uds + d\overline{s}$$

Each side contains one u quark and two d quarks, which don't change in the reaction. Removing these "spectator" quarks from each side of the reaction, we are left with the remaining transformation:

$$\overline{u} + u \to s + \overline{s}$$

The u and $\overline{u}$ annihilate, and from the resulting energy s and $\overline{s}$ are created.

(*b*) The π^+ has the quark composition $u\overline{d}$. There are no quarks in the final state ($\mu^+ + \nu_\mu$), so we must find a way to get rid of the quarks. One possible way is to change the u quark into a d quark: $u \to d + W^+$. The net process can thus be written as

$$u\overline{d} \to d + \overline{d} + W^+$$

with the products then undergoing the following processes to produce the final observed particles:

$$d + \overline{d} \to \text{energy} \quad \text{and} \quad W^+ \to \mu^+ + \nu_\mu$$

You may have noticed that some of the heavier mesons listed in Table 14.5 were not included in Figure 14.11, and they cannot be accounted for among the quark–antiquark combinations listed in Table 14.8. Where do these particles fit in our scheme?

In 1974, a new meson called J/ψ was discovered at a rest energy of 3.1 GeV. (It was given different names J and ψ by the two competing experimental groups that first reported its discovery.) This new meson was expected to decay to lighter mesons in a characteristic strong interaction time of around 10^{-23} s. Instead, its lifetime was stretched by 3 orders of magnitude to about 10^{-20} s, and its decay products were e^+ and e^-, which are more characteristic of an electromagnetic process. Why is the rapid, strong interaction decay path blocked for this particle? This was soon explained by assuming the J/ψ to be composed of a new quark c, called the *charm* quark, and its antiquark $\bar{c}$. The existence of the c quark had been predicted 4 years earlier as a way to explain the failure to observe the decay $K^0 \rightarrow \mu^+ + \mu^-$, which violates no previously known law but is nevertheless not observed.

The c quark, which carries a charge of $+\frac{2}{3}e$, has a property, charm, that operates somewhat like strangeness. We assign a charm quantum number $C = +1$ to the c quark (and assign $C = -1$ to its antiquark $\bar{c}$). All other quarks are assigned $C = 0$. We can now construct a new set of mesons by combining the c quark with the $\bar{u}, \bar{d}$, and $\bar{s}$ antiquarks and by combining the $\bar{c}$ antiquark with the u, d, and s quarks. Instead of nine spin-0 mesons, there are now 16, and the two-dimensional graphs of Figures 14.11 and 14.13 must be extended to a third dimension to show the C axis (Figure 14.15). All of these new mesons, called D, have been observed in high-energy collision experiments. Baryons containing this new quark have also been discovered, analogous to the Λ, Σ, Ξ, and Ω particles but with an s quark replaced by a c quark.

In 1977, the same sequence of events was repeated with another meson, Υ (upsilon). The rest energy was determined to be about 9.5 GeV, and again its decay was slowed to about 10^{-20} s and occurred into $e^+ + e^-$ rather than into mesons. Once again, a new quark was postulated: the b (bottom) quark with a new quantum "bottomness" number $B = -1$ and a charge of $-\frac{1}{3}e$. (The letter B is used to represent baryon number as well as bottomness. It should always be apparent from the discussion which one is meant.) The Υ is assigned as the combination $b\bar{b}$. Many new particles containing the b quark have been discovered, including B mesons (in which a b quark is paired with a different antiquark) and baryons similar to Λ, Σ, and Ξ with a b quark replacing one of the s quarks.

A sixth quark was discovered in 1994 in proton–antiproton collisions at Fermilab. These collisions created this new quark and its antiquark, both of which decayed into a shower of secondary particles (as in Figure 14.5). By measuring the energy and momentum of the secondary particles, the experimenters were able to determine the mass of the new quark to be 172 GeV

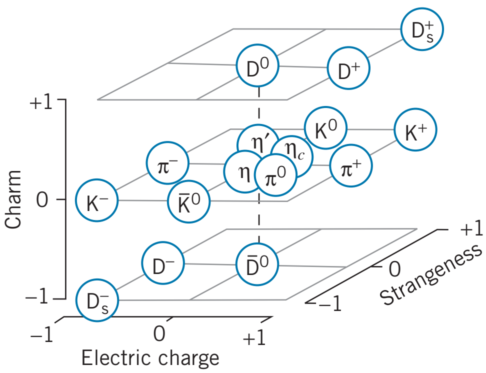


FIGURE 14.15 The relationship among electric charge, strangeness, and charm for the spin-0 mesons.

TABLE 14.10 Properties of the Quarks

Type	Symbol	Antiparticle	Charge (e)	Spin ($\hbar$)	Baryon Number	Rest Energy (MeV)	Properties C	S	T	B
Up	u	$\bar{u}$	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	330	0	0	0	0
Down	d	$\bar{d}$	$-\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	330	0	0	0	0
Charm	c	$\bar{c}$	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	1500	+1	0	0	0
Strange	s	$\bar{s}$	$-\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	500	0	−1	0	0
Top	t	$\bar{t}$	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	172,000	0	0	+1	0
Bottom	b	$\bar{b}$	$-\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{3}$	4700	0	0	0	−1

(roughly the mass of a tungsten atom). This new quark is known as t (top) and has a new associated property of "topness" with a quantum number $T = +1$.

