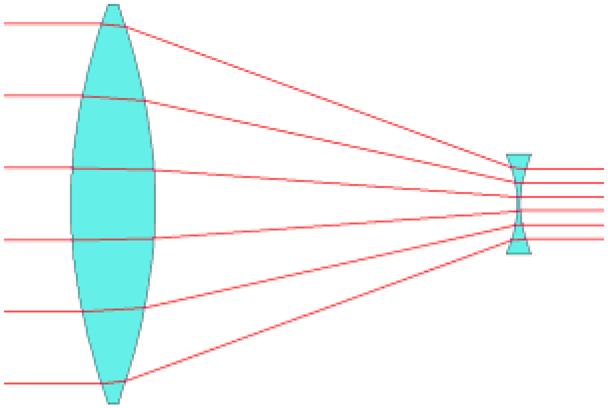
OPAC 101
Introduction to Optics

Topic 4

Aberrations



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Dec 2020

Content

Introduction

- In an ideal optical system, all rays of light from a point in the object plane would converge to the same point in the image plane, forming a clear image.

- The influences which cause different rays to converge to different points are called **aberrations**.

- Aberration leads to blurring of the image produced by an image-forming optical system. It occurs when light from one point of an object after transmission through the system does not converge into (or does not diverge from) a single point.

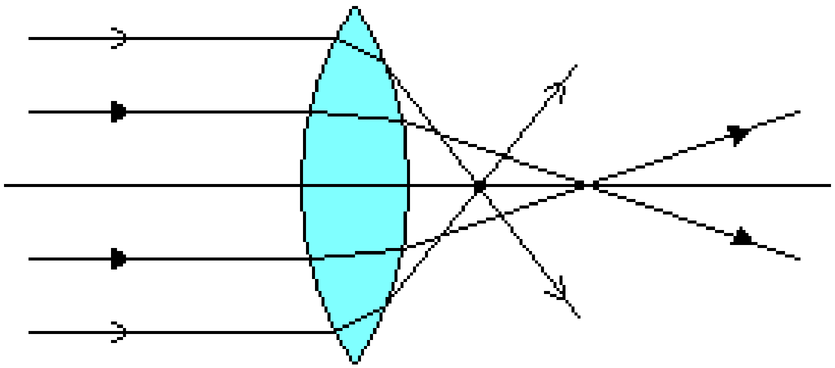
- Optical Instrument-makers need to correct optical systems to compensate for aberration.

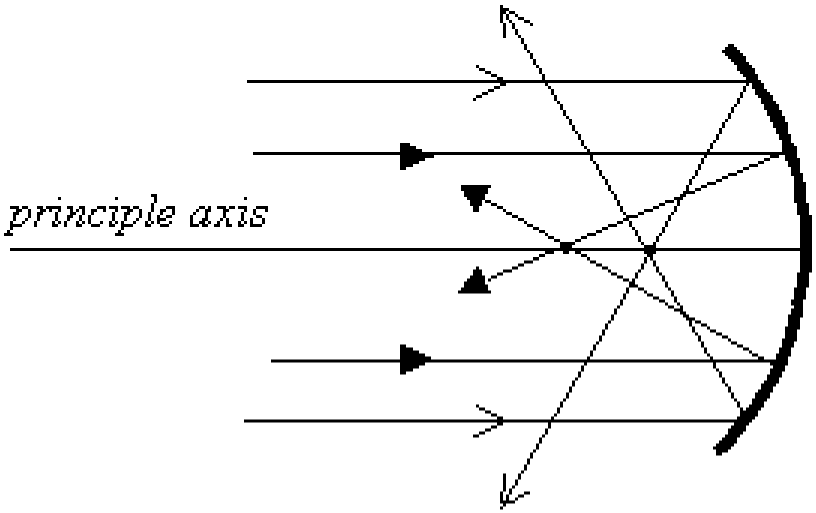
- In this chapter, we will consider spherical and chromatic aberrations and their corrections.

Spherical Aberrations (SA)

- Spherical aberrations occur because the focal points of rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis.

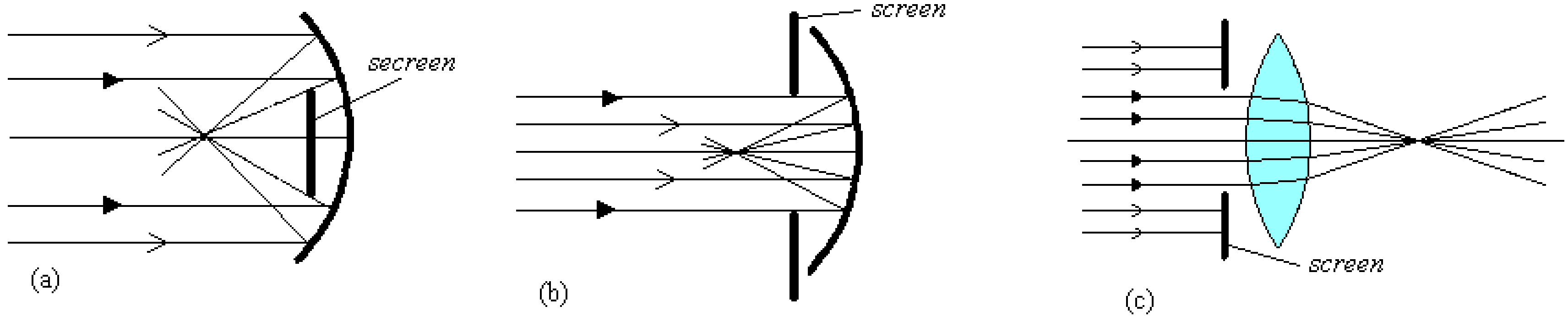
- Figures given below illustrate *spherical aberrations* for parallel rays passing through a converging lens, and reflecting from a concave mirror. Rays are converged to different points instead of a single focal point.





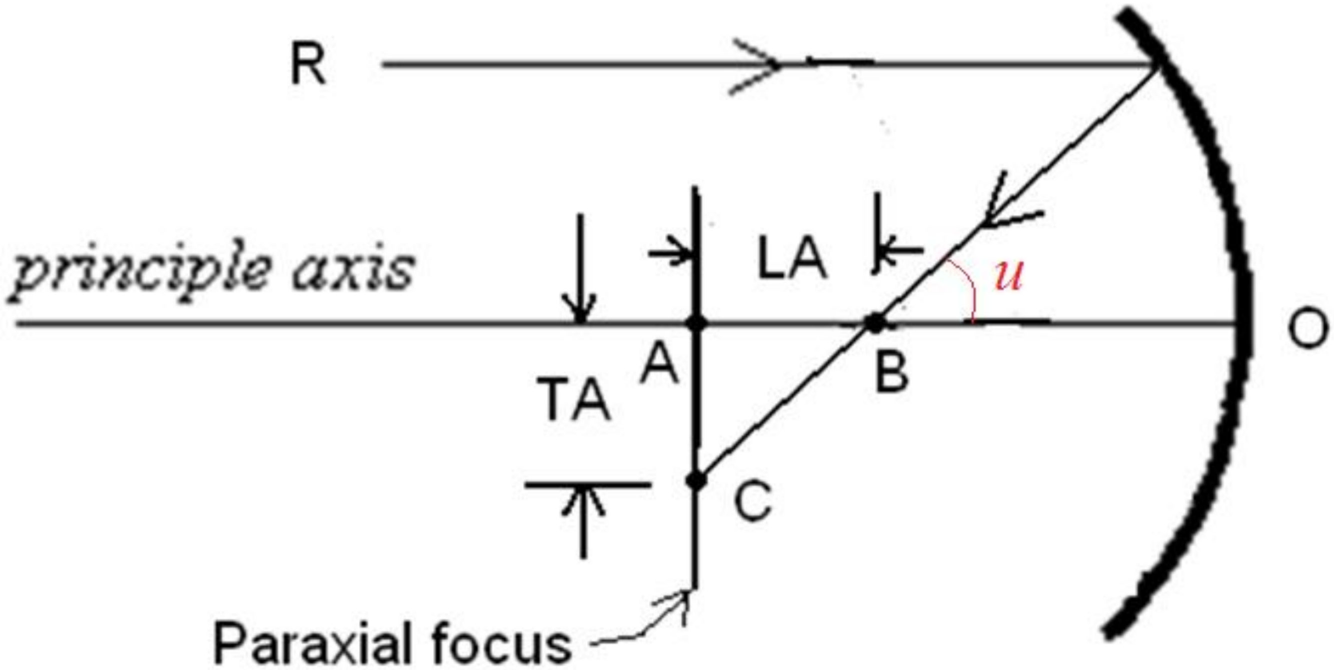
- Spherical aberration can be reduced by a screening method.

- To do this, a screen (pupil) can be placed in front of the mirror or lens allowing parallel rays to focus on a single point.



- NOTE THAT
  - Screening reduces the brightness (light intensity) of the image while it reduces the spherical aberrations.
  - Many cameras have an adjustable aperture to control light intensity and reduce spherical aberration.

