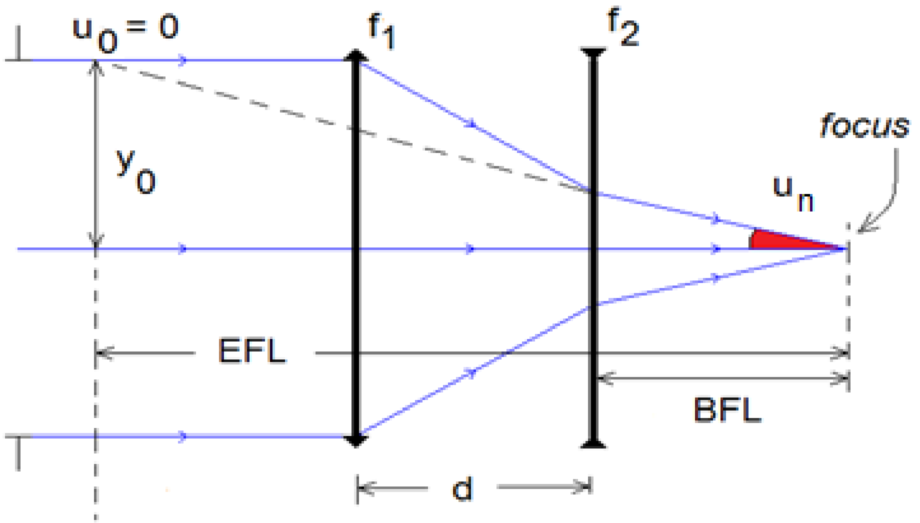
Lectures Notes on Optical Design using Zemax OpticStudio

Lecture 5

Ray Tracing



Ahmet Bingül

Gaziantep University
Department of Optical
Engineering

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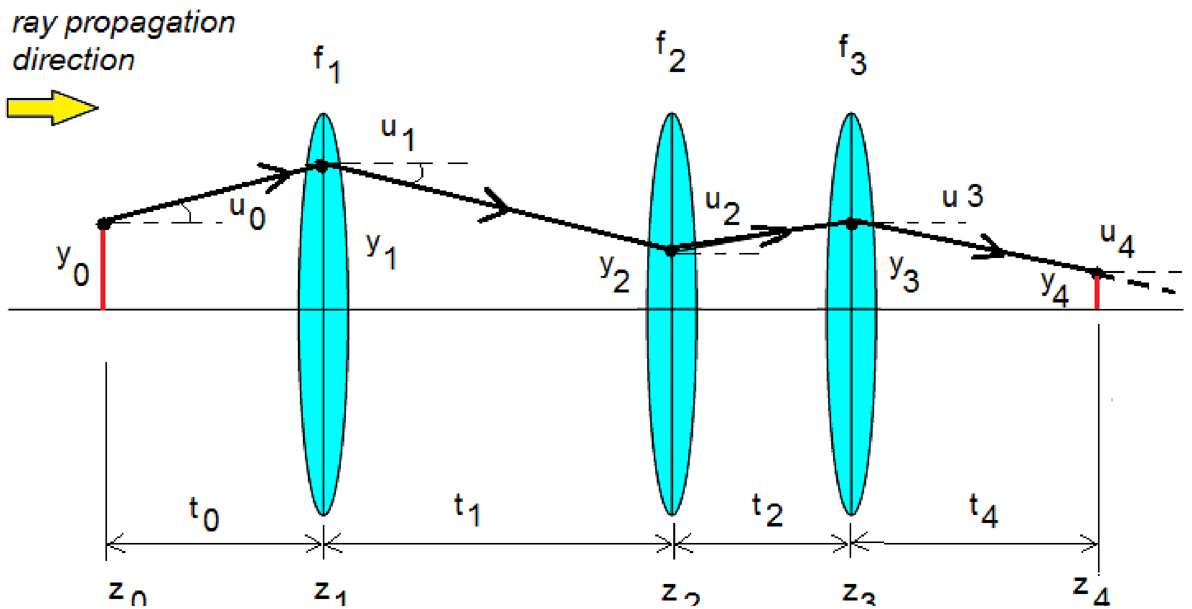
Introduction

- In this section, we will see a method for producing fast paraxial ray tracing on the system containing thin (and thick) lenses. The method is called the y-u trace.
- In a y-u trace, we will assume that the rays always propagate from left to right through the optical system. Also, the clock-wise (cw) direction for the angles is positive (ccw direction is negative).

