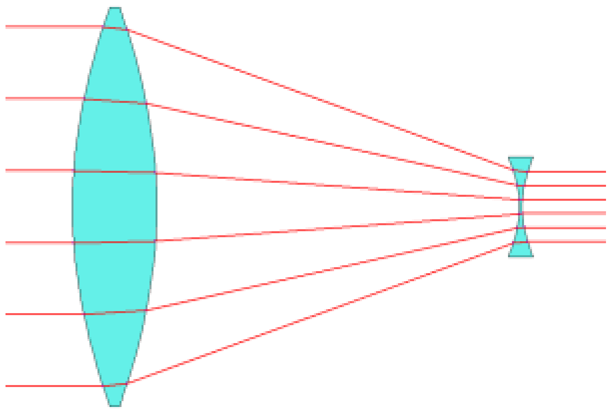
EP 324
Applied Optics

Topic 5

Inverse Square Law



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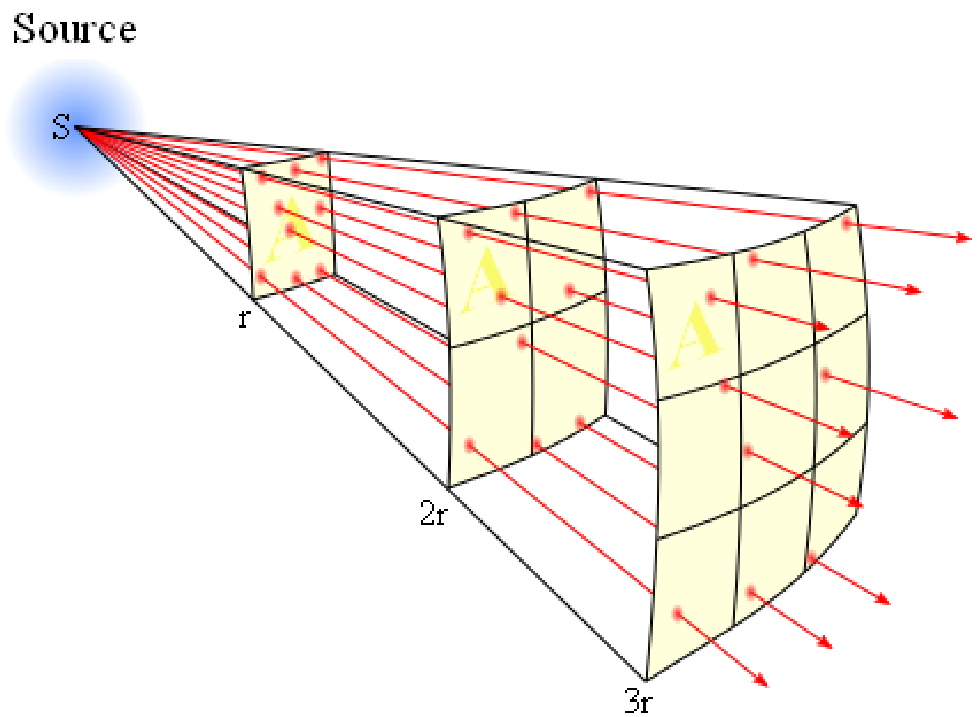
Inverse Square Law

states that:

a specified physical quantity or intensity is inversely proportional to the square of the distance from the source of that physical quantity.

$$\text{Intensity} \propto \frac{1}{\text{distance}^2}$$



- This is due to geometric dilution corresponding to point-source radiation into 3D space.
- The divergence of a vector field is the result of radial inverse-square law