Longitudinal & Transverse Aberrations in Mirror

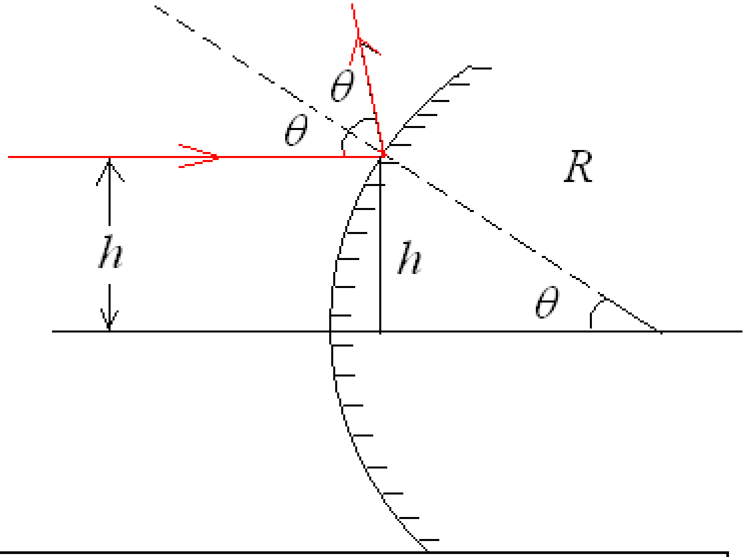


For ray R:

LA = Paraxial distance – Trigonometric distance = |OA| - |OB|

TA = LA tan(u)   [u is the angle with the principle axis].

Example 1

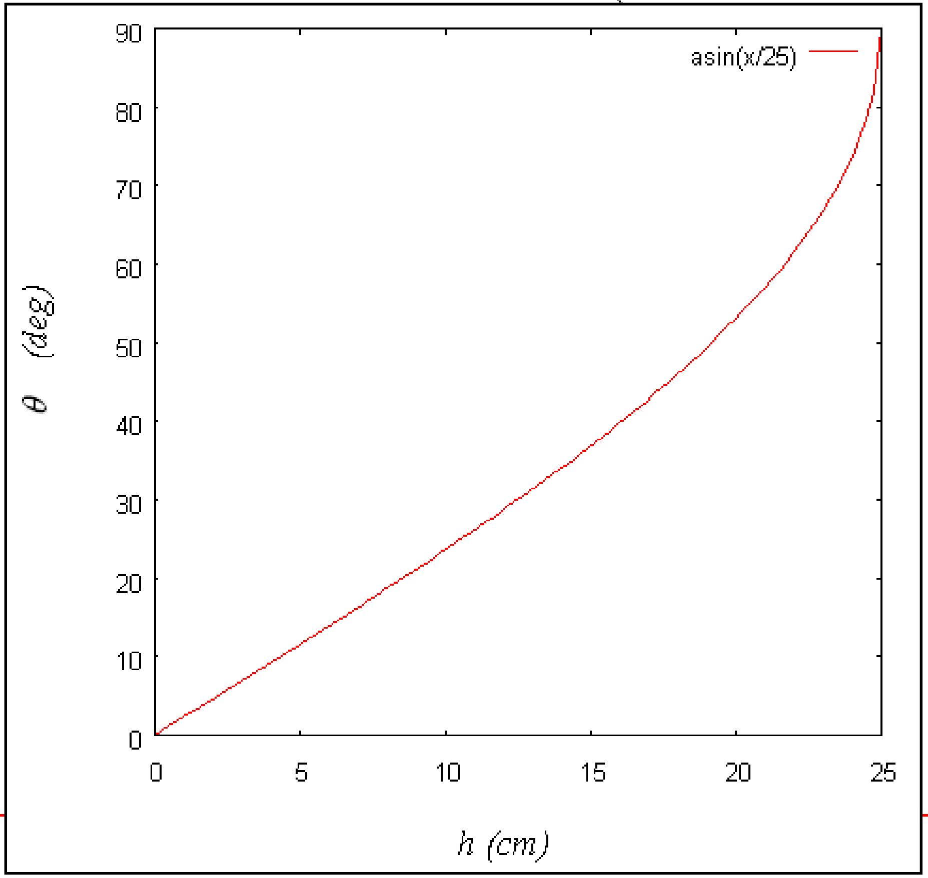


A ray traveling parallel to the principal axis strike a convex mirror whose radius of curvature is R = 25 cm as shown in Figure. Assume that the ray is at distance h = 10 cm.

(a) Find the angle θ.

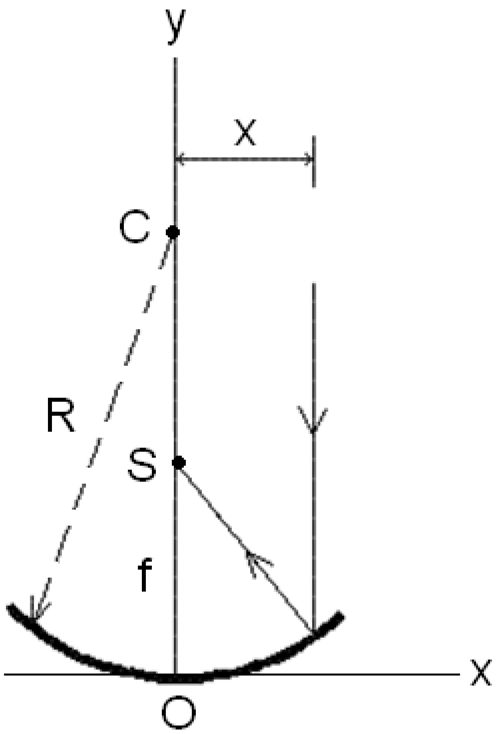
(b) Find LA and TA

SOLUTION



Example 2

A light ray traveling parallel to the principal axis at a distance *x* from the principal axis strike a concave mirror having a radius of curvature *R* as shown.



The ray is focused on point S such that |OS| = *f*.

(a) Find the distance *f* if x = 2 cm and *R* = 10 cm.

(b) Find LA and TA

SOLUTION

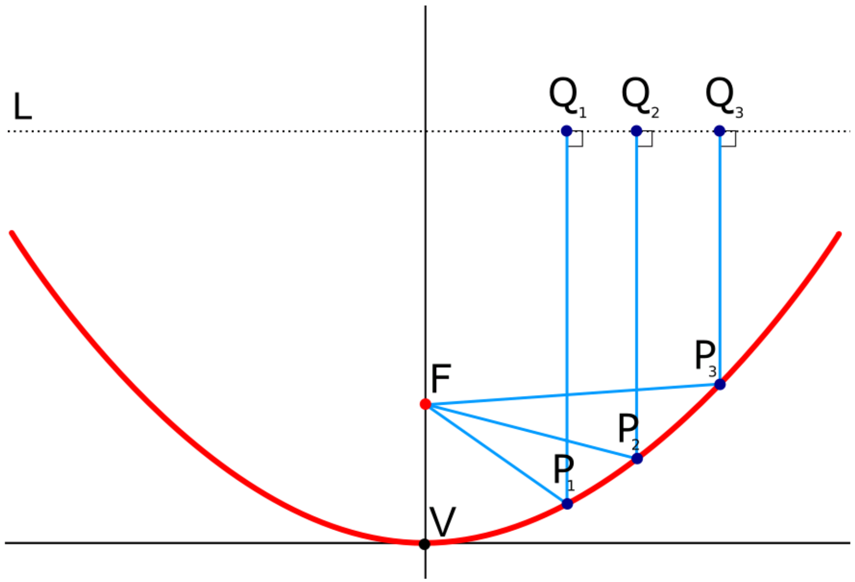
- To overcome the spherical aberration in mirrors, parabolic reflecting surface rather than a spherical surface is used.

- Parabolic surfaces are not used often **-->** *very expensive to make*.

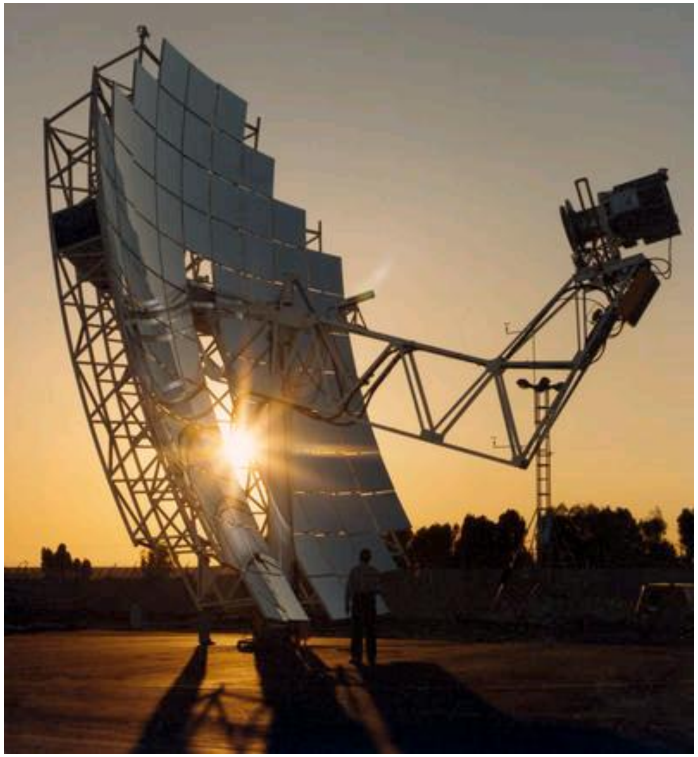
- Parallel light rays incident on a parabolic surface focus at a common point, regardless of their distance from the principal axis.

