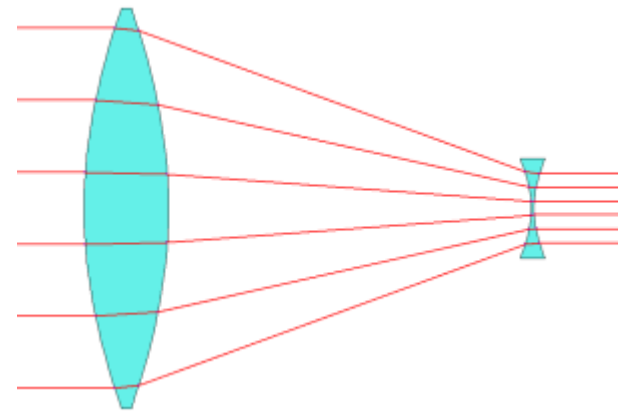




# EP 324 Applied Optics

## *Topic 2*

## Reflection, Refraction and Prisms



Department of  
Engineering of Physics  
Gaziantep University

Oct 2015

# Geometric Optics

The field of Geometric Optics involves the study of the propagation of light with the assumption that *the light travels in a fixed direction in a straight line called ray.*

- Rays are defined to propagate in a rectilinear path as they travel in a homogeneous medium.
- Rays bend (and may split in two) at the interface between two dissimilar media.
- Rays may curve in a medium where the refractive index changes.
- Rays may be absorbed and reflected.

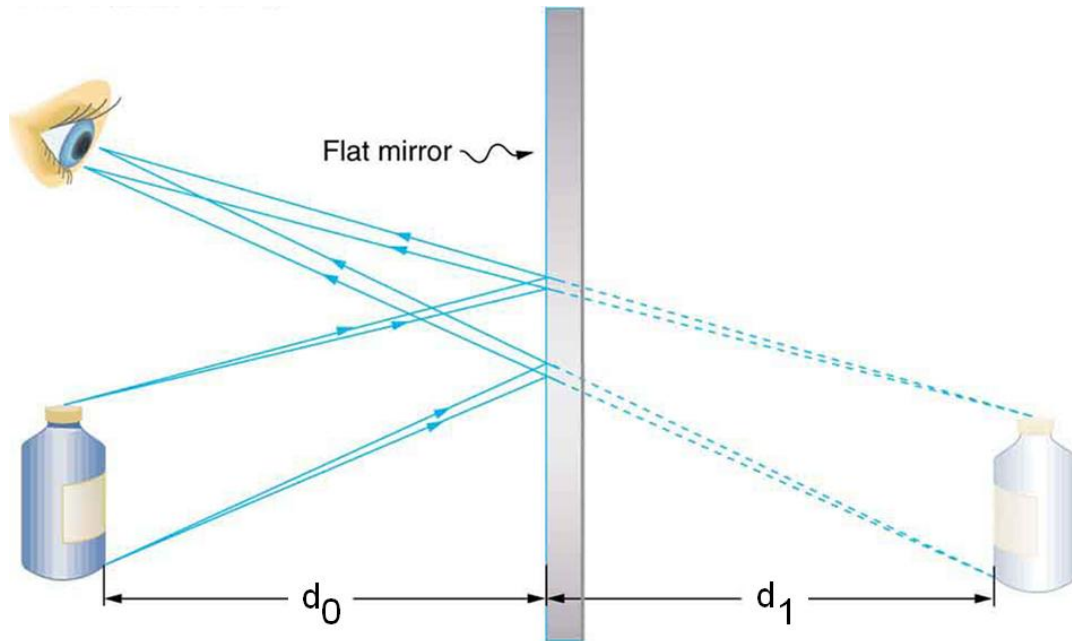
# **PART I**

## ***REFLECTION & REFRACTION***

# Reflection

Shining surfaces (e.g. mirrors) reflect the light in a simple way:

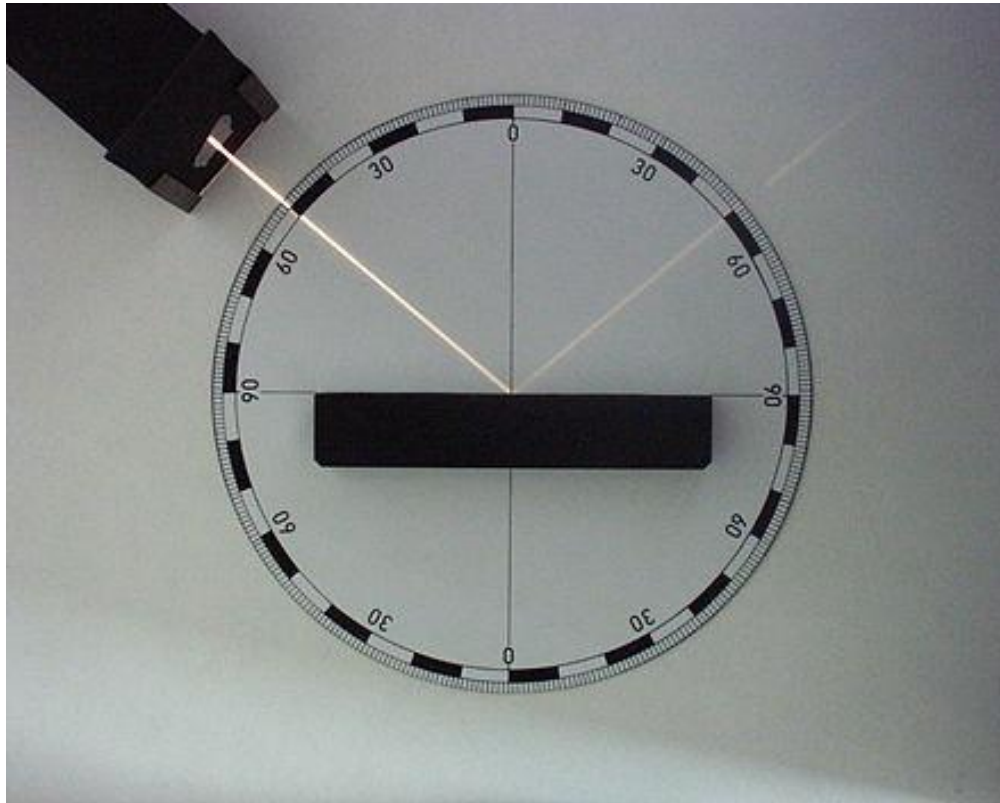
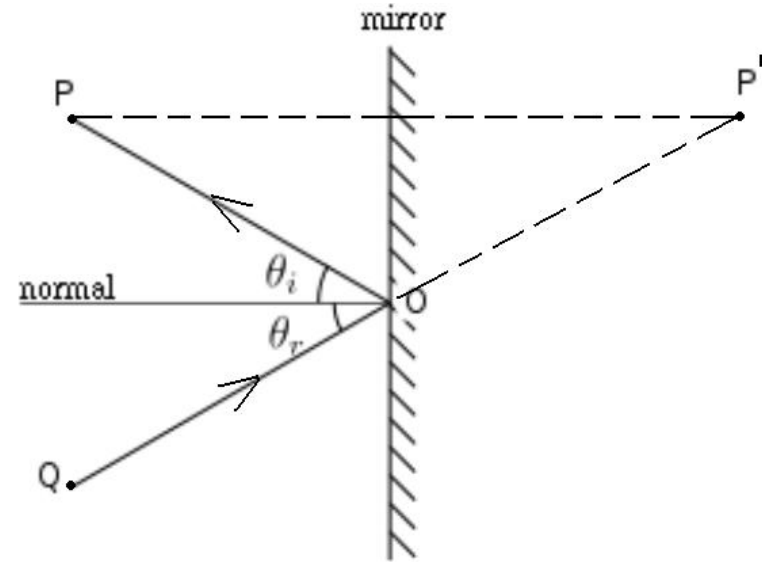
$$d_0 = d_1$$



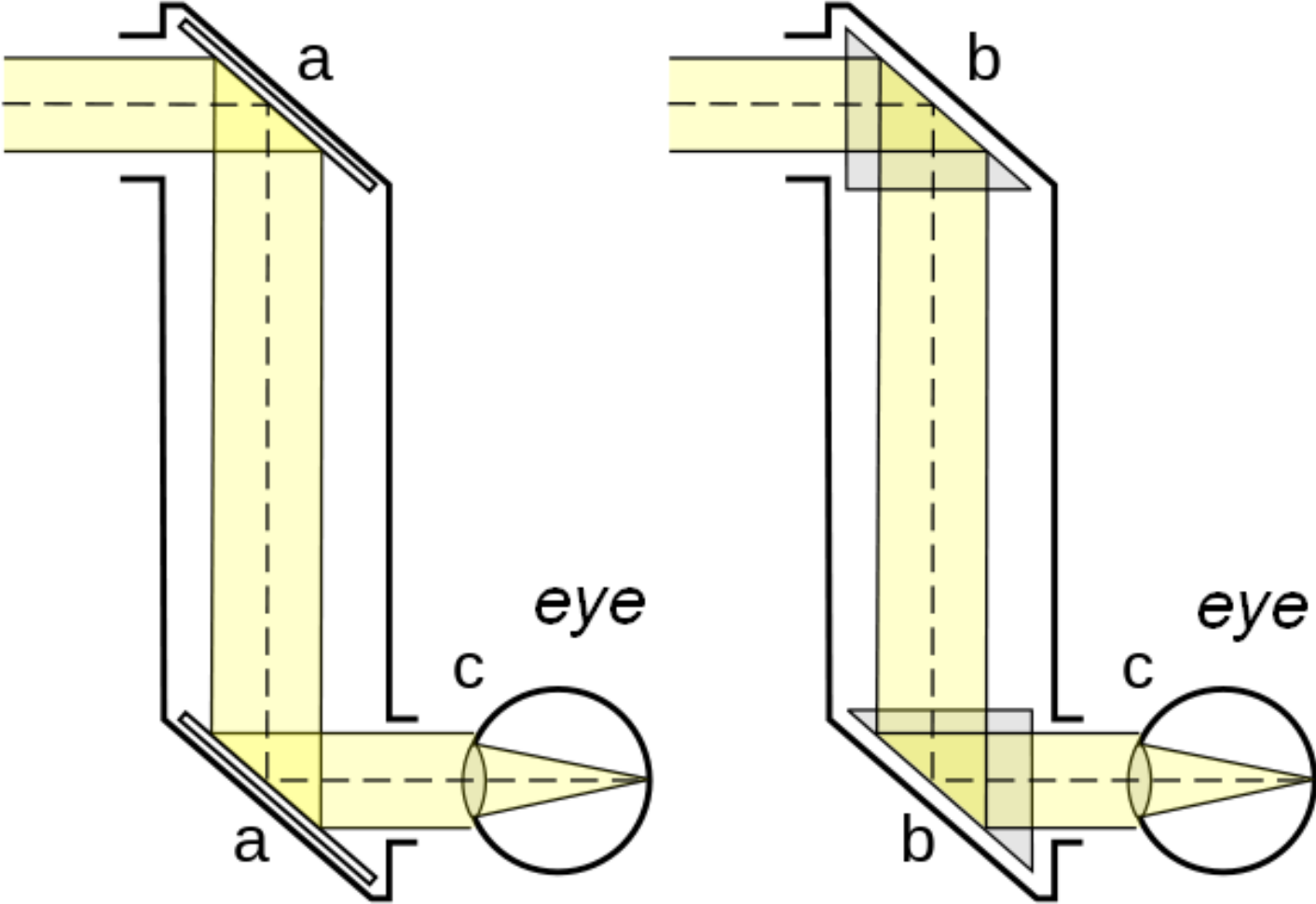
# Reflection

The law of reflection:

