EP 228 Particle Physics, Lecture Notes on Experimental Concepts, Dec 2020

Elementary Interactions

There are three experimental probes of elementary particle interactions:

- Bound states [such as: $\pi^+(\bar{u}d),\ p(uud),\ J/\psi(c\bar{c})$]
- Decays [such as: $\pi^+ \to \mu^+ + \nu_\mu,\ \ \pi^0 \to \gamma + \gamma,\ \ \Lambda^0 \to \pi^- + X$,]
- Scattering [such as: $p + p \to p + p,\ \ p + p \to p + p + p + \bar{p}$]

Nonrelativistic quantum mechanics (in Schrodinger's formulation) is particularly well adapted to handle bound states By contrast, the relativistic theory (in Feynman's formulation) is especially well suited to describe decays and scattering. In this chapter, some definitions used in experiments will be discussed.

Decay Rate and Lifetime

Consider a particle (say muon) is produced at a given space-time. *Lifetime* of the muon is the time elapsed before it disintegrates. One can define a critical parameter called the decay rate, Γ, the *probability per unit time* that any given particle will decay. Imagine we have a large collection of muons say $N(t)$, at time t, at rest. Then $N\Gamma\, dt$ of them will decay in the next instant dt. Hence the number remaining muons are $dN = -N\Gamma\, dt$.

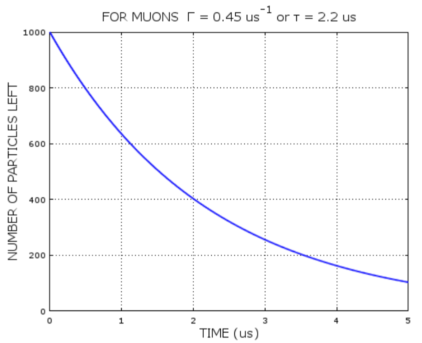
Solving for N yields:

$$N(t) = N_0 e^{-\Gamma t}$$

Here N_0 is the number of muons at $t = 0$. It is clear that the number of muons left decrease exponentially with time (see figure right).

The *mean lifetime* of muons can be found from:

$$\tau = \frac{\int_0^\infty tN dt}{\int_0^\infty N dt} = \frac{\int_0^\infty tN_0 e^{-\Gamma t} dt}{\int_0^\infty N_0 e^{-\Gamma t} dt} = \frac{1}{\Gamma}$$



For muons at rest $\Gamma = 0.45$ and $\tau = 1/\Gamma = 2.2\ \mu s$.

Actually, most particles can decay by several different ways. For example, $D^{*+}(2010)$ decays to three different channels as follows:

Mode	Channel	Fraction Γ_i / Γ
Γ_1	$D^{*+} \to D^0\pi^+$	0.6770 ± 0.005
Γ_2	$D^{*+} \to D^+\pi^0$	0.3070 ± 0.005
Γ_3	$D^{*+} \to D^+\gamma$	0.0160 ± 0.004

The total decay rate is the sum of the individual decay rates: $\Gamma = \sum \Gamma_i$
and lifetime $\tau = \sum \tau_i = 1/\Gamma$
and Branching Ratio: $BR_i = \Gamma_i/\Gamma$

See Problem 1.

Fixed Target vs Colliding Beam Experiments

In nuclear and particle physics, most of our knowledge have been obtained either *from the bombardment of the stable target nuclei with energetic incident beam of particles* or *from the head on collision of particles*. There are two cases:

- Fixed target experiments where center of mass energy is $E_{cm} \propto \sqrt{E}$
- Colliding beam experiments where center of mass energy $E_{cm} \propto E$

Here the *center of mass energy* E_{cm} is the energy can be used for generation of new particles.
See Problem 2, 3, and 4.

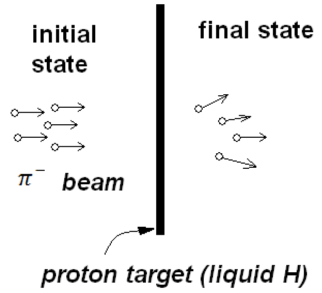
Threshold Energy

This is the energy required to start a reaction. Consider the reaction $p + p \to p + p + p + \bar{p}$ where one of the proton is at rest. This is a typical fixed target experiment. The threshold energy for the reaction is $E = 7m_p c^2$. That is the kinetic energy incident proton must be at least around 6 GeV. See problem 5, 6, 7.

Cross Section

As we said before, most of our knowledge of particle physics has been obtained from scattering experiments. In a scattering experiment, one of the important quantity that experimentalist measure and the theorist calculate is called the *cross-section*. In a typical scattering experiment, a mono-energetic beam of particles (such as pions) are directed on to a target (such as liquid Hydrogen) and production rates of various particles are measured.

For each initial state a number of final state is possible:



For example:
$\pi^- + p \to \pi^- + p$
$\pi^- + p \to K^0 + \Lambda$
$\pi^- + p \to \pi^- + \pi^+ + n$
$\pi^- + p \to \pi^- + \pi^+ + \pi^0 + n$
⋮
etc.

