

Lecture 8 Monochomatic Aberrations



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What is aberration?

- Paraxial approximations result in perfect image!
- Imperfect images caused by geometric factors are called aberrations.
- Aberration leads to blurring of the image produced by an image-forming optical system.



Aberration Types

Spherical aberration
 Coma
 Astigmatism
 Field Curvature
 Distortion
 Chromatic Aberration
 due to dispersion of optical material

Origin of Aberrations

Snell's law of refraction:

Taylor series of expansion:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Taking only first term

Including third order terms

- ➔ We arrive <u>first order</u> optics which is the study of perfect optical systems without aberrations.
- → We arrive <u>third order</u> optics.
- In 3rd order optics, we have set of equations for describing lens aberrations as departures from paraxial theory.
- These equations are called Siedel Aberrations.

Seidel Coefficients

Consider a ray originates from object point at O(0, -h, 0). The ray hits image surface at I(x', y', z). Question: What are the mathematical relations between these two points?



Seidel Coefficients

The solution is given below. The coefficients of

- 1^{st} order terms: A_1 , A_2 are related to perfect imaging.
- 3rd order terms: B₁, B₂, B₃, B₄, B₅ are Seidel Coefficients. They are related to 3rd order departure from perfect imaging. (Higher order terms can also be included).

$$\begin{split} \mathbf{y}' &= A_1 s \, \cos \, \theta \, + \, A_2 h \\ &+ \, B_1 s^3 \, \cos \, \theta \, + \, B_2 s^2 h (2 \, + \, \cos \, 2\theta) \, + \, (3B_3 \, + \, B_4) s h^2 \cos \, \theta \, + \, B_5 h^3 \\ &+ \, C_1 s^5 \, \cos \, \theta \, + \, (C_2 \, + \, C_3 \, \cos \, 2\theta) s^4 h \, + \, (C_4 \, + \, C_6 \, \cos^2 \, \theta) s^3 h^2 \, \cos \, \theta \\ &+ \, (C_7 \, + \, C_8 \cos \, 2\theta) s^2 h^3 \, + \, C_{10} s h^4 \, \cos \, \theta \, + \, C_{12} h^5 \, + \, D_1 s^7 \, \cos \, \theta \, + \, \cdots \end{split}$$

$$\begin{split} x' &= A_1 s \, \sin \, \theta \\ &+ B_1 s^3 \, \sin \, \theta + B_2 s^2 h \, \sin 2\theta + (B_3 + B_4) s h^2 \sin \, \theta \\ &+ C_1 s^5 \, \sin \, \theta + C_3 s^4 h \, \sin 2\theta + (C_5 + C_6 \cos^2 \theta) s^3 h^2 \, \sin \theta \\ &+ C_9 s^2 h^3 \, \sin 2\theta + C_{11} s h^4 \, \sin \theta + D_1 s^7 \, \sin \theta + \, \cdots \end{split}$$

Wave Aberration

Wave aberration function **W** is the optical length, measured along a ray, from the aberrated wavefront to the reference sphere. The distance ε is called the transverse ray error.



Monochromatic aberrations can also be described by expanding **W** in a power series of aperture and field coordinates, ρ , θ and H:

 $W_{IJK} \Rightarrow H^I \rho^J \cos^K \theta$

$$W(H,\rho,\theta) = W_{020}\rho^{2} + W_{111}H\rho\cos\theta + W_{040}\rho^{4} + W_{131}H\rho^{3}\cos\theta + W_{222}H^{2}\rho^{2}\cos^{2}\theta + W_{220}H^{2}\rho^{2} + W_{311}H^{3}\rho\cos\theta + O(6)$$

 W_{020} : Defocus W_{111} : Wavefront tilt W_{040} : Spherical aberration W_{131} : Coma W_{222} : Astigmatism W_{220} : Field curvature W_{311} : Distortion



Aberration Plots & Seidel Coefficients

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Monochromatic Aberration Demo in Zemax

λ = 550 nm, ENPD = 25 mm, SFOV = 0° and 10°.