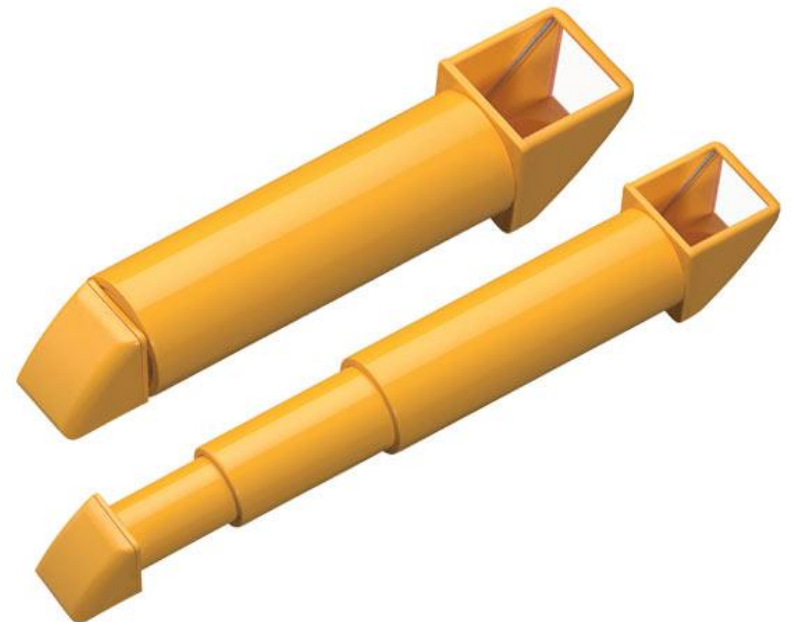
$$\theta_i = \theta_r$$



# Periscope



# Periscope



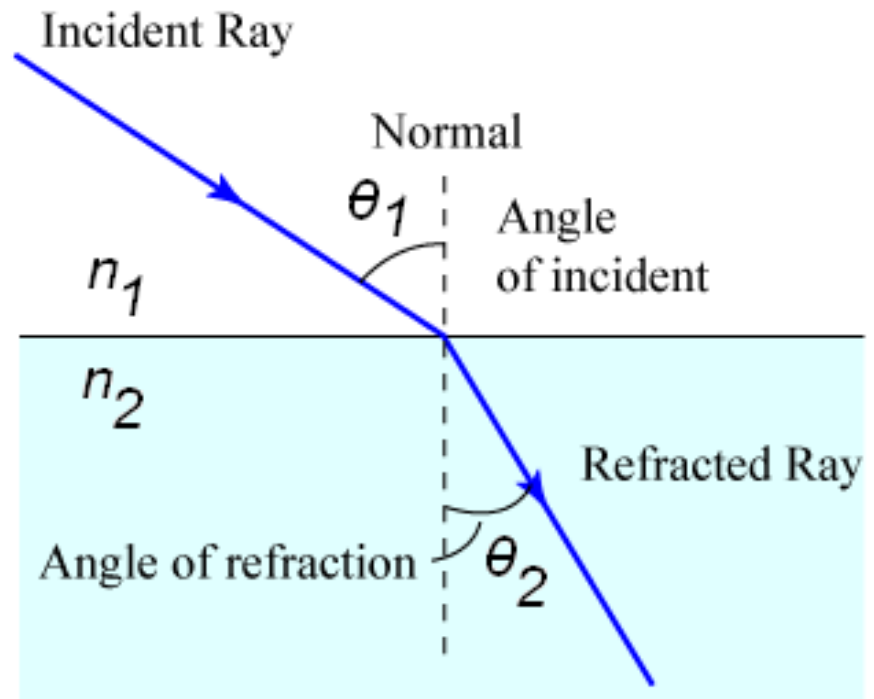
# Refraction

Refraction is the change in direction of a ray due to a change in its transmission medium.



Snell's Law of refraction:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

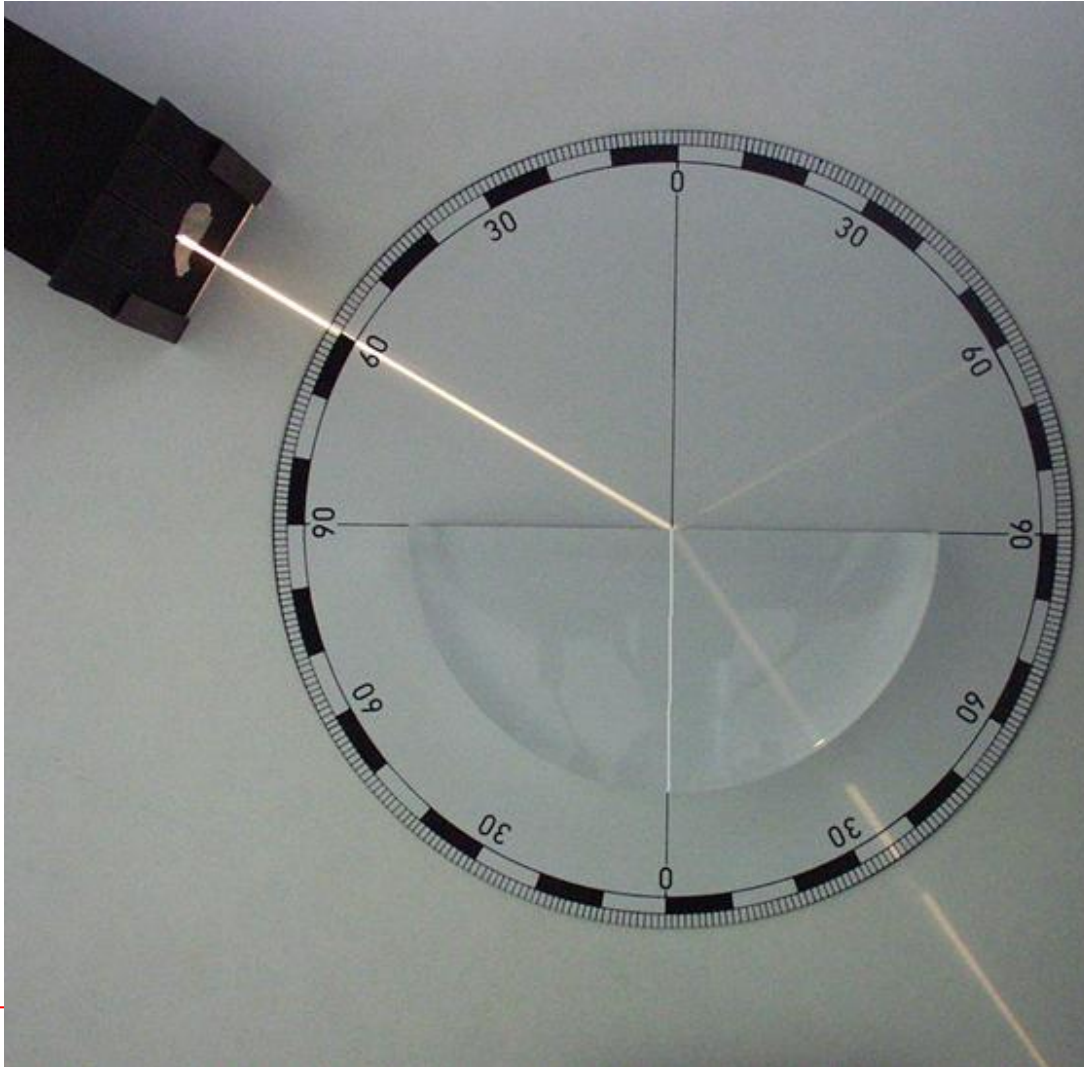




# Example

Determine

- (a) the index of refraction of the material and
- (b) the speed of light in the material as shown in Figure.



# Example

For commercial purposes, perfume bottles are made from thick glasses.



Consider a concentric cylindrical bottle:

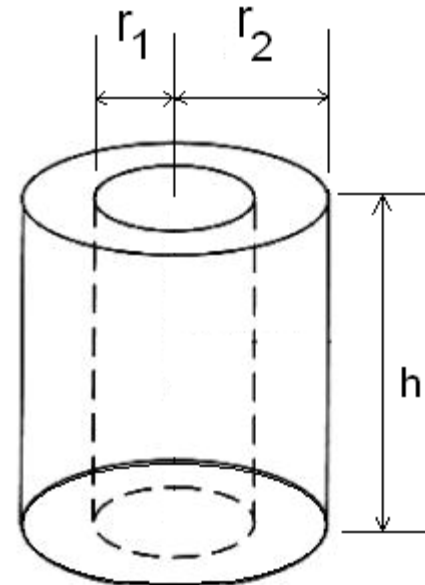
Innder radius  $r_1 = 1\text{ cm}$

Outer radius  $r_2 = 2\text{ cm}$

Refractive index  $n = 1.33$

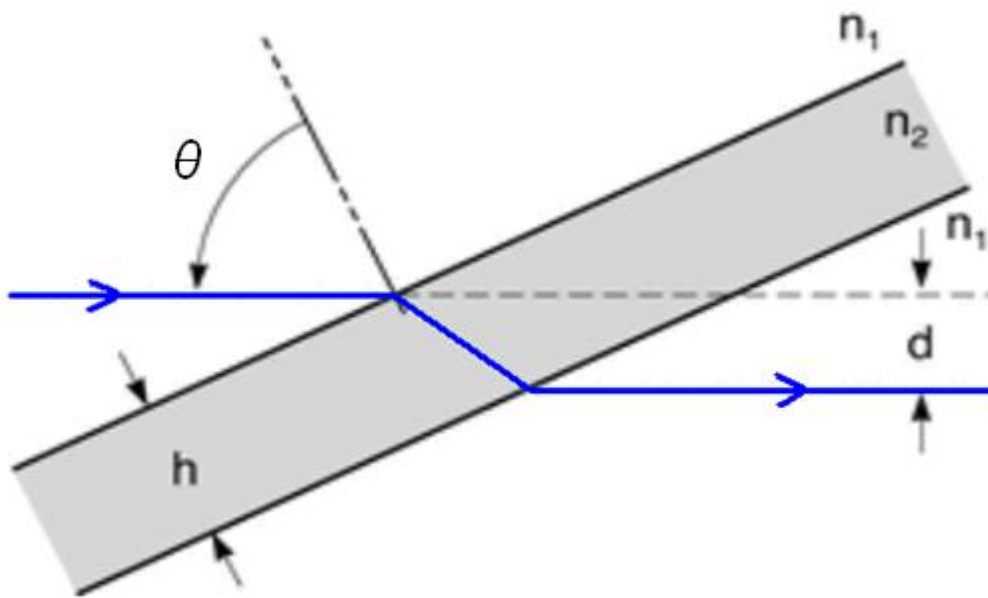
Height  $h = 5\text{ cm}$

Compute the real volume and seen volume of the liquid in the bottle.