- Parabolic reflecting surfaces are used in many astronomical telescopes to enhance image quality.

- One can prove why we need parabolic reflecting surface (See Appendix at the end)



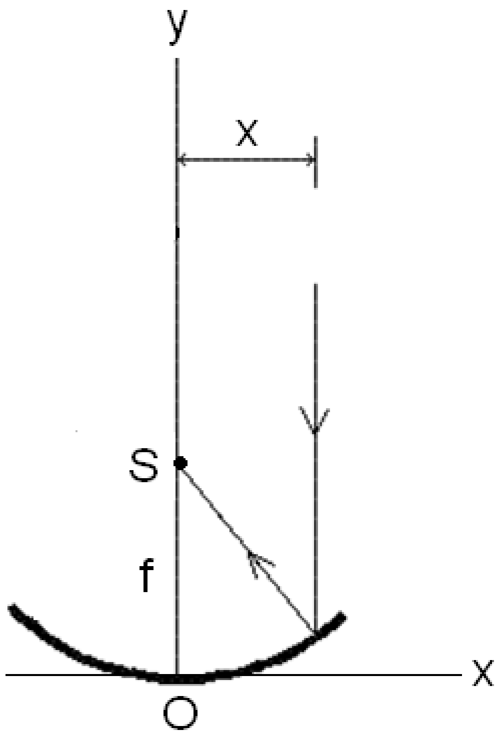
A parabolic solar dish



The mirror of the Hubble Space Telescope

Example 3

A light ray traveling parallel to the principal axis at a distance *x* from the principal axis strike a parabolic mirror. Find *f* if the equation is $y = 2x^2$. Here both x and y are measured in cm.

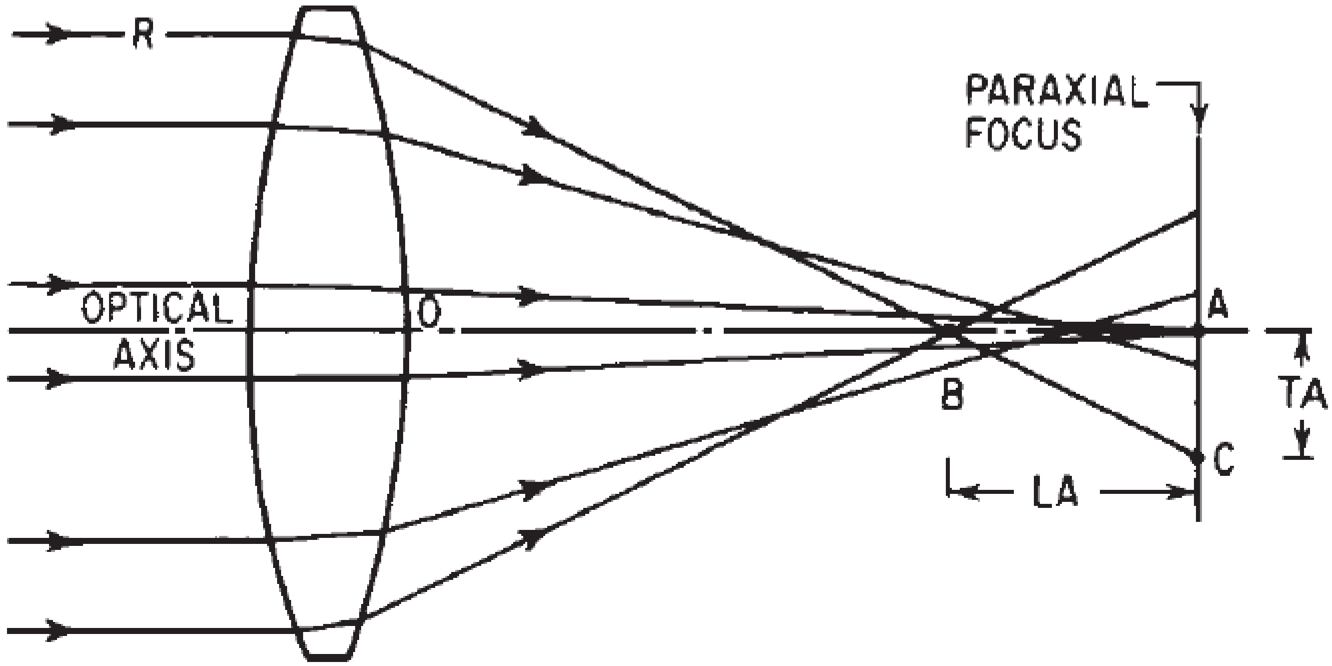


See Appendix.

SOLUTION

$$f = \frac{x^2}{4y} = \frac{x^2}{4(2x^2)} = \frac{1}{8} = 0.125\,\text{cm}$$

Longitudinal & Transverse Aberrations in Lens



For ray R:

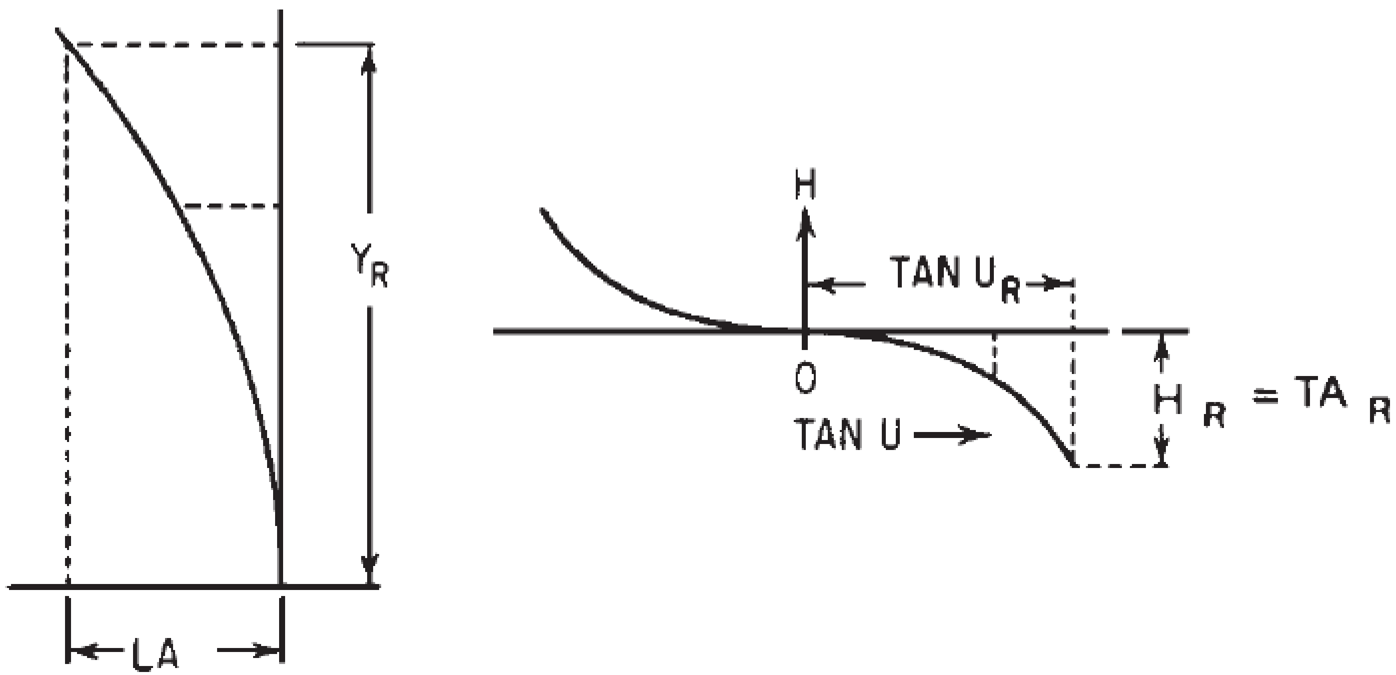
LA = Paraxial distance – Trigonometric distance = |OA| - |OB|

TA = LA tan(u)   [u is the angle with the principle axis].