Extracted from: *https://en.wikipedia.org/wiki/Inverse-square_law*

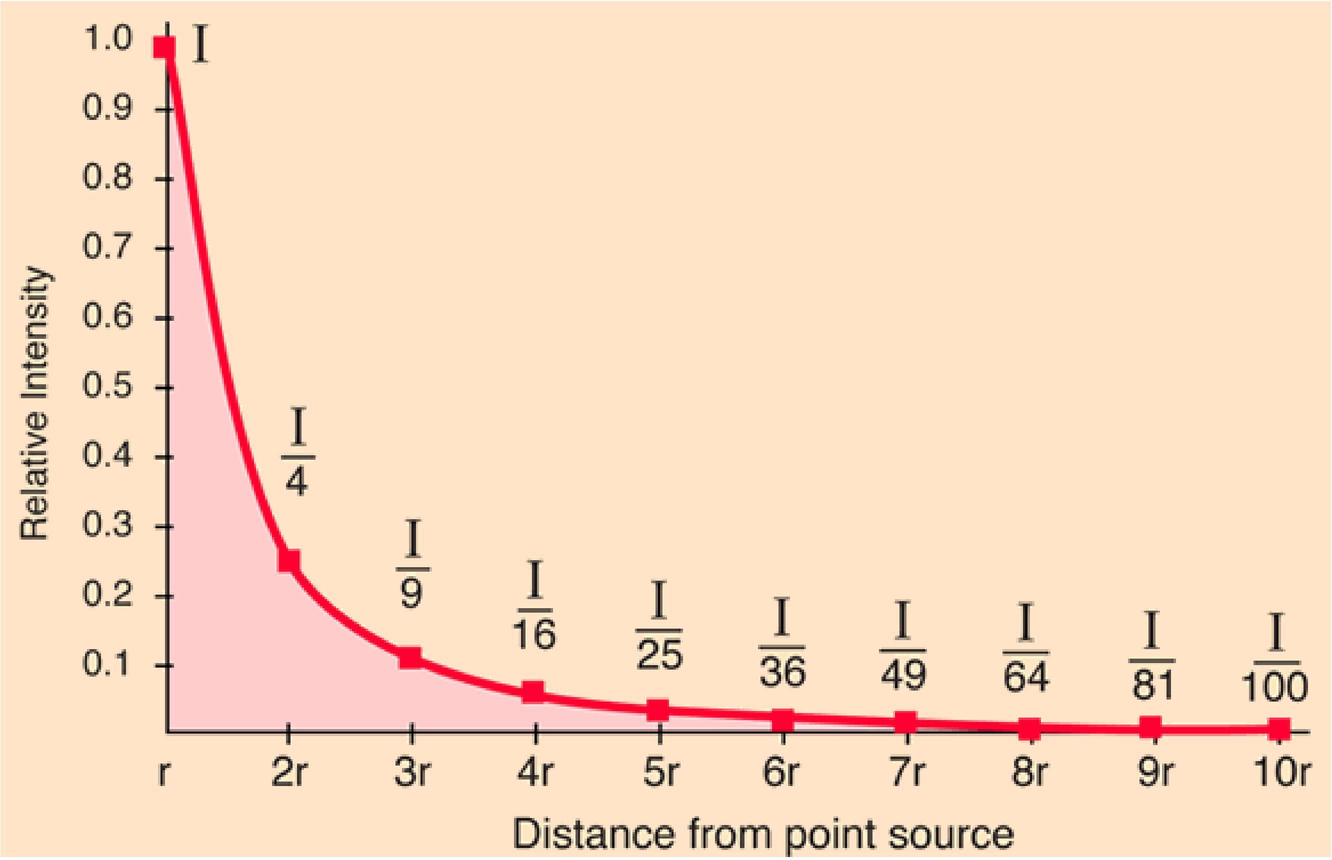
Inverse Square Law

can be applied to many areas:

- Newton's law of universal gravitation
- Coulombs Law
- Light Intensity
- Sound Intensity
- Radiation phenomena

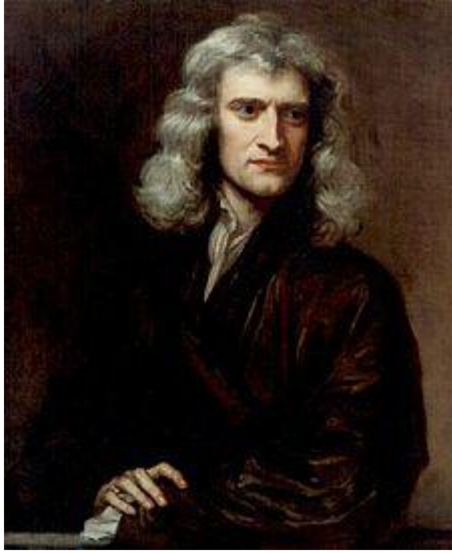
$$I \propto \frac{1}{r^2}$$

Inverse Square Law



$$I \propto \frac{1}{r^2}$$

Newton's Law of Gravitation(1687)

