

Lecture 4 First Order Optics



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Content

In this chapter, we will briefly see some elements of an ideal optical system.

- Spherical Refracting Surfaces
- Thin Lenses
- Thick Lenses
- Mirrors

Paraxial Optics

Snell's law of refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Taylor series of expansion:

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Taking only first term \rightarrow we arrive <u>first order</u> optics (or paraxial optics) Including third order terms \rightarrow we arrive <u>third order</u> optics. etc

Note that: For paraxial rays, the angle θ is small. In this case $\tan \theta \approx \sin \theta \approx \theta$ and $\cos \theta \approx 1$

where θ is in radian.

Paraxial Lens and Real Lens

In paraxial optics there are no aberrations of any kind => image is perfect.



Spherical Refracting Surface

Spherical Refracting Surface

Follow a ray from O to I.

Snell's law:

Paraxial rays:

Angles:

Distances:



Combining results yields:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$



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Quantity	Positive When	Negative When
Object location (<i>p</i>)	object is in front of surface (real object).	object is in back of surface (virtual object).
Image location (q)	image is in back of surface (real image).	image is in front of surface (virtual image).
Image height (h')	image is upright.	image is inverted.
Radius (R)	center of curvature is in back of surface.	center of curvature is in front of surface.

Sign Conventions for Refracting Surfaces



Thin Lenses

What is a Thin lens?

- The term lens originates from lentil.
- A lens can be constructed from intersection of two spherical surfaces.
- If center thickness of the lens is much smaller than size of the lens or its focal length, we call it a thin lens.





Various Lens Shapes and Equivalent Picture



DIVERGING LENSES



Lensmakers Formula

If we apply the following equation twice:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

for $n_1 = 1$ and $n_2 = n$, We will have:

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



The focal length *f* of a thin lens is the image distance that corresponds to an infinite object distance. Namley, if $p \rightarrow \infty$ then $q \rightarrow f$. Thus, we arrive at **lensmakers equation**:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Image Formation

- **Object** is light source or reflected light from a body that you want to see
- Image is a scaled copy of the object produced by your optical system.
- To form image, we need at least 2 rays from object.

If rays intersect a real image is formed.

If extention of rays intersect a **virtual image** is formed.



Image Formation for Thin Lens

Following image formation equations can also be derived From basic geometry:

Focal length:

 $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

Magnification:

$$m = \frac{\text{image height}}{\text{omject height}} = \frac{h_i}{h_o} = -\frac{q}{p}$$



lens

Image Formation for two Thin Lenses

For given f_1, f_2, p_1 and d one can find image locations q_1 and q_2 as follows:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1} \qquad \qquad \frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{d - q_1} + \frac{1}{q_2}$$

To obtain system's **back focal length** (bfl), we need to evaluate the following limit:

$$bfl = \lim_{p_1 \to \infty} q_2 = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$$

Also, system's **effective focal length** (f) can be found from:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$



Magnifications:

 $m_1 = -\frac{q_1}{p_1}$ $m_2 = -\frac{q_2}{p_2}$

$$m = m_1 m_2$$

Example: Paraxial Lens in Zemax

Zemax allows user to work with paraxial thin lenses.

To do that, user should replace the Surface Type with Paraxial in LDE.

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Marginal & Chief Rays



Aperture Stop and Field Stop

- AS: controls number of rays from object to image plane.
- FS: do or do not obstruct these rays entirely.



Aperture Stop and Field Stop

• Fields 0, 2 and 4 degress

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Pupils



- The image of the aperture stop as seen from object space is called the Entrance Pupil (EnP) of the system.
- The image of the aperture stop as seen from image space is known as the Exit Pupil (ExP) of the system.
- If there are no lenses between object plane and AS, then AS is EnP, (figure left).
- If there are no lenses between image plane and AS, then AS is ExP, (figure right).



Example: Pupil Positions

Given lens data.

Find position and diameter of entrance (EnP) and exit pupil (ExP) of the system.

Surface	Focal length	Distance	Diameter
OBJ	\inf	100	40
AS	\inf	50	10
LENS	30	90	30
IMG	\inf	0	40



Solution

EnP is the image of AS as seen from object space.