For details and derivations, see Chapter 2 of the book

D.C. O'Shea, Elements of Modern Optical Design, John Wiley & Sons Inc.

Thin Lens Ray Tracing (y-u method)



Using paraxial rays ($\tan u \approx u$) in Figure the ray transfer equation from one lens to another is given by:

$$y_{k+1} = y_k + u_k t_k \tag{5.1}$$

slope angle equation can be obtained from

$$u_{k+1} = u_k - y_{k+1} p_{k+1} \tag{5.2}$$

and ray position along z-axis:

$$z_{k+1} = z_k + t_k \tag{5.3}$$

Starting with an initial ray position (z_0, y_0, u_0), this iterative procedure is used to obtain final ray position (z_n, y_n, u_n).

Here

- $k = 0, 1, 2, \cdots, n$
- y_k = ray height at k^{th} lens (surface)
- u_k = ray slope (angle in radian) at k^{th} lens (surface)
- t_k = distance between k^{th} and $(k+1)^{\text{th}}$ lens (surface)
- z_k = z-position of the k^{th} lens (surface). Usually we start with $z_0 = 0$.
- $p_k = 1/f_k$ is the power of the k^{th} lens whose focal length is f_k. Note that for object (OBJ) and image (IMG) planes $p_k = 0$.

Focal Length and Magnification

By using a ray which is parallel to optical axis, namely $(z_0, y_0, u_0) = (0, y_0, 0)$, the effective focal length and back focal length of the optical system can be obtained from:

$$\text{efl} = f = -\frac{\text{initial ray height}}{\text{final ray angle}} = -\frac{y_0}{u_n} \quad (7.3)$$

and

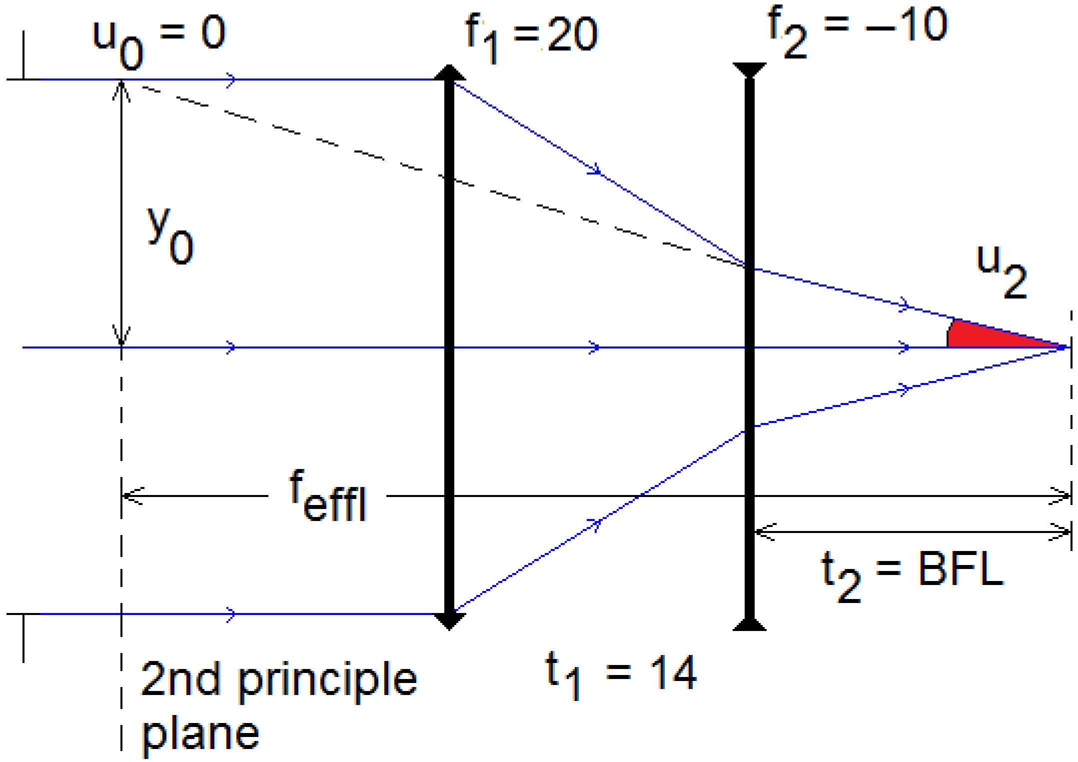
$$\text{bfl} = -\frac{\text{final lens surface ray height}}{\text{final ray angle}} = -\frac{y_{n\text{-}1}}{u_n} \quad (7.4)$$

Also, for parallel incident rays, the magnification may be defined as:

$$m = \frac{y_n}{y_0} \quad (7.5)$$

Example

Figure shows a simple telephoto lens. Calculate effective focal length and back focal length of the system using y-u trace.



