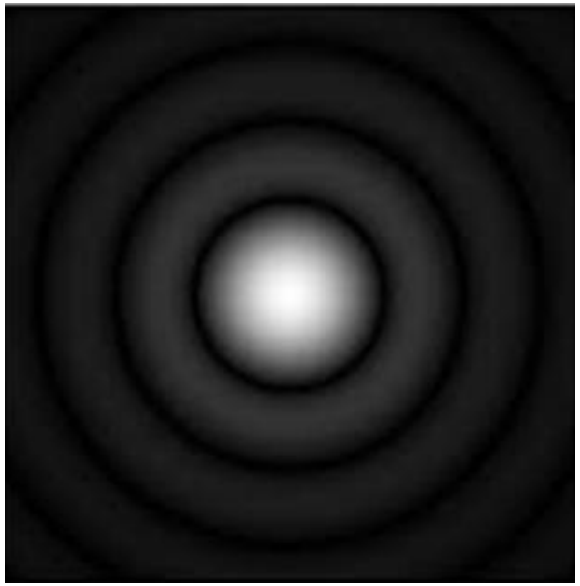
Lectures Notes on Optical Design using Zemax OpticStudio

Lecture 10

Diffraction, OPD and MTF



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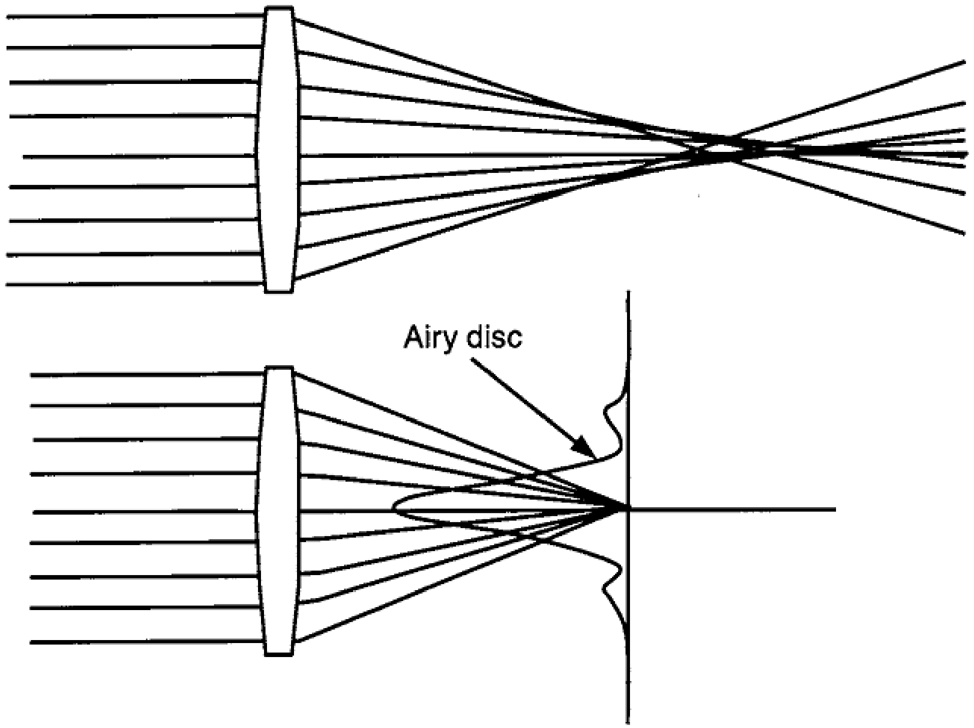
Content

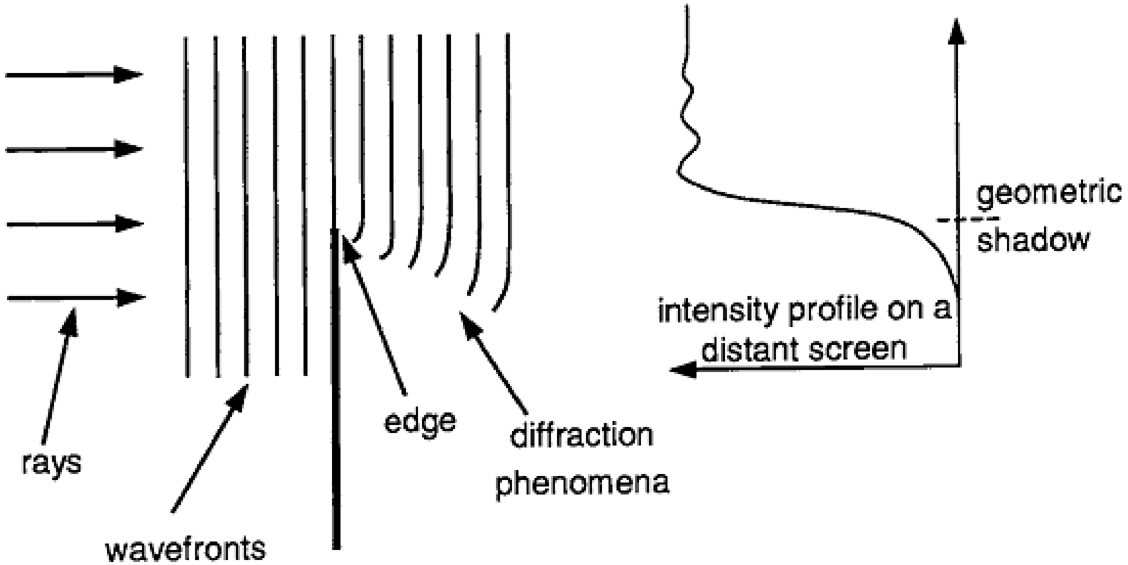
Image Quality

- **Image quality is never perfect**. It would be very nice if the image of a point object could be formed as a perfect point image. This is impossible!
- Image quality is suffered by both
  - geometrical aberrations and
  - diffraction



Diffraction

Diffraction is an effect resulting from the interaction of light wave with the sharp limiting edge or aperture of an optical system.



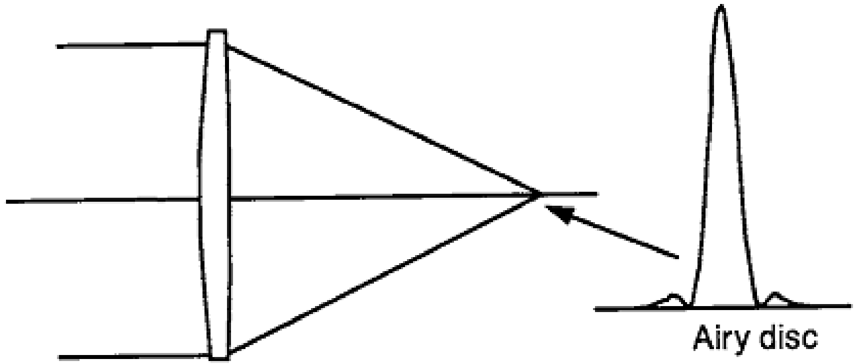
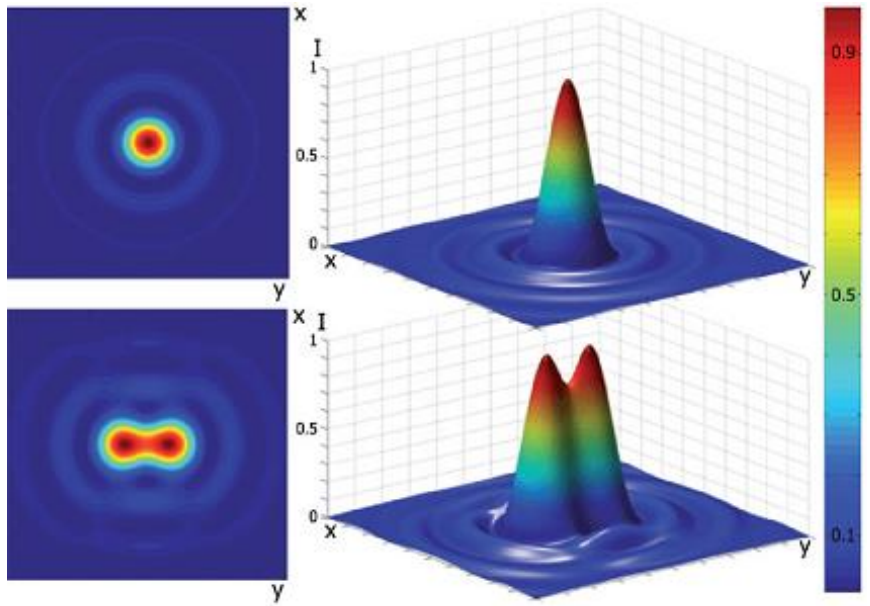