It may now seem that we are losing sight of our goal to achieve simplicity (to add the "bottomness" axis to Figure 14.15, we would need to depict a four-dimensional space!) and that we are moving toward replacing a complicated array of particles with an equivalently complicated array of quarks. However, there is good reason to believe that there are no more than six fundamental quarks. In the next section, we discuss how we are indeed on the path to a simple explanation of the fundamental particles.

Table 14.10 shows the six quarks and their properties. The masses of the quarks cannot be directly determined, because a free quark has yet to be observed. The rest energies shown in Table 14.10 are estimates based on the "apparent" masses that quarks have when bound in various particles. For example, the observed rest energy of the proton is the sum of the rest energies of its three quark constituents less the binding energy of the quarks. Since we don't know the binding energy, we can't determine the rest energy of a free quark. The rest energies shown in Table 14.10 are often called those of *constituent* quarks.

The quark model does a great deal more than allow us to make geometrical arrangements of particles such as Figure 14.15. It can be used to explain many observed properties of the particles, such as their masses and magnetic moments, and to account for their decay lifetimes and reaction probabilities. Nevertheless, a free quark has never been observed, despite heroic experiments to search for them. How can we be sure that they exist? In experiments that scatter high-energy electrons from protons, we observe more particles scattered at large angles than we would expect if the electric charge of the proton were uniformly distributed throughout its volume, and from the analysis of the distribution of the scattered electrons we conclude that inside the proton are three point-like objects that are responsible for the scattering. This experiment is exactly analogous to Rutherford scattering, in which the presence of the nucleus as a compact object inside the atom was revealed by the distribution of scattered alpha particles at angles larger than expected. Like Rutherford's experiment, the observed cross section depends on the electric charge of the object doing the scattering, and from these experiments we can deduce charges of magnitude $\frac{1}{3}e$ and $\frac{2}{3}e$ for these point-like objects. These experiments give clear evidence for the presence of quarks inside the proton.

We don't yet know why free quarks have not been observed. Perhaps they are so massive that no accelerator yet built has enough energy to liberate one. Perhaps the force between quarks increases with distance (in contrast with electromagnetism or gravitation, which *decreases* with separation distance), so that an infinite amount of energy would be required to separate a quark from a nucleon. Or (as is now widely believed) perhaps the basic theory of quark structure forbids the existence of free quarks.

Quarkonium

Physicists often find it helpful to begin to understand new discoveries by comparison with well-known systems having similar properties. In the case of the quark structure of mesons, it turns out there are great similarities between the properties of a meson-sized combination of a quark and its antiquark, called *quarkonium*, and the properties of another particle–antiparticle combination, the atom-sized electron–positron structure called *positronium*. The structure of positronium can be analyzed by solving the Schrödinger equation exactly as we did for the hydrogen atom to obtain a series of energy levels labeled with the principal quantum number n. From more detailed considerations based on the fine structure, we can find small corrections to the energies depending on whether the spins of the electron and positron combine to give a total spin of either $S = 1$ (parallel combination) or $S = 0$ (antiparallel combination), in a way similar to the singlet and triplet states of the two electrons in helium (Figure 8.17), and also how the spin and orbital angular momentum combine to give different values of the total angular momentum.

The quark–antiquark combinations of the light quarks (u, d, s) cannot be used for this comparison, because the binding energies and thus the kinetic energies in the bound state are large compared with their masses, so relativistic effects would be expected to have a large effect on the levels. For the more massive quarks (c, b, t), the binding energies are small compared with the rest energies, so a nonrelativistic approach can be used, at least as a starting point.

Figure 14.16 compares the energy levels of positronium with the measured levels of two quarkonium combinations: charmonium ($c\overline{c}$) and bottomonium ($b\overline{b}$). (Because of its large mass the top quark is much harder to produce and decays very quickly, so the $t\overline{t}$ states have not been observed.) As with positronium, the quarkonium states can be labeled with the total spin ($S = 0$ or $S = 1$), the orbital angular momentum of the combination (states are shown with $L = 0$ and $L = 1$), and the principal quantum number n. The lowest $S = 1$, $L = 0$ states in quarkonium are the spin 1 mesons J/ψ and Υ listed in Table 14.5, which also have excited states of larger principal quantum number, just like positronium (and hydrogen). The triplet of states with $S = 1$ and $L = 1$ corresponds to the three ways of coupling those angular momenta to give totals of 0, 1, or 2. Although the energy scales differ by a factor of 10^8, there is a great similarity between the energy levels of positronium and quarkonium.The similarities between the $c\overline{c}$ and $b\overline{b}$ states are surprising, because the energy levels expected from the Schrödinger equation depend on the reduced mass of the system, and the $b\overline{b}$ combination is about three times as massive as $c\overline{c}$.

The next step in the analysis would be to try to reproduce the quarkonium spectrum by solving the Schrödinger equation. The rough resemblance to the positronium states might suggest trying a $1/r$ potential energy, but it can't be a Coulomb interaction because the electric potential energy of two quarks