Solution

$$y_{k+1} = y_k + u_k t_k$$

$$u_{k+1} = u_k - y_{k+1} p_{k+1}$$

Given: $f_1 = 20$, $f_2 = -10$, $t_1 = 14$

$p_0 = 0$, $p_1 = 1/20$, $p_2 = -1/10$, $p_3 = 0$

Let's start with $y_0 = 1$, $u_0 = 0$, $t_0 = 1$

$y_1 = y_0 + u_0 t_0 = 1 + (0)(1) = 1.0$

$u_1 = u_0 - y_1/f_1 = 0 - 1/20 = -0.05$

$y_2 = y_1 + u_1 t_1 = 1 + (-0.05)(14) = 0.3$

$u_2 = u_1 - y_2/f_2 = -0.05 - 0.3/(-10) = -0.02$

$y_3 = y_2 + u_2 t_2 = 0.3 + (-0.02)(t_2) = 0.0$ => $t_2 = 15$

$u_3 = u_2 - y_3/f_3 = -0.02 - 0.10/(-\text{inf}) = -0.02$

BFL $= -y_2/u_3 = -0.3/-0.02 = 15$ mm

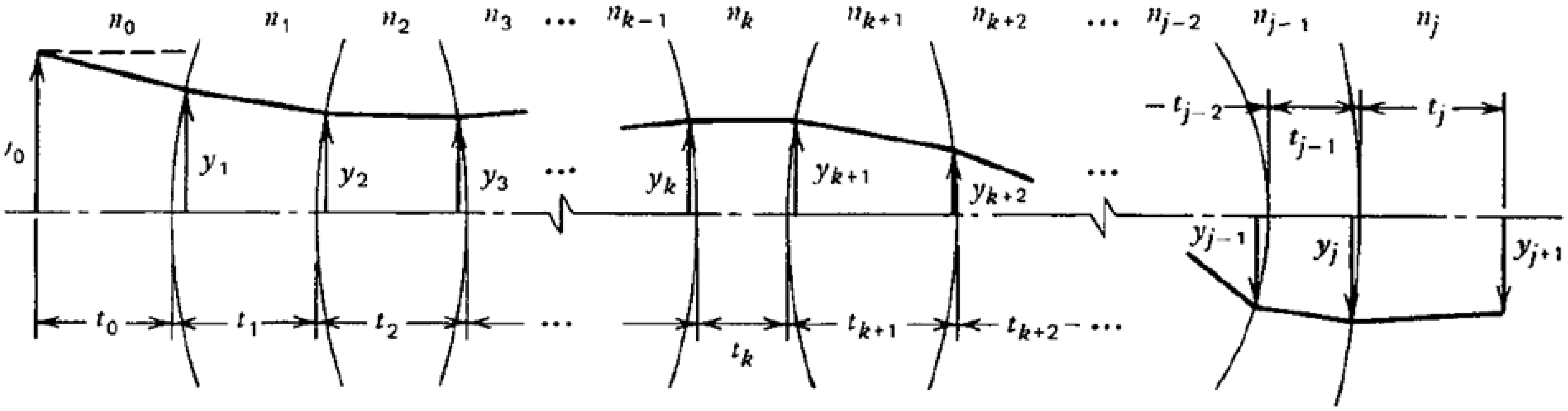
EFFL $= -y_0/u_3 = -1.0/-0.02 = 50$ mm

Telephoto ratio $= \text{EFFL}/(t_1+t_2) = 50/(14+15) = 1.7$

Summary of y-u trace:

z	y	u
0.0000	1.0000	0.0000
1.0000	1.0000	-0.0500
15.0000	0.3000	-0.0200
29.0000	0.0000	-0.0200

Thick Lens Ray Tracing (y-nu method)



As in the y-u trace, the ray transfer equation is as follows:

$$y_{k+1} = y_k + u_k t_k \tag{7.1}$$

Then, the slope angle (or refraction) equation is given by:

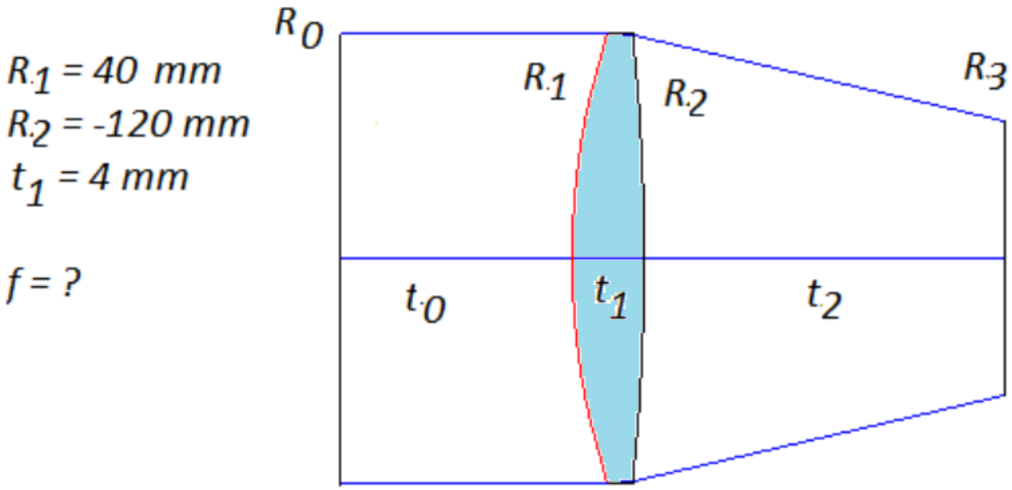
$$n_{k+1} u_{k+1} = n_k u_k - (n_{k+1} - n_k) \frac{y_{k+1}}{R_{k+1}} \tag{7.2}$$

n = index of refraction

R = radius of curvature

EXAMPLE: EFFL of a Single Lens

Using y-nu method for $(y_0, u_0) = (1, 0)$, calculate the effective focal length for a lens of crown glass (n = 1.517) of a thickness of 4 mm, with radii of curvatures, R_1 = 40 mm and R_2 = -120 mm. Assume that the lens is in air.



Center thickness of the lens is given by $t_1 = 4$ mm. The values of t_0 and t_2 can be selected arbitrarily. Let

thickness vector: $t = [t_0, t_1, t_2] = [1, 4, 1]$

radius vector: $R = [R_0, R_1, R_2, R_3] = [\infty, 40, -120, \infty]$

index vector: $n = [n_0, n_1, n_2] = [1, 1.517, 1]$

EXAMPLE: Solution

$$y_1 = y_0 + u_0 t_0 = 1 + (0)(1) = 1.0000$$
$$u_1 = (n_0 u_0 - (n_1 - n_0) y_1 / R_1)/n_1 = ((1)(0) - (1.517 - 1)1/40)/1.517 = -0.0085$$

$$y_2 = y_1 + u_1 t_1 = 0.9660$$
$$u_2 = (n_1 u_1 - (n_2 - n_1) y_2 / R_2)/n_2 = -0.0171$$

$$y_3 = y_2 + u_2 t_2 = 0.9575$$
$$u_3 = (n_1 u_1 - (n_2 - n_1) y_2 / R_2)/n_2 = -0.0171$$

The result is summarized below:

k	z_k	y_k	u_k
0	0.0000	1.0000	0.0000
1	1.0000	1.0000	−0.0085
2	5.0000	0.9660	−0.0171
3	7.0000	0.9575	−0.0171

Hence, EFFL i: $f = \frac{-y_0}{u_n} = \frac{-y_0}{u_3} = \frac{-1}{-0.0171} \rightarrow \boxed{f = +58.4795 \text{ mm}}$.

y-nu Trace with Reflecting Surfaces

Between interfaces, from surface k to surface k + 1, so far, we have used only refracting surfaces. How is a refecting surface handled in a paraxial ray trace? To account for a large number of possible sign changes, all conventions in terms of height and angles can be preserved if only two things are changed. After a reflection, the signs of

- all refractive indices (n_k) are reversed
- all spacing and thicknesses (t_k) are reversed.