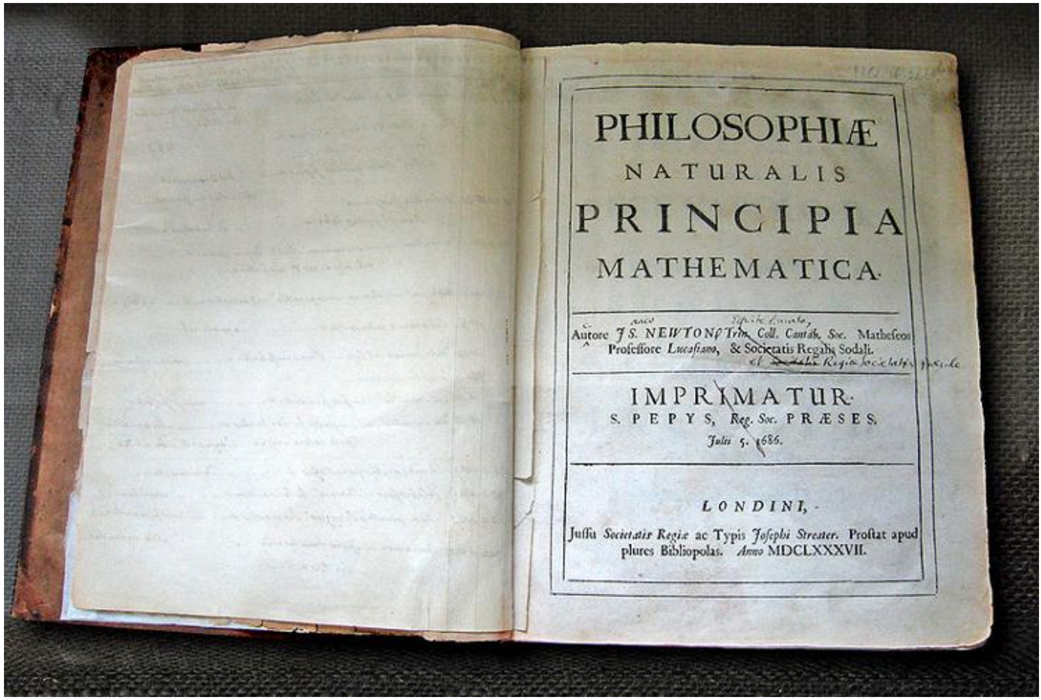


Mathematical Principles of Natural Philosophy

http://archive.org/details/philosophiaenatu00newt

Mentions about:

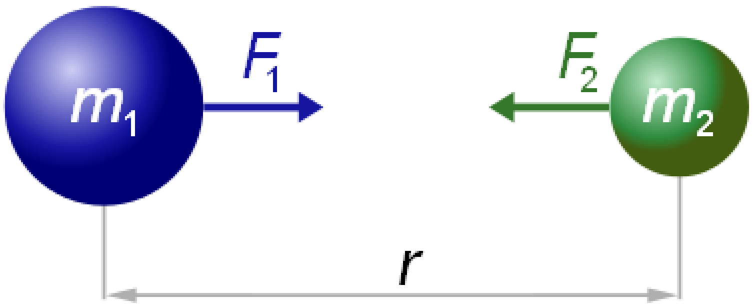
* Three laws of motion
* Universal law of gravitation



Newton's Law of Gravitation

Gratitational force between two objects:

- is proportional to masses and
- is inversely proportional to distance-square



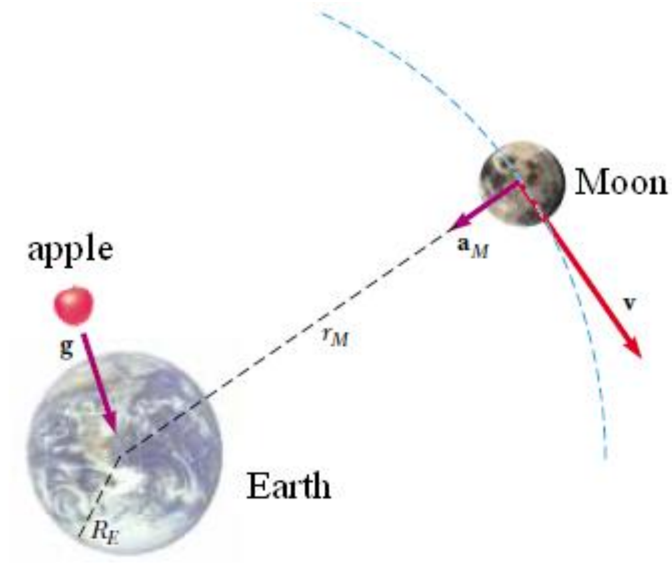
$$F = G\frac{m_1 m_2}{r^2}$$

$$F_1 = F_2 = F$$

$G = 6.673 \times 10^{-11}\ N.m^2/kg^2$

$$F = \frac{K}{r^2}$$

Newton's Law of Gravitation



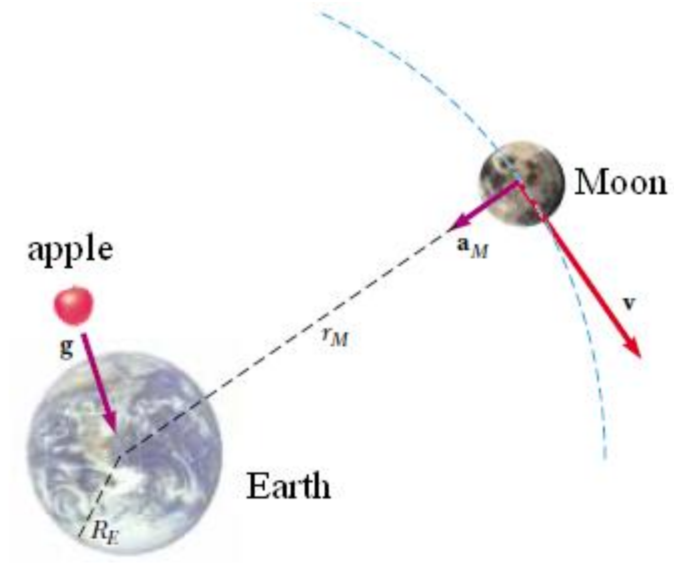
$$\frac{\text{Acceleration of Moon}}{\text{Acceleration on Earth}} = \frac{a}{g} = \frac{k/r_M^2}{k/r_E^2} = \left(\frac{r_E}{r_M}\right)^2$$

$$= \left(\frac{6.37\times10^6 m}{3.84\times10^8 m}\right)^2$$

$$= 2.75\times10^{-4}$$

$$a = (2.75\times10^{-4})(9.8\ \text{m/s}^2) = 2.70\times10^{-3}\ \text{m/s}^2$$

Newton's Law of Gravitation



Period of the moon: 27.3 days:

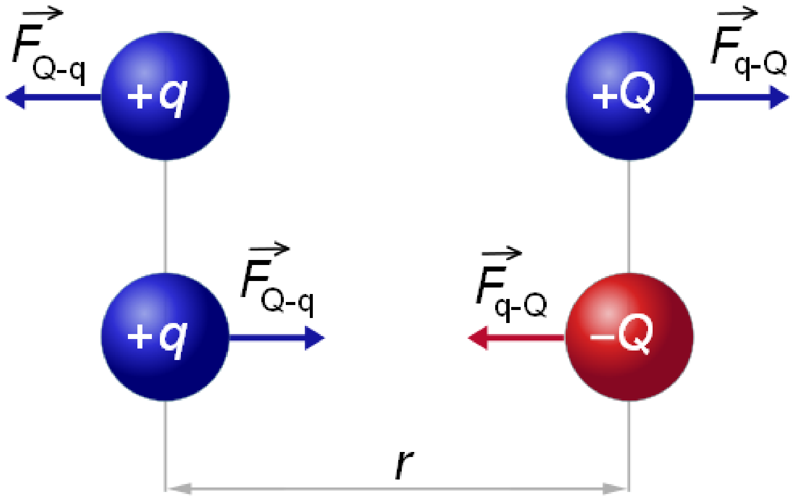
$$a = \frac{v^2}{r_M} = \cdots = 2.72 \times 10^{-3} \text{ m/s}^2$$

Percentage error:

$$\frac{2.72 \times 10^{-3} - 2.70 \times 10^{-3}}{2.70 \times 10^{-3}} \times 100 = \%\, 0.7$$

Couloms Law (1784)