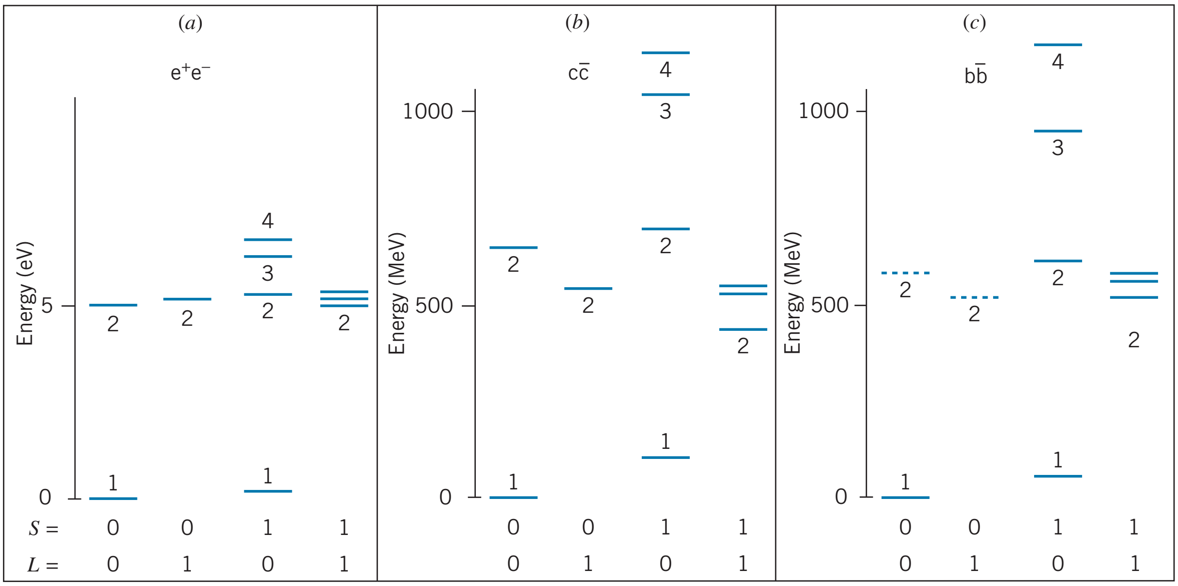


FIGURE 14.16 The energy levels of (*a*) positronium (e^+e^-), (*b*) $c\bar{c}$ quarkonium, and (*c*) $b\bar{b}$ quarkonium. The atom-like states are labeled with the value of the principal quantum number n. The zero of the energy scale is at 2980 MeV for $c\bar{c}$ and 9389 MeV for $b\bar{b}$. The $n = 2$ states with $S = 0, L = 0$ and $S = 0, L = 1$ in $b\bar{b}$, which are shown with dashed lines in part (*c*), have not yet been discovered.

separated by a typical meson size of 0.5 fm would be about 1 MeV, far too small to account for the separation of the excited states of hundreds of MeV in Figure 14.16. Moreover, the $1/r$ potential energy gets weaker as the separation increases, while the apparent inability to produce a free quark suggests that the interaction should grow stronger as the separation increases. The simplest potential energy that satisfies these criteria would be

$$U(r) = -\frac{a}{r} + br \tag{14.8}$$

The Schrödinger equation can be solved numerically for this potential energy, with the constants a and b adjusted to give best agreement with experiment. The constant b turns out to have a value of about 1 GeV/fm. This large value is consistent with the failure to observe a free quark—to separate the quarks in a meson even to an atom-sized distance would require about 10^5 GeV, far greater than the beam energy of any accelerator.

14.8 THE STANDARD MODEL

Ordinary matter is composed of protons and neutrons, which are in turn composed of u and d quarks. Ordinary matter is also composed of electrons. In the radioactive decay of ordinary matter, electron-type neutrinos are emitted. Our entire world can thus be regarded as composed of four spin-$\frac{1}{2}$ particles (and their antiparticles), which can be grouped into a pair of leptons and a pair of quarks:

$$(e, \nu_e) \qquad \text{and} \qquad (u, d)$$

Within each pair, the charges of the two particles differ by one unit: -1 and 0, $+\frac{2}{3}$ and $-\frac{1}{3}$.

When we do experiments with high-energy accelerators, we find new types of particles: muons and muon neutrinos, plus mesons and baryons with the new properties of strangeness and charm. We can account for the structure of these particles with another pair of leptons and another pair of quarks:

$$(\mu, \nu_\mu) \quad \text{and} \quad (\text{c}, \text{s})$$

Once again, the particles come in pairs differing by one unit of charge.

At even higher energies, we find a new generation of particles consisting of another pair of leptons (tau and its neutrino) and a new pair of quarks (top and bottom), which permits us to continue the symmetric arrangement of the fundamental particles in pairs:

$$(\tau, \nu_\tau) \quad \text{and} \quad (\text{t}, \text{b})$$

Is it possible that there are more pairs of leptons and quarks that have not yet been discovered? At this point, we strongly believe the answer to be "No." Every particle so far discovered can be fit into this scheme of 6 leptons and 6 quarks. Furthermore, the number of lepton generations can be determined by the decay rates of the heaviest particles, and a limit of 3 emerges from these experiments. Finally, according to present theories the evolution of the universe itself would have proceeded differently if there had been more than three types of neutrinos. For these reasons, it is generally believed that there are no more than three generations of particles.

The strong force between quarks is carried by an exchanged particle, called the *gluon*, which provides the "glue" that binds quarks together in mesons and baryons. (There are actually eight different gluons in the model.) A theory known as *quantum chromodynamics* describes the interactions of quarks and the exchange of gluons. In this theory, the internal structure of the proton consists of three quarks "swimming in a sea" of exchanged gluons. Like the quarks, the gluons cannot be observed directly, but there is indirect evidence of their existence from a variety of experiments.

The theory of the structure of the elementary particles we have described so far is known as the *Standard Model*. It consists of 6 leptons and 6 quarks (and their antiparticles), plus the field particles (photon, 3 weak bosons, 8 gluons) that carry the various forces (Figure 14.17). It is remarkably successful in accounting for the properties of the fundamental particles, but it lacks the unified treatment of forces we would expect from a complete theory.