Image Evaluation of a Point Object

- The ideal image of a point object is a point image.
- Observation of the images of point objects produced by aberration free lens systems with circular entrance pupils shows that the images are not pointy, but have a light distribution known as the **Airy pattern, A(r).**
- A(r), is described mathematically by the function:

$$A(r) = \left(\frac{2J_1(\pi r)}{\pi r}\right)^2$$

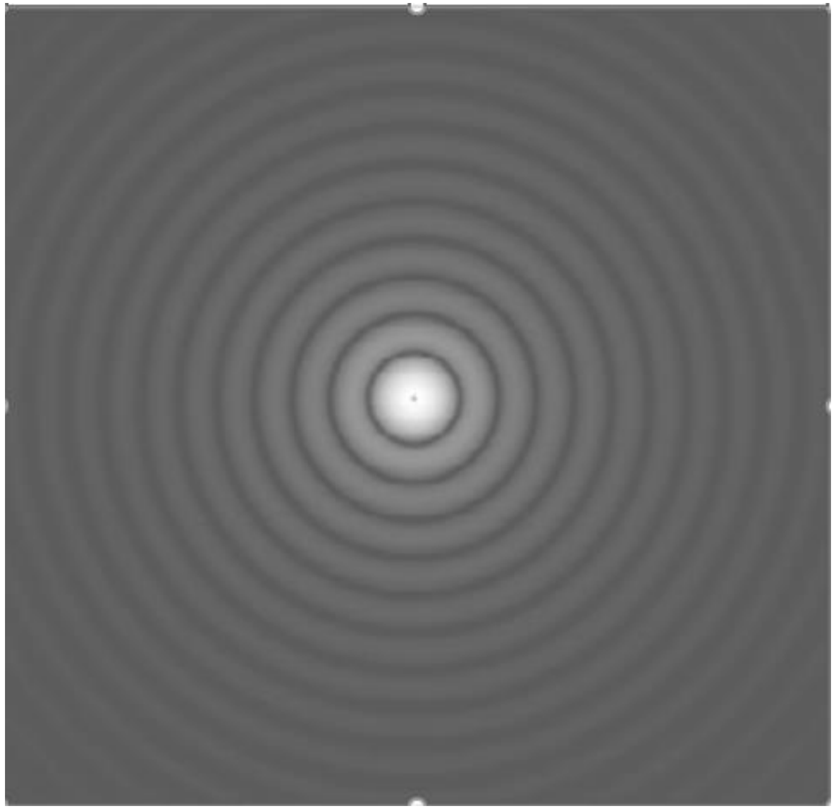
$J_1(r)$ = 1st order Bessel Function of first kind



The radial locations of the first three zeros in the Airy pattern take place at approximately

- r =1.22 => A(r) = 0
- r = 2.23 => A(r) = 0
- r = 3.24 => A(r) = 0

$$A(r) = \left(\frac{2J_1(\pi r)}{\pi r}\right)^2$$



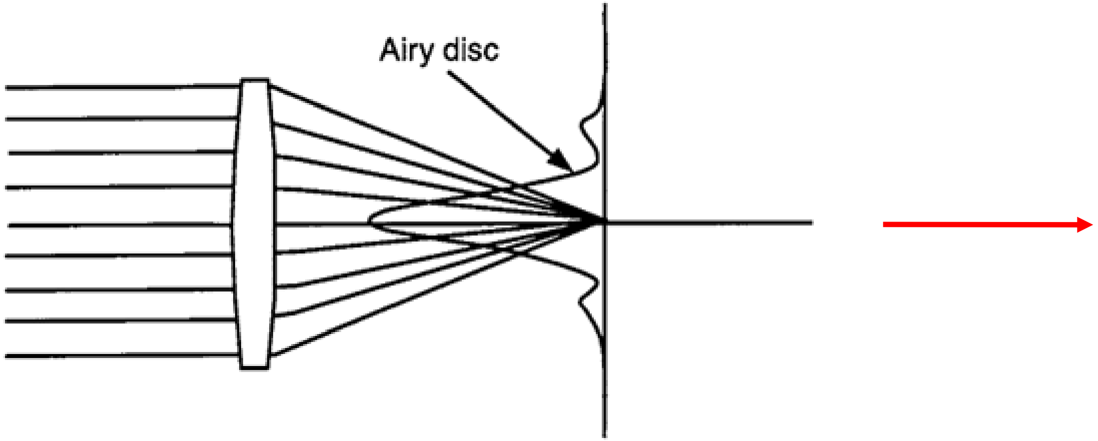
These locations corresponds to dark rings.

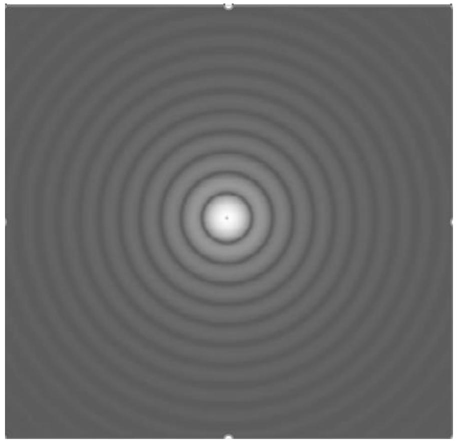
Airy Disk

If the geometrical aberrations are significantly smaller than the diffraction blur, the image is well represented by the Airy disk.

This form of optics is called diffraction-limited optics.

Radius of Airy disk (Radius of first dark ring) is: $r_A = 1.22\,\lambda\,(f/\#)$

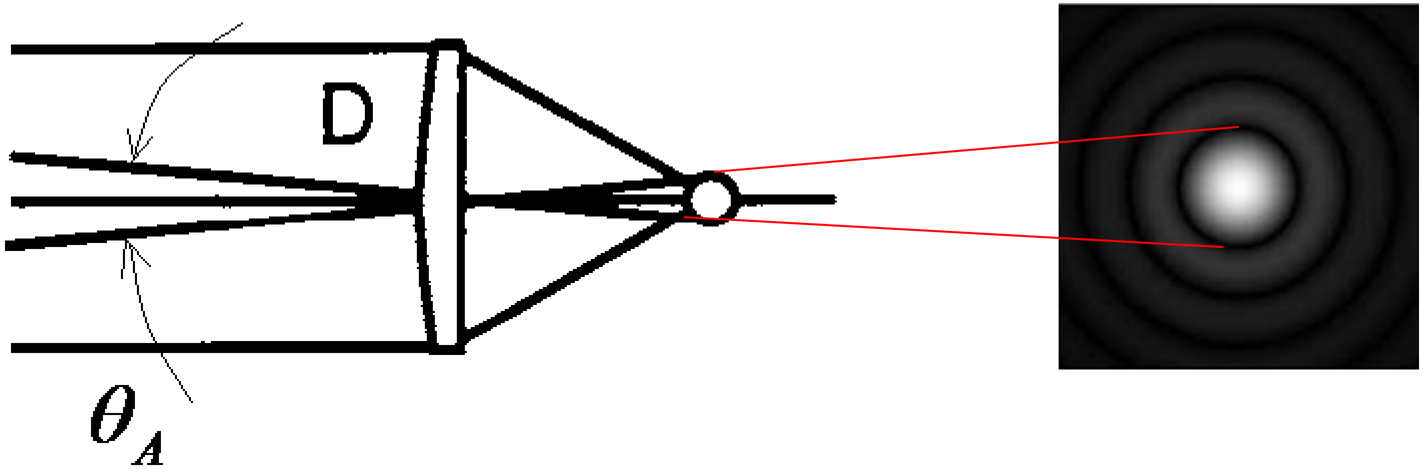




About 83% of energy is encircled within the first dark ring called the Airy Disk

Airy disk is the smallest point to which a beam of light can be focused.