# Optical Slab

$$d = h \sin \theta \left[ 1 - \frac{\cos \theta}{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta}} \right]$$



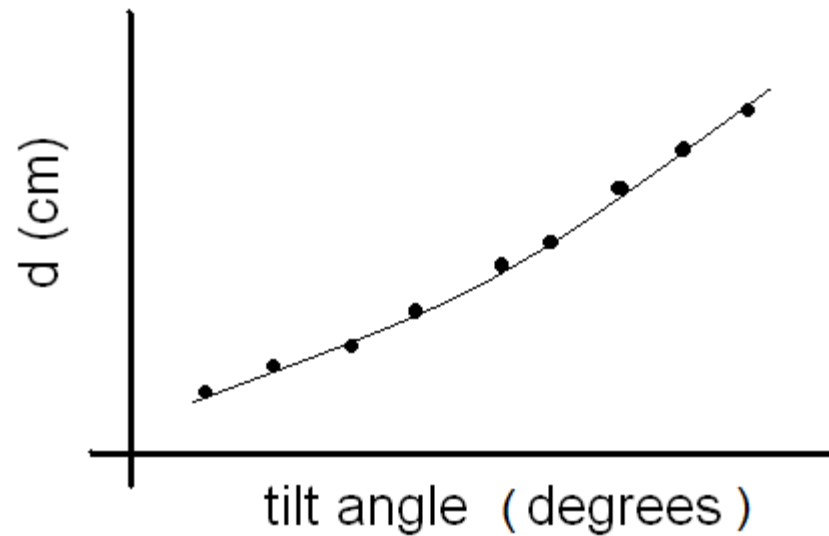
# HW

On a millimeter paper, plot the tilt angle (from 0 to 90 degrees with step 10 degrees) vs  $d$  for

$$h = 1.0 \text{ cm}$$

$$n_1 = 1.0$$

$$n_2 = 1.5$$



# Fermat's Principle of Least Time

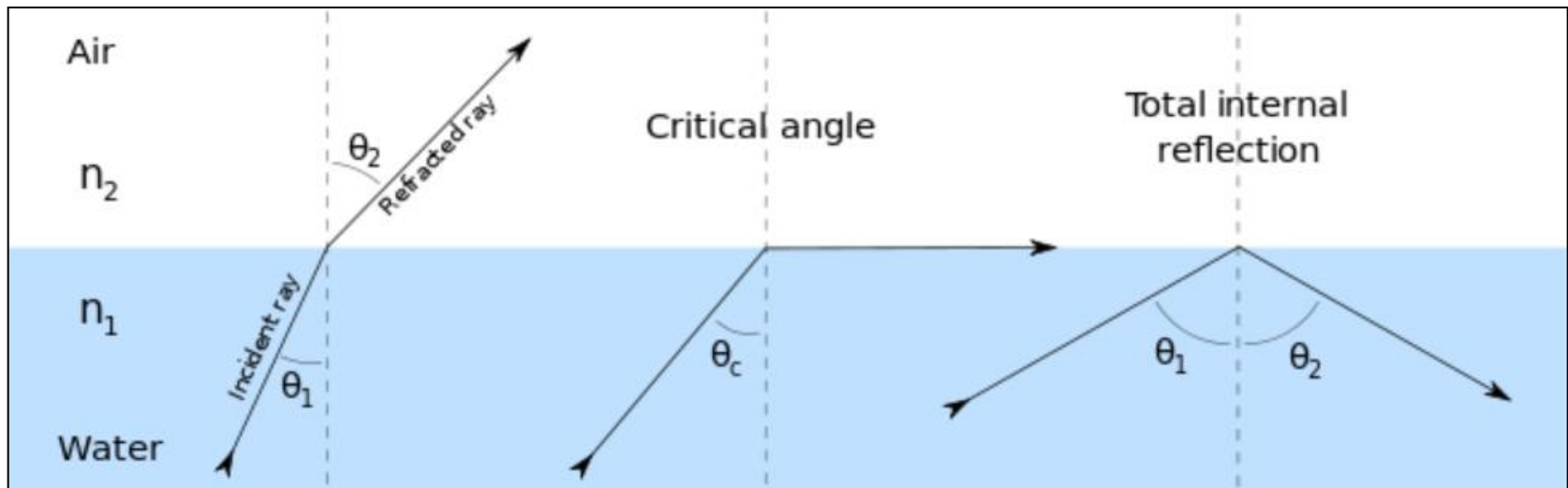
Light takes the path which requires the shortest time.

*Derivations for  
the law of reflection and the law of refraction  
will be given in the lecture.*

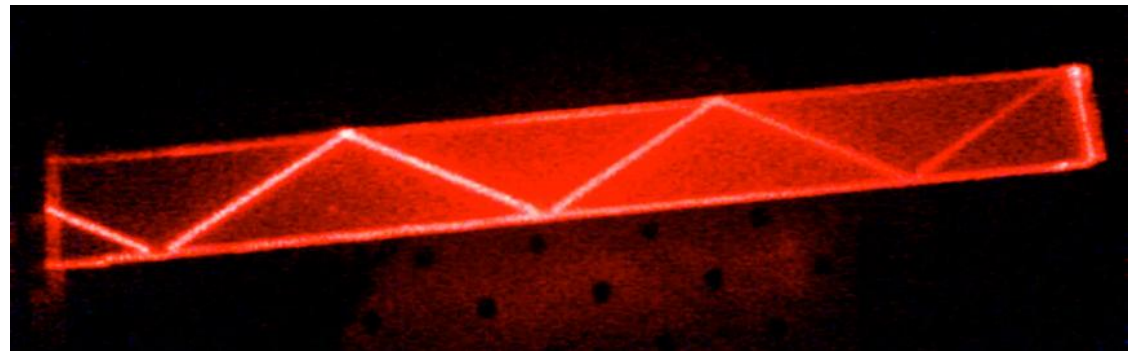
# Total Internal Reflection (TIR)

TIR happens when a ray strikes a medium boundary at an angle larger than a particular critical angle given by:

$$\sin \theta_c = n_2 / n_1$$

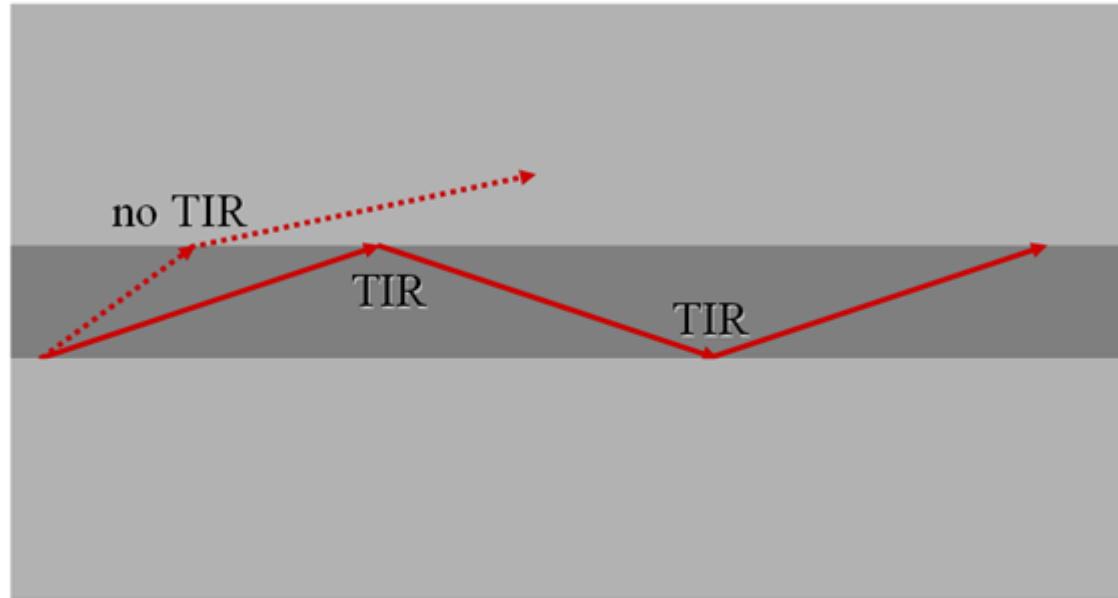


*TIRs in a block of acrylic* ==>



# Total Internal Reflection (TIR)

## Optical waveguides



“planar” waveguide: high-index dielectric material sandwiched between lower-index dielectrics

# Applications of TIR

## Optical Fibers

- flexible, transparent fiber made of extruded glass or plastic, slightly thicker than a human hair.
- used in fiber-optic communications



## Endoscope

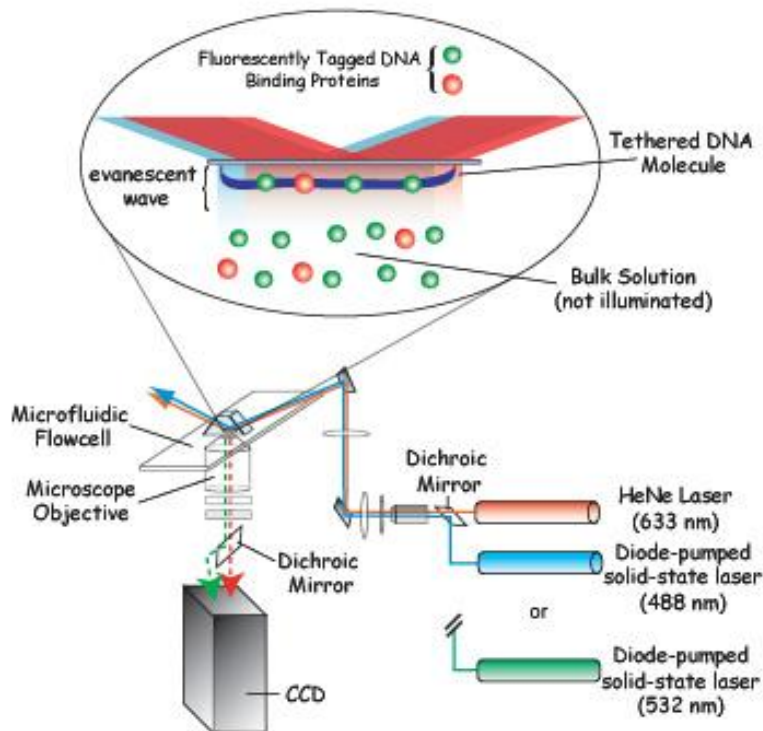
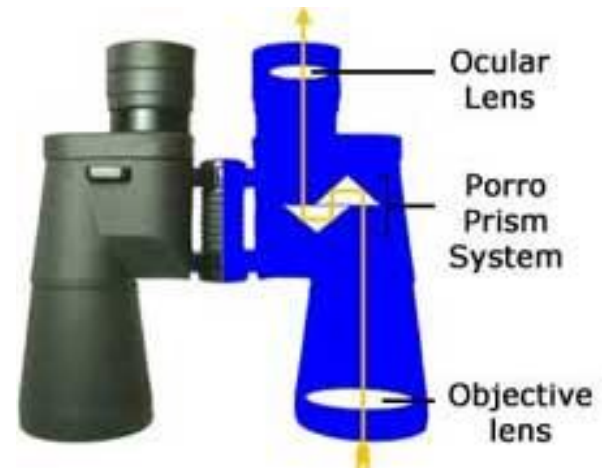
An instrument used to examine the interior of a hollow organ or cavity of the body.





# Applications of TIR

## Prisms in binoculars



## Total internal reflection fluorescence microscope (TIRFM)

# **PART II**

## ***PRISMS***

# *Transparent medium between two planes is called a prism*

## **Deflecting Prisms**

*Wedge prism*

*Anamorphic Prism Pairs*

## **Reflective prisms**

*Dove prism*

*Porro prism*

*Porro–Abbe prism*

*Amici roof prism*

*Pentaprism*

*Abbe–Koenig prism*

*Schmidt–Pechan prism*

*Bauernfeind prism*

*Retroreflector prism*

## **Polarizing Prisms**

*Nicol prism*

*Wollaston prism*

*Nomarski prism*

*Rochon prism*

*Senarmont prism*

*Glan–Foucault prism*

*Glan–Taylor prism*

*Glan–Thompson prism*

**Prism**

## **Dispersive Prisms**

*Triangular prism*

*Abbe prism*

*Pellin–Broca prism*

*Roof prism*

*Compound prism*

# Prisms

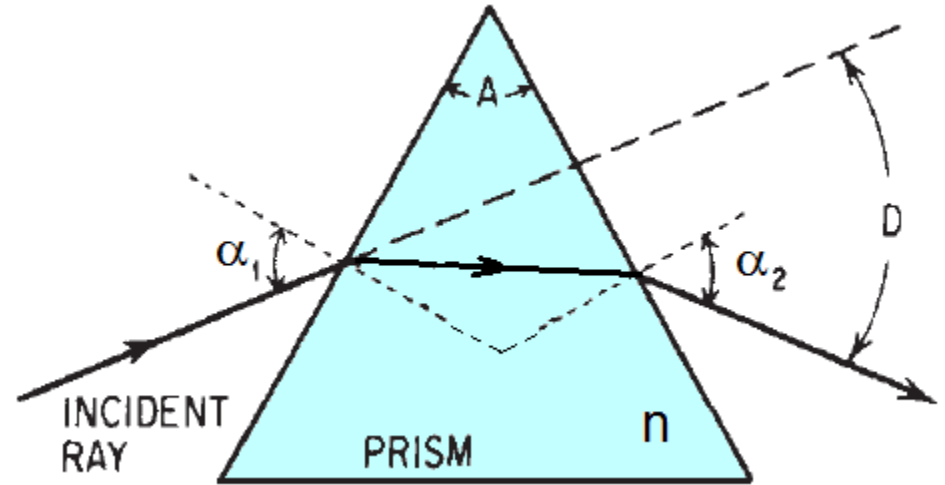
In a typical dispersing prism:

$\alpha_1$  = angle of incidence

$\alpha_2$  = outgoing angle

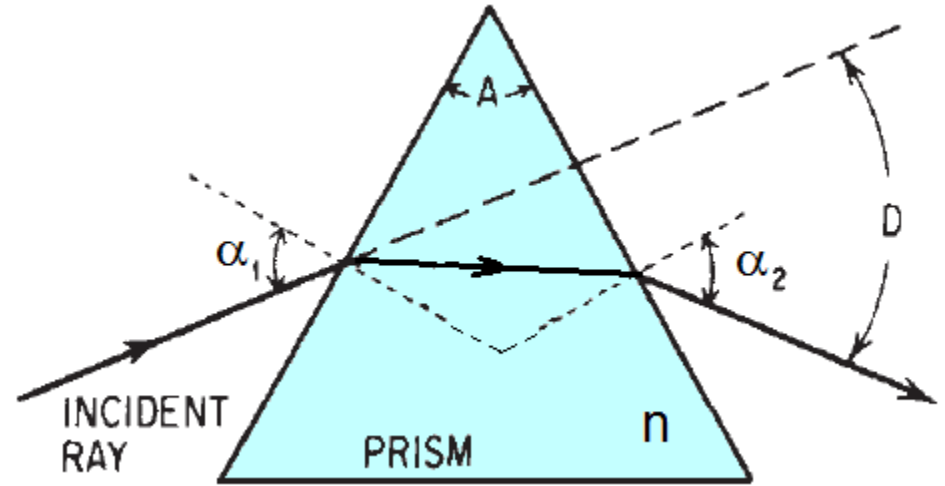
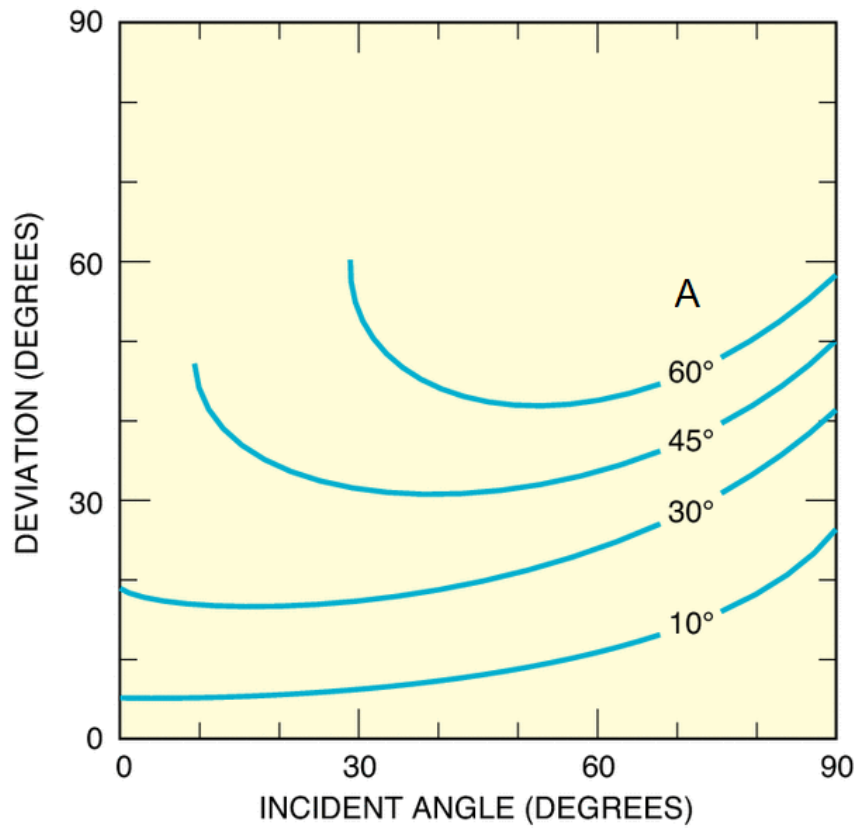
$A$  = apex angle

$D$  = angle of deviation



$$D = \alpha_1 + \alpha_2 - A$$

# Prisms



$$D = \alpha_1 + \alpha_2 - A$$

# HW

(a) Show that the deviation angle  $D$  is given by:

$$D = \alpha_1 + \sin^{-1}[(n^2 - \sin^2 \alpha_1)^{1/2} \sin A - \cos A \sin \alpha_1] - A$$

(b) By using calculus, show that minimum deviation occurs when  $\alpha_1 = \alpha_2$ .

(c) On a millimeter paper, plot the incident angle  $\alpha_1$  (from 0 to 90 degrees with step 10 degrees) vs deviation angle for  $n = 1.5$ ,  $A = 45$  degrees.

# Minimum Deviation

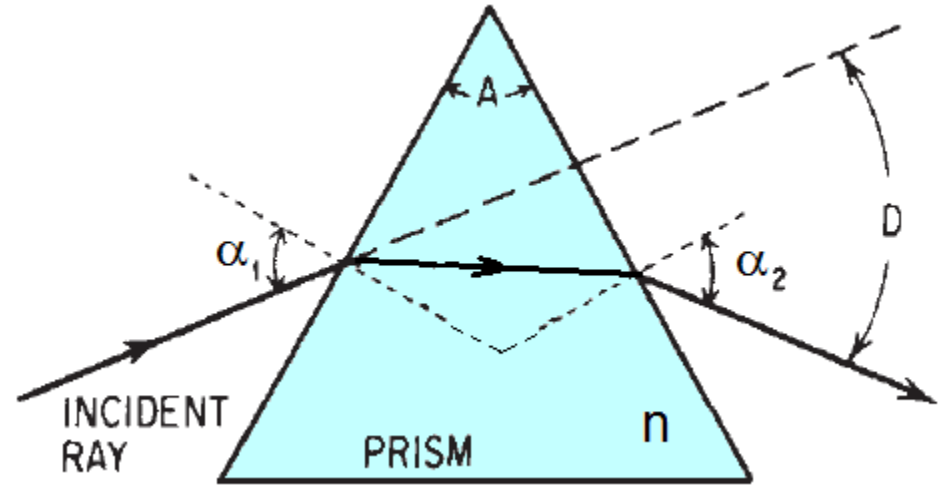
Minimum deviation occurs when  $\alpha_1 = \alpha_2$ .

$$D_{\min} = 2\alpha_1 - A$$

$$n = \frac{\sin[(A + D_{\min}) / 2]}{\sin[A / 2]}$$

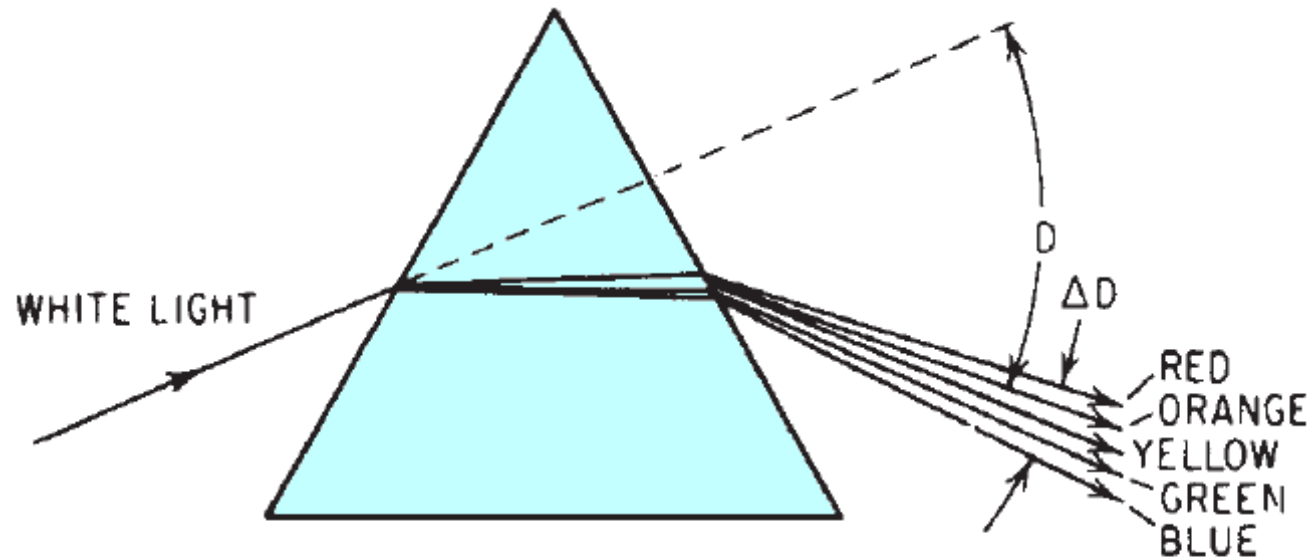
Note that for a thin prism ( $A$  is small)

$$D \approx (n - 1)A$$



# Dispersion

*Refractive index  
is function of  
the wavelength*



The dispersion of white light into its component wavelengths by a refracting prism (highly exaggerated).

$$\text{dispersion} = \frac{dn}{d\lambda}$$

For a thin prism:

$$D = A(n - 1) \quad \implies$$

where  $V$  is the Abbe number.

$$\begin{aligned} dD &= Adn \\ &= D \frac{dn}{n-1} \\ &= \frac{D}{V} \end{aligned}$$



# Dispersion Measure

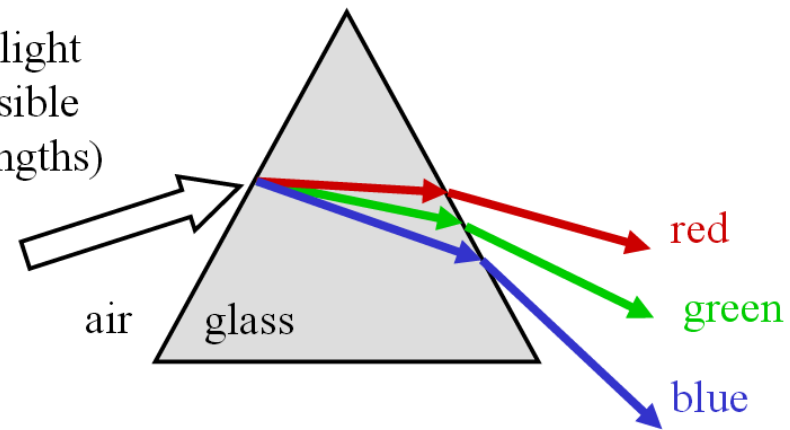
Reference color lines:

C (H-  $\lambda=656.3\text{nm}$ , red),

D (Na- $\lambda=589.2\text{nm}$ , yellow),

F (H -  $\lambda=486.1\text{nm}$ , blue)

white light  
(all visible  
wavelengths)



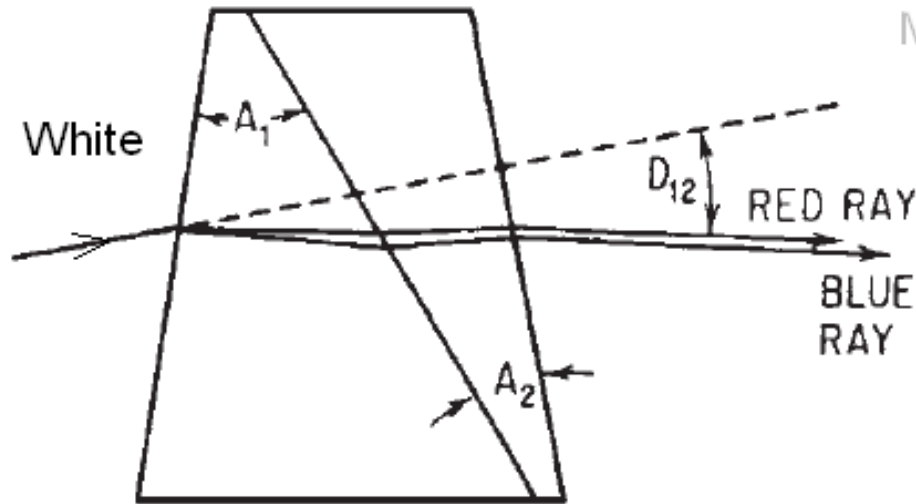
Newton's prism

$$\text{Abbe value} = V = \frac{n_D - 1}{n_F - n_C}$$

For a crown glass:

$$V_{\text{crown}} = \frac{1.5233 - 1.0000}{1.5290 - 1.5204} = 60.8$$

# Types of Prisms: Achromatic Prism



Modern Optical Engineering, W.J.Smith

**Figure 4.3** An achromatic prism. The red and blue rays emerge parallel to each other; no chromatic dispersion is introduced by the deviation.