The first step toward unification was taken in 1967 with the development of the *electroweak* theory by Stephen Weinberg and Abdus Salam. In this theory, the weak and electromagnetic interactions are regarded as separate aspects of the same basic force (the electroweak force), just as electric and magnetic forces are distinct but part of a single phenomenon, electromagnetism. The theory predicted the existence of the W and Z particles; their discovery in 1983 provided a dramatic confirmation of the theory.

The next-higher level of unification would be to combine the strong and electroweak forces into a single interaction. Theories that attempt to do this are called *Grand Unified Theories* (GUTs). By incorporating leptons and quarks into a single theory, the GUTs explain many observed phenomena:

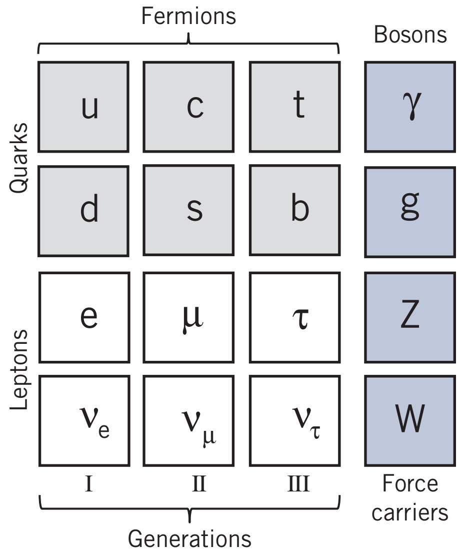
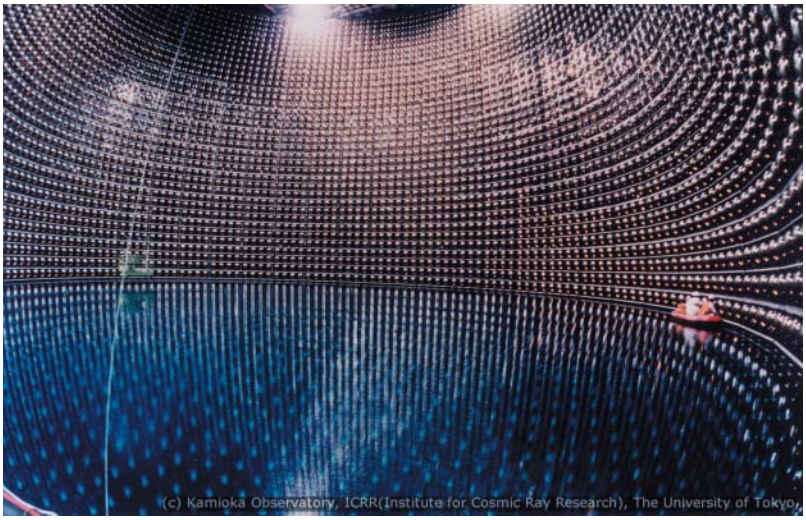


FIGURE 14.17 The Standard Model of elementary particles.

the fractional electric charge of the quarks and the difference of one unit of charge between the members of the quark and lepton pairs within each generation. The GUTs also predict new phenomena, such as the conversion of quarks into leptons, which would permit the proton (which we have so far assumed to be an absolutely stable particle) to decay into lighter particles with a lifetime of at least 10^{31} y. Searches for proton decay (by looking for evidence of decays in a large volume of matter; see Figure 14.18) have so far been unsuccessful and have placed lower limits on the proton lifetime of at least 10^{34} y, which excludes several possible GUTs that make smaller predictions.

Although the electroweak theory was otherwise successful, it had one glaring defect: it predicted that the field particle carrying the electromagnetic force (the photon) and those carrying the weak force (the W and Z bosons) should all be massless. The photon is of course massless, but the masses of the W and Z are considerable, roughly that of a midsized atom. To avert this problem, UK physicist Peter Higgs suggested in 1964 that the mass symmetry of the electroweak force carriers could be broken by the existence of a new force field that permeates the entire universe (even in vacuum). According to the theory, interactions with this field not only give the W and Z bosons their mass but also give mass to the quarks and leptons as well. The question of why the particles have their particular values of mass is then switched to a question that can be tested in the laboratory: to determine the strength of the interaction of the particles with the Higgs field. Somewhat like particles traveling through a viscous medium, the various leptons and quarks interact differently with the Higgs field and thus acquire their unique masses. Like the other force fields, the Higgs field should be carried by its own boson, and for nearly 50 years, the existence of the Higgs boson was one of the main unsolved issues of particle physics. The search for the Higgs boson was carried out unsuccessfully at the major particle accelerators until 2012, when the Large Hadron Collider announced the discovery of a particle with



Courtesy of Kamiokande Observatory, Institute of Cosmic Ray Research, The University of Tokyo

FIGURE 14.18 The Superkamiokande detector system in Japan was designed to search for proton decay. The water tank, 40 m in diameter and located 1000 m underground, holds 50,000 tons of water. The tank is lined with more than 10,000 photomultiplier detectors that respond to flashes of light that would be emitted when one of the protons in the water decayed. Here the tank has been partly emptied so that the technicians (in the boat) can service the photomultipliers.

mass 125 GeV/c^2, whose spin and decay properties were consistent with the predicted properties of the Higgs boson. For the prediction of this mechanism for acquiring mass, Peter Higgs and François Englert (a Belgian physicist who made an independent prediction of the effect in 1964) shared the 2013 Nobel Prize in physics.