Angular Diameter of Airy Disk

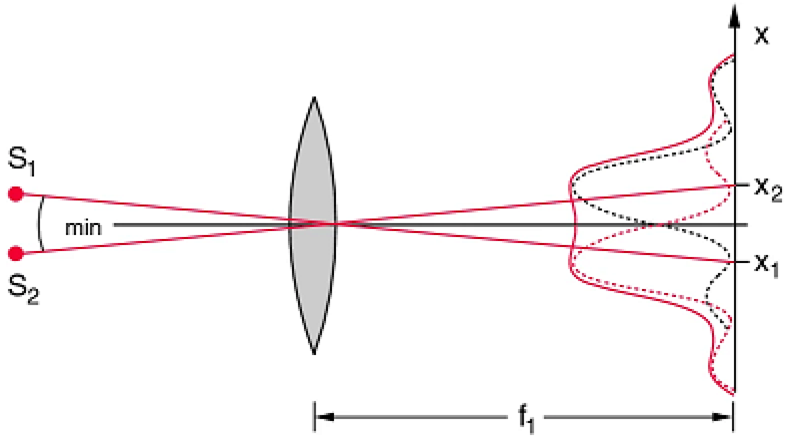


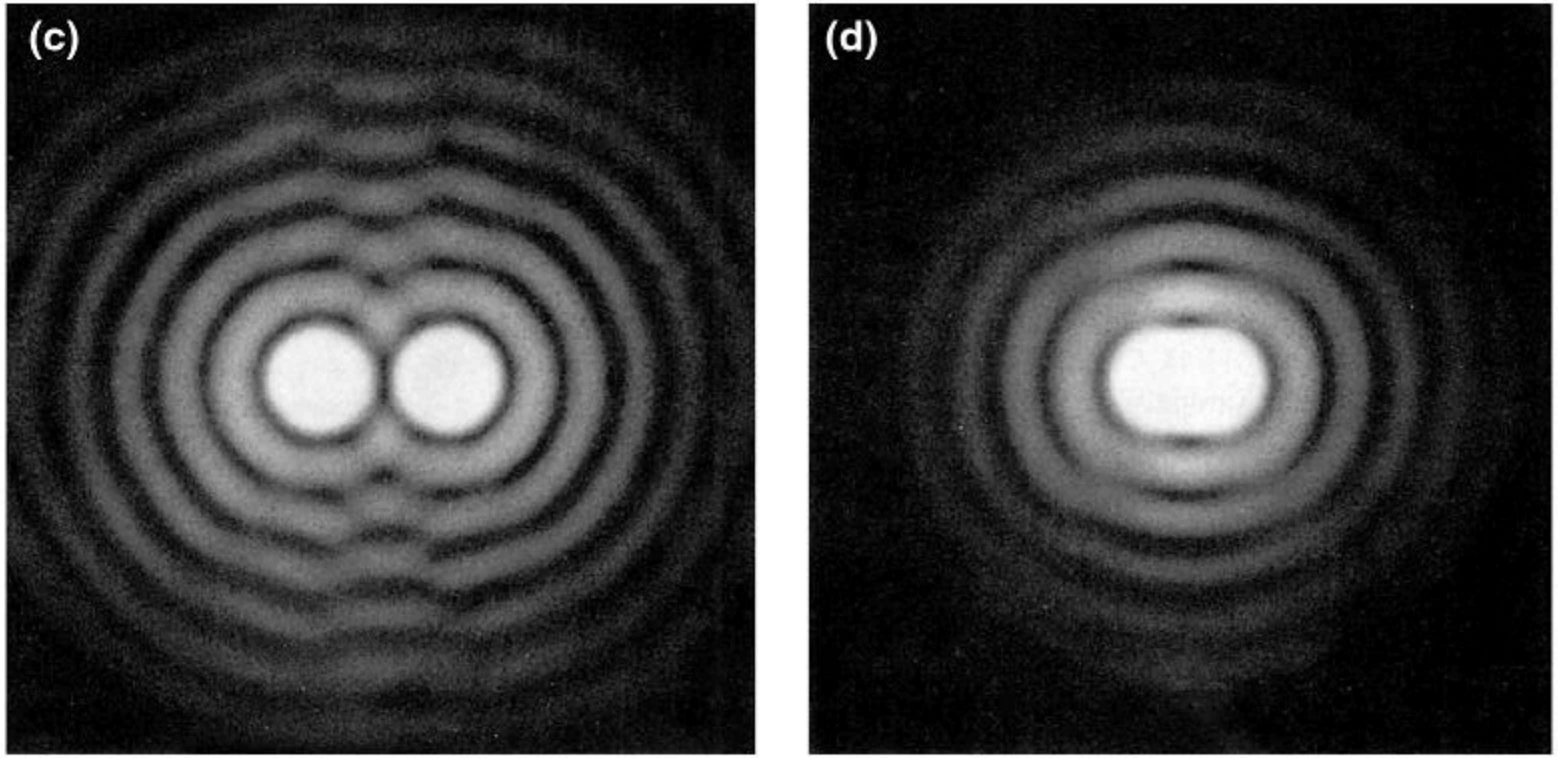
$$\theta_A = \frac{2.44\lambda}{D}$$

θ_A = Angular diameter of Airy disk

λ = Wavelength used in operation

D = Clear aperture diameter is the diameter of light entering to lens. Usually, it is not the full diameter of the lens.

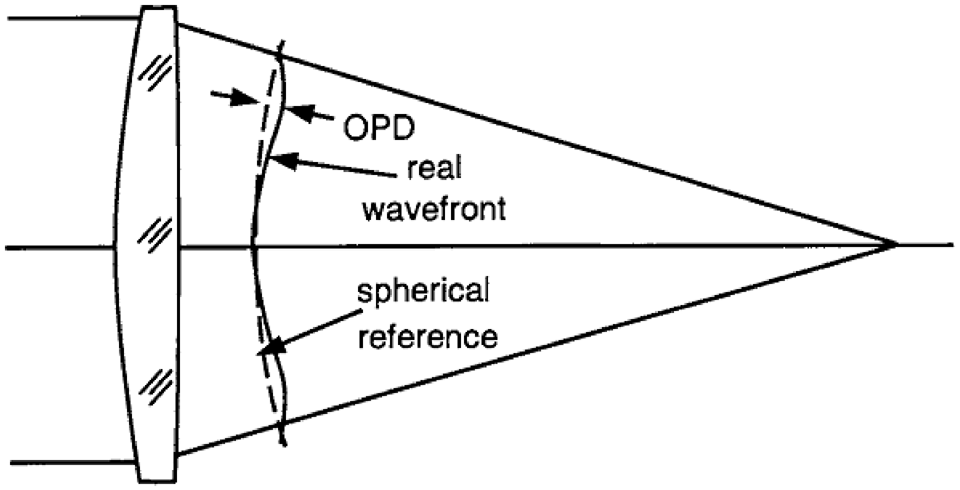

x
S_1
x_2
min
x_1
S_2
f_1

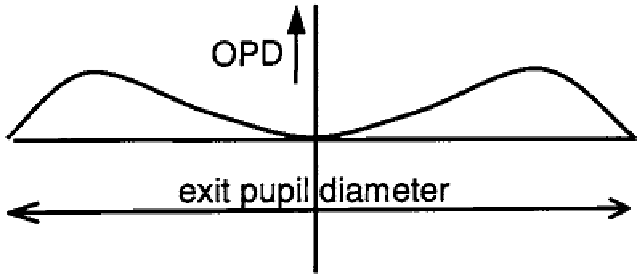

(c)
(d)

Optical Path Difference

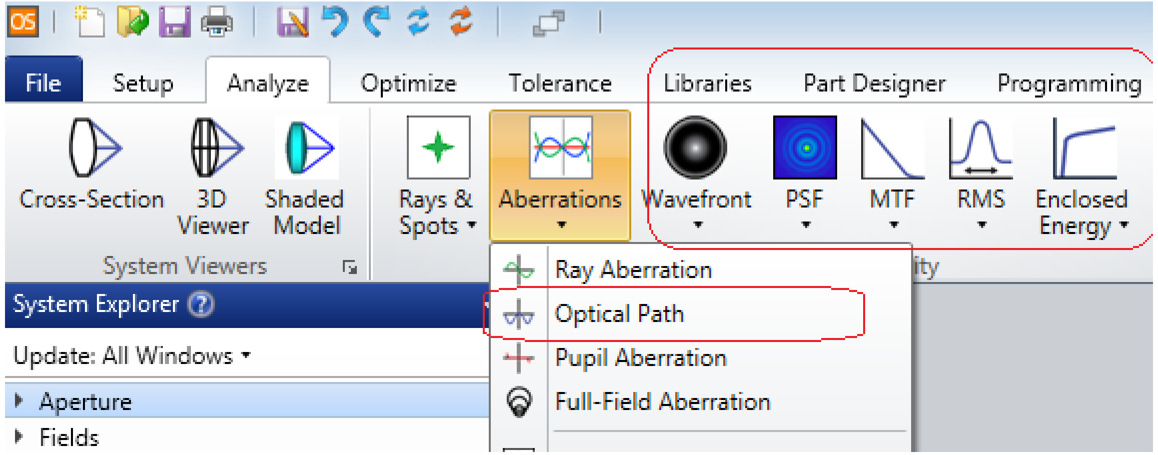
- **OPD** is useful measure of performance of an imaging optical system.