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Spherical Aberration

SA is occurs only on-axis. We have two types of spherical aberrations:

Longitudinal Aberration (L.SA)

Transverse Aberration (T.SA)





Ray Fan Plot



Aspherical Surfaces

Using Aspherical surfaces one can reduce S.A.

- Aspherical surface is relatively harder to make and measure.
- Aspheric lenses improve image quality and reduce the number of required optical elements.

An important property of an optical surface is <u>sag</u> defined by:

$$z = \frac{Cy^2}{\sqrt{1 + (1+k)C^2y^2}} + A_2y^2 + A_4y^4 + A_6y^6 + \cdots$$

z = sag of surface parallel to the optical axisy = radial distance from the optical axisC = curvature, inverse of radius (C = 1/R)k = conic constant $A_i = i^{th}$ order aspheric coefficient



Geometric meaning of conic constant:

k = 0	=> circle
k = -1	=> parabola
k < -1	=> hyperbola
k > 0	=> ellipse

-1 < k < 0 => ellipse



Suggestions in use of Aspheric Surfaces

- 1. If possible, optimize your design first using spherical surfaces, and then use the conic and/or aspheric coefficients in the final stages of optimization. This may help in keeping the asphericities to a more manageable level.
- 2. Conic surfaces can be used for correcting third-order spherical aberration and other low-order aberrations.
- 3. If you have a nearly flat surface, then use A_4 and higher-order terms rather than a conic.

How to Get Rid Off Spherical Aberration

To reduce spherical aberration:

- Reduce size (diameter) of the lens
- Change bending (radii of curvatures) of the lens
- Use more than one spherical lens
- Use aspherical surface(s)

Example 1: Three Lenses to reduce SA

In this example, we will use three N-BK7 glasses separated by 5 mm and 7 mm. ENPD = 25 mm, F/# = 4 and λ = 550 nm. Diameter of each lens is D = 30 mm and ct₁ = ct₂ = ct₃ = 6 mm (*Note that for optomechanical reasons center thickness must satisfy ct > D/10*).

An example recipie is as follows:

Step 1: We have only one lens. Do not insert other lenses.

 R_{11} = 90 mm and R_{12} is variable.

Optimize (min spot) such that focal length of the lens is $f_1 = 120$ mm.

Step 2: Insert new lens 5 mm away from first lens. Now, we have two lenses.

 R_{11} , R_{12} are fixed. R_{21} and R_{22} are variable.

Optimize (min spot) such that focal length of the two lenses is $f_{12} = 80$ mm.

Step 3: Insert new lens 7 mm away from second lens. Now, we have three lenses.

 R_{11} , R_{12} , R_{21} , R_{22} are fixed. R_{31} and R_{32} are variable.

Optimize (min spot) such that focal length of lenses is $f_{123} = 100$ mm.

Step 4: Set all 6 radii variables.

Optimize (min spot) such that focal length of lenses is $f_{123} = 100$ mm.









Example 2: Even Asphere Surface

In some cases, use of the conic constant may not be enough to remove S.A. An alternative way is to use even aspheric surface which is a standard surface plus polynomial asphere terms (See Page 13). In Zemax OpticStudio, this surface is defined as **Even Asphere**. In this example, we'll consider a plano convex aspherical lens whose focal length is 100 mm and ENPD = 25 mm.

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After optimization, we have a perfect form. Compare the solution with Example 1.

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Coma

Coma is similar to SA but in addition the rays come from off-axis points. Coma increases rapidly as the third power of the lens aperture.



The term comes from comet due to the shape of the spot diagram.

Coma

To correct coma,

- Change bending of the lens or
- Change the position and/or diameter of the aperture stop



Astigmatizm

- Astigmatism is another off-axis aberration.
- Tangential and Sagital rays for oblique rays are focused at dierent points.



Astigmatizm

To correct or reduce astigmatizm

- Change the shape of the lens
- Change its distance from the aperture stop
- Use spaced doublet lens with stop in center
- Place a weak menuscus lens before image plane



Field Curvature

- For a light ray comes from off-axis, a positive lens appears to be thicker than really it is.
- As a results, for oblique rays we have different focal lengths.
- The points on a plane object, then form a curved image, a deciency is called **field curvature.**
- This aberration is important in camera systems and projectors: **image is expected to be flat**.





