EP375 Computational Physics

Topic 7
SYSTEM OF LINEAR EQUATIONS

Department of Engineering Physics
University of Gaziantep

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Content

1. Linear Algebraic Equations
2. Solutions in MATLAB
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Linear Algebraic Equations

- Mathematical formulations of engineering problems often lead to sets of simultaneous linear equations of the form:

\[
\begin{align*}
a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n &= b_1 \\
a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n &= b_2 \\
& \vdots \\
a_{n,1}x_1 + a_{n,2}x_2 + \cdots + a_{n,n}x_n &= b_n
\end{align*}
\]
This system can be written in matrix form:

$$
\begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix}
\begin{pmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{pmatrix}
=
\begin{pmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_n
\end{pmatrix}
$$

or simply:

$$A \mathbf{x} = \mathbf{b}$$

Here

- **A** is called the “coefficient matrix”
- **x** is the “unknown vector”
- **b** is the “constant vector” or “right hand side vector”
The solution of the system
\[ A \mathbf{x} = \mathbf{b} \]
is
\[ \mathbf{x} = A^{-1} \mathbf{b} \]
where \( A^{-1} = \text{adj}(A) / |A| \)
Here
\[ \text{adj}(A) \] is adjoint of the matrix \( A \)
\[ |A| \] is the determinant of \( A \).

Note that to get the solution the condition:
\[ |A| \neq 0 \]
must be satisfied.
Example 1: Consider the equation:

\[ x + y = 4 \]
\[ x - 3y = 0 \]

which can be written in matrix form:

\[
\begin{pmatrix}
1 & 1 \\
1 & -3
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
4 \\
0
\end{pmatrix}
\]

The solution is:

\[
\begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix}
1 & 1 \\
1 & -3
\end{pmatrix}^{-1} \begin{pmatrix}
4 \\
0
\end{pmatrix}
\]

Here:

\[
\begin{pmatrix}
1 & 1 \\
1 & -3
\end{pmatrix}^{-1} = \frac{1}{-4} \begin{pmatrix}
-3 & -1 \\
-1 & 1
\end{pmatrix} = \begin{pmatrix}
3/4 & 1/4 \\
1/4 & -1/4
\end{pmatrix}
\]

\[
\begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix}
3/4 & 1/4 \\
1/4 & -1/4
\end{pmatrix} \begin{pmatrix}
4 \\
0
\end{pmatrix} = \begin{pmatrix}
3 \\
1
\end{pmatrix} \rightarrow x = 3, y = 1
\]
MATLAB solution:

\[
\begin{pmatrix}
1 & 1 \\
1 & -3
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
4 \\
0
\end{pmatrix}
\]

\begin{verbatim}
>> A = [1 1; 1 -3];  % coefficient matrix
>> b = [4 ; 0];  % row vector
>> x = A \ b  % solution vector
x =
3
1
\end{verbatim}
Example 2: Consider the equation:

\[
\begin{align*}
x + y + z &= 6 \\
-2x + y &= 0 \\
3x + 2y + z &= 10
\end{align*}
\]

which can be written in matrix form:

\[
\begin{pmatrix}
1 & 1 & 1 \\
-2 & 1 & 0 \\
3 & 2 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
=
\begin{pmatrix}
6 \\
0 \\
10
\end{pmatrix}
\]

\[
\text{>> } A = \begin{bmatrix}
1 & 1 & 1 \\
-2 & 1 & 0 \\
3 & 2 & 1
\end{bmatrix};
\]

\[
\text{>> } b = \begin{bmatrix}
6 \\
0 \\
10
\end{bmatrix}';
\]

\[
\text{>> } x = A\backslash b
\]

\[
x = 
\begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}
\]
Example 3: Consider the circuit:

Two batteries with $\varepsilon_1 = 3$ V and $\varepsilon_2 = 5$ V are connected with three resistors $R_1 = 10$ Ω, $R_2 = 20$ Ω and $R_3 = 30$ Ω.

Using Kirchhoff's laws, find the currents $i_1$, $i_2$ and $i_3$ passing through the resistors.
## Eigenvalues

```matlab
>> A = [1 3 -2; 3 5 1; -2 1 4]
A =
    1     3    -2
    3     5     1
   -2     1     4

>> [v d] = eig(A)

v =
    -0.8184   -0.3153   -0.4804
     0.4347     0.2071   -0.8764
    -0.3758     0.9261     0.0324

d =
     -1.5120     0      0
     0     4.9045     0
     0      0    6.6076
```
HW 1: Consider the circuit:

Write a MATLAB function of the form

\[
\text{function potdiff}(a,b)\\
\]

to find (return) the potential difference between point \(a\) and \(b\) where \(a, b = 1, 2, 3, 4, 5\) or \(6\). For example,

\[
\begin{align*}
\text{>> } V &= \text{potdiff}(1,6) \\
\text{>> } V &= 200
\end{align*}
\]
HW 2: Consider the circuit:

Find the current flowing in each branch of this circuit.
HW 3:
An upward force of 25 N is applied at the top of a tripod as shown in figure. Determine the forces in the legs of the tripod.
References