- **Rayleigh Criteria**:
  If the OPD is less than or equal to one-quarter of a wave, then the performance will be almost indistinguishable from perfect.

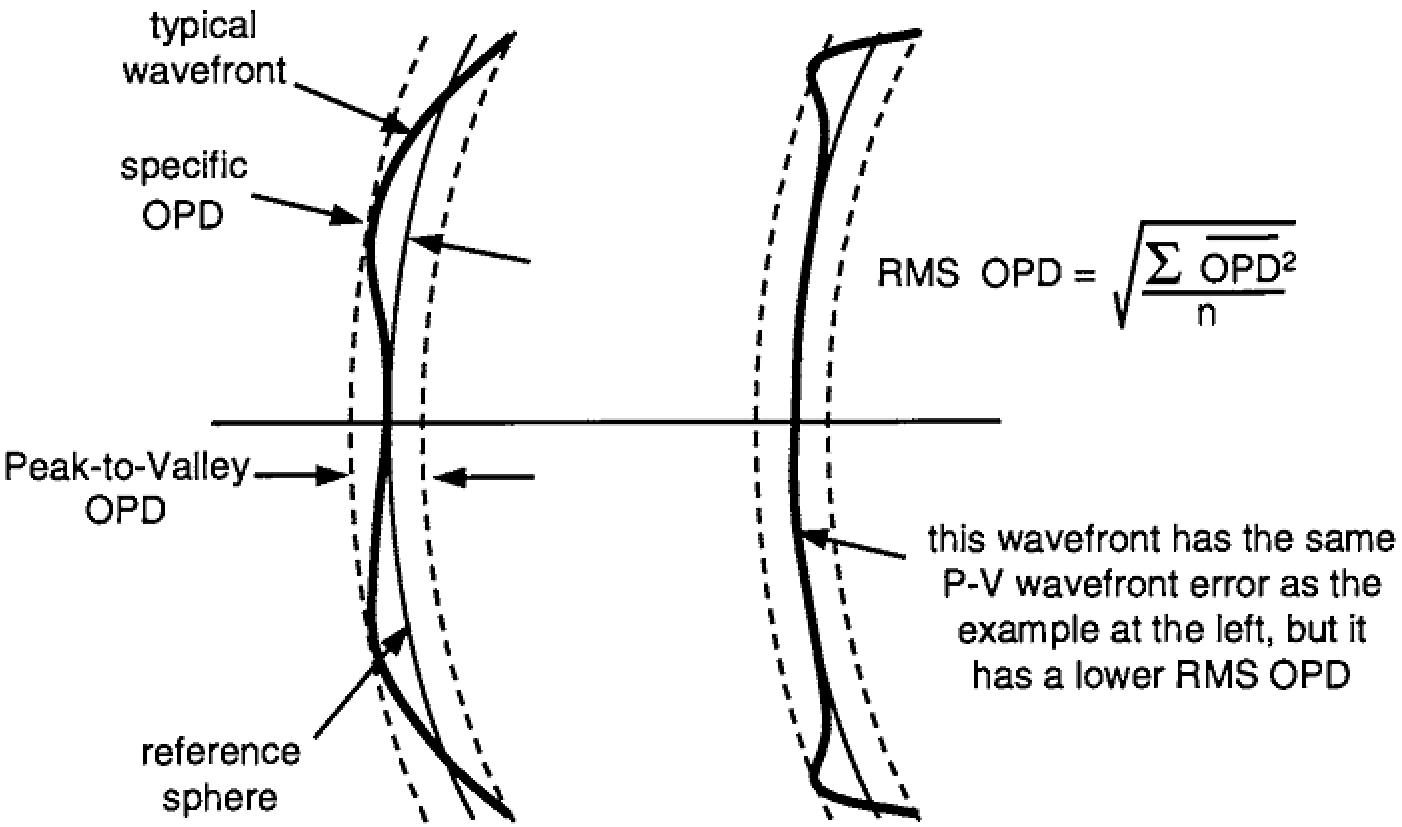




- Zemax menu



Peak-to-Valey OPD

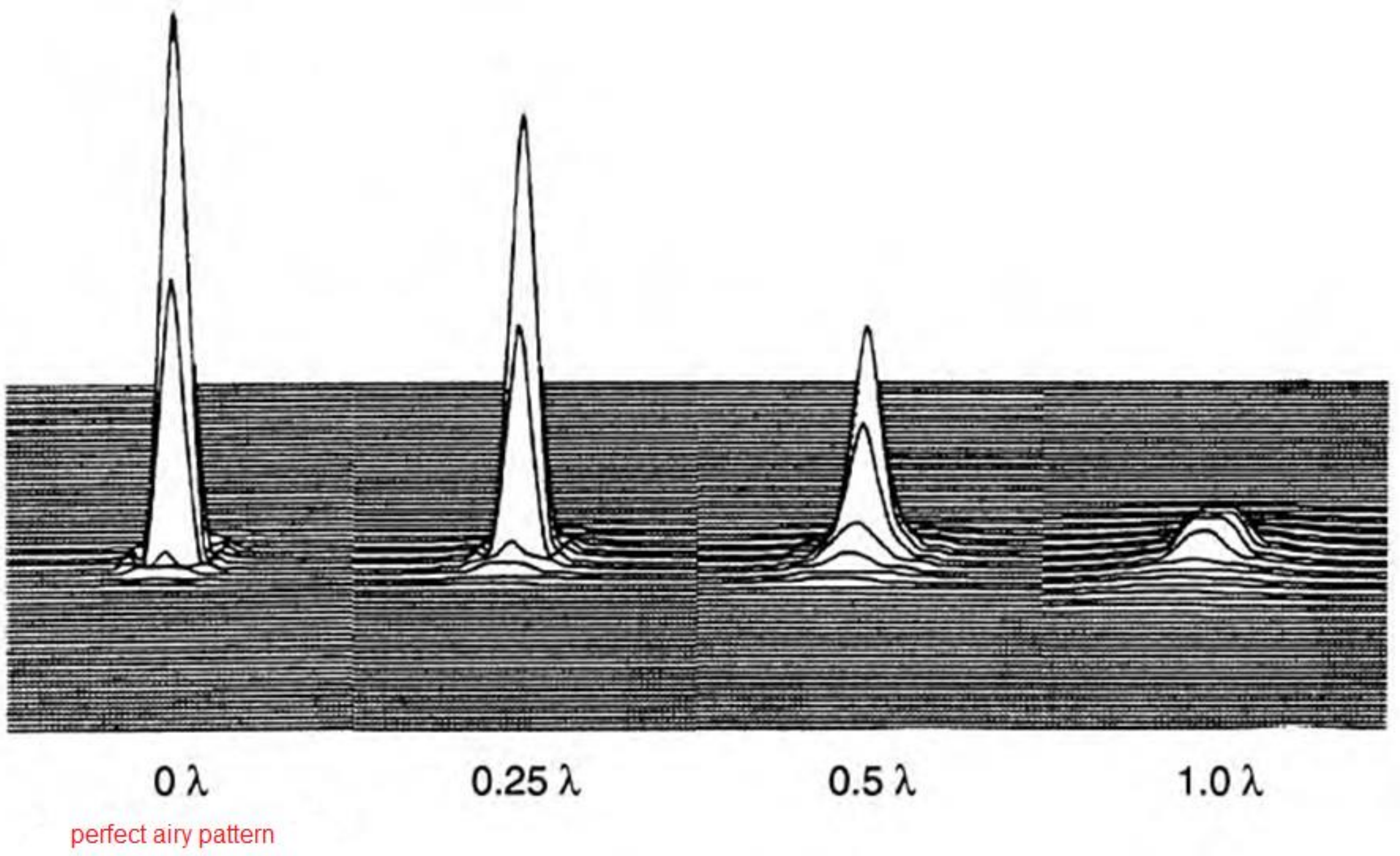


$$\text{RMS OPD} = \sqrt{\frac{\sum \overline{\text{OPD}^2}}{n}}$$

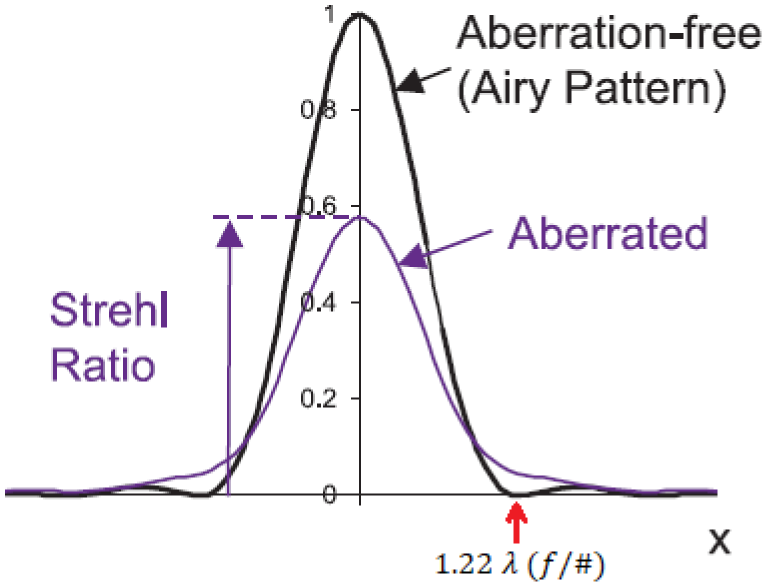
RMS wavefront error = square root of the sum of the squares of the OPDs as measured from a best-fit reference spherical wavefront over the total wavefront area

PSF (Point Spread Function)

Figure left shows the appearance of the image of a point source, which is known as a PSF, for optical path differences of 0 wave, 0.25 wave, 0.5 wave, and 1.0 wave.



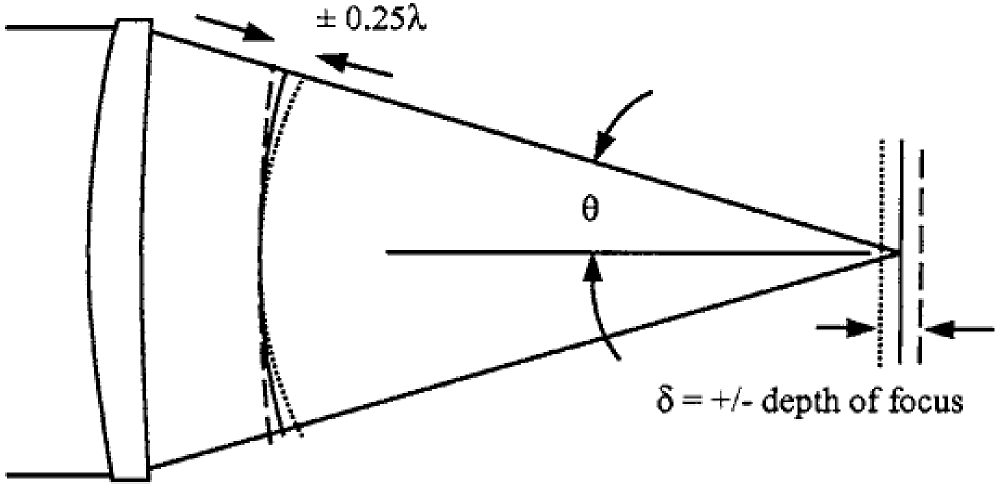
The **Strehl ratio** is defined as the ratio of the peak intensity of an aberrated lens to the peak intensity of an aberration-free lens.

