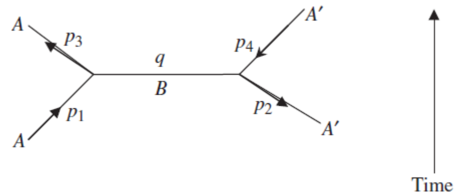
EP523 Introduction to QFT I

Topic 0

INTRODUCTION TO COURSE



Department of Engineering Physics

University of Gaziantep

September 2011

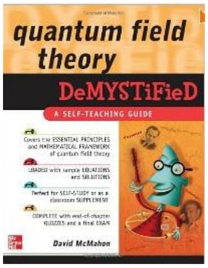
Content

- Introduction
- Review of SR, QM, RQM and EMT
- Lagrangian Field Theory
- An Introduction to Group Theory
- Discrete Symmetries and Quantum Numbers
- The Dirac Equation
- Scalar Fields
- Spin 1/2 Fields
- Spin 1 Fields

Resources for the Course

- **Course web page**
  http://www.gantep.edu.tr/~bingul/ep523

- **Course Book**



McGraw-Hill (2008)
David McMahon
Amazon:
http://www.amazon.com/Quantum-Field-Theory-Demystified-McMahon/dp/0071543821

- **Additional Web Resources:**
  quantumfieldtheory.info
  quantumfieldtheory.org
  www.damtp.cam.ac.uk/user/tong/qft.html
  Wikipedia

Lectures, Attendance, Exams

- **Fridays**
  three-hour lecture
  09:00-12:00 Engineering of Physics S?

- **Attendance**
  You must attend <u>all</u> of the lectures

- **Exams**
  - **Written Homeworks 70%**
  - **Written Final Exam 30% December 2011**

Introduction

Quantum Field Theory (QFT) is a theoretical framework that combines:

* Quantum Mechanics (QM) and
* Special Relativity (SR).

- **QM**: is a theory describing the behavior of small systems, such as atoms and individual electrons.
- **SR**: is the study of high energy physics, the motion of particles and systems at velocities near the speed of light (but without gravity).

QM

There are three key ideas we want to recall from QM

- physical observables are mathematical operators
  e.g. Hamiltonian (energy) of a SHO

$$\hat{H} = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$$

- the uncertainty relations
  between the position & momentum operators and energy & time

$$\Delta\hat{x}\,\Delta\hat{p} \geq \frac{\hbar}{2} \qquad \Delta E\,\Delta t \geq \frac{\hbar}{2}$$

- the commutation relations. In particular,

$$[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$$

SR

Energy and mass relation:

$$E = mc^2$$

- if there is enough energy—that is, enough energy proportional to a given particle's mass—then we can "produce" the particle.
- Due to conservation laws, we actually need twice the particle's mass, so that we can produce a particle and its antiparticle.

So in high energy processes,

- Particle number is not fixed.
- The types of particles present are not fixed.

The last two facts are in direct conflict with nonrelativistic Quantum Mechanics !

Schrödinger Equation

In Non-Relativistic Quantum Mechanics (NRQM) we describe the dynamics of a system with the Schrödinger equation, which for a particle moving in one dimension with a potential $V = V(x)$ is

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

- We can extend this formalism to treat the case when several particles are present. However, the number and types of particles are absolutely fixed.

- The Schrödinger equation cannot in any shape or form handle changing particle number or new types of particles appearing and disappearing as relativity allows.

Klein-Gordon Equation

Early attempts to merge QM and SR focused on generating a relativistic version of the Schrödinger equation known as Klein-Gordon Equation (KGE):

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}-\frac{\partial^2\varphi}{\partial x^2}=\frac{m^2c^2}{\hbar^2}\varphi$$

- Schrödinger discarded KGE, because it gave the wrong fine structure for the hydrogen atom.
- KGE appears to give negative probabilities, something that obviously contradicts the spirit of quantum mechanics.
- KGE allows negative energy states!

Dirac Equation

The next attempt at a relativistic quantum mechanics was made by Dirac. His famous equation is

$$i\hbar\frac{\partial\psi}{\partial t}=-i\hbar c\vec{\alpha}\cdot\vec{\nabla}\psi+\beta mc^2\psi$$

- Here α and β are actually matrices.
- Dirac Equation (DE) resolves some of the problems of the KGE but also allows for negative energy states.

Why QFT?

As we have mentioned, in QM

- we deal with a single particle such as an electron in a potential
  e.g. square well, harmonic oscillator, etc.

- the particle retains its integrity
  e.g. an electron remains an electron throughout the interaction.

- there is no general way to treat interactions between particles
  e.g. a particle and its antiparticle annihilating one another to yield neutral particles such as:

$$e^{+} + e^{-} \rightarrow \gamma + \gamma$$

- there is no way to describe the decay of an elementary particle
  e.g a muon decay

$$\mu^{-} \rightarrow e^{-} + \bar{\upsilon}_{e} + \upsilon_{\mu}$$

Q.F.T.

- provides a means whereby particles can be annihilated, created, and transmigrated from one type to another.

- is a relativistic theory, and thus more all encompassing.

- is an extrapolation of non-relativistic quantum mechanics (NRQM) to relativistic quantum mechanics (RQM).

QFT Approach

- The problem was to get a relativistic wave equation.
- Part of the problem with these relativistic wave equations is in their interpretation.
- In order to be truly compatible with special relativity, we need to discard the notion that φ and ψ in KGE and DE describe single particle states. In their place, we propose the following new ideas:
  - The wave functions φ and ψ are not wave functions at all, instead they are *fields*.
  - The fields are operators that can create new particles and destroy particles.

