

Lectures Notes on Optical Design using Zemax OpticStudio

Lecture 10 Diffraction, OPD and MTF



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Image Quality

- Image quality is never perfect. It would be very nice if the image of a point object could be formed as a perfect point image. This is impossible!
- Image quality is suffered by both
 - geometrical aberrations and
 - diffraction



Diffraction

Diffraction is an effect resulting from the interaction of light wave with the sharp limiting edge or aperture of an optical system.





Image Evaluation of a Point Object

- The ideal image of a point object is a point image.
- Observation of the images of point objects produced by aberration free lens systems with circular entrance pupils shows that the images are not pointy, but have a light distribution known as the Airy pattern, A(r).
- A(r), is described mathematically by the function:

$$A(r) = \left(\frac{2J_1(\pi r)}{\pi r}\right)^2$$

 $J_1(r) = 1$ st order Bessel Function of first kind



The radial locations of the first three zeros in the Airy pattern take place at approximately

- r =1.22 => A(r) = 0
- r = 2.23 => A(r) = 0
- r = 3.24 => A(r) = 0

These locations corresponds to dark rings.

$$A(r) = \left(\frac{2J_1(\pi r)}{\pi r}\right)^2$$



Airy Disk

If the geometrical aberrations are significantly smaller than the diffraction blur, the image is well represented by the Airy disk.

This form of optics is called <u>diffraction-limited</u> optics.

Radius of Airy disk (Radius of first dark ring) is: $r_A = 1.22 \lambda (f/\#)$



About 83% of energy is encircled within the first dark ring called the Airy Disk

Airy disk is the smallest point to which a beam of light can be focused.

Angular Diameter of Airy Disk



- θ_A = Angular diameter of Airy disk
- λ = Wavelength used in operation
- D = Clear aperture diameter is the diameter of light entering to lens.Usually, it is not the full diameter of the lens.

Optical Resolution

Rayleigh's Criterion:

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved.

So, for a circular aperture, two point objects are just resolvable if they are separated by the angle:

$$\theta_{\min} = \frac{\theta_A}{2} = \frac{1.22\lambda}{D}$$



Example 1

The primary mirror of the orbiting Hubble Space Telescope has a diameter of 2.40 m. Being in orbit, (a) what is the angle between two just-resolvable point light sources (two stars)?

$$\theta_{\min} = \frac{1.22\lambda}{D} = \frac{1.22 \times 550 \times 10^{-9}}{2.4} = 2.8 \times 10^{-7}$$

(b) If these two stars are at a distance of 2 million light-years, which is the distance of the Andromeda Galaxy, how close together can they be and still be resolved?

$$s = d \times \theta_{\min} = (2.0 \times 10^6)(2.8 \times 10^{-7}) = 0.56$$
 ly

Optical Path Difference

- **OPD** is useful measure of performance of an imaging optical system.
- Rayleigh Criteria:

If the OPD is less than or equal to <u>one-quarter of a wave</u>, then the performance will be almost indistinguishable from perfect.



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Peak-to-Valey OPD



RMS wavefront error = square root of the sum of the squares of the OPDs as measured from a best-fit reference spherical wavefront over the total wavefront area

PSF (Point Spread Function)

Figure left shows the appearance of the image of a point source, which is known as a PSF, for optical path differences of 0 wave, 0.25 wave, 0.5 wave, and 1.0 wave.



Depth of Focus

- Rayleigh Criteria can be used to determine just how much defocus is tolerable to maintain diffraction limited performance.
- Depth of focus corresponding to $OPD = \lambda / 4$ is

$$\delta = \pm 2 \,\lambda (f/\#)^2$$





Difraction Limited Optics

Diffraction-limited lens is limited solely by diffraction and refers to a zero-wave RMS OPD; however, these criteria are rarely the intent of a "diffraction-limited" specification.

Common approximations for diffraction-limited performance include:

- peak-to-valley OPD < λ / 4</p>
- RMS OPD < 0.07 λ</p>
- Strehl ratio > 0.8

(Rayleigh criterion), or (Maréchal criterion), or

Example 2: Simple LWIR Objective Design

Consider an objective used in the long-wave infrared (LWIR) range.

- * Spectral band: 8 μm to 12 μm
- * We need to resolve: 0.25 mrad in object space
- * Detector used: CCD with 50 µm pixel size (pitch)

Determine f/#, clear aperture diameter, focal length and depth of focus.