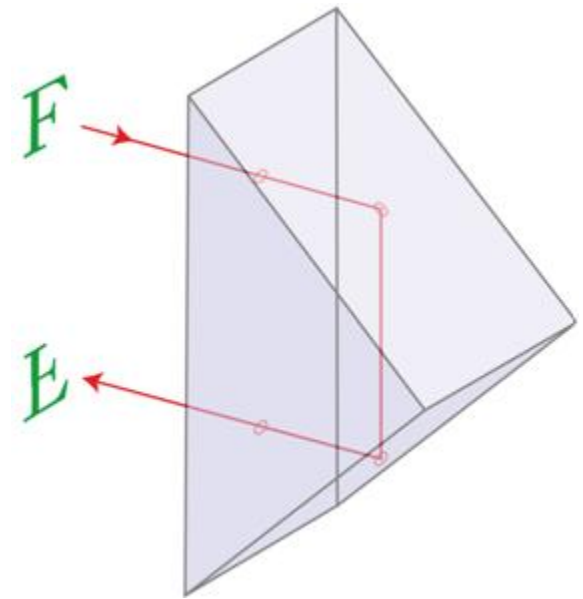
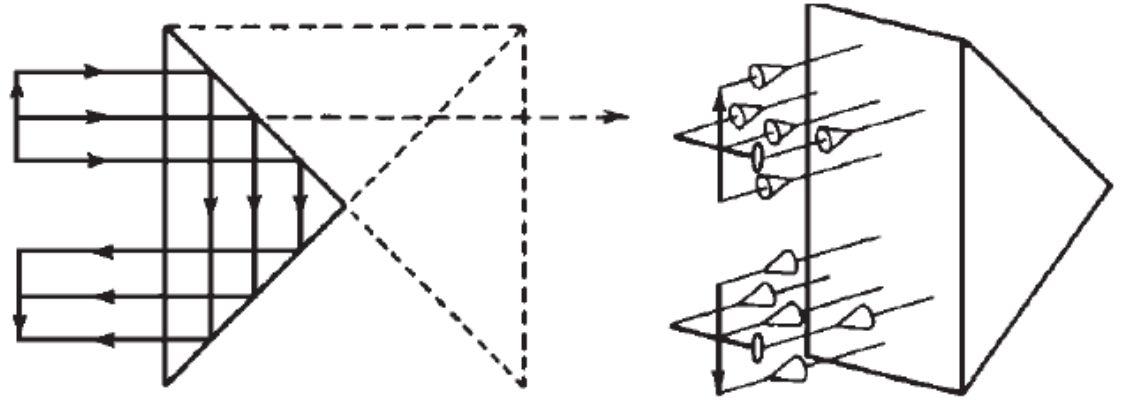
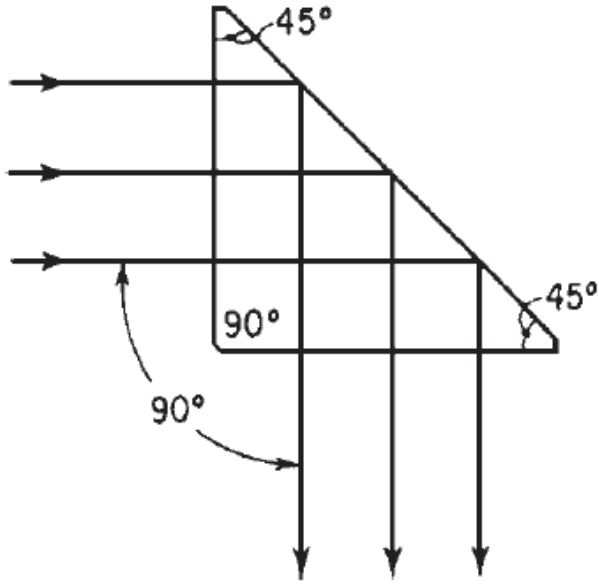
The aim:

Angular deviation of a light beam without introducing any chromatic dispersion. For thin prisms, we have two equations:

$$D_1 + D_2 = D_{12}$$

$$dD_1 + dD_2 = 0$$

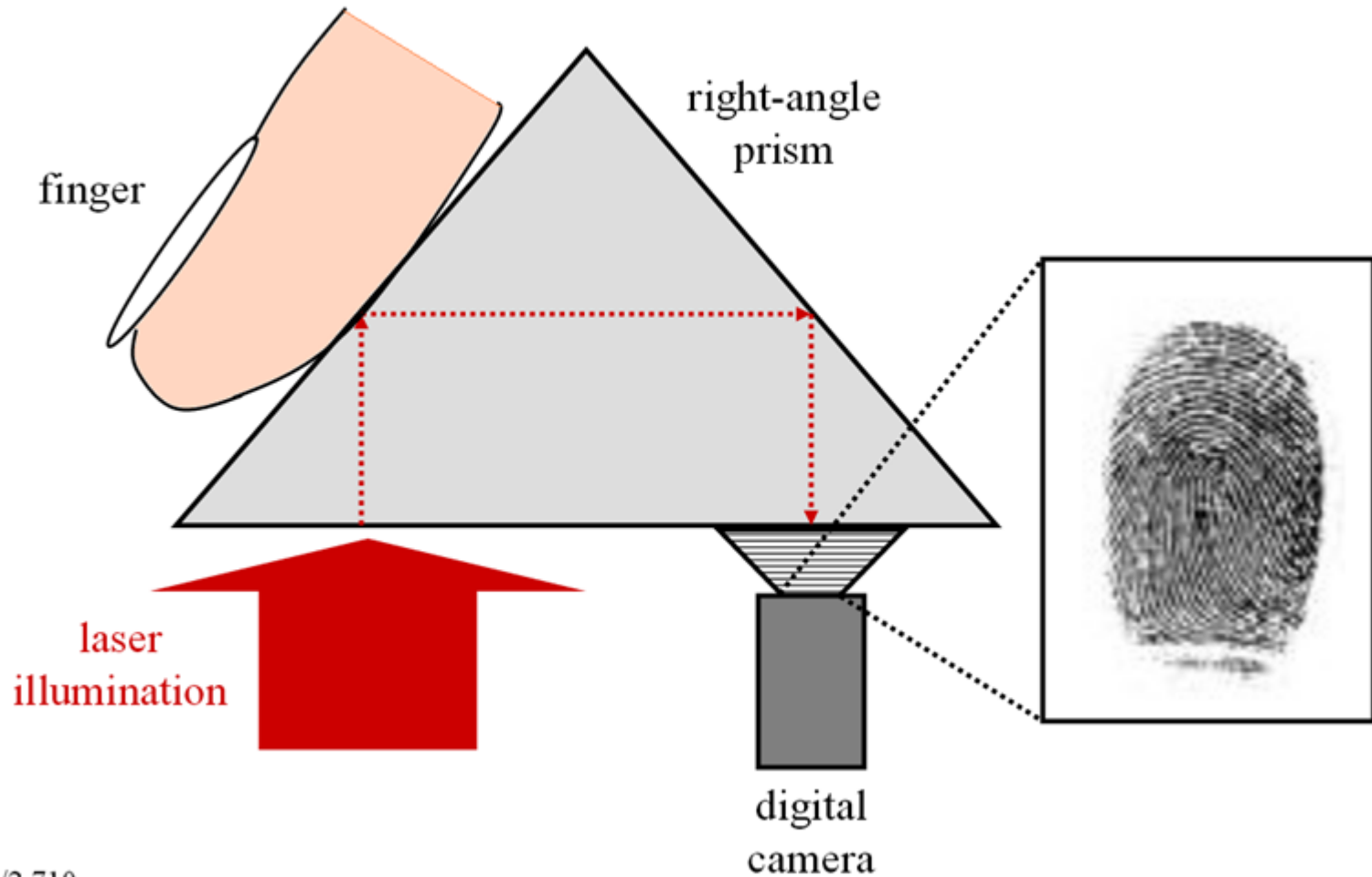
# Right-Angle Prism



Modern Optical Engineering, W.J. Smith

# Right-Angle Prism

## Fingerprint sensors

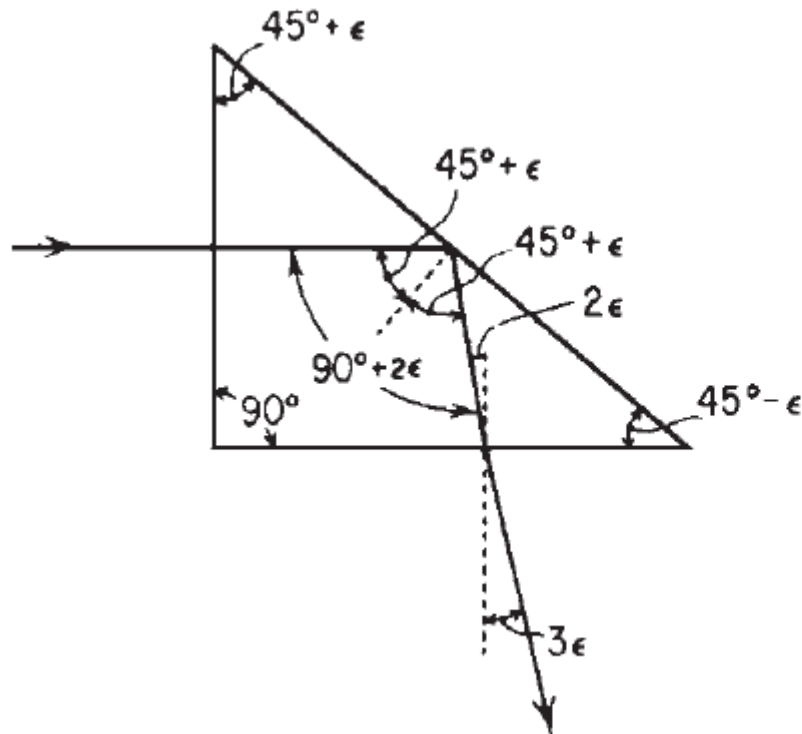


# Fabrication Errors

Due to manufacturing tolerances, the prism angles can be produced with certain errors.

Assume that the upper angle is  $45 + \epsilon$  degrees.

Final outgoing ray will deviate at angle  $3\epsilon$  w.r.t normal.