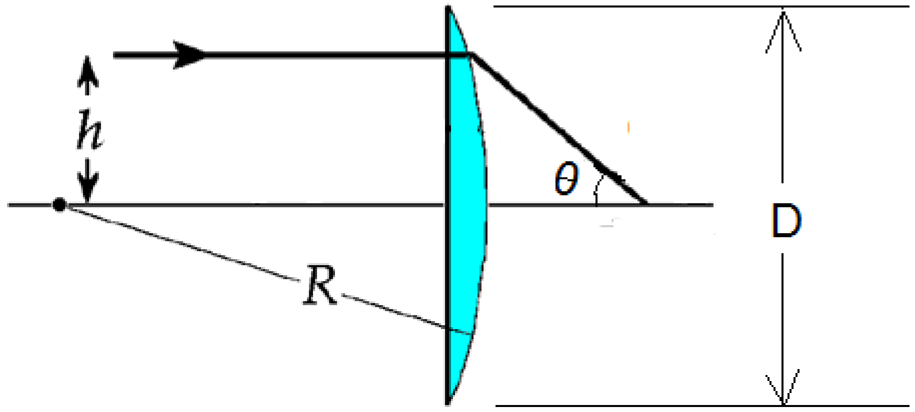
Longitudinal & Transverse Aberrations

The spherical aberration of a system is usually represented graphically.

- Longitudinal spherical is plotted against the ray height.
- Transverse spherical is plotted against the final slope of the ray,



Example 4



Plano-convex lens:

R = 40.0 cm

D = 12.0 cm

h = 4.0 cm

n = 1.5

(a) Find the effective focal length (paraxial focal length)

(b) Find longitudinal and transverse spherical aberrations.

(c) Plot h vs LA and h vs TA graphs.

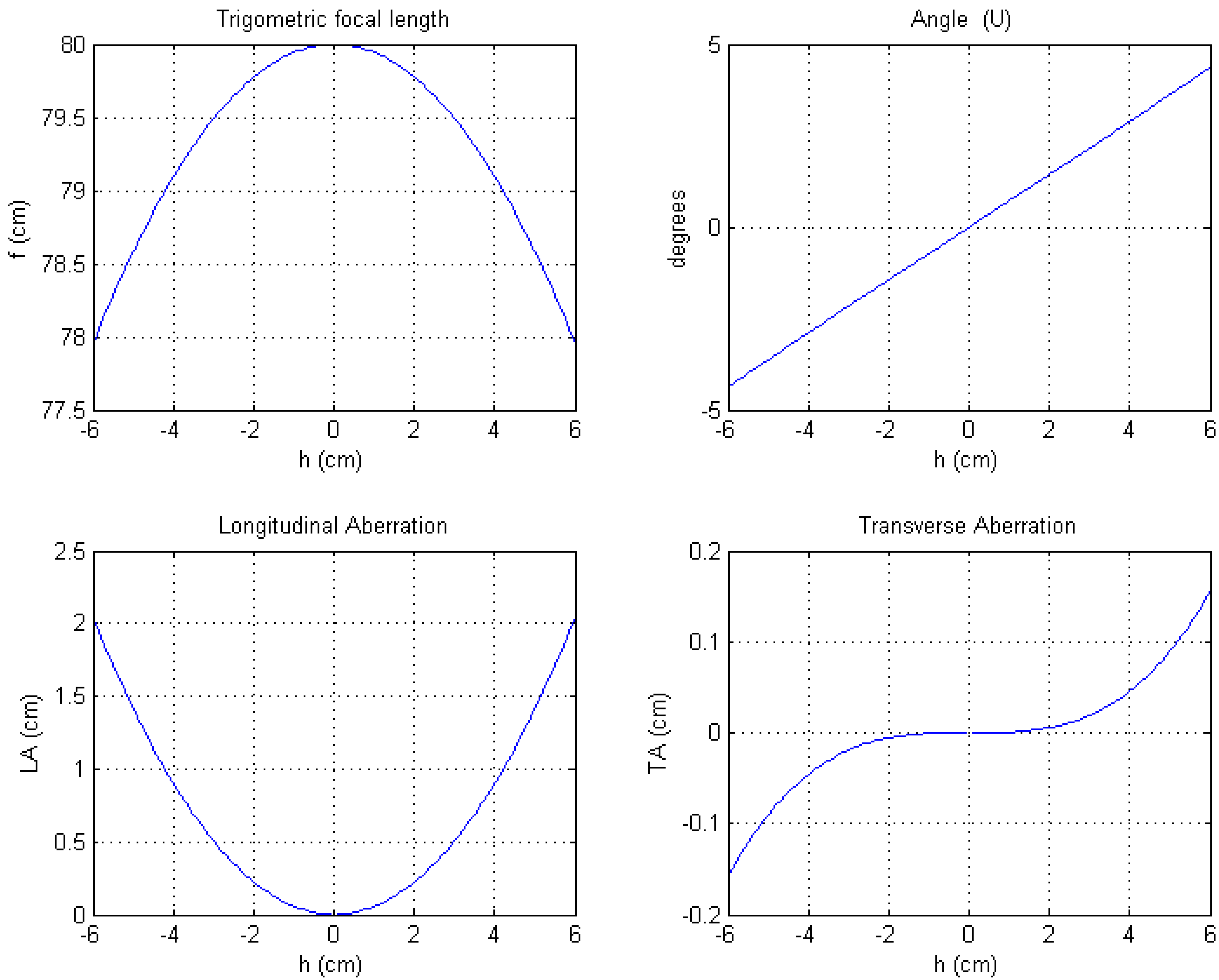
[Ans: f = 80 cm, LA = 0.9040 cm, TA = 0.0456 cm].

What if you change the orientation of the lens?

(That is to say, curved face will look at the incoming ray)

Repeat case (a), (b) and (c).

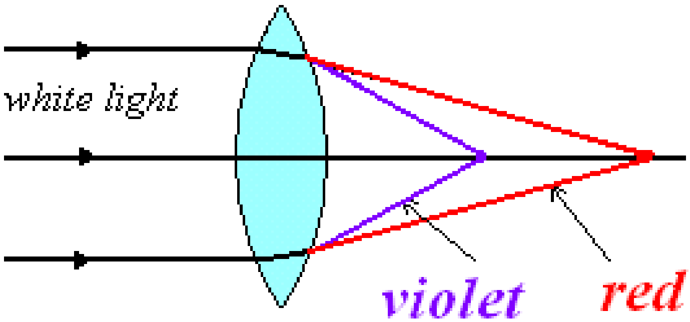
Example 4 (Plots)

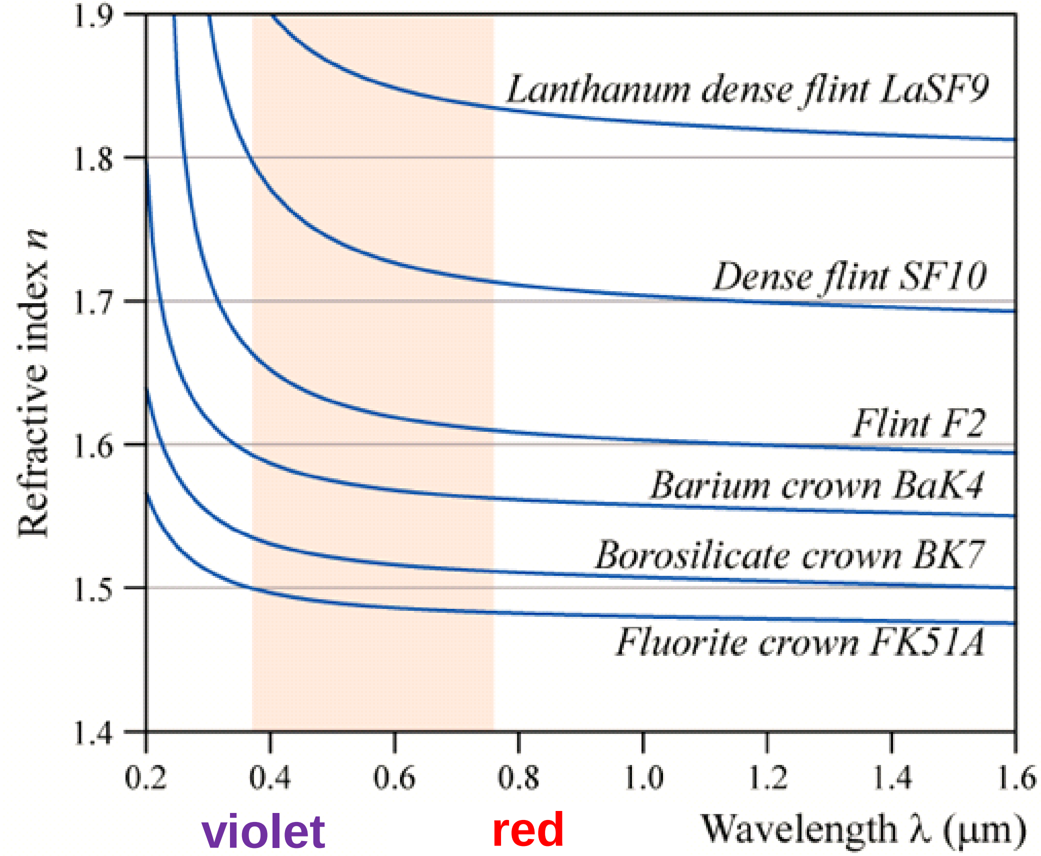


Chromatic Aberrations (CA)

- A lens will not focus different colors in exactly the same place.