 $\delta = \pm 2 \; (10 \; \mu \text{m}) \; (2.2)^2 \approx \pm 100 \; \mu \text{m}$



Example 3: Zemax Implementation of Example 2

Before Optimization

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ENPD = 100 mm EFFL = 220 mm $\lambda = 8$, **10**, 12 µm FOV = 0, 3 deg ct = [8, 15] mm

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After Local Optimization

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DRI

How far can a camera see an object?

What does it mean to see an object clearly in an electro-optical system (night vision, thermal etc.)?

In optical and imaging systems,

DRI stands for **Detection**, **Recognition**, and **Identification**.

These terms represent different levels of information acquisition in surveillance, target acquisition, and other imaging applications.

DRI distance is the universal standard for describing both spatial domain and frequency domain approaches to analyze the ability of observers to perform visual tasks using image intensifier technology.

Johnson's Criteria

The minimum required resolution according to **Johnson's criteria** are expressed in terms of line pairs of image resolution across a target in terms of several tasks:

Criteria Name	Description	Number of lines pairs (lp)	# of pixel on on target
Detection	an object is present	1.0 ± 0.25	2
Recognition	the type object can be discerned, <i>a person versus a car</i>	4.0 ± 0.8	8
Identification	a specific object can be discerned, <i>a woman versus a man,</i> <i>a specific car</i>	6.4 ± 1.5	12.8

Recognition

Identification









Vehicle







Boat







Detection, Recognition and Identification



DRI Range

Consider an imaging optical system.

Using similar triangles, range of target can be calculated as:



 $r = \frac{T}{h}$

In image plane, if we have a CCD sensor with pixel pitch p, then image height is $h = N \times p$ where N is number pixel on target (see table). Hence, DRI range can be computed as:

$$r = \frac{Tf}{Np}$$

Here DRI range (r) is the minimum distance required to see target clearly (with certain DRI value).

Example 4: DRI Calculation

Compute the DRI ranges for the optical system given in Example 2 for the target about 2.5 m wide.

Detection:
$$r = \frac{(2500 \text{ mm})(220 \text{ mm})}{(2)(0.050 \text{ mm})} = 5500 \text{ m}$$

Recognition: $r = \frac{(2500 \text{ mm})(220 \text{ mm})}{(8)(0.050 \text{ mm})} = 1375 \text{ m}$
Identification: $r = \frac{(2500 \text{ mm})(220 \text{ mm})}{(12.8)(0.050 \text{ mm})} = 860 \text{ m}$

Example

Consider human target with dimensions 1.8 m x 0.5 m.

Compute the DRI ranges for each dimensions. Assume f = 100 mm and $p = 30 \mu \text{m}$.

```
%-----% dri.m
% all values are in mm
%-----
clear; clc;
T = 1800; % target size
f = 100; % focal length
p = 0.03; % pixel pitch
N = [2, 4, 12.8]; % DRI values, Johnson's criteria
r = T*f./(N*p)/1000; % in meters
```

fprintf('Detection : r = %5d m\n',round(r(1)))
fprintf('Recognition : r = %5d m\n',round(r(2)))
fprintf('Identification: r = %5d m\n',round(r(3)))

For 1.8 m Detection : r = 3000 m Recognition : r = 1500 m Identification: r = 469 m

For 0.5 m

Detection :	r	=	833	m
Recognition :	r	=	417	m
Identification:	r	=	130	m

MTF

The most used metric for characterizing the optical system's performance is the **Modulation Transfer Function** (MTF)*.

MTF is a measure of how well a lens relays contrast from object to image.

The spatial frequency (line-pair per millimeter (lp/mm)) is the standard unit of measurement for resolution. A line pair consists of one black line and one white line.

Contrast is difference between a black line and a white line. The <u>constrast modulation</u> is defined as:

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

 I_{max} = maximum intensity level (of bright value) I_{min} = minimum intensity level (of dark value)

* MTF is the magnitude of the complex optical transfer function (OTF)

RESOLVING POWER TEST TARGET 2 -2 -1 2 -1 3 -1 3 -1 4 -1 4 -1 5 -1 5 -1 6 -1 USAF · 1951

Contrast Transfer

The contrast in object plane can be transferred to image plane by a transfer function called <u>Modulation Transfer Function (MTF)</u>.



Perfect line edges before (left) and after (right) passing through a low resolution imaging lens

Diffraction-limited MTF of a Circular Lens

MTF is a function of <u>spatial frequency</u> (s):s = number of lines within a given length.Usually we used *lines / mm* or *line pairs / mm* unit.