Describes the electrostatic interaction between electrically charged particles:



$$F = k\frac{qQ}{r^2}$$

$k = 9 \times 10^9$ N.m^2/C^2

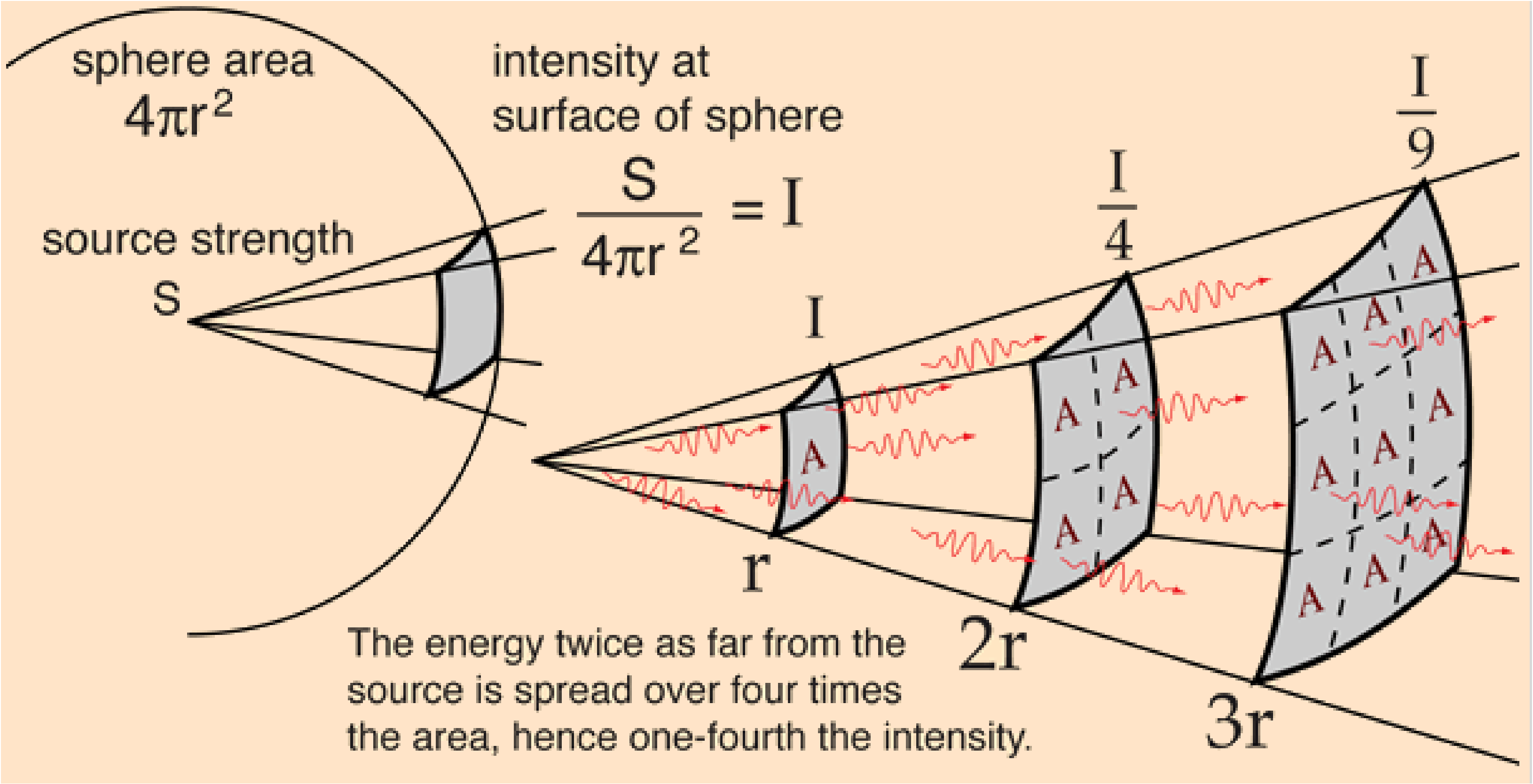
$$F = \frac{K}{r^2}$$

Radiadion

a point radiation source can be characterized by inverse square law.