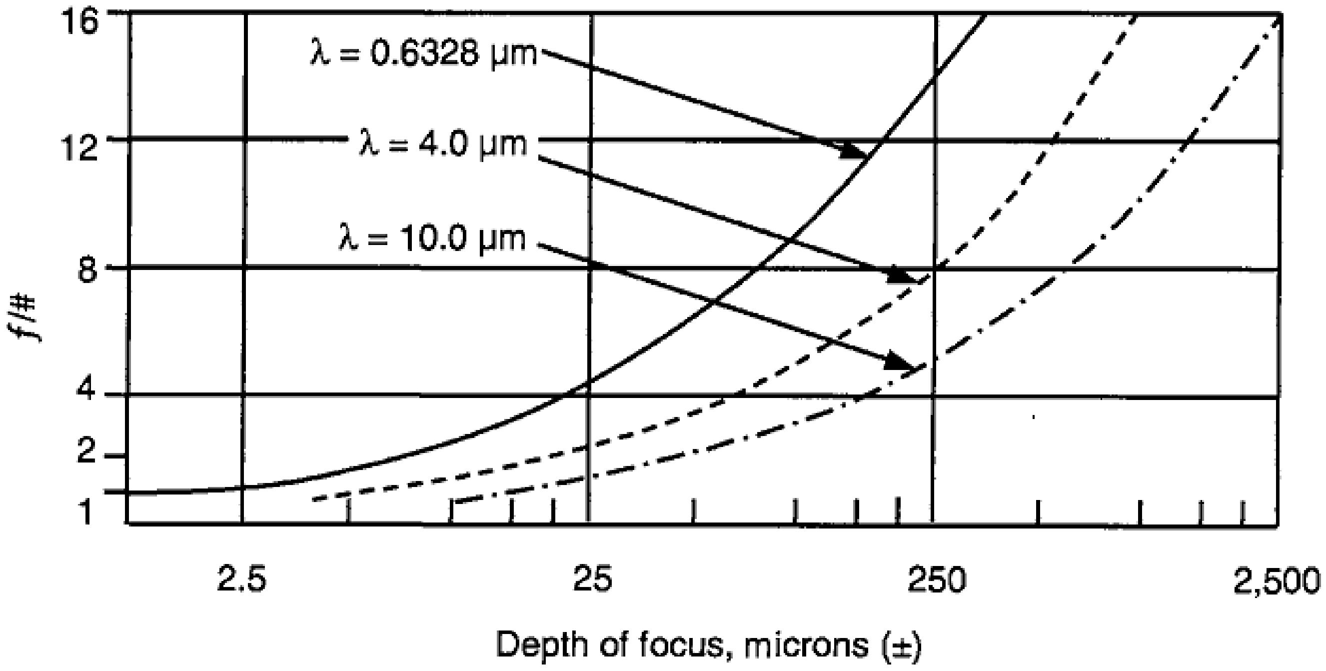


Depth of Focus

- Rayleigh Criteria can be used to determine just how much defocus is tolerable to maintain diffraction limited performance.
- Depth of focus corresponding to OPD = λ / 4 is

$$\delta = \pm 2\,\lambda (f/\#)^2$$





Difraction Limited Optics

Diffraction-limited lens is limited solely by diffraction and refers to a zero-wave RMS OPD; however, these criteria are rarely the intent of a “diffraction-limited” specification.

Common approximations for diffraction-limited performance include:

- **peak-to-valley OPD < λ / 4** (Rayleigh criterion), or
- **RMS OPD < 0.07 λ** (Maréchal criterion), or
- **Strehl ratio > 0.8**

Example 1: Simple LWIR Objective Design

Consider an objective used in the long-wave infrared (LWIR) range.