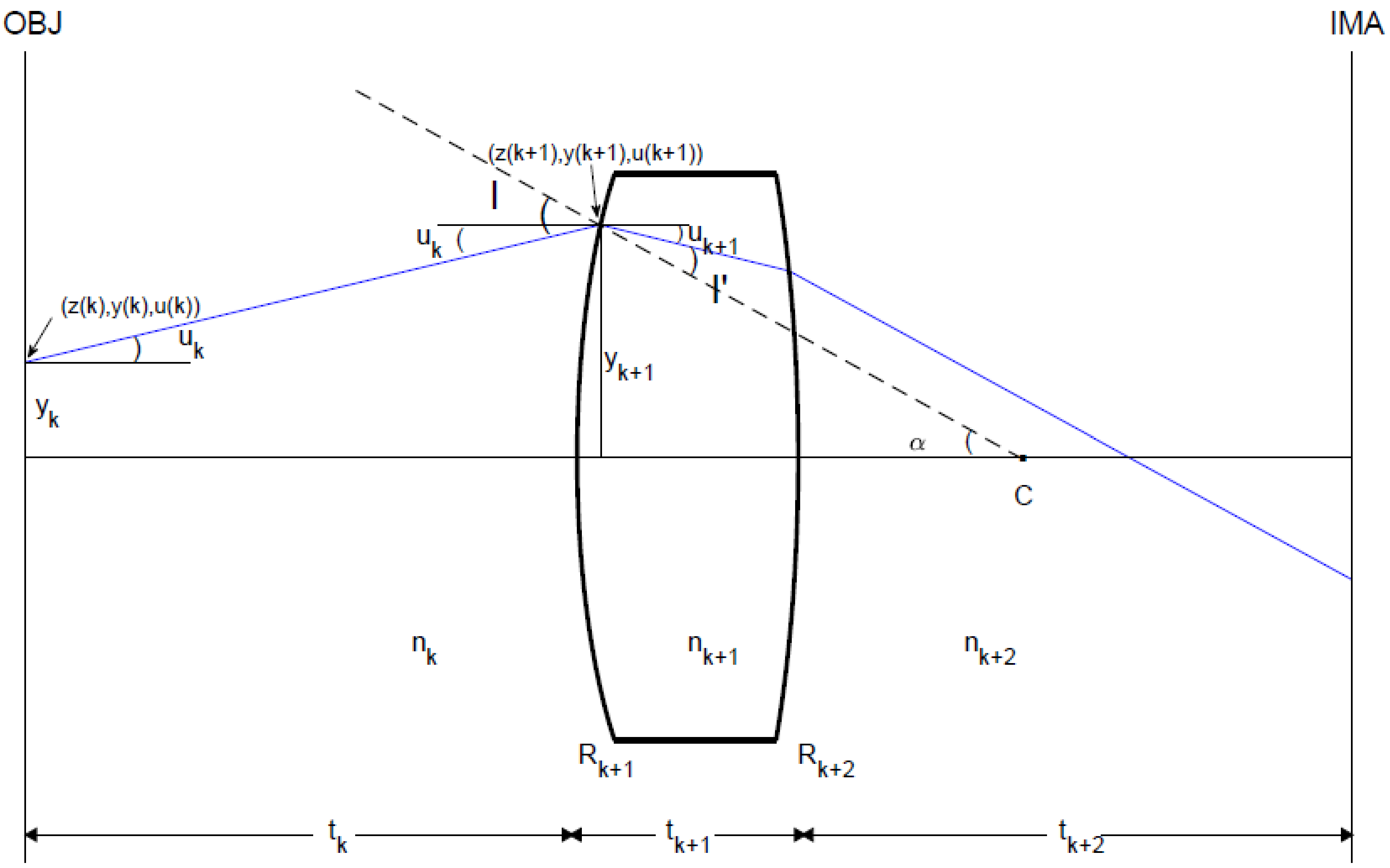
Therefore, after one reflection, all distances and surface separations, are negative.

Exact Ray Tracing

Paraxial theory demonstrates perfect imagery by optical systems since all of rays each point on the object combine same image point. In fact, the paraxial image is not a true representation of the object.