Another shortcoming of the Standard Model is that it is based on massless neutrinos. Although the upper limit (see Table 14.4) on the mass of the electron neutrino is very small (2 eV), the limits on the other neutrino masses are much larger. Measurements of the flux of neutrinos reaching Earth from the Sun, produced in the fusion reactions discussed in Chapter 13, have consistently revealed a large deficit—the intensity of electron neutrinos observed on Earth is only about $\frac{1}{3}$ of what is predicted based on models of how fusion reactions occur in the Sun's interior. Recent measurements have revealed that, although the intensity of electron neutrinos from the Sun is only $\frac{1}{3}$ of the expected value, the total intensity of *all* neutrinos (including muon and tau neutrinos) reaching us from the Sun agrees with the predicted rate. This is very puzzling, because the fusion reactions in the Sun should produce only electron neutrinos; the reacting particles in the solar interior are not sufficiently energetic to produce mu and tau leptons. This mystery has been explained by proposing that the electron neutrinos are produced in the solar interior at the expected rate, but that during their journey from the Sun to Earth, the purely electron neutrinos become a mixture of roughly equal parts electron, muon, and tau neutrinos. This nicely explains why the rate of electron neutrinos from the Sun appears to be only about $\frac{1}{3}$ of what is expected (the other $\frac{2}{3}$ of the electron neutrinos having been converted into muon or tau neutrinos). This phenomenon of *neutrino oscillation* (which refers to neutrinos oscillating from one type to another) can occur only if the neutrinos have mass. The required masses are very small, well within the experimental limits, but the neutrino masses are definitely not zero. The Standard Model must be extended to include nonzero neutrino masses, and the rules for conservation of lepton number must be modified to allow one type of neutrino to transform into another.

The search for a consistent explanation of the elementary particles has led physicists to work with exotic theories. In *string theory*, the particles are replaced by tiny (10^{-33} cm) strings, whose vibrations give rise to the properties we observe as particles. These theories exist in spacetimes with 10 or more dimensions, and at present seem to be far beyond any possible experimental test. Another extension of the Standard Model is called *supersymmetry*; this theory proposes that there is a higher symmetry between the spin-$\frac{1}{2}$ particles (such as the quarks and leptons) and particles with integral spin, so that under this theory there would be electrons and quarks with a spin of 0 and W and Z particles and photons with a spin of $\frac{1}{2}$. The masses of these supersymmetric particles are estimated to be very much larger than their ordinary partners, perhaps in the range of 100 GeV/c^2, but even in this range they should be observable through experiments currently planned at the Large Hadron Collider.

There is so far no conclusive verification for any of the GUTs, nor is there a successful theory that incorporates the remaining force, gravity, into a unified theory. The quest for unification and its experimental tests remain active areas of research in particle physics.

Chapter Summary

		Section
Forces	Strong, electromagnetic, weak, gravitational	14.1
Field particles	Gluon (g), photon (γ), weak boson ($W^{\pm}, Z^0$), graviton	14.1
Leptons	$e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau$	14.2
Mesons	$\pi^{\pm}, \pi^0, K^{\pm}, K^0, \overline{K}^0, \eta, \rho^{\pm}, \eta'$, $D^{\pm}, \psi, B^{\pm}, \Upsilon, \ldots$	14.2
Baryons	$p, n, \Lambda^0, \Sigma^{\pm,0}, \Xi^{-,0}, \Omega^-, \ldots$	14.2
Conservation of lepton number L	*In any process,* L_e, L_μ, *and* L_τ *remain constant.*	14.3

		Section
Conservation of baryon number B	*In any process, B remains constant.*	14.3
Conservation of strangeness S	*In strong and electromagnetic processes, S remains constant; in weak processes,* $\Delta S = 0$ or ± 1.	14.3
Q value in decays or reactions	$Q = (m_i - m_f)c^2$	14.5, 14.6
Threshold energy in reactions	$K_{th} = -Q(m_1 + m_2 + m_3 + m_4 + m_5 + \cdots)/2m_2$	14.6
Quarks	u, d, c, s, t, b	14.7