* Spectral band: 8 μm to 12 μm

* We need to resolve: 0.25 mrad in object space

* Detector used: CCD with 50 μm pixel size (pitch)

Determine f/#, clear aperture diameter, focal length and depth of focus.

$$\textbf{Diameter of Airy disk} = \textbf{CCD pixel size}$$

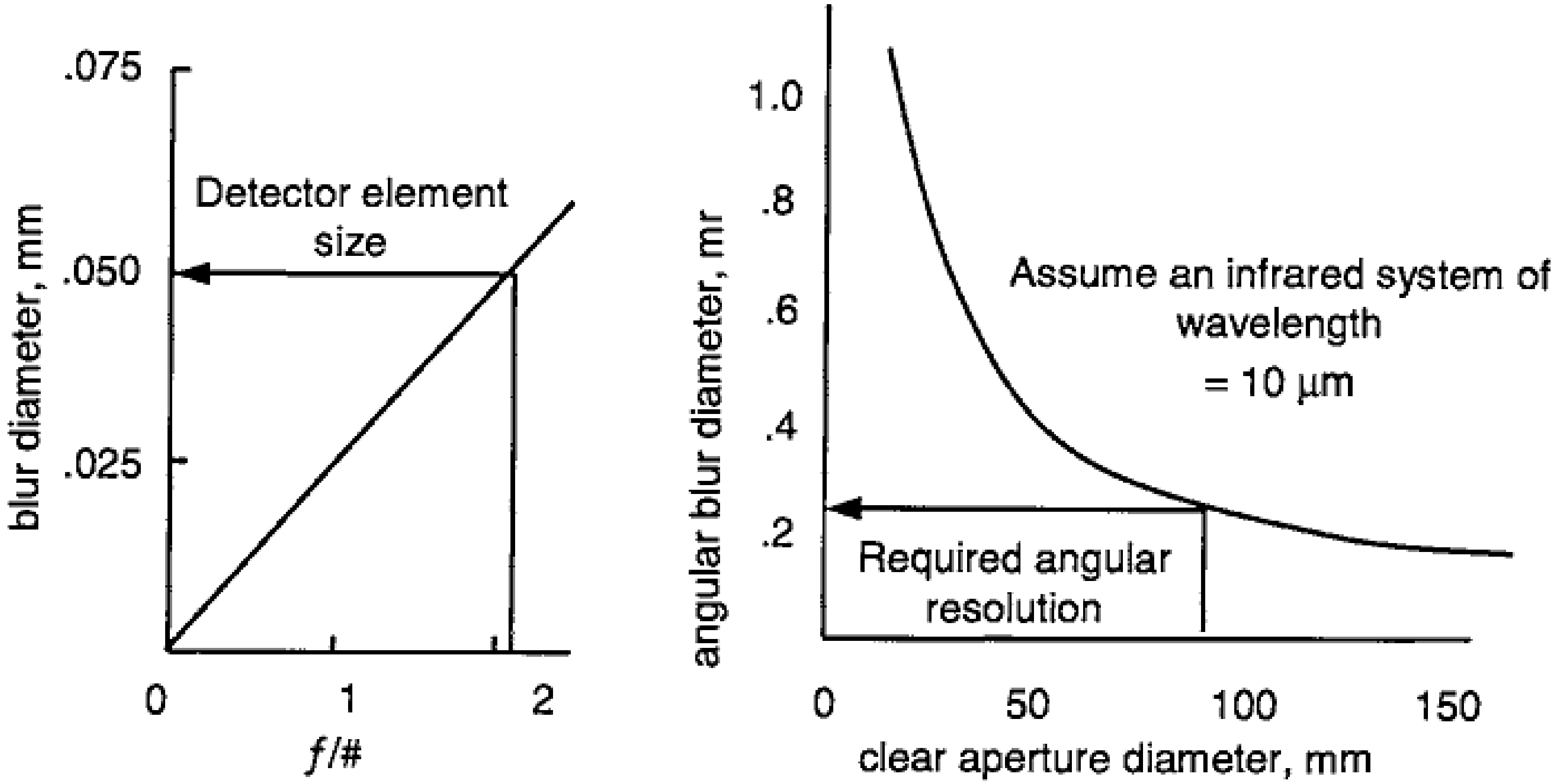
$$2.44\,\lambda\,(f/\#) = \textbf{CCD pixel size}$$

$$2.44\,(10\ \mu\text{m})\,(f/\#) = 50\ \mu\text{m} \longrightarrow \boxed{f/2.2}$$

$$\boxed{f/2.2} \longrightarrow \boxed{f = 220\ \text{mm}} \longleftarrow \boxed{D = 100\ \text{mm}}$$

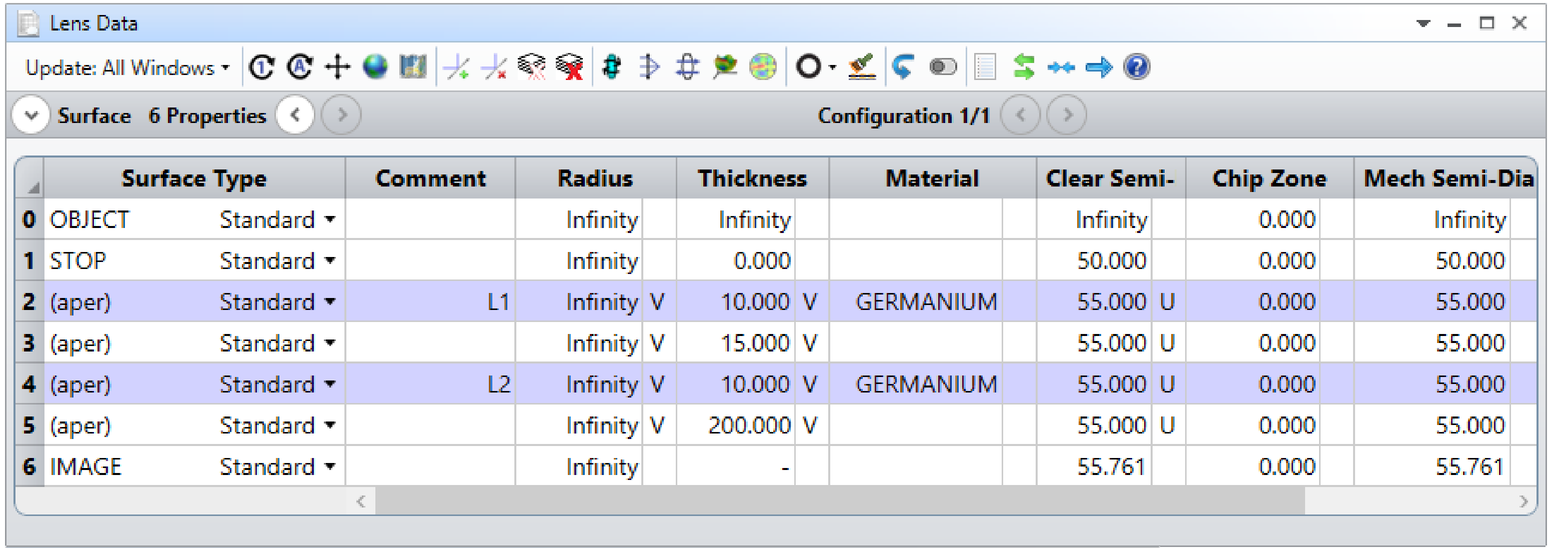
$$D = \frac{2.44\lambda}{\theta_A} = \frac{2.44(10\ \mu\text{m})}{0.25\times 10^{-3}} = 100\ \text{mm} \longrightarrow \boxed{D = 100\ \text{mm}}$$

$$\delta = \pm 2\,(10\ \mu\text{m})\,(2.2)^2 \approx \pm 100\ \mu\text{m}$$


.075
.050
.025
blur diameter, mm
Detector element size
0
1
2
f/#
1.0
.8
.6
.4
.2
angular blur diameter, mr
Assume an infrared system of wavelength = 10 µm
Required angular resolution
0
50
100
150
clear aperture diameter, mm

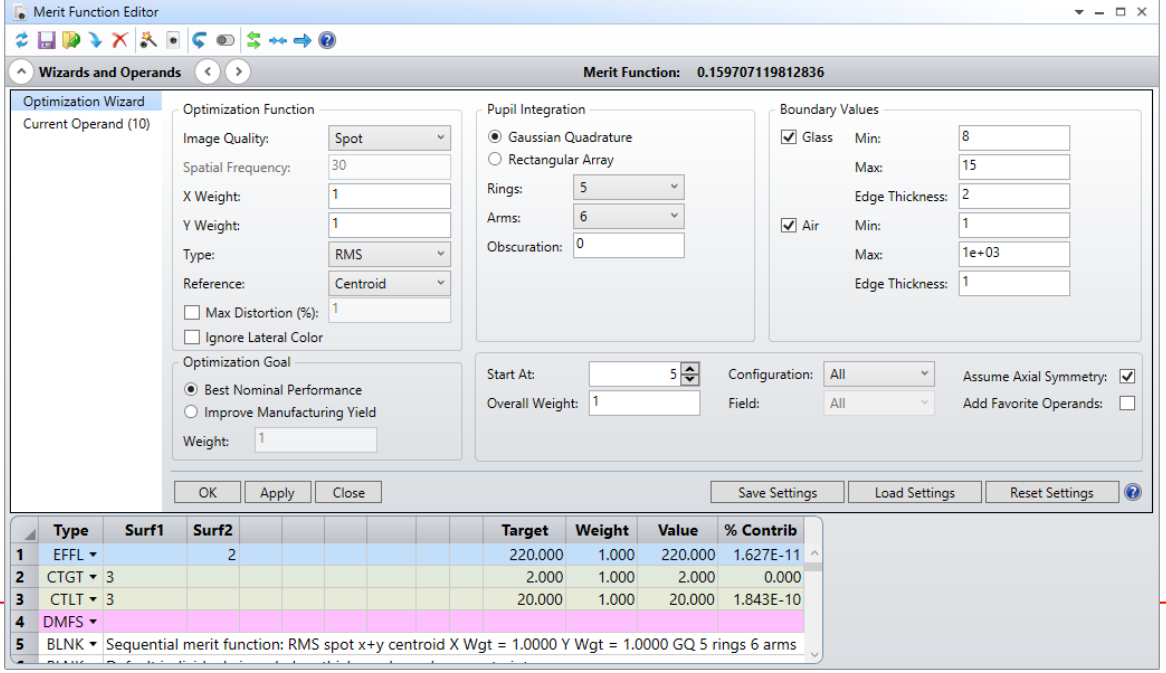
Example 2: Zemax Implementation of Example 1