Field Curvature

To correct curvature of field (known as Field flattening)

- Change the position and/or diameter of the aperture stop.
- Try to obey the Petzval Condition in your system, that is:

Petzval sum =
$$P = n_m \sum_{i=1}^m \frac{n_i - n_{i-1}}{n_i n_{i-1} R_i} = 0$$

- n_{i-1} and n_i are refractive indexes before and after the surface number i.
- R_{i} is the radius of curvature of the surface number i.
- is the surface number of image surface. \boldsymbol{m}
- is the index of image surface n_m

For a system of thin lenses of focal lengths f_i the Petzval sum is given by: $P = \sum_{i=1}^{n} \frac{1}{n_i f_i} = 0$



In Zemax help, see operands: **PETC** and **PETZ** and topic **Petzval Radius**.

Distortion

- Distortion occurs as variation in the lateral magnication.
- If the magnication decreases with distance from the axis, the images appears as barrel distortion.
- If the magnication increases with the from axis, the image appears as pincushion distortion.
- Distortion increases with the cube of the field of view.
- Distortion is defined as

$$D = \frac{y - y_p}{y_p}$$

y is the height in the image plane y_p is the paraxial height Generally, distortion in the order of 2-3% is acceptable visually.



Distortion



Distortion

To correct or reduce distortion

- Change the shape of the lens.
- Change the position of the aperture stop.
- Use spaced doubled lens system after placing aperture stop in the center of the system.
- Use image processing softwares in digital systems.

Typical Spot Diagrams



Sasián, J. (2012). Ray aberrations. In *Introduction to Aberrations in Optical Imaging Systems* (pp. 100-118). Cambridge: Cambridge University Press. doi:10.1017/CB09780511795183.012



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Example 3: Aperture Size and Position

In this example, we will investigate the effect of <u>aperture size and postion</u> on the monochromatic aberrations for the demo example.

 $(\lambda = 550 \text{ nm}, \text{ENPD} = 25 \text{ mm}, \text{SFOV} = 0^{\circ} \text{ and } 10^{\circ}).$

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First, change the positon of STOP from 0 to 50 mm with step 10 mm. Extract the Seidel coeffcients (Total value on image plane).

position	SPHA	COMA	ASTI	FCUR	DIST
0	0.083460	-0.044210	0.045543	0.033177	0.004688
10	0.083460	-0.032437	0.034731	0.033177	0.026236
20	0.083460	-0.020664	0.027241	0.033177	0.043912
30	0.083460	-0.008891	0.023071	0.033177	0.059120
40	0.083460	0.002882	0.022224	0.033177	0.073267
50	0.083460	0.014655	0.024697	0.033177	0.087758



Second, change the diameter of STOP from 5 to 25 mm with step 5 mm for the stop position 30 mm in front of the lens. Extract the Seidel coeffcients (Total value on image plane).

diameter	SPHA	COMA	ASTI	FCUR	DIST
5	0.000134	-0.000071	0.000923	0.001327	0.011824
10	0.002137	-0.000569	0.003691	0.005308	0.023648
15	0.010816	-0.001921	0.008306	0.011944	0.035472
20	0.034185	-0.004552	0.014766	0.021233	0.047296
25	0.083460	-0.008891	0.023071	0.033177	0.059120
30	0.173063	-0.015364	0.033223	0.047775	0.070944



Summary of Monochomatic Aberrations

Summary of Third-Order Monochromatic Aberration Dependence on Aperture and Field Angle

Aberration	Aperture Dependence	Field Dependence
Spherical	Cubic	_
Coma	Quadratic	Linear
Astigmatism	Linear	Quadratic
Field curvature	Linear	Quadratic
Distortion	_	Cubic

Aberration Performance Plots in Zemax OpticStudio

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Zernike Polynomials

Zernike polynomials are a set of circularly symmetric orthogonal basis functions defined over a unit circle. They are 2D functions of both radial and azimuthal coordinates. In optical design, Zernikes are used to describe either surface irregularity or system wavefront (measured in the pupil).



http://www1.gantep.edu.tr/~bingul/opac202/docs/A1-zernike_polynomials.pdf