Real rays will not intersect at same point at image plane.
Therefore, real ray tracing reveal aberration in an image.
Accordingly, image will be blurred or distorted → Aberrations

Exact Ray Tracing



Exact Ray Tracing

- Ray transfer equation:

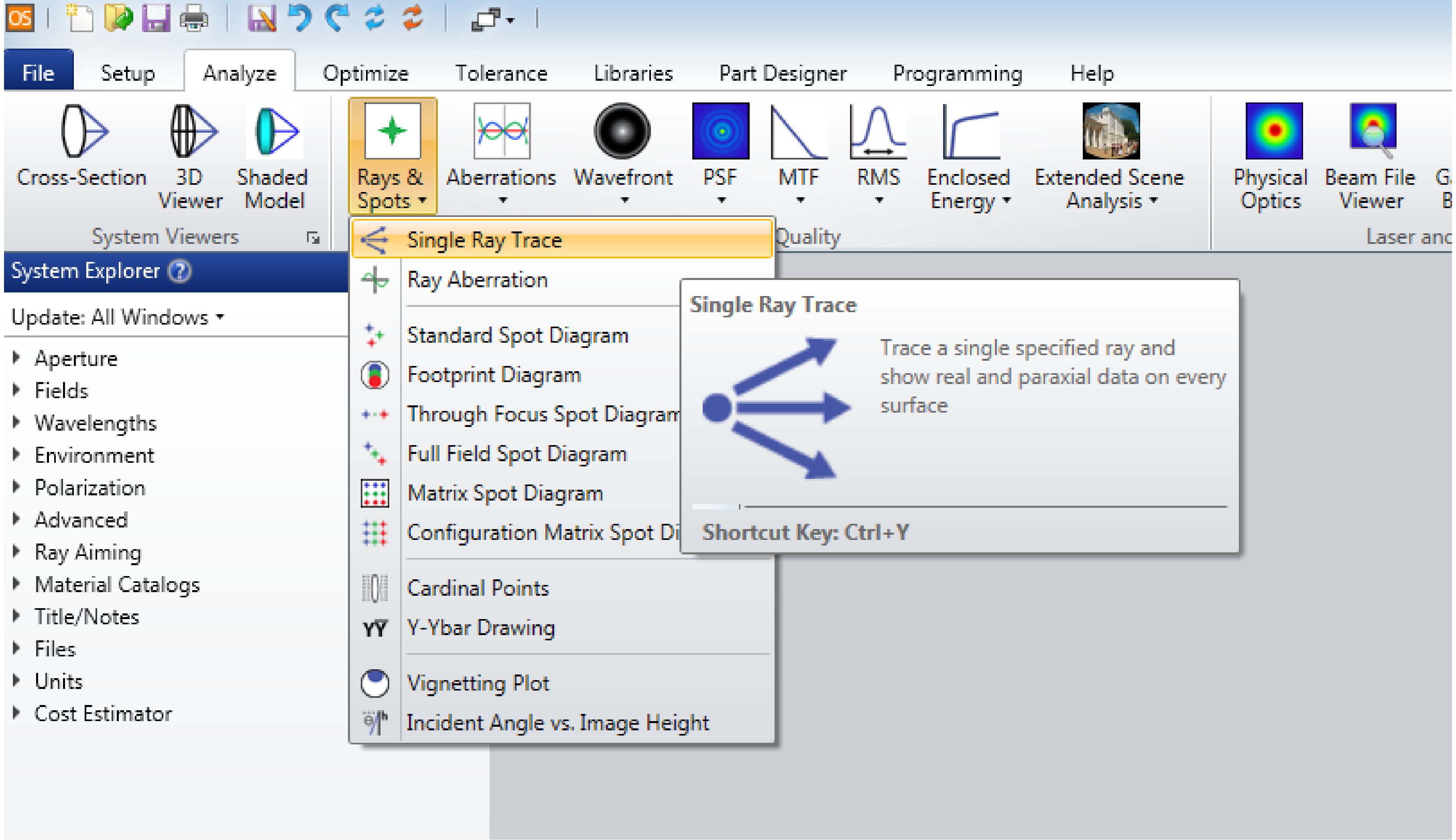
$$y_{k+1} = y_k + \tan u_k (z_{k+1} - z_k)$$

- Apply Snell's law of refraction at each surface. For example, for the first surface:

$$\begin{aligned} \sin\alpha &= \frac{y_{k+1}}{R_{k+1}} \\ I &= u_k + \alpha \\ \alpha &= I' - u_{k+1} \end{aligned}$$

$$n_{k+1}\ \sin I' = n_k\ \sin(u_k + \alpha)$$

Single Ray Tracing in Zemax



Example: Real Ray Tracing

Consider the zemax sample file **Double Gauss 28 degree field** saved at:

C:\<Zemax>\Samples\Sequential\Objectives

Using Zemax, investigate ray tracing (both paraxial and real) for top maginal ray for F, d and C lines.