Before Optimization



Configuration 1/1

	Surface Type		Comment	Radius	Thickness	Material	Clear Semi-	Chip Zone	Mech Semi-Dia
0	OBJECT	Standard		Infinity	Infinity		Infinity	0.000	Infinity
1	STOP	Standard		Infinity	0.000		50.000	0.000	50.000
2	(aper)	Standard	L1	Infinity V	10.000 V	GERMANIUM	55.000 U	0.000	55.000
3	(aper)	Standard		Infinity V	15.000 V		55.000 U	0.000	55.000
4	(aper)	Standard	L2	Infinity V	10.000 V	GERMANIUM	55.000 U	0.000	55.000
5	(aper)	Standard		Infinity V	200.000 V		55.000 U	0.000	55.000
6	IMAGE	Standard		Infinity	-		55.761	0.000	55.761



Merit Function: 0.159707119812836

Optimization Function
Image Quality: Spot
Spatial Frequency: 30
X Weight: 1
Y Weight: 1
Type: RMS
Reference: Centroid
☐ Max Distortion (%): 1
☐ Ignore Lateral Color

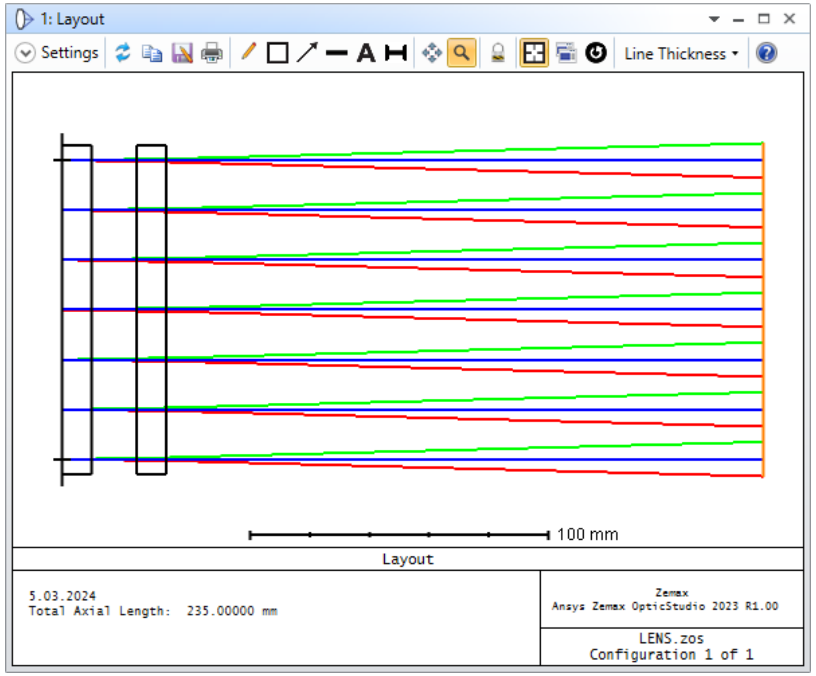
Optimization Goal
● Best Nominal Performance
○ Improve Manufacturing Yield
Weight: 1

Pupil Integration
● Gaussian Quadrature
○ Rectangular Array
Rings: 5
Arms: 6
Obscuration: 0

Boundary Values
☑ Glass Min: 8
Max: 15
Edge Thickness: 2
☑ Air Min: 1
Max: 1e+03
Edge Thickness: 1

Start At: 5
Overall Weight: 1
Configuration: All
Field: All
Assume Axial Symmetry: ☑
Add Favorite Operands: ☐

	Type	Surf1	Surf2						Target	Weight	Value	% Contrib
1	EFFL		2						220.000	1.000	220.000	1.627E-11
2	CTGT	3							2.000	1.000	2.000	0.000
3	CTLT	3							20.000	1.000	20.000	1.843E-10
4	DMFS											
5	BLNK	Sequential merit function: RMS spot x+y centroid X Wgt = 1.0000 Y Wgt = 1.0000 GQ 5 rings 6 arms										



5.03.2024
Total Axial Length: 235.00000 mm

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LENS.zos
Configuration 1 of 1