Poisson Brackets

- Classical particle theories contain rarely used entities call *Poisson brackets*.
- Poisson brackets are mathematical manipulations of certain pairs of properties (*dynamical variables like position and momentum*)
- Poisson brackets for two objects *A* and *B* is represented by

$$\{A, B\}$$

For position *x* and its conjugate momentum p_x

$$\{x, p_x\}$$

- Following rules are valid

$$\{x, p_x\} = \{y, p_y\} = \{z, p_z\} = 1$$

$$\{x, p_y\} = \{x, p_z\} = 0$$

$$\{y, p_x\} = \{y, p_z\} = 0$$

$$\{z, p_x\} = \{z, p_y\} = 0$$

in general:

$$x_1 = x \quad \text{and} \quad p_x = p_1$$

$$x_2 = y \quad \text{and} \quad p_y = p_2$$

$$x_3 = z \quad \text{and} \quad p_z = p_3$$

$$\{x_i, p_j\} = \delta^i_j$$

First Quantization (Particle Theories)

Shortly after NRQM theory had been worked out, theorists, led by Paul Dirac, realized that *for each pair of quantum operators that had non-zero (zero) commutators, the corresponding pair of classical dynamical variables also had non-zero (zero) Poisson brackets*.

1. Assume the quantum particle Hamiltonian has the same form as the classical particle Hamiltonian.

2. Replace the classical Poisson brackets for conjugate properties with commutator brackets:

$$\{x_i, p_j\} = \delta^i_j \rightarrow [x_i, p_j] = i\hbar\delta^i_j$$

the classical properties (dynamical variables) of position and its conjugate 3-momentum become quantum non-commuting operators.

We will see later…

Second Quantization (Particle Theories)

In QFT, since we have promoted the fields to the status of operators, they must satisfy commutation relations.

1. Assume the quantum field Hamiltonian *density* has the same form as the classical field Hamiltonian *density*.

2. Replace the classical Poisson brackets for conjugate property densities with commutator brackets:

$$\{x_i, p_j\} = \delta^i_j \rightarrow [x_i, p_j] = i\hbar\delta^i_j \rightarrow [\hat{\phi}(x,t), \hat{\pi}(y,t)] = i\hbar\delta(x-y)$$

* $\hat{\pi}(y,t)$ (another field) is the conjugate momentum *density* of the field φ and plays the role of momentum in QFT.

* *x* and *y* are two points in space. if two fields are spatially separated they cannot affect one another

We will see later…

In QM,

position x is an operator while time t is just a parameter.

In SR,

position and time are on a similar footing.

In QFT,

> Fields ϕ and ψ are operators.
> They are parameterized by spacetime points (x, t).
> Position x and time t are just numbers that fix a point in spacetime—they are not operators.
> Momentum continues to play a role as an operator.

We will see later…

Comparison of QM, RQM and QFT

- In NRQM & RQM solutions are states (particles such as an electron)
- In QFT solutions are operators that produce and destroy states and QFT accommodates multi-particle states.
- QFT can handle transmutation of particles from one kind into another (e.g., muons into electrons, by destroying the original muon and creating the final electron), whereas NRQM and RQM can not.
- The problem of negative energy state solutions in RQM does not appear in QFT.
- And in both RQM and QFT (as well as NRQM), operators act on states in similar fashion. Expected energy measurement is determented same way in both theories:

$$< E >= \langle \phi \,|\, H \,|\, \phi \rangle = \int \phi^{*} H \phi \, dV$$

	NRQM	RQM	QFT
Wave equation	Schroedinger	Klein-Gordon (spin 0) Dirac (spin ½) Proça (spin 1) Special case of Proça: Maxwell (spin 1 massless)	Same as RQM at left
Solutions to wave equation	States	States	Operators that create and destroy states
Negative energy states?	No	Yes	No
Particles per state	Single*	Single*	Multi-particle
Expectation values	$\bar{\mathcal{O}} = \langle\phi\|\mathcal{O}\|\phi\rangle$	As at left, but relativistic.	As at left in RQM.
Phenomena:			
1. bound states	Yes, non-relativistic	Yes, relativistic	Yes (usually not studied in introductory courses)
2. scattering			
a. elastic	a. Yes	a. Yes	a. Yes
b. inelastic (transmutation)	b. No (though some models can estimate)	b. Yes and no. (i.e., cumbersome, but can be done, though only for particle/antiparticle creation & destruction.)	b. Yes
3. decay			
a. composite particles	a. Yes (tunneling)	a. Yes	a. Yes
b. elementary particles	b. No	b. No	b. Yes
4. vacuum	No	Yes and no.	Yes

Lagrangian Field Theory

In QFT, we frequently use tools from classical mechanics to deal with fields. Specifically, we often use the Lagrangian:

$$L = T - V$$

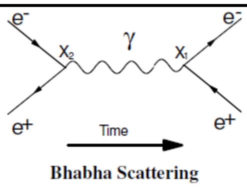
The Lagrangian is important because symmetries (such as rotations) leave the form of the Lagrangian invariant.

The classical path taken by a particle is the one which minimizes the action:

$$S = \int L dt$$

We will see how these methods are applied to fields in later chapters.

Main Aim of QFT



We want to understand Nature.

- To do so, we need to be able to predict the outcomes of particle accelerator scattering experiments and elementary particle half lives.
- To do this, we need to be able to calculate probabilities for scattering, and decay, to occur.
- To do that, we need to be able to calculate transition amplitudes for specific elementary particle interactions.
- And for that, we need first to master a fair amount of theory, based on the postulates of quantization.

References

[1]. David McMahon, *Quantum Field Theory Demistified*, (2008), The McGraw-Hill Companies.

[2]. www.quantumfiledtheory.info