S can be grams of a radioactive isotope or

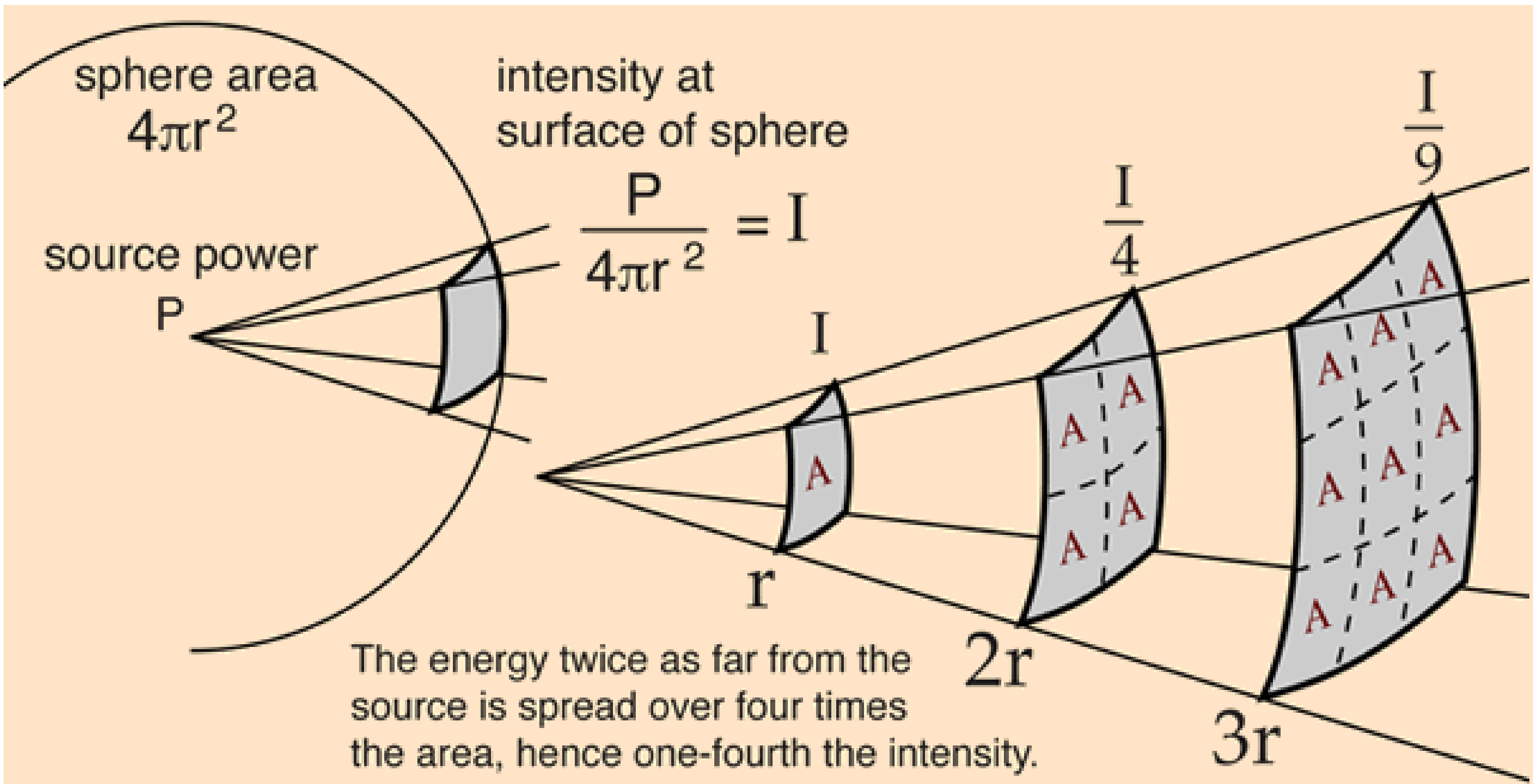
S can be in Curies.



http://hyperphysics.phy-astr.gsu.edu/hbase/forces/isq.html

Sound Intensity

The sound intensity from a point source of sound will obey the inverse square law if there are no reflections or reverberation.



Sound Intensity

Sound intensity is defined as the sound power (P) per unit area.

$$I = \frac{P}{4\pi r^2}$$

Many sound intensity measurements are made relative to a standard threshold of hearing intensity I_0:

$$I_0 = 10^{-12} \text{ W/m}^2$$

Decibel (dB)

Sound intensity measurement uses the decibel scale:

$$I(\mathrm{dB}) = 10\log(I/I_0)$$

Since sound is the pressure wave, sound intensity can be given by:

$$I(\mathrm{dB}) = 10\log(I/I_0) = 20\log(p/p_0)$$

Here, the pressure *p* is the amplitude of the pressure wave and

$$p_0 = 2\mathrm{x}10^{-5}\ \mathrm{N/m}^2$$

Common microphones produce a voltage which is proportional to the sound pressure. Change in sound intensity on the microphone is:

$$\Delta I(\mathrm{dB}) = 20\log(V_2/V_1)$$

where V_1 and V_2 are the measured voltage amplitudes.