Questions

1. Some conservation laws are based on fundamental properties of nature, while others are based on systematics of decays and reactions and have as yet no fundamental basis. Give the basis for the following conservation laws: energy, linear momentum, angular momentum, electric charge, baryon number, lepton number, strangeness.
2. Does the presence of neutrinos among the decay products of a particle always indicate that the weak interaction is responsible for the decay? Do all weak interaction decays have neutrinos among the decay products? Which decay product indicates an electromagnetic decay?
3. Do all strongly interacting particles also feel the weak interaction?
4. In what ways would physics be different if there were another member of the lepton family less massive than the electron? What if there were another lepton more massive than the tau?
5. Suppose a proton is moving with high speed, so that $E \gg mc^2$. Is it possible for the proton to decay, such as into $\mathrm{n} + \pi^+$ or $\mathrm{p} + \pi^0$?
6. On planet anti-Earth, antineutrons beta decay into antiprotons. Is a neutrino or an antineutrino emitted in this decay?
7. List some experiments that might distinguish antineutrons from neutrons. Among others, you might consider (*a*) neutron capture by a nucleus; (*b*) beta decay; (*c*) the effect of a magnetic field on a beam of neutrons.
8. The Σ^0 can decay to Λ^0 without changing strangeness, so it goes by the electromagnetic interaction; the charged $\Sigma^\pm$ decay to p or n by the weak interaction in characteristic lifetimes of 10^{-10} s. Why can't $\Sigma^\pm$ decay to Λ^0 by the strong interaction in a much shorter time?
9. The Ω^- particle decays to $\Lambda^0 + \mathrm{K}^-$. Why doesn't it also decay to $\Lambda^0 + \pi^-$?
10. Explain why we do not account for the number of mesons in decays or reactions with a "meson number" in analogy with lepton number or baryon number.
11. Consider that leptons and baryons both obey conservation laws and are both fermions; mesons do not obey a conservation law and are bosons. Can you think of another particle (other than a meson) that has integral spin and can be emitted or absorbed in unlimited numbers?
12. Can antibaryons be produced in reactions between baryons and mesons?
13. List some similarities and differences between the properties of photons and neutrinos.
14. Is it reasonable to describe a resonance as a definite particle, when its mass is uncertain (and therefore variable) by 20%?
15. Why are most particle physics reactions endothermic ($Q < 0$)?
16. Although doubly charged baryons have been found, no doubly charged mesons have yet been found. What would be the effect on the quark model if a meson with charge $+2e$ were found? How could such a meson be interpreted within the quark model?
17. All direct quark transformations must involve a change of charge; for example, $\mathrm{u} \rightarrow \mathrm{d}$ is allowed (accompanied by the emission of a W^-), but $\mathrm{s} \rightarrow \mathrm{d}$ is not. Can you suggest a two-step process that might permit the transformation of an s quark into a d?
18. The decay $\mathrm{K}^+ \rightarrow \pi^+ + \mathrm{e}^+ + \mathrm{e}^-$ is at least five orders of magnitude less probable than the decay $\mathrm{K}^+ \rightarrow \pi^0 + \mathrm{e}^+ + \nu_\mathrm{e}$. Based on Question 17, can you explain why?
19. The D mesons decay to π and K mesons with a lifetime of 10^{-13} s. (*a*) Why is the lifetime so much slower than a typical strong interaction lifetime? Is a quantum number not conserved in the decay? (*b*) What interaction is responsible for the decay?
20. The Δ^* baryons are found with electric charges $+2, +1, 0$, and -1. Based on the quark model, why do we expect no Δ^* with charge -2?
21. Although we cannot observe quarks directly, indirect evidence for quarks in nucleons comes from the scattering of high-energy particles, such as electrons. When the de Broglie wavelength of the electrons is small compared with the size of a nucleon (~ 1 fm), the electrons appear to be scattered from massive, compact objects much smaller than a nucleon. To which phenomenon discussed previously in this text is this similar? Can the scattering be used to deduce the mass of the struck object? How does the scattering depend on the electric charge of the struck object? What would be the difference between scattering from a particle of charge e and one of charge $\frac{2}{3}e$?

Problems

14.1 The Four Basic Forces

1. Identify the interaction responsible for the following decays (approximate half-lives are given in parentheses):

(*a*) $\Delta^* \rightarrow \mathrm{p} + \pi$ (10^{-23} s)

(*b*) $\eta \rightarrow \gamma + \gamma$ (10^{-18} s)

(*c*) $\mathrm{K}^+ \rightarrow \mu^+ + \nu_\mu$ (10^{-8} s)

(*d*) $\Lambda^0 \rightarrow \mathrm{p} + \pi^-$ (10^{-10} s)

(*e*) $\eta' \rightarrow \eta + 2\pi$ (10^{-21} s)

(*f*) $\mathrm{K}^0 \rightarrow \pi^+ + \pi^-$ (10^{-10} s)

2. What is the range of the W^- particle that is responsible for the weak interaction of a proton and a neutron?

14.2 Classifying Particles

3. Give one possible decay mode of the following mesons:
(*a*) π^- (*b*) ρ^- (*c*) D^- (*d*) $\overline{K}^0$
4. Give one possible decay mode of the following antibaryons:
(*a*) $\overline{n}$ (*b*) $\overline{\Lambda}^0$ (*c*) $\overline{\Omega}^-$ (*d*) $\overline{\Sigma}^0$
5. Suggest a possible decay mode for the K^0 meson that involves the emission of
(*a*) ν_e (*b*) $\overline{\nu}_e$ (*c*) ν_μ (*d*) $\overline{\nu}_\mu$
Is it possible to have a decay mode of the K^0 that involves the emission of ν_τ or $\overline{\nu}_\tau$?

14.3 Conservation Laws

6. Name the conservation law that would be violated in each of the following decays:
(*a*) $\pi^+ \to e^+ + \gamma$
(*b*) $\Lambda^0 \to p + K^-$
(*c*) $\Omega^- \to \Sigma^- + \pi^0$
(*d*) $\Lambda^0 \to \pi^- + \pi^+$
(*e*) $\Lambda^0 \to n + \gamma$
(*f*) $\Omega^- \to \Xi^0 + K^-$
(*g*) $\Xi^0 \to \Sigma^0 + \pi^0$
(*h*) $\mu^- \to e^- + \gamma$
7. Each of the following reactions violates one (or more) of the conservation laws. Name the conservation law violated in each case:
(*a*) $\nu_e + p \to n + e^+$
(*b*) $p + p \to p + n + K^+$
(*c*) $p + p \to p + p + \Lambda^0 + K^0$
(*d*) $\pi^- + n \to K^- + \Lambda^0$
(*e*) $K^- + p \to n + \Lambda^0$
8. Supply the missing particle in each of the following decays:
(*a*) $K^- \to \pi^0 + e^- +$
(*b*) $K^0 \to \pi^0 + \pi^0 +$
(*c*) $\eta \to \pi^+ + \pi^- +$
9. Each of the reactions below is missing a single particle. Supply the missing particle in each case.
(*a*) $p + p \to p + \Lambda^0 +$
(*b*) $p + \overline{p} \to n +$
(*c*) $\pi^- + p \to \Xi^0 + K^0 +$
(*d*) $K^- + n \to \Lambda^0 +$
(*e*) $\overline{\nu}_\mu + p \to n +$
(*f*) $K^- + p \to K^+ +$

14.4 Particle Interactions and Decays

10. Carry out the calculations of mc^2 for the three decays of Figure 14.6.
11. Determine the energy uncertainty or width of (*a*) η; (*b*) η'; (*c*) Σ^0; (*d*) Δ^*.