We have an aperture stop in front of first lens.

EnP position = AS position = 100 mm (50 mm left of the lens).

EnP diamater = AS diameter = 10 mm.

ExP is the image of AS as seen from image space. From Eqn 6.1, 1/50 + 1/(ExP) = 1/30, we have ExP = 75 mm (on the right of the lens). Hence, its position is z = 225 mm from the origin. ExP diameter can be found from the magnification. $m = D_{ExP}/D_{AS} = ExP/AS$ or $D_{ExP}/10 = 75/50$. Hence, $D_{ExP} = 15$ mm.

Thick Lenses

Cardinal Points

- There are six cardinal points which are widely used to approximate the behavior of real optical systems.
 - > the first and second system focal points; F_1 and F_2 .
 - > the first and second principal points; H_1 and H_2 .
 - > the first and second nodal points; N_1 and N_2 .



Image Formation for a Thick Lens

EFL is measured from second principle plane. Back Focal Length (BFL) is measred from the second vertex.

Effective Focal Length (EFL) of a lens in air is calculated from:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)t}{nR_1R_2}\right]$$

Location of H_1 and H_2 can be found from:

$$r = -\frac{f(n-1)t}{nR_2}$$
$$w = -\frac{f(n-1)t}{nR_1}$$



 R_1 , R_2 are radius of curvatures t is the center thickness n is the refractive index of the lens.

Example

Find the image distance for an object positioned 30 cm from the vertex of a double convex lens having radii 20 cm and 40 cm, a thickness 1 cm, and index of $n = n_L = 1.5$.

$$\frac{1}{f} = (n_L - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_L - 1)t}{n_L R_1 R_2} \right) \longrightarrow f = 26.8 \text{ cm}$$

$$r = v = -\frac{f(n_L - 1)t}{n_L R_2} = -\frac{26.8(1.5 - 1)1}{1.5(-40)} = +0.22 \text{ cm}$$

$$s = w = -\frac{f(n_L - 1)t}{n_L R_1} = -\frac{26.8(1.5 - 1)1}{1.5(20)} = -0.44 \text{ cm}$$

That is to say, H_1 is to the right of V_1 , and H_2 is to the left of V_2 .

$$bfl = 26.8 - 0.44 = 26.36 \text{ cm}$$

$$ffl = 26.8 - 0.22 = 26.58 \text{ cm}$$
Finally, $s_o = 30 + 0.22 = 30.22 \text{ cm}$

$$\frac{1}{f_i} = \frac{1}{p_i} + \frac{1}{q_i}$$

$$\frac{1}{26.8} = \frac{1}{30.22} + \frac{1}{s_i}$$

solving for $s_i = 238$ cm measured from H_2 .

Image Formation via two Thick Lenses

For the first lens:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$$

For the second lens:

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{d - q_1} + \frac{1}{q_2}$$

System's Effective Focal Length

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$





Image Formation for a Thick Lens

For most applications, it is necessary that the mirrors be first-surface mirrors, as opposed to ordinary second-surface mirrors.

The first-surface mirror is usually preferable because it does not produce a *ghost* image as does the second-surface mirror.



First-surface mirrors are usually made with vacuum deposited aluminum films protected by a thin transparent over coating of silicon monoxide (SiO) or magnesium fluoride (MgF₂).

RetroReflectors

This is a surface that reflects light back to its source with a minimum of scattering.



RetroReflectors on Moon



Apollo 11





These mirrors are used to calculate moon's distance accurately via high power lasers.

See also video: https://www.youtube.com/watch?v=IGpNiRkmxSA



Example: Fold Mirror in Zemax

You need to use **coordinate break**.

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Spherical Mirrors

A curved mirror surface has a focal length and is capable of forming images just as a lens does.







Image Formation using a Spherical Mirror

Focal Length: f = R/2

Image location:
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Magnification:
$$m = \frac{h_i}{h_o} = -\frac{q}{p}$$

- f + for converging mirror - for diverging mirror
- p + for real object
 for virtual object
- q + for real image - for virtual image