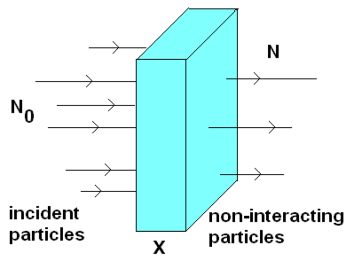
Whatever the reaction we chose, the production rate will be proportional to:

- Number of particles in the target
- Number of particles in beam per unit area per unit time

A proper way to define the reaction probability is cross-section. Now, consider we have a slab of material as shown in Fig. We have N_0 particles at the entrance, then number of surviving particles at a distance x is (See Problem 8):

$$N(x) = N_0 e^{-n\sigma x}$$

where
n = number of atoms (targets) per unit volume
σ = cross sectional area of target



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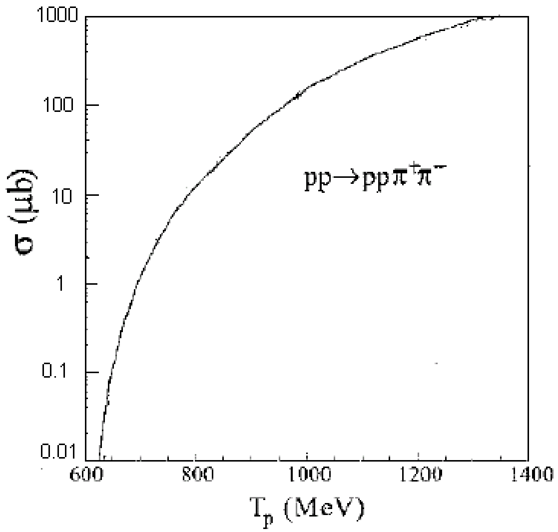
SI unit of cross section is m^2. In general, barn unit is used:

$$1\ \text{b} = 10^{-28}\ \text{m}^2 = 10^{-24}\ \text{cm}^2$$

The cross section of a reaction can be a function of the incident energy. Figure below shows the production cross section as a function of energy for the reaction $p + p \rightarrow p + p + \pi^+ + \pi^-$.



The probability of an interaction in distance Δx is: $P = n\,\sigma\,\Delta x$. The mean free path (path traversed by particle without doing any interaction) is defined as (See Problem 9):

$$l = \frac{\Delta x}{P} = \frac{1}{n\sigma}$$

In particle physics, *luminosity* (L) is defined as the number of beam particles per unit area per unit time. The *production rate* (R) of an event is the number of events per unit time and defined as:

$$R = \sigma\ L$$

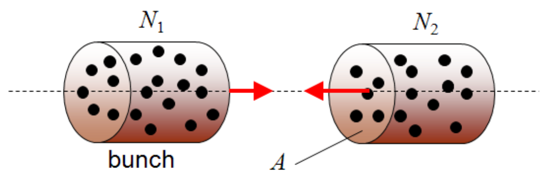
Reaction rate = Cross-section * Luminosity

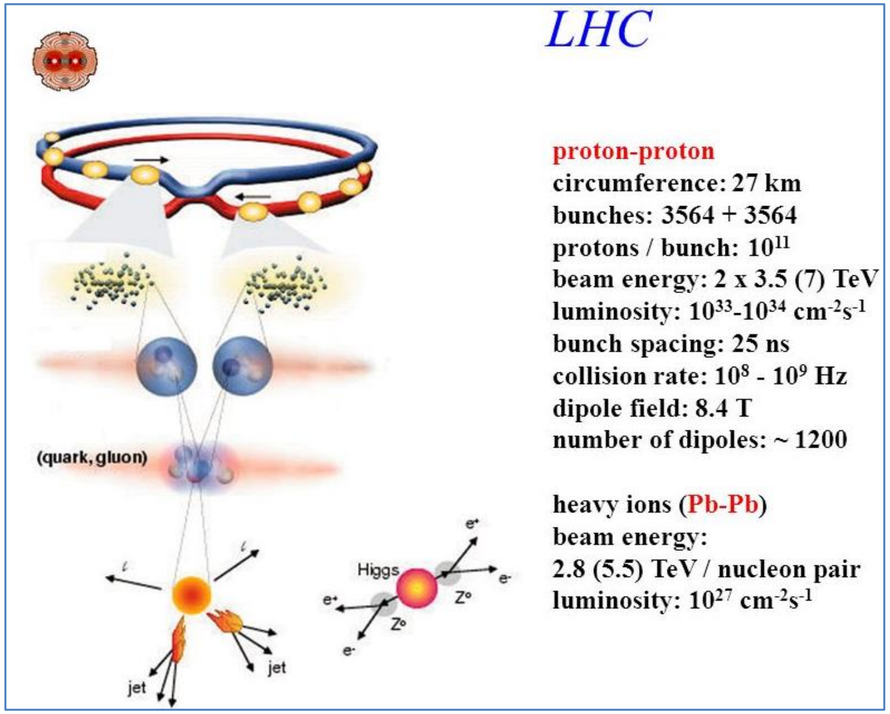
See Problems 10 and 11.