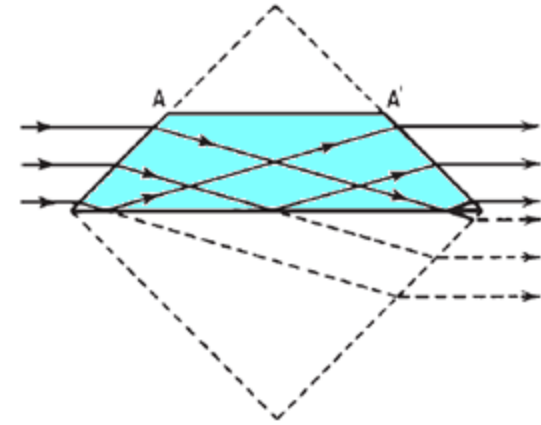
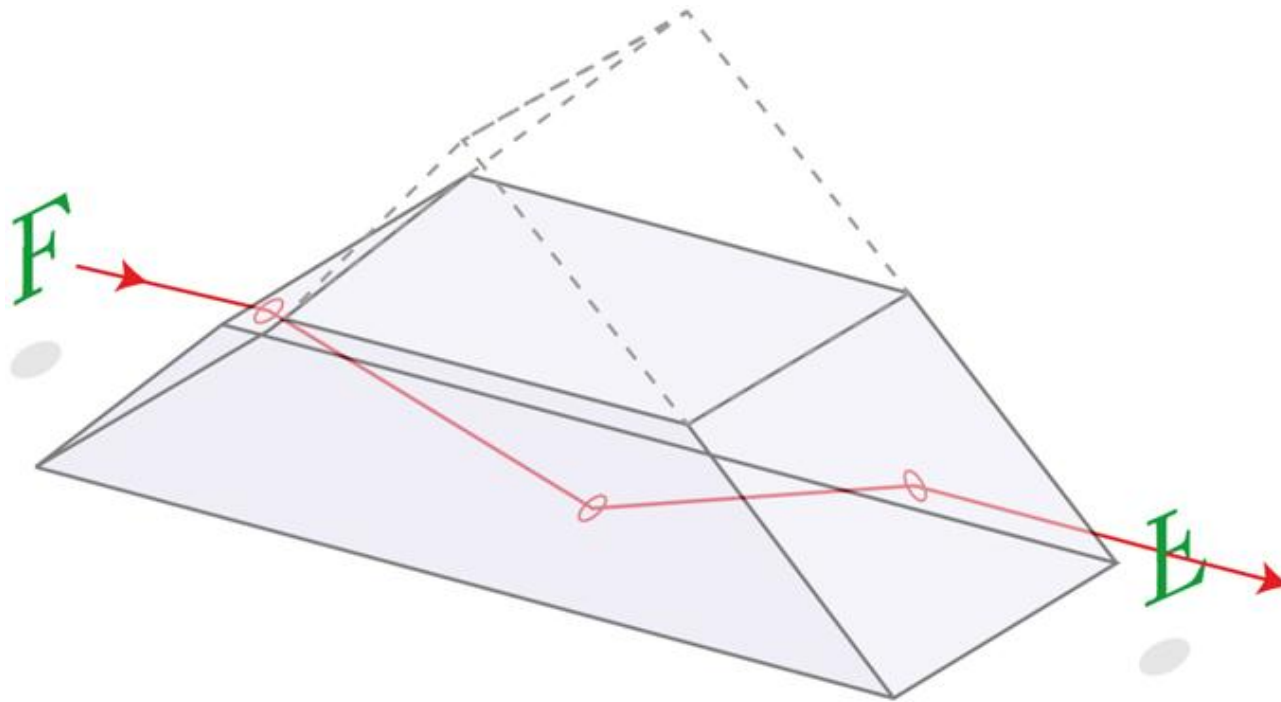
Modern Optical Engineering, W.J.Smith

**Figure 4.40** The passage of a ray through a right-angle prism whose hypotenuse face is tilted from its proper position by a small angle  $\epsilon$ . After reflection, the ray is deviated by  $2\epsilon$ ; this is increased to  $3\epsilon$  (or  $2n\epsilon$ ) by refraction at the exit face.

# Dove Prism

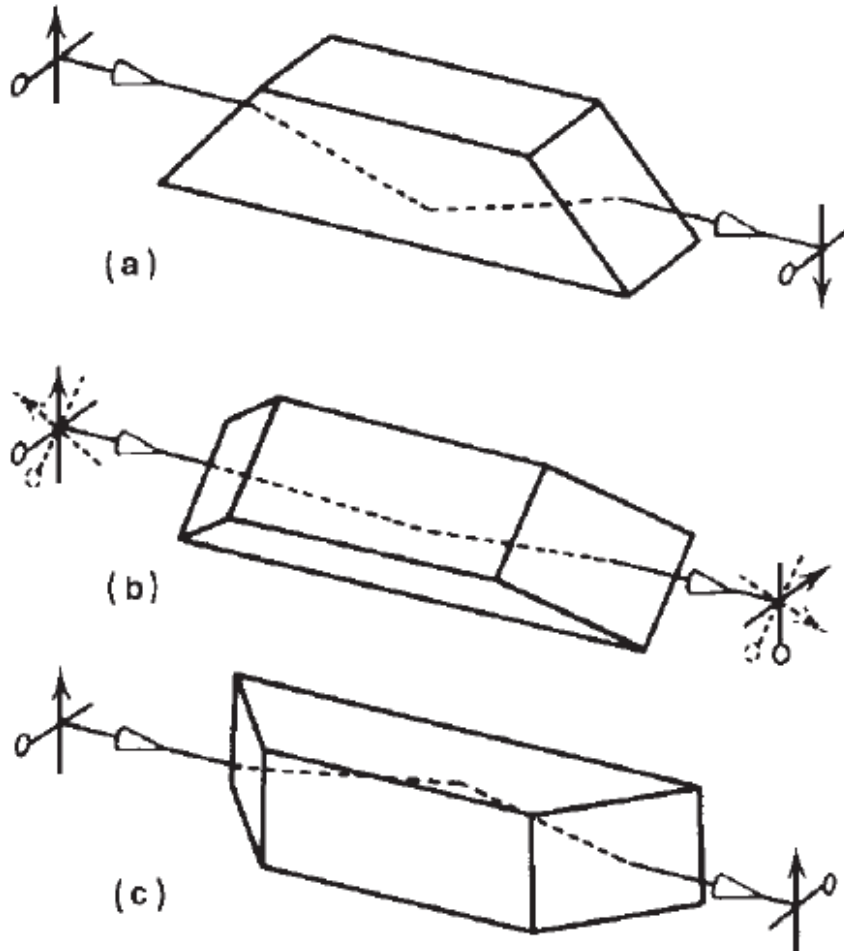
Dove prism is used to invert an image.

Dove prisms are shaped from a truncated right-angle prism.



# Dove Prism as Beam Rotator

- *When a Dove Prism is rotated along its longitudinal axis, the transmitted image rotates at twice the rate of the prism.*
- *This property is used in astronomy and pattern recognition.*

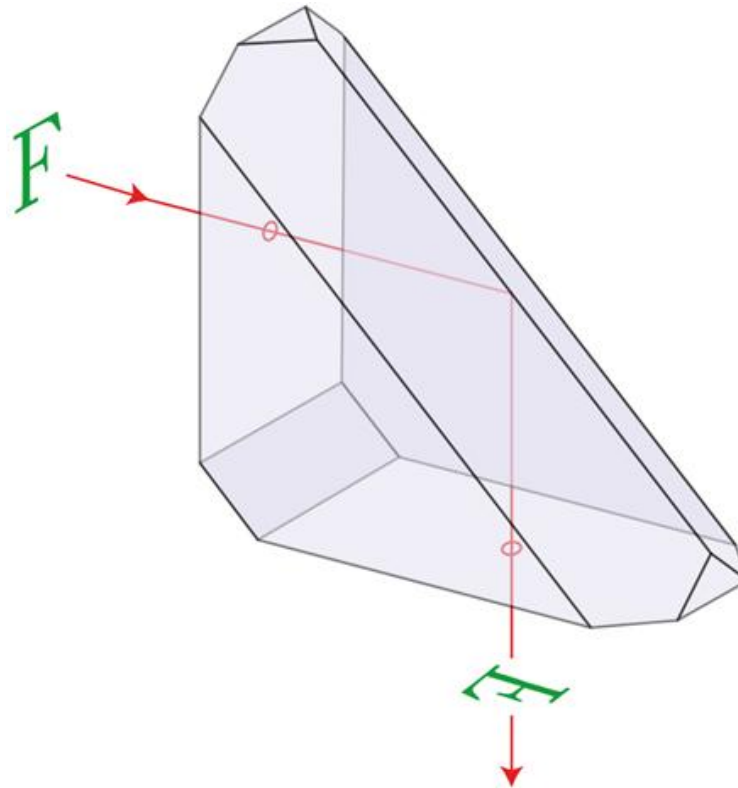


Modern Optical Engineering, W.J.Smith

**Figure 4.20** The orientation of an image by a Dove prism. (a) Original position. (b) Prism rotated  $45^\circ$ ; image is rotated  $90^\circ$  (c) Prism rotated  $90^\circ$ ; image is rotated  $180^\circ$ . Note that the dotted arrow and crossbar in (b) is oriented so that the dotted arrow is in the plane of incidence to simplify the analysis of the image orientation.

# Roof Prism

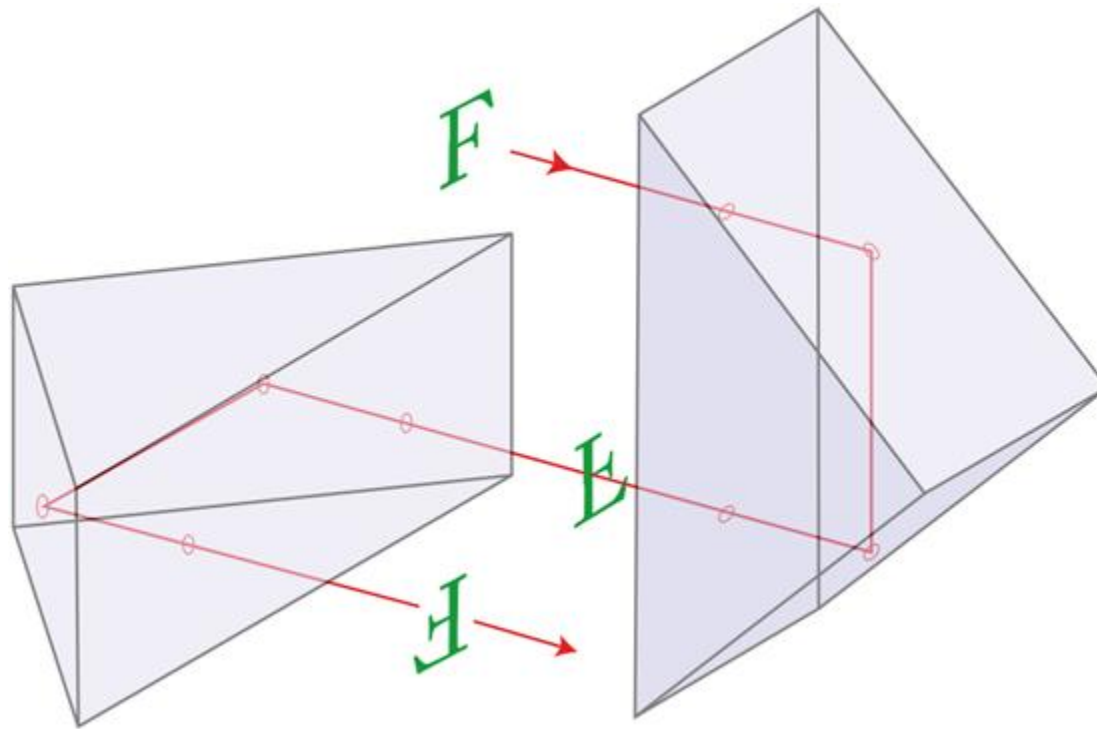
- A roof prism is a reflective optical prism containing a section where two faces meet at a 90° angle.
- If there is an error in the roof angle (90°), the beam is split into two beams which diverge at an angle which is six times the error.