- CA occurs because lenses have a different refractive index for different wavelengths of light (the dispersion of the lens).



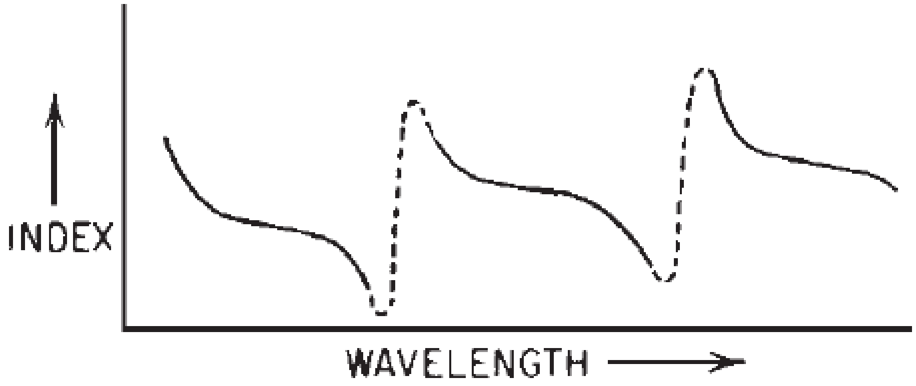


$$v = \frac{c}{n(\lambda)}$$

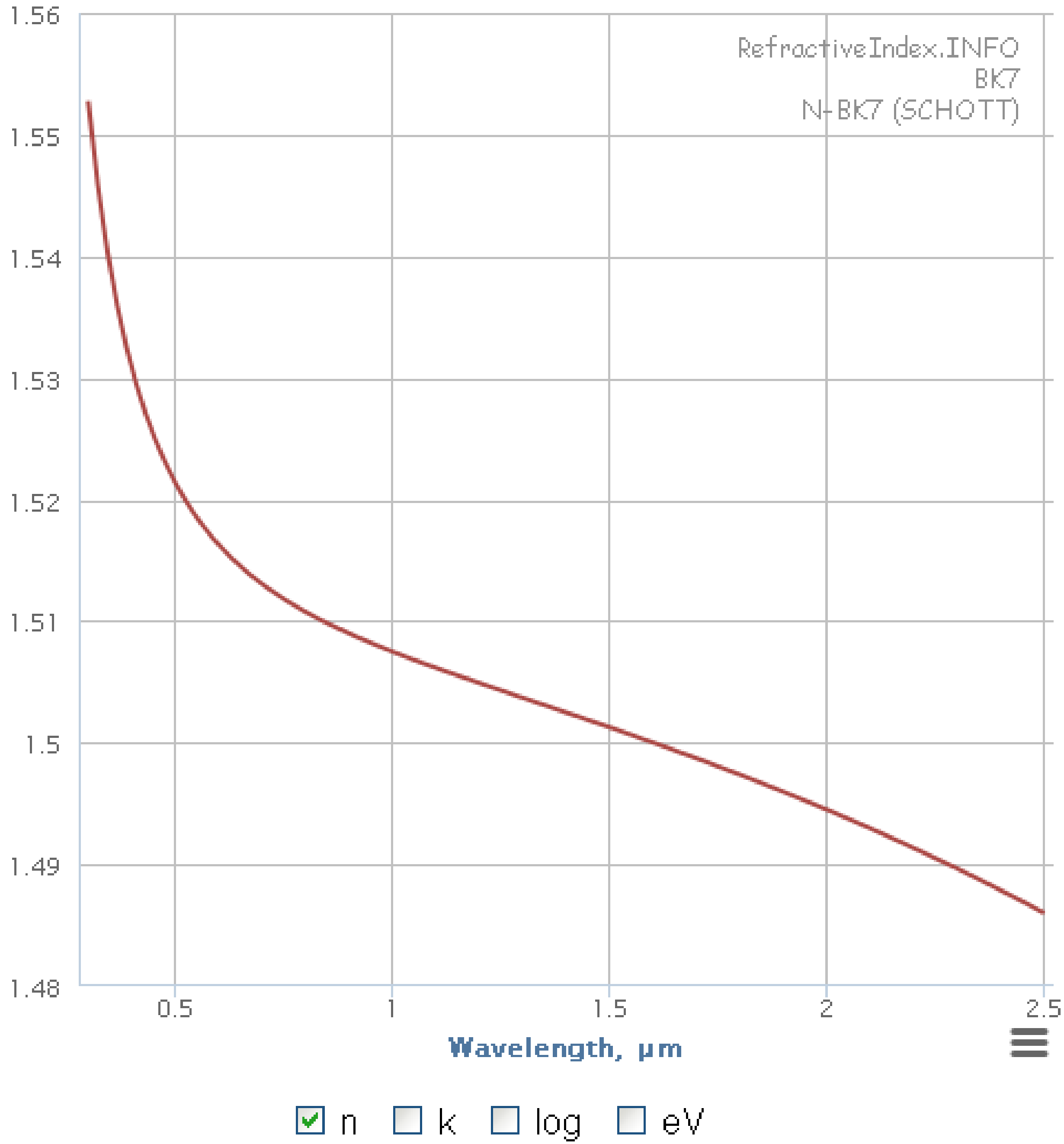
$$1 < n(\lambda_{\text{red}}) < n(\lambda_{\text{yellow}}) < n(\lambda_{\text{blue}})$$

Index Dispersion

- The index of refraction of an optical material varies with wavelength.
- The dashed portions of the curve represent absorption bands.



- In optical engineering, useful spectral region lies between the UV and infrared.



RefractiveIndex.INFO
BK7
N-BK7 (SCHOTT)
1.56
1.55
1.54
1.53
1.52
1.51
1.5
1.49
1.48
0.5
1
1.5
2
2.5
Wavelength, µm
n
k
log
eV

Dispersion Formulae

(a, b, c, ... are constants)

Cauchy
$$n(\lambda) = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \cdots$$

Hartmann
$$n(\lambda) = a + \frac{b}{(c-\lambda)} + \frac{d}{(e-\lambda)}$$

Conrady
$$n(\lambda) = a + \frac{b}{\lambda} + \frac{c}{\lambda^{3.5}}$$

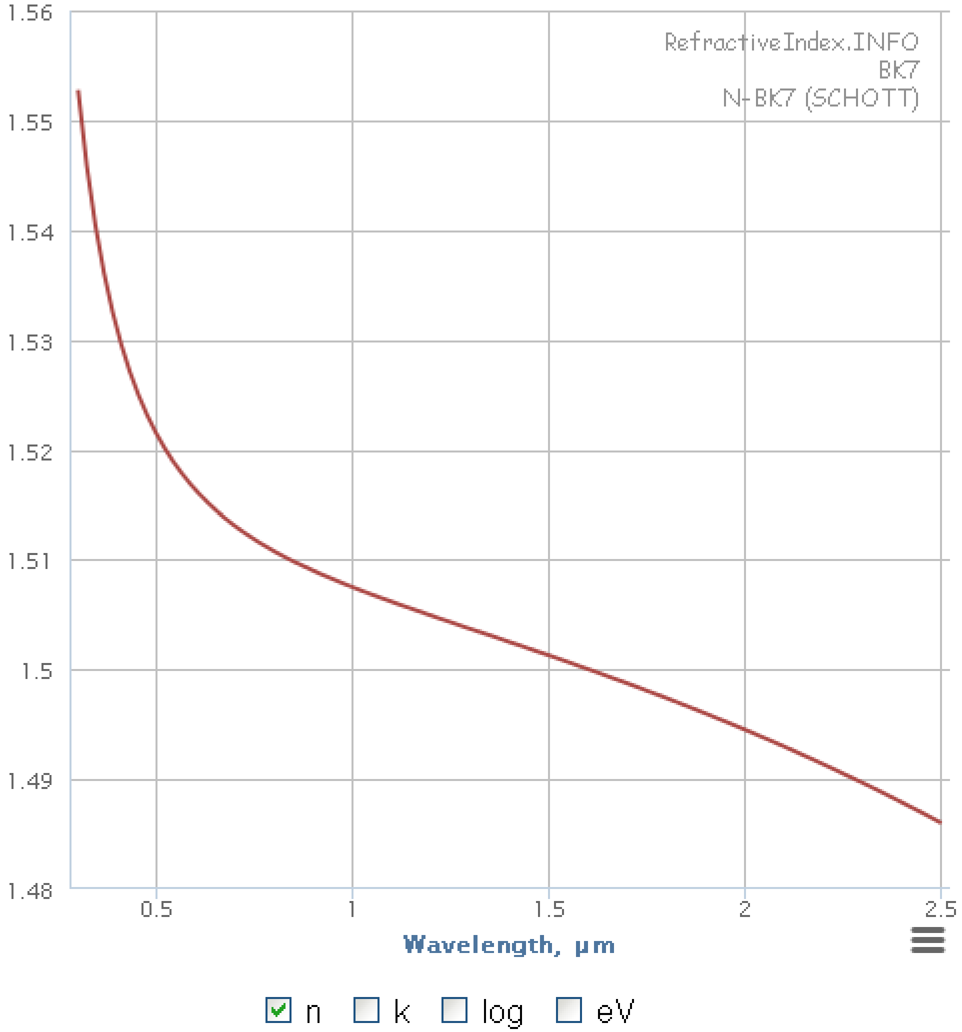
Kettler-Drude
$$n^2(\lambda) = a + \frac{b}{c-\lambda^2} + \frac{d}{e-\lambda^2} + \cdots$$