The diffraction-limited incoherent MTF for a lens having a circular pupil is given by:

$$MTF(s) = \frac{2}{\pi} \left\{ a\cos\left(\frac{s}{s_c}\right) - \frac{s}{s_c} \sqrt{1 - \left(\frac{s}{s_c}\right)^2} \right\}$$

The cut-off resolution can be found by: $s_c = \frac{1}{\lambda \times (f/\#)}$



Example

A target contains a series of black lines, each line 8 mm wide and seperated from the next line by an interval also 8 mm. What is the spatial frequency?

$$s = \frac{1 \text{ line pair}}{16 \text{ mm}} = 0.0625 \text{ mm}^{-1}$$







Nyquist Frequency

The Nyquist frequency (f_N) is referred to as the **sampling frequency**. In a digital system f_N is calculated as:

$$f_N = \frac{1}{2p}$$

p = the pixel size in mm of the sensor.

Performance of the MTF is subsequently evaluated using this value as a reference parameter.

For the sensor in Example 1,

$$p = 50 \ \mu m \Rightarrow f_N = 10 \ lp/mm.$$

and
cutoff resolution $= \frac{1}{\lambda(f/\#)} = 57 \ lp/mm$



Example

Consider photographs are taken from a high-altitude aircraft of a cruise ship. Assume that the MTF = 0.8 (typical camera lens).

Let $I_{max} = 5$ and $I_{min} = 2$.

Find M_{obj} and M_{img} so that ship's image is 0.5 mm wide in image plane.

Solution

Contrast at object plane:

$$M = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{5 - 2}{5 + 2} = 0.43 = 43\%$$

Since width of the ship is desired to be 0.5 mm, resolution is $s = 1 \text{ mm}^{-1}$ and MTF = 0.8. Therefore:

$$M_{img} = MTF \times M_{obj} = 0.8 \times 0.43 = 0.34$$

This is enough for visual observations.



Example

What is the power of a lens that has an angular resolution of 0.05° and resolves details in the focal plane that have a spatial resolution of 7 mm⁻¹?

Solution

The scale of the image formed in the focal plane of a lens of focal length f can be geometrically determined. When the object is seen at the angle θ , it forms an image of height s:

 $h = f \tan \theta \approx f \ \theta$

Angular res.: $\theta = 0.05^{\circ} = 8.73 \ 10^{-4} \ rad$

Spatial res.:
$$s = \frac{1}{h} = 7$$
 lines/mm

Hence the focal length and power can be found as follows:

$$f = \frac{h}{\theta} = \frac{1}{s\theta} = 163.7 \ mm \rightarrow P = 6.1 \ D$$



MTF Analysis in Zemax



Measuring MTF via USAF Target



Using the Air Force Target to Test a Camera Lens

https://harvestimaging.com/blog/?p=1294

Measuring MTF via Slanted Edge



See related scientific paper from web page.

Measuring MTF via Slanted Edge

MTF is calculated from ESF and LSF using FFT. Slanted angle is usally selected in the range [3°, 7°].

- Edge Spread Function (ESF) is measured using a half-Moon target which is tilted a few degrees. This
 target gives differences between light source (blackbody) and background in a single image. SFR
 measurement readings is a step-function whose derivative is Line Spread Function (LSF).
- LSF is a function of the angle of view which describes the sharpness of the camera.
- MTF comes as a result of the **Fast Fourier Transform** of the LSF.



Evaluating MTF using MATLAB

You may use the following functions implemented in **image processing tool**:

<pre>imread()</pre>	open an image file
imshow()	display an image file
rgb2gray()	convert rgb image to grayscale
<pre>imrotate()</pre>	rotate an image
<pre>imcrop()</pre>	crop an image
fft()	Fast Fourier Transform function

System MTF

Each factor influencing performance should be reviewed to determine the MTF budget of the optical system.

```
MTF_{system} = MTF_{atmosphere} * MTF_{optics} * MTF_{detector} * \cdots
```

For example, MTF operation for satellite system.



Signal-to-Noise Ratio (SNR)

- SNR measures the radiometric performance of the captured image.
- SNR is computed for the light path from the source to the imager.
- SNR is one important performance evaluation parameter for
 - > space-borne electro-optic imagers
 - > Thermal Camera systems.
- SNR is the ratio of the imaging signal to the total noise:

$$SNR = rac{S_{total}}{\sigma_{total}}$$

See paper 'Signal-to-noise ratio model in Python for high-resolution space-borne electrooptic imagers' at: https://doi.org/10.1117/1.JRS.17.014508