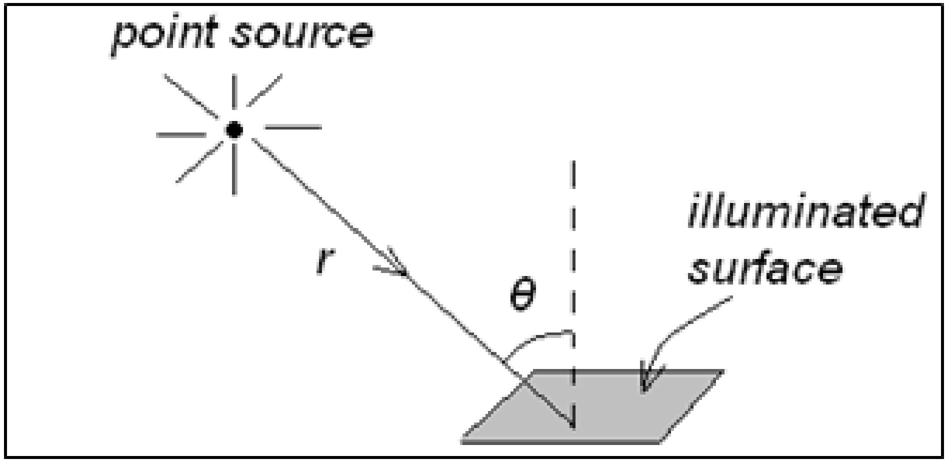
Light Intensity

Light intensity is the power output in Watts of a light source divided by the area over which the light is spread:

$$I = \frac{P}{4\pi\, r^2} = \frac{K}{r^2}$$

If the radiation direction makes an angle θ with the normal of the illuminated (irradiated) surface then the intensity is given by:

$$I = \frac{K}{r^2}\cos(\theta)$$



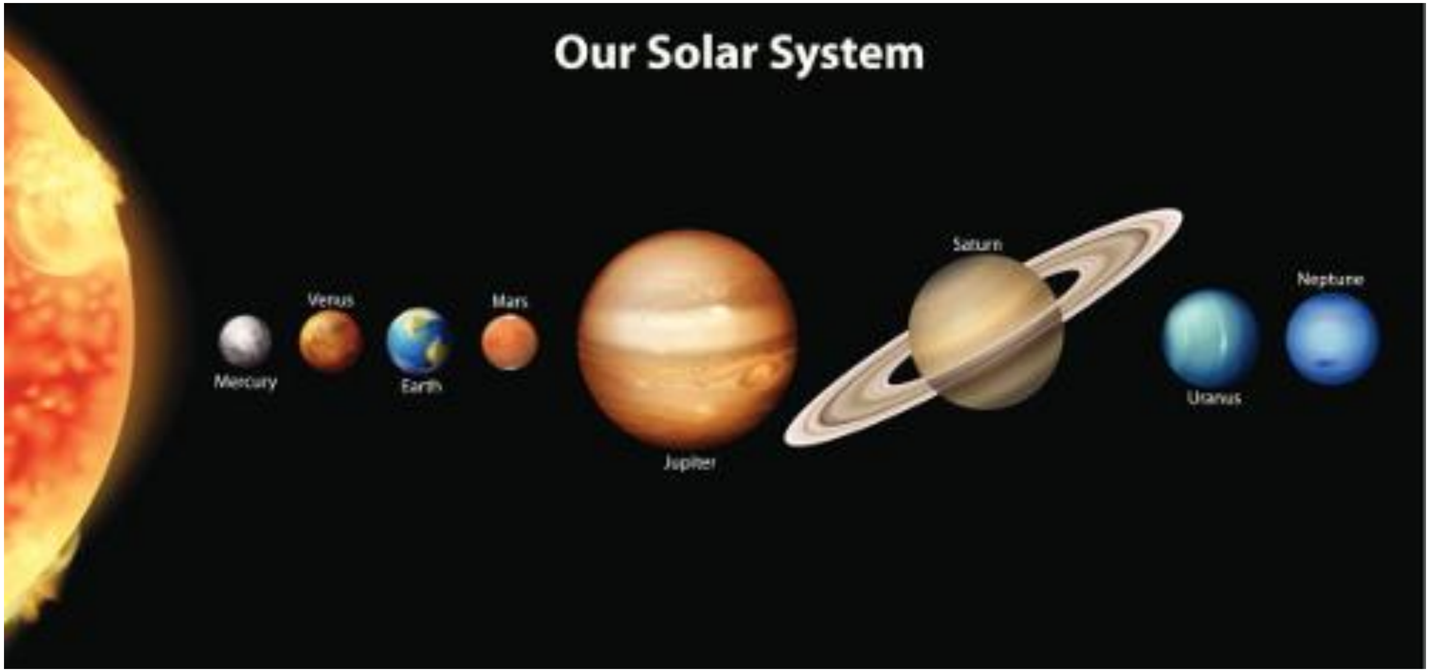
Light Intensity

The intensity of radiation from the Sun is

* 9126 watts per square meter at the distance of Mercury (0.387 AU)