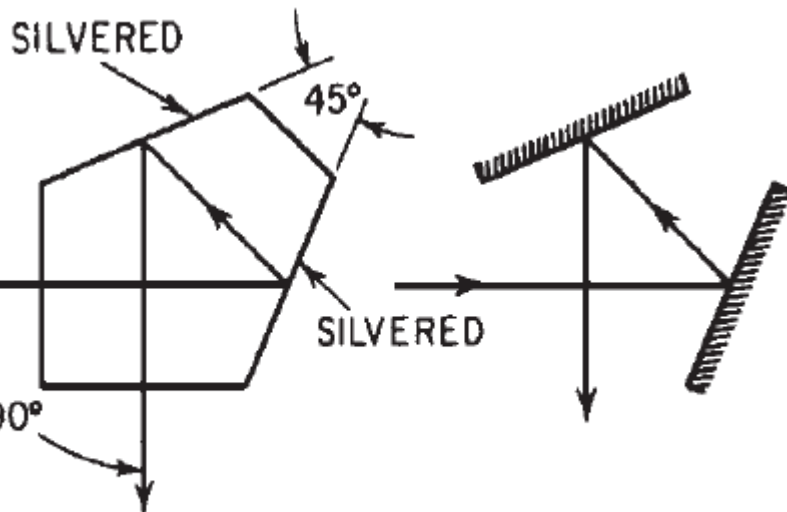
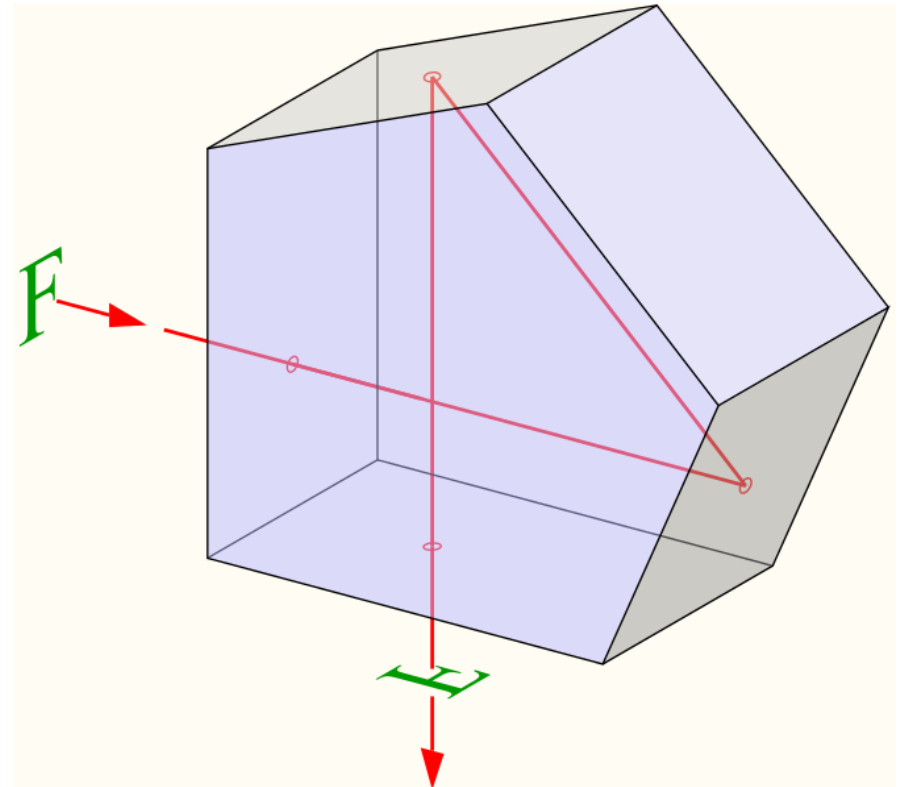
# Porro Prism

- Porro prism is used to alter the orientation of an image.
- Porro prism systems are used in small optical telescopes to re-orient an inverted.



# Penta Prism

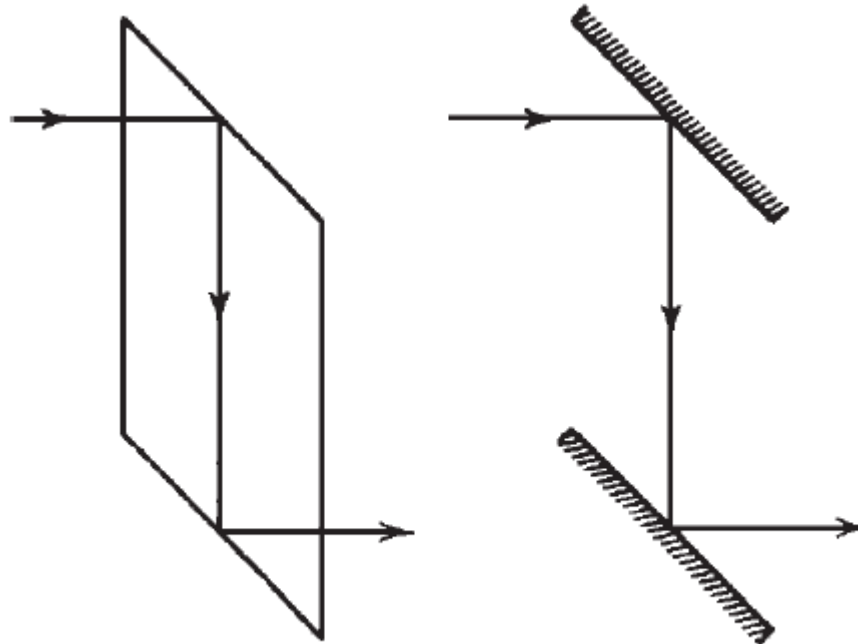
- Neither invert nor reverse the image.
- The function is to deviate the line of sight by  $90^\circ$ .
- Commonly used in the viewfinder of single-lens reflex cameras.



Penta prism and its mirror equivalent

# Rhomboid Prism

The rhomboid prism displaces the ray without affecting the orientation of the image or deviating the line of sight.

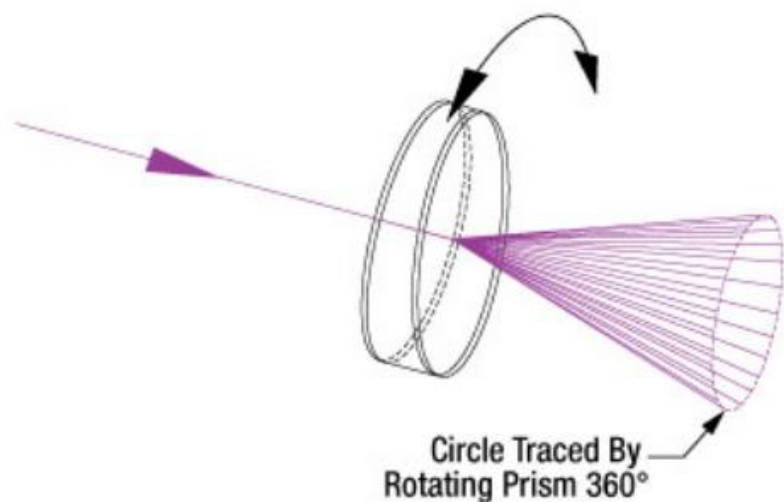


Rhomboid prism and its mirror equivalent

# Wedge Prisms

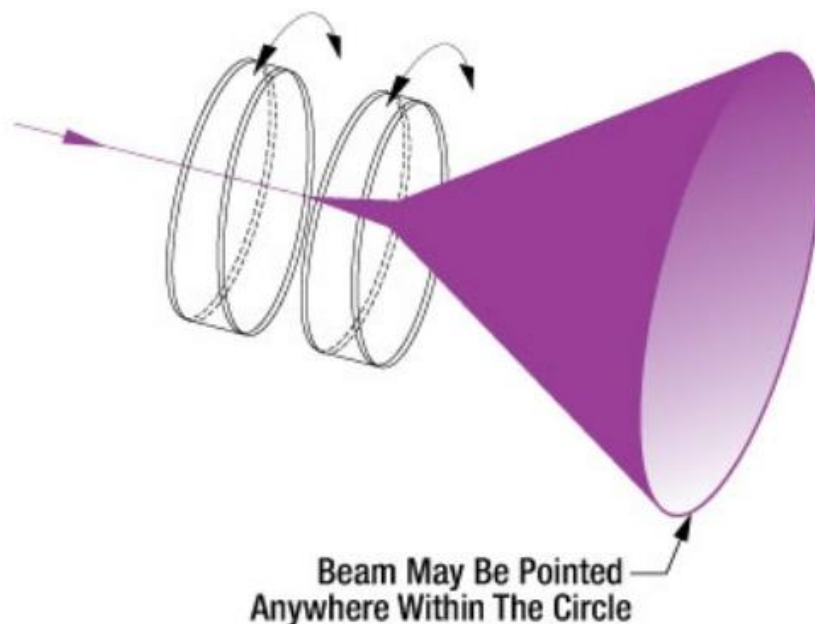
- Wedge Prisms are ideal for laser beam steering applications.
- Deflect a beam normal to the prism's perpendicular surface through an angular deviation ranging from  $2\theta$  to  $10\theta$ .

[www.thorlabs.com](http://www.thorlabs.com)



[Click to Enlarge](#)

The drawing above depicts a single wedge prism and an incident beam of light. The incident light is refracted at the specified deviation angle. As the wedge is rotated, the deviated beam traces out a circle defined by an angle equal to two times the specified deviation angle.



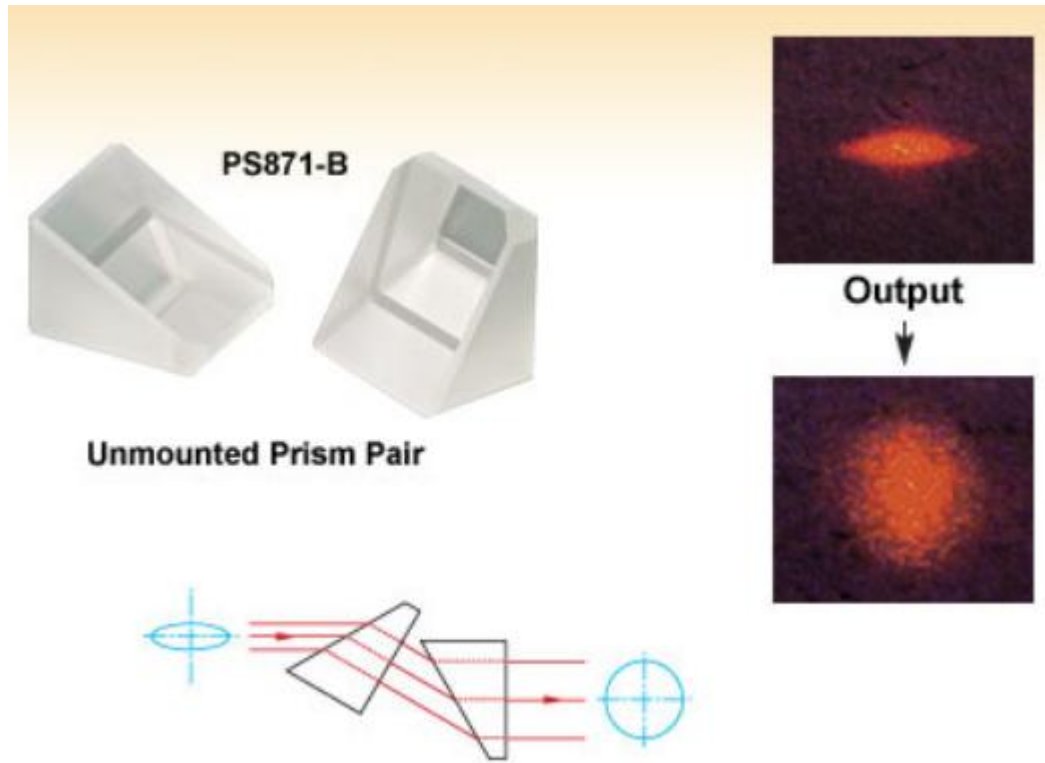
[Click to Enlarge](#)

The drawing above depicts two wedge prisms and an incident beam of light. Since each individual prism can trace out a circle of two times the deviation angle, the total deviation by two prisms will be four times the deviation angle. By controlling the angle of each prism independently, the beam can be positioned at any point within the circle.