Consider two beams of particles consisting of n-bunches with N_1 and N_2 particles in each. The bunches traverse a circular path and collide head on. In this case luminosity is given by:

$$L = nf\frac{N_1 N_2}{A}$$

where
n = number of bunches
f = collision frequency
A = cross-sectional area of beams





Solved Problems

[1]. According to Grand Unified Theory, a proton can decay to channel $p \to e^{+} + \pi^{0}$ with lifetime $\tau = 10^{31}\ y$. Experiments are under way to detect the proton decay by monitoring and underground cave containing full of water such as Kamiokande.

(a) Estimate how much pure water is needed to observe one decay per hour. [Ans: 2.6 x 10^{9} kg.]

(b) Estimate how long you have to wait for one proton decay in a 1 kg of water to decay. [Ans: 3x10^{4} y]

[2]. Show that the center of mass energy in a Fixed Target Experiment is $E_{cm} \propto \sqrt{E}$ where the target particle is at rest and the projectile particle has energy E.

[3]. Show that the center of mass energy in a Colliding Beam experiment is $E_{cm} \propto E$ where both particles (each has energy E) are they make head on collision.

[4]. Suppose the production of Z boson at LEP. $e^- + e^+ \rightarrow Z$ (m_z = 91.2 GeV/c^2). Find the required total beam energy to generate a Z boson at E_{cm} = 91.2 GeV

(a) in a fixed target experiment [Ans: E = 8 TeV]

(b) in a colliding beam experiment [Ans: E = 45.6 GeV]

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[5]. The Bevatron at Berkeley was built with the idea of producing antiprotons by the reaction

$$p + p \to p + p + p + \bar{p}$$

That is, a high-energy proton strikes a proton at rest, creating (in addition to the original particles) a proton-antiproton pair. What is the threshold energy for this reaction (i.e., the minimum energy of the incident proton)? Let mass of a proton is m. (Ans: $E = 7mc^2 \approx 7$ GeV).

[6]. Consider the reaction where m_2 is at rest:

$$m_1 + m_2 \to M_1 + M_2 + \cdots + M_n$$

Find the threshold *kinetic energy* of particle m_1.

[7]. Consider the reaction where proton is at rest $\pi^- + p \rightarrow \pi^- + \pi^+ + \pi^0 + n$.
Find the threshold *kinetic energy* of incident pion to start the reaction.

[8]. Derive the relation: $N(x) = N_0 e^{-n\sigma x}$.

[9]. Total cross-section for a neutrino interacting with iron is about $\sigma = 10^{-47}$ m^2. Calculate the mean free path of neutrinos in an iron block. (M_{Fe} = 55.9 g/mole, d_{Fe} = 7.8 g/cm^3) [Ans l=1.2x10^{18} m = 130 ly]

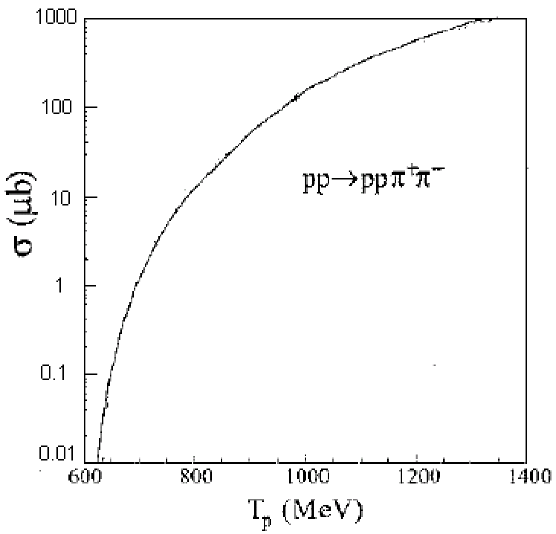
[10]. The mean free path of fast neutrons in lead is about 5 cm. Find the total neutron cross section of lead. (atomic mass number ~ 200, density ~ 10 g/cm^3). [Ans: σ = 6.64 b].

[11]. See figure below.

(a) Find the threshold K.E. for the reaction: $p + p \to p + p + \pi^+ + \pi^-$

(b) For $L = 10^{26}$ 1/(cm^{2}s), calculate the production rate of this reaction if incident proton has KE of T = 1000 MeV. [Ans: R = 0.01/s]

(c) For case (b) compute how many reactions are produced per hour? and per day?



[12]. The LHC at CERN is the largest particle accelerator with circumference of 27 km. It collides opposing protons bunches (each contains $1.0\text{x}10^{11}$ protons) at the center of mass energy of 14 TeV. For the operation, the collision frequency is designed to be nf = 40 MHz and each bunch has the geometric cross-sectional area of $A = \pi\,(35\ \mu\text{m})^2$. The primary task of the LHC will be the detection and the study of the Higgs Boson (H) whose mass is m_H = 125 GeV/c^2. The cross-section of the Higgs production is 3 fb (1 fb = 10^{-15} b).

(a) Calculate the luminosity of LHC in 1/(s.cm^2) unit.

(b) Calculate the Higgs production rate at LHC at the luminosity in case (a).

(c) How many Higgs bosons will be produced per day at LHC at the luminosity in case (a)?