* 1367 watts per square meter at the distance of Earth (1 AU)

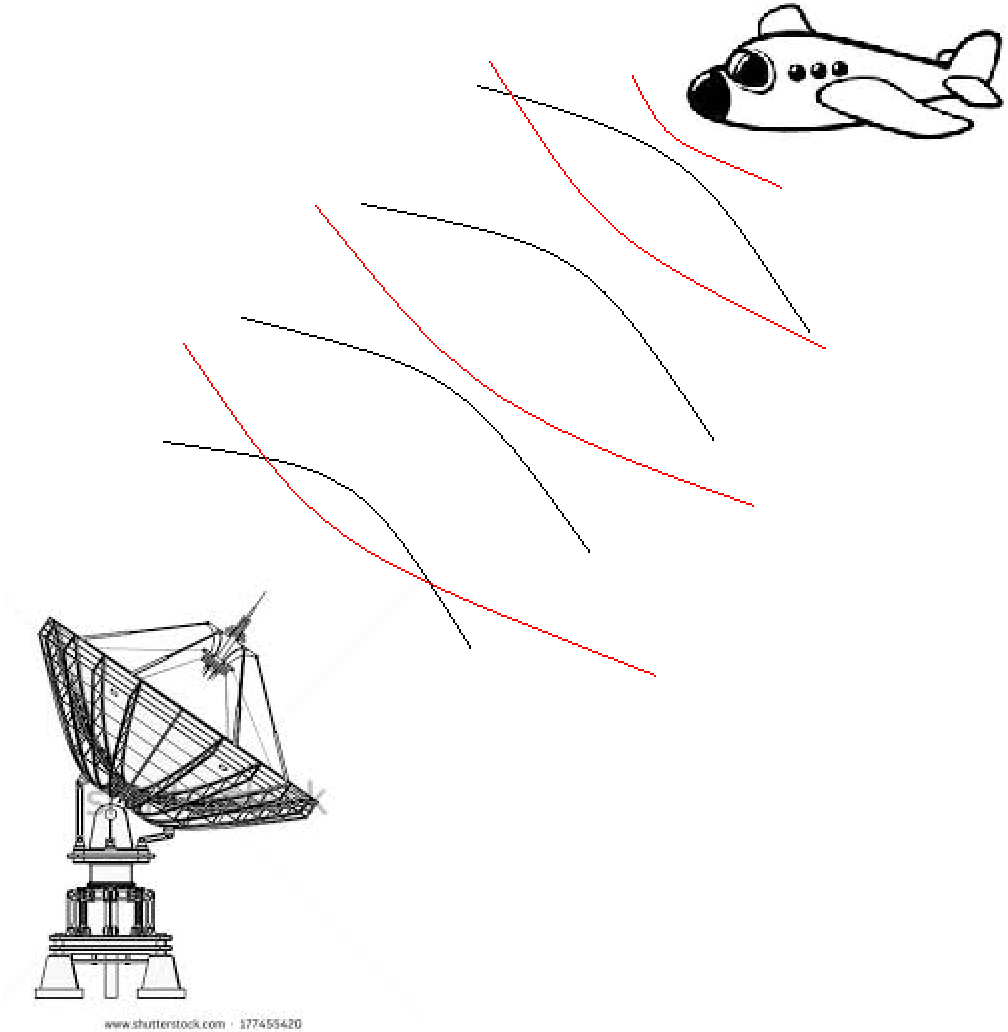
an approximate threefold increase in distance results in an approximate ninefold decrease in intensity of radiation.



Inverse Square Law (Light)

Radar will receive energy

according to $1/r^4$, why?



Light Intensity vs Angle

For a fixed (or constant) distance *r* from the point source then:

$$I = I_{\max} \cos(\theta)$$

Where $I_{max} = K/r^2$. This equation is called the **Lambert's cosine law** which says that

the intensity observed from an ideal

point source is proportional to the cos(θ).