14.5 Energy and Momentum in Particle Decays

12. A Σ^- baryon is produced in a certain reaction with a kinetic energy of 3642 MeV. If the particle decays after one mean lifetime, what is the longest possible track this particle could leave in a detector?
13. Repeat the calculation of Example 14.5 for the case in which the π meson has zero kinetic energy, and show that the electron energy in this case is less than the maximum value.
14. Find the Q values of the following decays:
(*a*) $\pi^0 \to \gamma + \gamma$
(*b*) $\Sigma^+ \to p + \pi^0$
(*c*) $D^+ \to K^- + \pi^+ + \pi^+$
15. Find the Q values of the following decays:
(*a*) $\pi^- \to \mu^- + \overline{\nu}_\mu$
(*b*) $K^0 \to \pi^+ + \pi^-$
(*c*) $\Sigma^0 \to \Lambda^0 + \gamma$
16. Find the kinetic energies of each of the two product particles in the following decays (assume the decaying particle is at rest):
(*a*) $K^0 \to \pi^+ + \pi^-$ (*b*) $\Sigma^- \to n + \pi^-$
17. Find the kinetic energies of each of the two product particles in the following decays (assume the decaying particle is at rest):
(*a*) $\Omega^- \to \Lambda^0 + K^-$ (*b*) $\pi^+ \to \mu^+ + \nu_\mu$
18. A Σ^- with a kinetic energy of 0.250 GeV decays into $\pi^- + n$. The π^- moves at 90° to the original direction of travel of the Σ^-. Find the kinetic energies of π^- and n and the direction of travel of n.
19. A K^0 with a kinetic energy of 276 MeV decays in flight into π^+ and π^-, which move off at equal angles with the original direction of the K^0. Find the energies and directions of motion of the π^+ and π^-.
20. (*a*) What interaction is responsible for the decay $\Omega^- \to \Lambda^0 + K^-$? (*b*) Find the kinetic energies of the product particles for Ω decays at rest. (*Hint*: Check the total kinetic energy available for the two product particles to see if it is a good approximation to use nonrelativistic kinematics.)
21. (*a*) In the decay $\Sigma^0 \to \Lambda^0 + \gamma$ with the Σ^0 initially at rest in the laboratory, what are the kinetic energy of the Λ^0 and the energy of the γ ray? (*b*) If you were to measure the energy of this γ ray, what would you expect the width of the peak to be? Would the peak be considered sharp or broad?

14.6 Energy and Momentum in Particle Reactions

22. Show Equation 14.6 reduces to Equation 13.14 in the nonrelativistic limit.
23. Determine the Q values of the following reactions:
(*a*) $K^- + p \to \Lambda^0 + \pi^0$
(*b*) $\pi^+ + p \to \Sigma^+ + K^+$
(*c*) $p + p \to p + \pi^+ + \Lambda^0 + K^0$
24. Determine the Q values of the following reactions:
(*a*) $\gamma + n \to \pi^- + p$
(*b*) $K^- + p \to \Omega^- + K^+ + K^0$
(*c*) $p + p \to p + \Sigma^+ + K^0$

25. Find the threshold kinetic energy for the following reactions. In each case the first particle is in motion and the second is at rest.
(*a*) $p + p \rightarrow n + \Sigma^+ + K^0 + \pi^+$
(*b*) $\pi^- + p \rightarrow \Sigma^0 + K^0$
26. Find the threshold kinetic energy for the following reactions. In each case the first particle is in motion and the second is at rest.
(*a*) $p + n \rightarrow p + \Sigma^- + K^+$
(*b*) $\pi^+ + p \rightarrow p + p + \overline{n}$
27. In the reaction $p + p \rightarrow n + \Lambda^0 + K^+ + \pi^+$, find the threshold energy if (*a*) a beam of protons strikes a fixed target of protons, and (*b*) two beams of protons collide head-on with equal momenta.