Sellmeier
$$n^2(\lambda) = a + \frac{b\lambda^2}{c-\lambda^2} + \frac{d\lambda^2}{e-\lambda^2} + \frac{f\lambda^2}{g-\lambda^2} + \cdots$$

Herzberger
$$n(\lambda) = a + b\lambda^2 + \frac{e}{(\lambda^2 - 0.035)} + \frac{d}{(\lambda^2 - 0.035)^2}$$

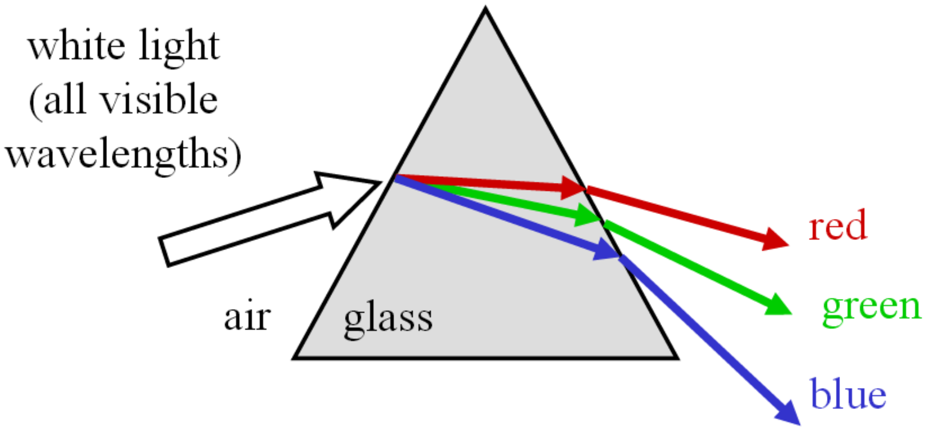
Old Schott
$$n^2(\lambda) = a + b\lambda^2 + \frac{c}{\lambda^2} + \frac{d}{\lambda^4} + \frac{e}{\lambda^6} + \frac{f}{\lambda^8}$$

Dispersion formula for BK7



$$n^2 - 1 = \frac{1.03961212\lambda^2}{\lambda^2 - 0.00600069867} + \frac{0.231792344\lambda^2}{\lambda^2 - 0.0200179144} + \frac{1.01046945\lambda^2}{\lambda^2 - 103.560653}$$

Dispersion Measure



Newton's prism

Reference color lines:

C (λ=656.3 nm, red),

D (λ=589.2 nm, yellow),

F (λ=486.1 nm, blue)

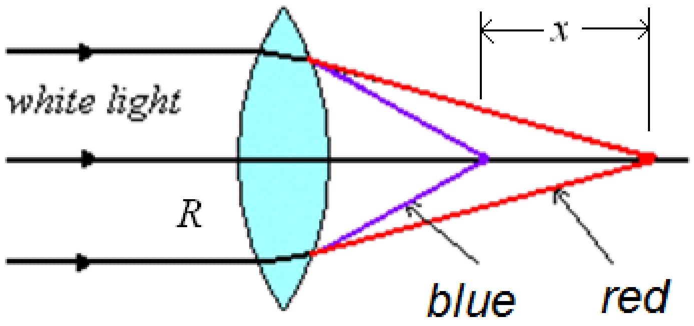
$$\text{Abbe value } = V = \frac{n_D - 1}{n_F - n_C}$$

For a crown glass:

$$V_{crown} = \frac{1.5233 - 1.0000}{1.5290 - 1.5204} = 60.8$$

Example 5

Figure shows a bi-convex lens of same radius (*R = 10 cm*) made up of a BK7. Determine the distance (*x*) between paraxial focal lengths for the blue (n=1.5308) and red (n=1.5131) lights if the lens is illuminated by a white light.



SOLUTION

Focal length as a function of curvature R and refractive index (n) is:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) \quad \rightarrow \quad f = \frac{R}{2(n-1)}$$

The distance between focal lengths:

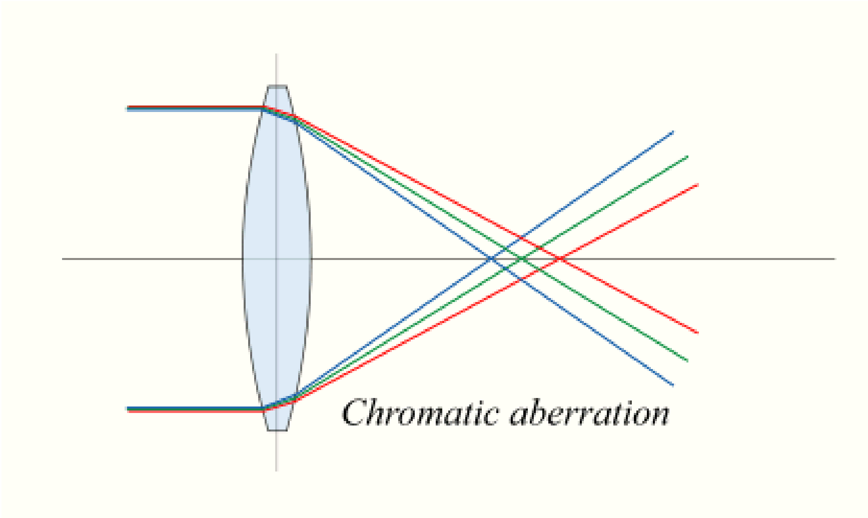
$$x = f^{red} - f^{blue} = \frac{R}{2(n^{red}-1)} - \frac{R}{2(n^{blue}-1)}$$

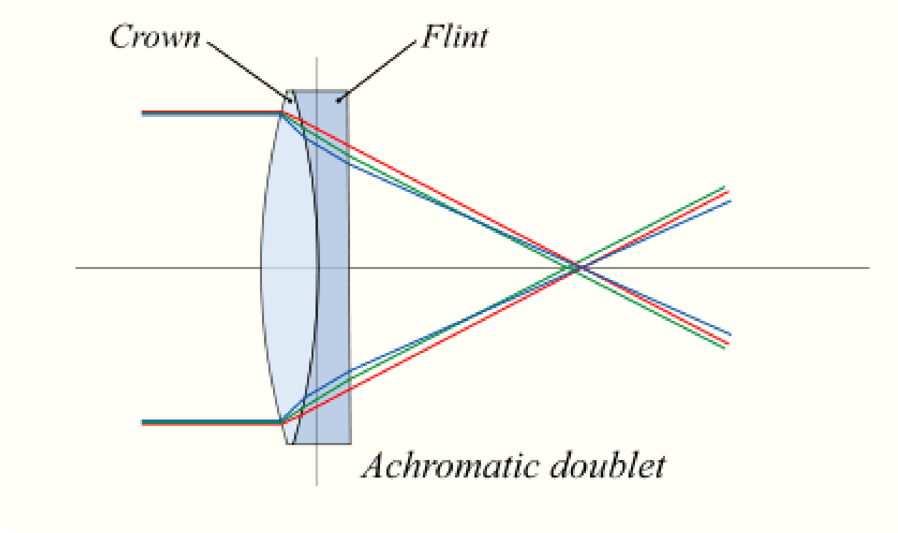
$$= \frac{10}{2(1.5131-1)} - \frac{10}{2(1.5308-1)} = 0.3250\,\mathrm{cm}$$

- One way to minimize this aberration is to use glasses of different dispersion in a doublet or other combination.

- The use of a strong positive lens made from a low dispersion glass like **crown glass (*n*<1.6)** coupled with a weaker high dispersion glass like **flint glass (*n* > 1.6)** can correct the *chromatic aberration* for two colors, e.g., red and blue.

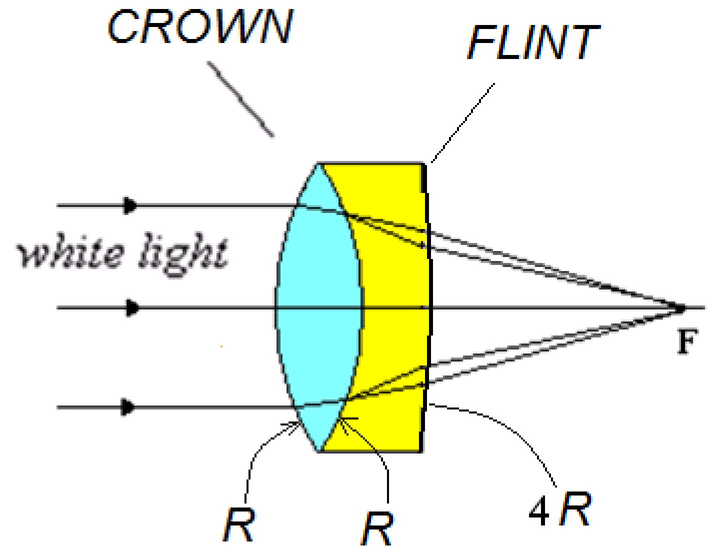
- Such doublets are often cemented together and called **achromat**.
- An achromatic lens (or achromat) is designed to limit the effects of chromatic and spherical aberration.