# Anamorphic Prism Pairs

Transform Elliptical Laser Diode Beams into Nearly Circular Beams

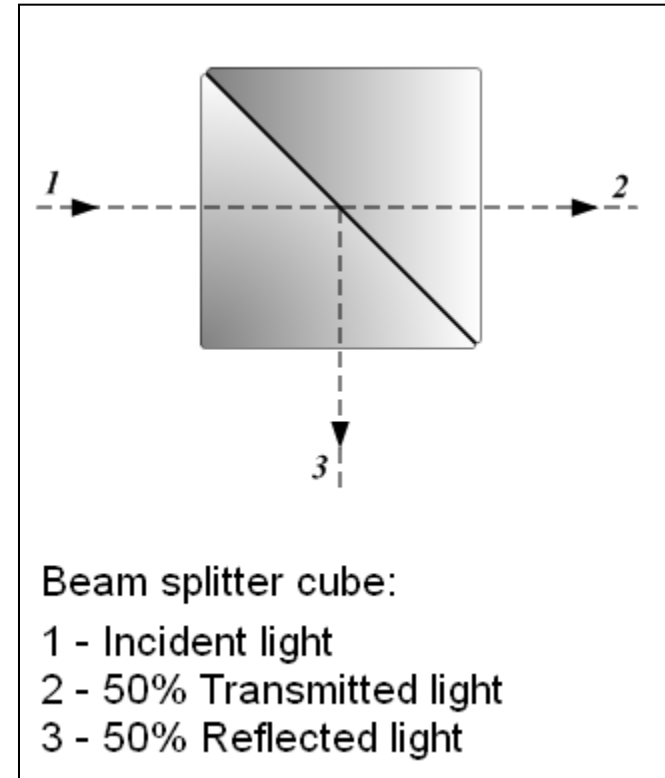
[www.thorlabs.com/Anamorphic\\_Prisms](http://www.thorlabs.com/Anamorphic_Prisms)



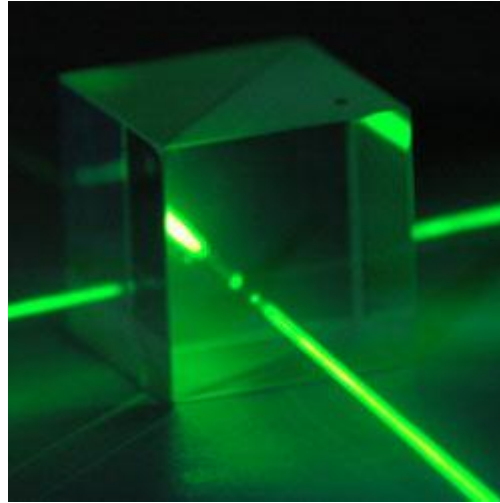
# Beam-Splitters

## A beamsplitter cube

- is composed of two right-angle prisms cemented together (The hypotenuse of one prism is coated with a semi-reflecting coating before cementing)
- is used for combining two beams (or images) into one
- is used for separating one beam into two
- is the crucial part of most interferometers



# Beam-Splitters



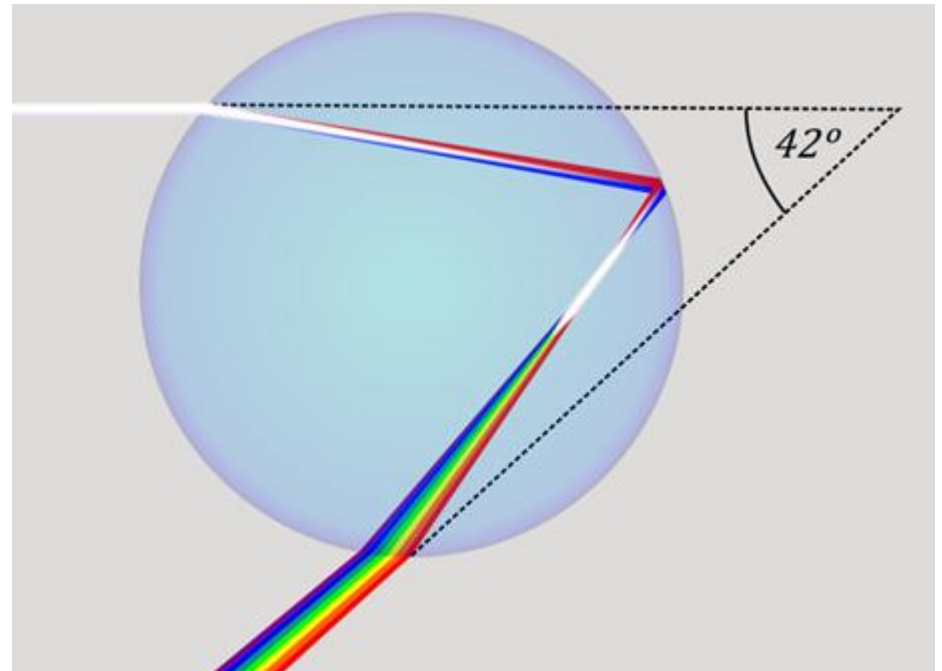
# **PART III**

## ***OPTICAL PHENOMENA***



# Rainbow

A rainbow is an optical phenomenon that is caused by both reflection and refraction of light in water droplets resulting in a spectrum of light appearing in the sky.



*See also the course web page.*

# Atmosphere Effect

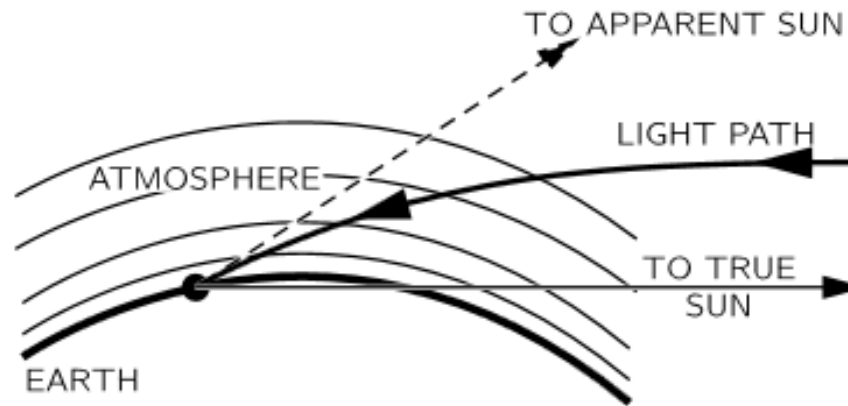


Fig. 26–7. Near the horizon, the apparent sun is higher than the true sun by about  $1/2$  degree.

The Feynman Lectures on Physics, Volume I

When we see the sun setting, it is already below the horizon!

*The earth's atmosphere is thin at the top and dense at the bottom. Light travels more slowly in air than it does in a vacuum, and so the light of the sun can get to point S beyond the horizon more quickly if, instead of just going in a straight line, it avoids the dense regions where it goes slowly by getting through them at a steeper tilt. When it appears to go below the horizon, it is actually already well below the horizon.*

# Mirage

A mirage is optical phenomenon in which light rays are bent to produce a displaced image of distant objects or the sky.



# Mirage

One sees “water” on the hot road, but when he gets there, it is as dry as the desert!.



Fig. 26–8. A mirage.

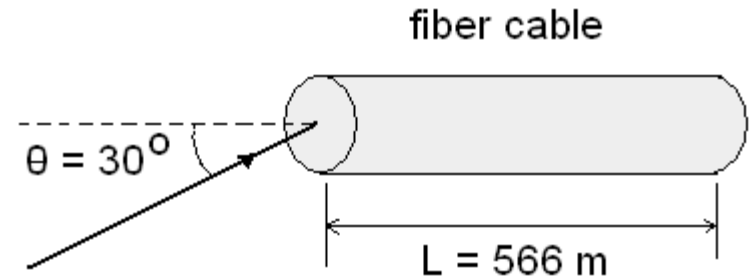
The Feynman Lectures on Physics, Volume I

*What we are really seeing is the sky light “reflected” on the road: light from the sky, heading for the road, can end up in the eye. The air is very hot just above the road but it is cooler up higher. Hotter air is more expanded than cooler air and is thinner, and this decreases the speed of light less. That is to say, light goes faster in the hot region than in the cool region. Therefore, instead of the light deciding to come in the straightforward way, it also has a least-time path by which it goes into the region where it goes faster for awhile, in order to save time. So, it can go in a curve.*

# Exercises

1. You might have noticed that emergency vehicles such as ambulances are often labeled on the front hood with reversed lettering (e.g., in Turkish SNALUBMA). Explain why this is so.
2. You run towards a plane mirror at a rate of 25 cm/s. What is the speed of your image w.r.t. mirror and w.r.t. you?

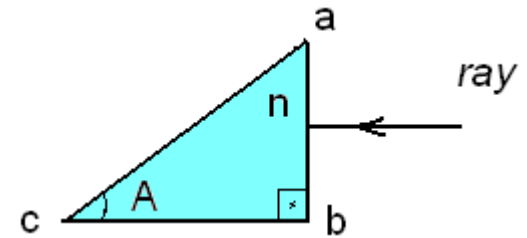
3. A light ray enters from the left into a fiber cable whose diameter is 2 mm and length is 566 m as shown. Determine  
(a) the number of TIRs that the ray can make in the cable.



- (b) the time required for the light ray to pass through the cable.

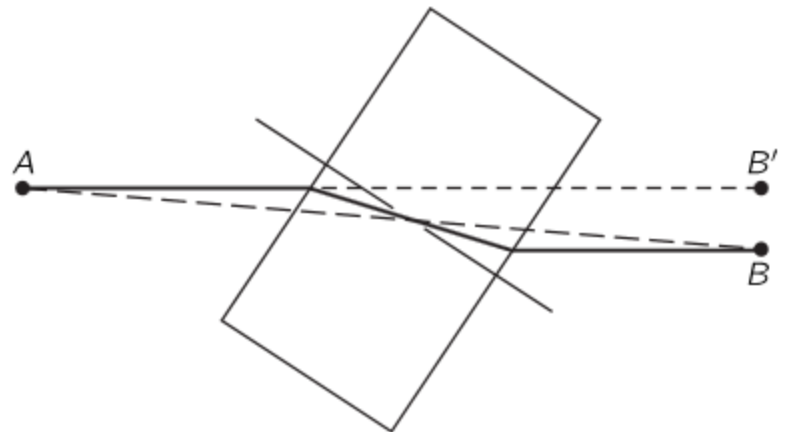
# Exercises

4. A light ray is incident normally on a face  $ab$  of a glass prism ( $n=1.52$ ) as shown.  
(a) Assume that the prism is immersed in air, find the largest value of the angle  $A$  so that the ray is totally reflected at face  $ac$ .  
(b) Find the angle  $A$  if the prism is immersed in water.



5. The atmosphere is a layer of gas whose index of refraction is  $n = 1.0003$  and width is  $D = 100$  km. The radius of Earth is approximately given by  $R = 6400$  km. Calculate the time difference in seconds for the sun rise at the sea level on the equator if the index of refraction was  $n = 1.0000$ ?

6. Using Fermat's Principle, analyze the path of the light passing through a block glass as shown.



# Exercises

7. The distance of a lightbulb from a large plane mirror is twice the distance of a person from the plane mirror. Light from the lightbulb reaches the person by two paths. It travels to the mirror at an angle of incidence  $A$  and reflects from the mirror to the person. It also travels directly to the person without reflecting off the mirror. The total distance traveled by the light in the first case is twice the distance traveled by the light in the second case. Find the value of the angle  $A$ .
8. A glass fiber ( $n = 1.50$ ) is submerged in water ( $n = 1.33$ ). What is the critical angle for light to stay inside the optical fiber?
9. A narrow beam of light is incident from air onto the surface of glass with index of refraction 1.50. Find the angle of incidence for which the corresponding angle of refraction is half the angle of incidence.

# References

1. Serway, Beichner, **Physics for Scientists and Engineers** 6th ed, Brooks/Cole
2. W.J.Simith, Modern Optical Engineering, 3rd Ed., McGraw-Hill
3. The Feynman Lectures on Physics, Volume I
4. <http://en.wikipedia.org/wiki/Prism>