14.7 The Quark Structure of Mesons and Baryons

28. Analyze the following reactions in terms of the quark content of the particles and reduce them to fundamental processes involving the quarks:
(*a*) $K^- + p \rightarrow \Omega^- + K^+ + K^0$
(*b*) $\pi^+ + p \rightarrow \Sigma^+ + K^+$
(*c*) $\gamma + n \rightarrow \pi^- + p$
29. Analyze the following reactions in terms of the quark content of the particles and reduce them to fundamental processes involving the quarks:
(*a*) $K^- + p \rightarrow \Lambda^0 + \pi^0$
(*b*) $p + p \rightarrow p + \pi^+ + \Lambda^0 + K^0$
(*c*) $\gamma + p \rightarrow D^+ + \overline{D}^0 + n$
30. Analyze the following decays in terms of the quark content of the particles and reduce them to fundamental processes involving the quarks:
(*a*) $\Omega^- \rightarrow \Lambda^0 + K^-$
(*b*) $n \rightarrow p + e^- + \overline{\nu}_e$
(*c*) $\pi^0 \rightarrow \gamma + \gamma$
(*d*) $D^+ \rightarrow K^- + \pi^+ + \pi^+$
31. Analyze the following decays in terms of the quark content of the particles and reduce them to fundamental processes involving the quarks:
(*a*) $K^0 \rightarrow \pi^+ + \pi^-$
(*b*) $\Delta^{*++} \rightarrow p + \pi^+$
(*c*) $\Sigma^- \rightarrow n + \pi^-$
(*d*) $\overline{D}^0 \rightarrow K^+ + \pi^-$
32. Based on Figure 14.15, give the quark content of the six D mesons.
33. (*a*) Arrange the 10 spin-$\frac{3}{2}$ baryons listed in Table 14.6 into a strangeness vs. electric charge diagram similar to Figures 14.11 and 14.12. (*b*) Arrange the 10 three-quark spin-$\frac{3}{2}$ combinations from Table 14.9 into a similar diagram.
34. (*a*) Suppose we add a plane above the spin-$\frac{1}{2}$ baryon diagrams of Figures 14.12 or 14.14 (similar to what is done in Figure 14.15) that would show the spin-$\frac{1}{2}$ baryons with a single charmed quark. How many particles would be in that plane and what would be their quark contents? (*Hint*: Review Example 14.10 about the number of particles associated with a combination of three different quarks.) (*b*) How many particles would appear in the next higher plane with doubly charmed baryons and what are their quark contents? (*c*) Recently the first doubly charmed baryon Ξ_{cc}^{++} with a charge of $+2e$ was discovered at the LHC. What is its quark content?

14.8 The Standard Model

General Problems

35. Table 14.5 lists the most likely decay mode of the K^+ meson; Example 14.5 gives another possible decay. List four other possible decays that are allowed by the conservation laws.
36. It is desired to form a beam of Λ^0 particles to use for the study of reactions with protons. The Λ^0 are produced by reactions at one target and must be transported to another target 2.0 m away so that at least half of the original Λ^0 remain in the beam. Find the speed and the kinetic energy of the Λ^0 for this to occur.
37. Find a decay mode, other than that listed in Table 14.6, for (*a*) Ω^-; (*b*) Λ^0; (*c*) Σ^+, that satisfies the applicable conservation laws.
38. Consider the reaction $p + p \rightarrow p + p + \pi^0$ discussed in Example 14.8, but viewed instead from a frame of reference in which the two protons collide head-on with equal velocities. (*a*) At threshold in this frame of reference, the product particles are formed at rest. Find the proton velocities in this case. (*b*) Use the Lorentz velocity transformation to switch to the laboratory frame of reference in which one of the protons is at rest, and find the velocity of the other proton. (*c*) Find the kinetic energy of the incident proton in the laboratory frame and compare with the value found in Example 14.8.
39. The D_s^+ meson (rest energy = 1969 MeV, $S = +1$, $C = +1$; see Figure 14.15) has a lifetime of 0.5×10^{-12} s. (*a*) Which interaction is responsible for the decay? (*b*) Among the possible decay modes are $\phi + \pi^+$, $\mu^+ + \nu_\mu$, and $K^+ + \overline{K}^0$. How do the S and C quantum numbers change in these three decays? (The ϕ meson has a spin of 1, a rest energy of 1020 MeV, and a quark content of $s\overline{s}$.) (*c*) Analyze the three decay modes according to the quark content of the initial and final particles. (*d*) Why is the decay into $K^+ + \pi^+ + \pi^-$ allowed, while the decay into $K^- + \pi^+ + \pi^+$ is forbidden?
40. In the decay $K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ with the initial K meson at rest, what is the maximum kinetic energy of the pi mesons?
41. A beam of π^- mesons with a speed of $0.9980c$ is incident on a target of protons at rest. The reaction produces 2 particles, one of which is a K^0 meson that is observed to travel with momentum 1561 MeV/c in a direction that makes an angle of 20.6° with the direction of the incident pions. (*a*) Find the momentum and the direction of the second product particle. (*b*) Find the energy of that

particle. (c) Find the rest energy of the second particle and deduce its identity.

42. (a) The Large Hadron Collider accelerates protons to an energy of 7 TeV (7×10^{12} eV). What is the speed of these protons? Express the result as the difference between the proton speed and the speed of light. (b) Suppose this beam were directed against a fixed target of protons to obtain the reaction $p + p \rightarrow p + p + X$, where X represents one or more new particles produced in the reaction. What is the maximum amount of energy available to produce the new particles? (c) At the LHC, two beams of 7 TeV protons collide head on, so the energy available for particles X is 14 TeV. What energy would be needed for a proton beam colliding with a fixed target to have 14 TeV available to produce new particles?

43. (a) In the reaction $\pi^- + p \rightarrow n + \pi^0$ with π mesons of momentum 1140 MeV/c incident on protons at rest, the neutron is observed to travel in the direction of the incident pions with very small momentum. (Why would you expect the neutron momentum to be small?) The π^0 decays quickly into two photons that make equal angles with the pion direction of travel. What are those angles? (b) The same reaction can produce a different product particle y in place of the π^0. Particle y also decays into two photons, which make equal angles of 28.6° with the direction of travel of the original particle. Find the mass of particle y and make a guess as to its identity.