Example 6

Figure shows an achromatic lens consisting of a thin bi-convex crown glass (BK7) and a thin bi-concave flint glass (SF5) where radius of curvature is R = 10 cm.



Find the distance between paraxial focal lengths for blue and red rays with and without flint glass.

Glass	Refractive Index 400 nm	700 nm
BK7 :	1.5308	1.5131
SF5 :	1.7130	1.6637

Ans:

Δf (BK7) = 0.32495 cm

Δf (BK7 + SF5) = 0.05657 cm

Example 6 (code)

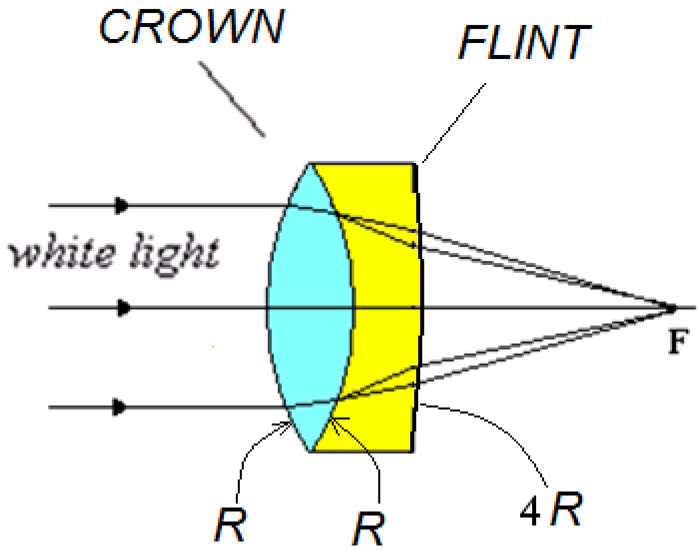
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% OCTAVE SCRIPT FILE                        %
% A simple achromatic lens calculations.    %
% The achromatic lens consists of           %
% a bi-convex converging lens (R1,R2) and   %
% a diverging lens (R2, R3).                %
% These two lenses are cemented together.   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The curvatures in cm are as follows:
R  = 10;
R1 = R;
R2 = -R;
R3 = -4*R;

% Refractive indices [blue red ]
n_bk7 = [1.5308  1.5131];
n_sf5 = [1.7130  1.6637];

disp('Focal length for BK7 only:')
f_bk7 = ((n_bk7-1) * (1/R1-1/R2)).^-1
f_sf5 = ((n_sf5-1) * (1/R2-1/R3)).^-1;
df = f_bk7(2) - f_bk7(1)

disp('')
disp('Focal length for BK7+SF5:')
f_sys = (1./f_bk7 + 1./f_sf5).^-1
df = f_sys(2) - f_sys(1)
```



PROGRAM OUTPUT

```
Focal length for BK7 only:
f_bk7 = 9.4197   9.7447
df    = 0.32495

Focal length for BK7+SF5:
f_sys = 18.981   18.924
df    = -0.056573
```

Optimum Doublet

Consider achromatic doublet in contact. If the focal lengths of the two (thin) lenses for light at the yellow Fraunhofer D-line (589.2 nm) are f_1 and f_2, then best correction occurs for the condition:

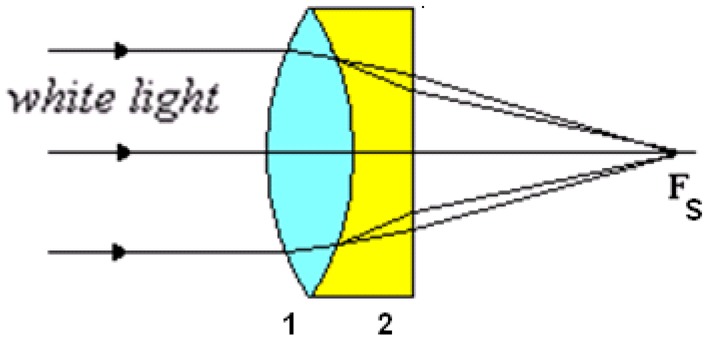
$$f_1 V_1 + f_2 V_2 = 0$$

where V_1 and V_2 are the Abbe numbers of the materials of the first and second lenses, respectively. Since Abbe numbers are positive, one of the focal lengths must be negative.

Note that if f_S is the focal length of the system then:

$$\frac{1}{f_1} = \frac{V_1}{V_1 - V_2} \frac{1}{f_S}$$

$$\frac{1}{f_2} = \frac{V_2}{V_2 - V_1} \frac{1}{f_S}$$



Example 7

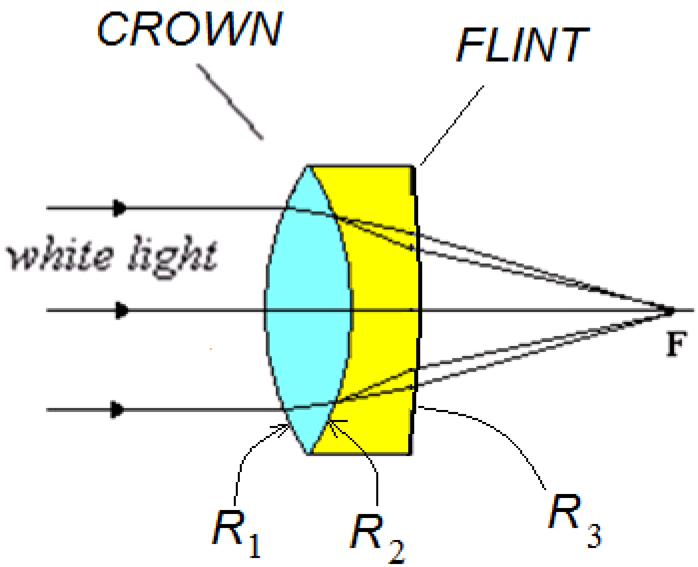


Figure shows an achromatic lens consisting of a thin bi-convex crown glass (BK7) and a thin bi-concave flint glass (SF5).
We want an optimum* doublet such that the system focal length to be f_s = 30 cm.
The radius of curvatures of BK7 are selected as $|R_1| = |R_2|$.

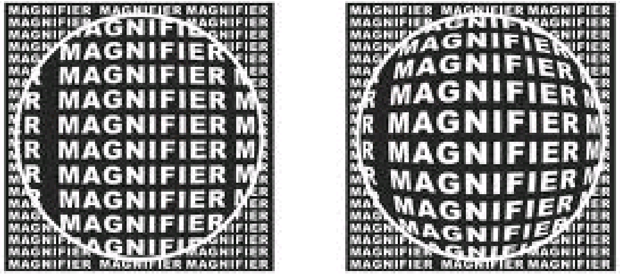
(a) Find the Abbe number of each lens.
(b) Find the focal length of each lens.
(c) Find the radii of curvatures R_1 , R_2 and R_3.
(d) Find the focal length of the doublet for each wavelength (color) given below.

Glass	n 486 nm	n 589 nm	n 656 nm
BK7	1.5224	1.5167	1.5143
SF5	1.6875	1.6726	1.6667

** Here, optimum means that focal length of the doublet for blue and red rays are the same.*

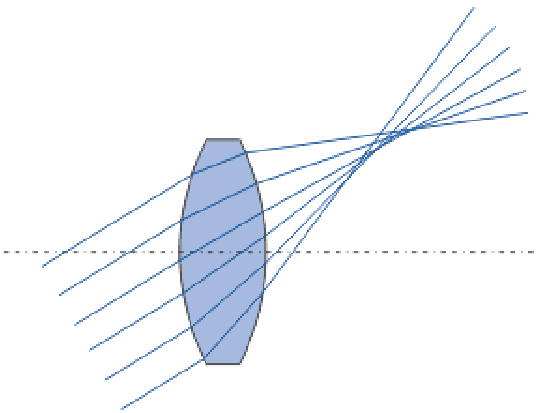
Other Types of Aberrations

- Distortion is a deviation from rectilinear projection.

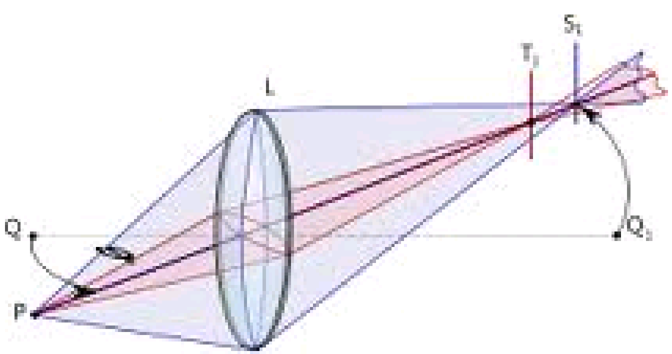


Distortion Free Image   Distorted Image

- Coma refers to aberration due to imperfection in the lens



- Astigmatism is one where rays that propagate in two perpendicular planes have different focal lengths.

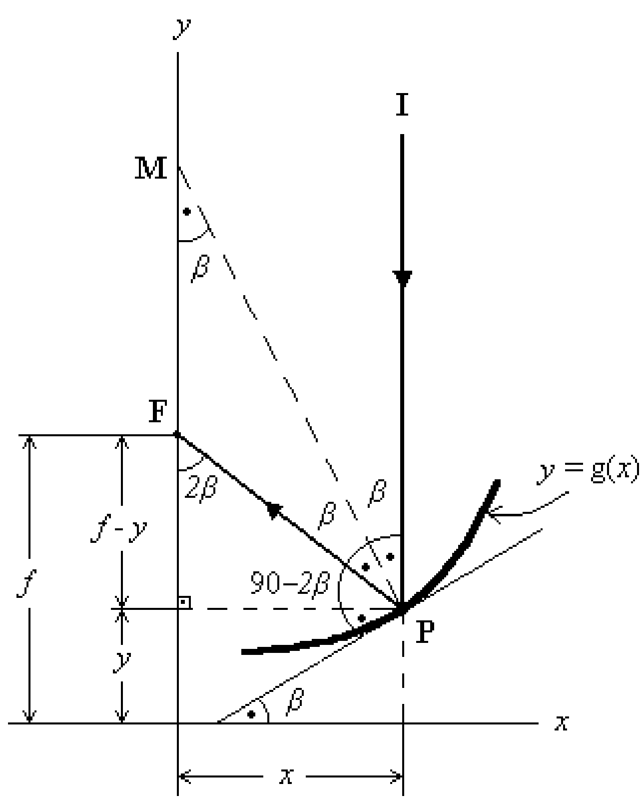


Appendix: Derivation of parabolic surface

1.1 Eğrisel Bir Yüzeyin Odak Uzaklığı

Bu kısımda herhangi bir eğrisel yüzeye paralel gelen ışınların yansıdıktan sonra asal ekseni kestiği noktanın (odak noktası) bulunuşu gösterilecektir. Basit ve daha anlaşılır olması açısından problem yüzey yerine bir eğri için çözülecektir.

Şekil 4'de denklemi $y = g(x)$ ile belli olan bir eğriye y eksenine paralel bir I ışını geldiği düşünelim. Bu ışın P noktasında yansıdıktan sonra y eksenini F noktasında kessin. Bu durumda ışının y eksenini kestiği nokta, f, odak uzaklığıdır.



Şekil 4: $y = g(x)$ eğrisine gelen bir ışının yansıması

Şekilden:

$$\tan 2\beta = \frac{x}{f - y} \qquad (2)$$

olduğu açıktır. Fonksiyonun P noktasındaki teğetinin eğimi o noktadaki türevine eşittir. Bundan dolayı:

$$\text{eğim} = \tan\beta = \frac{dy}{dx} = y' \qquad (3)$$

Diğer taraftan

$$\tan 2\beta = \frac{2\tan\beta}{1 - \tan^2\beta} \qquad (4)$$

dır. (2) ve (3) numaralı denklemler Denklem (4)' yerleştirildiğinde odak uzaklığı için:

$$f = \frac{x(1 - y'^2)}{2y'} + y \qquad (5)$$

bağıntısı elde edilir.

Denklem (5) f odak uzaklığının ile x, y ve y' değişkenlerine bağlı olarak hesaplanmasına imkan sağlar. Gerçekte bu denklem doğrusal olmayan bir diferansiyel denklemdir ve şöyle de yazılabilir:

$$\left(\frac{dy}{dx}\right)^2 + \frac{2(f - y)}{x}\left(\frac{dy}{dx}\right) = 1 \qquad (6)$$

Bu denklemin çözümü, $y = f(x)$, sonsuz uzaklıktan bir yüzeye paralel gelen ışınları tek bir sabit noktada, f, toplayan yüzeyin denklemini verecektir. $u = y / x$ dönüşümü yapıldıktan sonra denklemin çözümü:

$$y = \frac{x^2}{4f} \qquad (7)$$

dir. Bu bir p a r a b o l denklemidir. Öyle ise, sonsuzdan gelen ışınları ancak pabolik yüzeyler kusursuz olarak odaklayabilir.

Exercises

1. Does the image formed by a plane mirror suffer from any aberration?

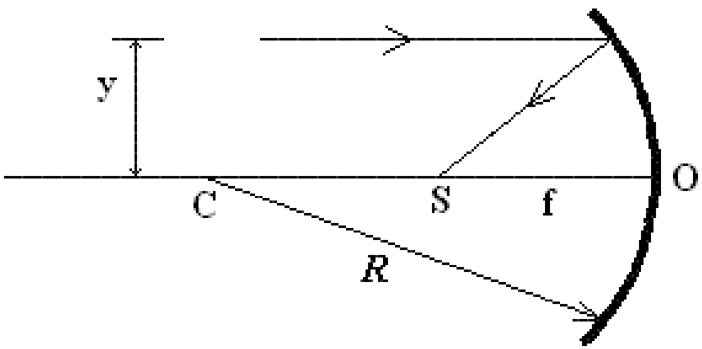
2. What are spherical and chromatic of aberrations?

3. Explain how can the spherical and chromatic aberrations be reduced?

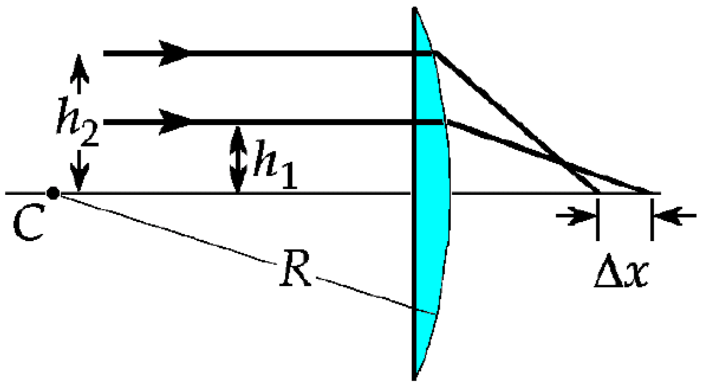
4. The magnitudes of the radii of curvature are 30 cm and 40 cm for the two faces of a biconcave lens. The glass has index of refraction 1.54 for violet light and 1.50 for red light. For a very distant object, locate and describe
   (a) the image formed by violet light, and
   (b) the image formed by red light.

5. Consider you have two thin lenses in contact to form a doublet. The system is illuminated by a mono-chromatic light at 589.2 nm. Given: f1 = +500 mm, V1 = 59.3 and V2 = 30.0.
   (a) Find the optimum focal length of the second lens that minimizes the chromatic aberration.
   (b) What is the focal length of this achromat at 589.2 nm?

6. A light ray traveling parallel to the principal axis at a distance *y* from the principal axis strike a concave mirror having a radius of curvature *R* as shown. The ray is focused on point S such that |OS| = *f*.
(a) Find an expression for *f* in terms of *y* and *R*.
(b) What is the value of *f* when *R* >> *y*?
Conclude the result. *Hint*: tan2θ = 2tanθ/(1−tan2θ)

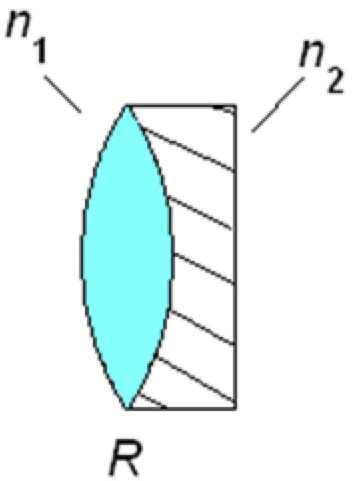


7. (Serway 6th Edition ref[1])
Two rays traveling parallel to the principal axis strike a large plano-convex lens having a refractive index of 1.6.
If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs). Assume this face has a radius of curvature of 20.0 cm and the two rays are at distances h1 = 0.5 cm and h2= 12.0 cm from the principal axis. Find the difference Δx in the positions where each crosses the principal axis.



8. Two lenses made of kinds of glass having different refractive indices n_1 and n_2 are cemented together to form what is called an optical doublet. Optical doublets are often used to correct chromatic aberrations in optical devices. The first lens of a doublet has one flat side and one concave side of radius of curvature R. The second lens has two convex sides of radius of curvature R. Show that the doublet can be modeled as a single thin lens with a focal length described by[1]:

$$f = \frac{R}{2n_1 - n_2 - 1}$$



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