ENPD = 100 mm

EFFL = 220 mm

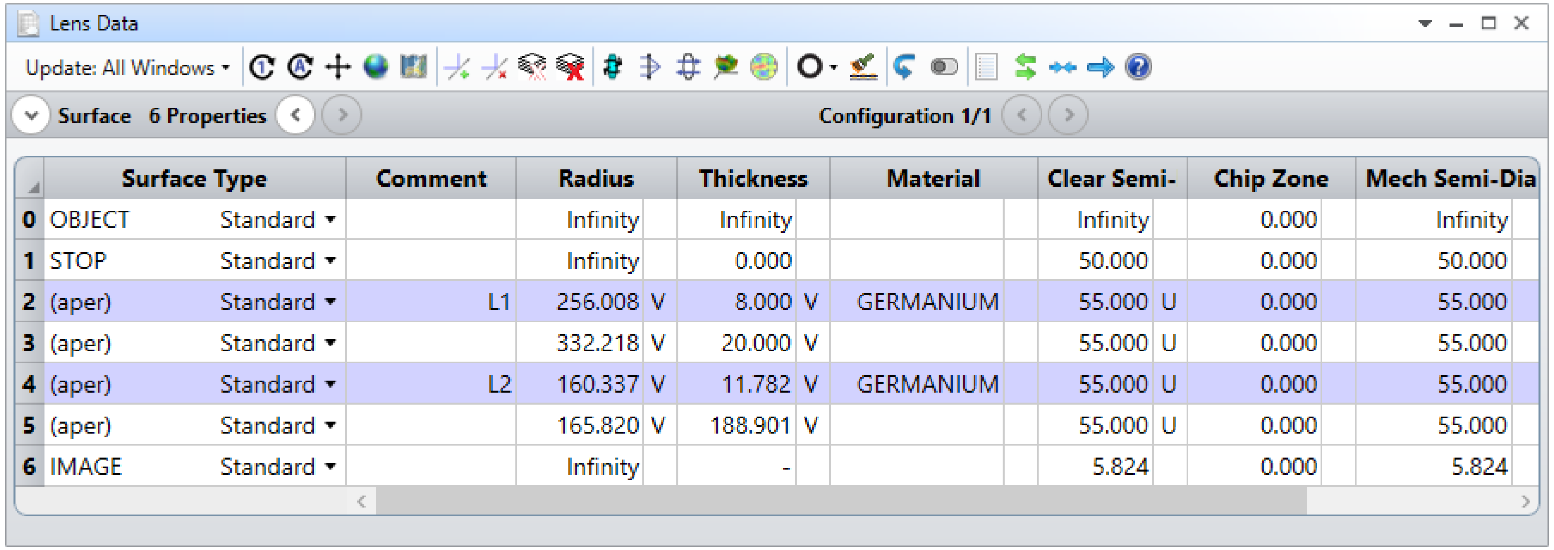
λ = 8, **10**, 12 μm

FOV = 0, 3 deg

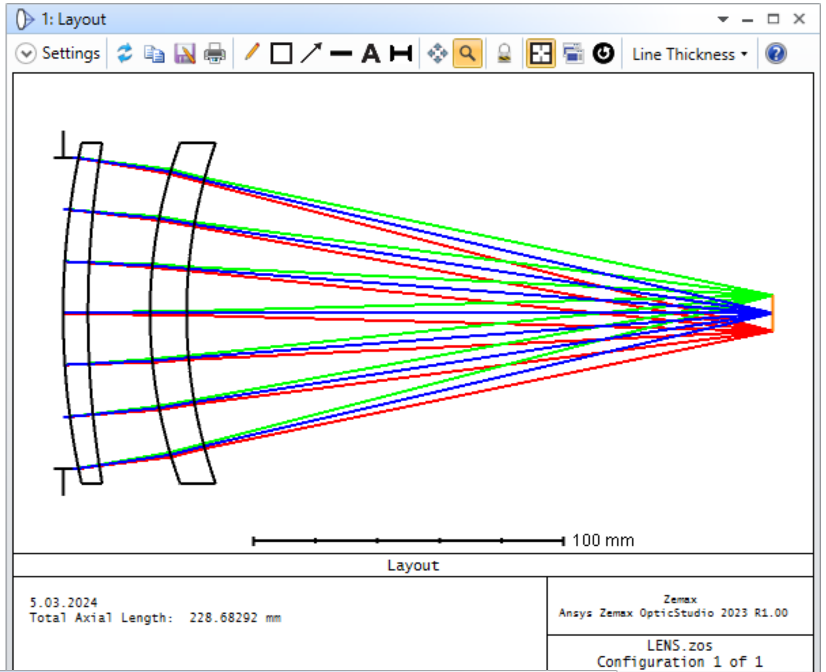
ct = [8, 15] mm

Example 2: Zemax Implementation

After Local Optimization



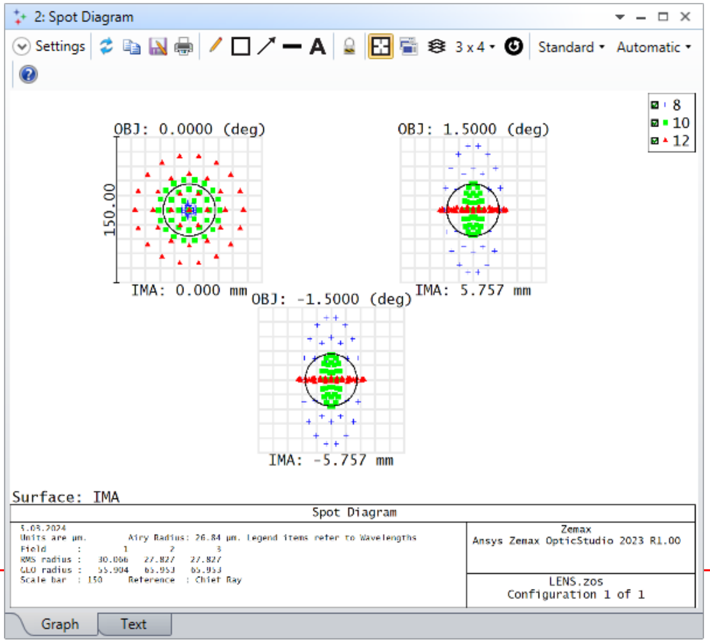
	Surface Type		Comment	Radius		Thickness		Material	Clear Semi-		Chip Zone	Mech Semi-Dia
0	OBJECT	Standard		Infinity		Infinity			Infinity		0.000	Infinity
1	STOP	Standard		Infinity		0.000			50.000		0.000	50.000
2	(aper)	Standard	L1	256.008	V	8.000	V	GERMANIUM	55.000	U	0.000	55.000
3	(aper)	Standard		332.218	V	20.000	V		55.000	U	0.000	55.000
4	(aper)	Standard	L2	160.337	V	11.782	V	GERMANIUM	55.000	U	0.000	55.000
5	(aper)	Standard		165.820	V	188.901	V		55.000	U	0.000	55.000
6	IMAGE	Standard		Infinity		-			5.824		0.000	5.824



5.03.2024
Total Axial Length: 228.68292 mm

Zemax
Ansys Zemax OpticStudio 2023 R1.00

LENS.zos
Configuration 1 of 1



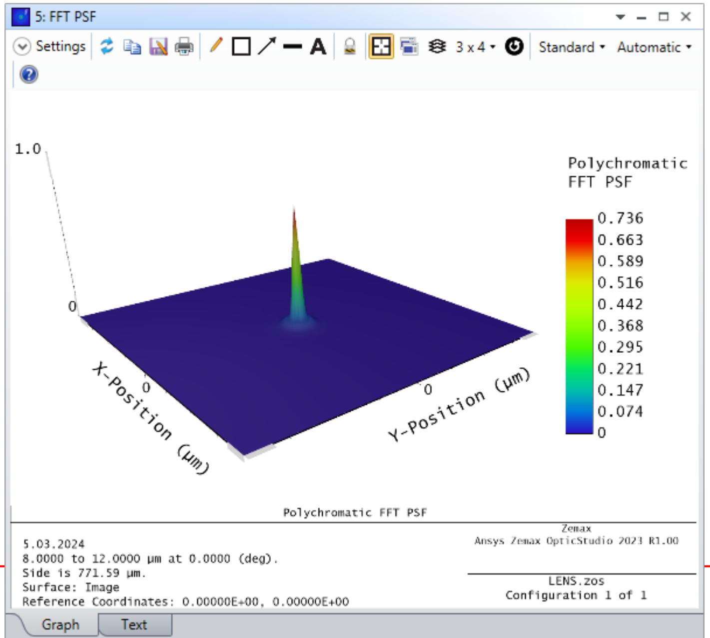
Surface: IMA

Spot Diagram

5.03.2024
Units are µm.   Airy Radius: 26.84 µm. Legend items refer to Wavelengths
Field   :   1   2   3
RMS radius :   30.066   27.827   27.827
GEO radius :   55.904   65.953   65.953
Scale bar : 150   Reference : Chief Ray

Zemax
Ansys Zemax OpticStudio 2023 R1.00

LENS.zos
Configuration 1 of 1

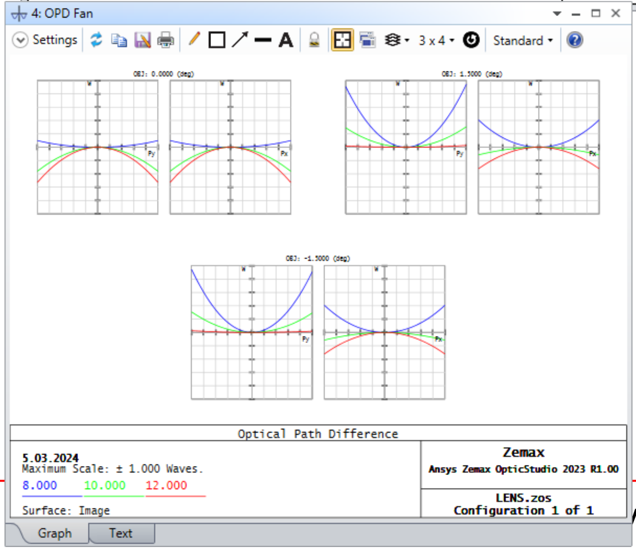


Polychromatic FFT PSF

5.03.2024
8.0000 to 12.0000 µm at 0.0000 (deg).
Side is 771.59 µm.
Surface: Image
Reference Coordinates: 0.00000E+00, 0.00000E+00

Zemax
Ansys Zemax OpticStudio 2023 R1.00

LENS.zos
Configuration 1 of 1



Optical Path Difference

5.03.2024
Maximum Scale: ± 1.000 Waves.
8.000 10.000 12.000
Surface: Image

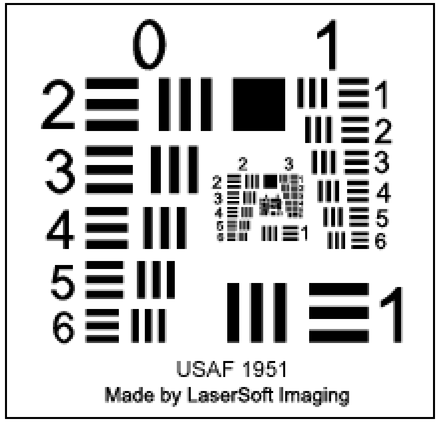
Zemax
Ansys Zemax OpticStudio 2023 R1.00

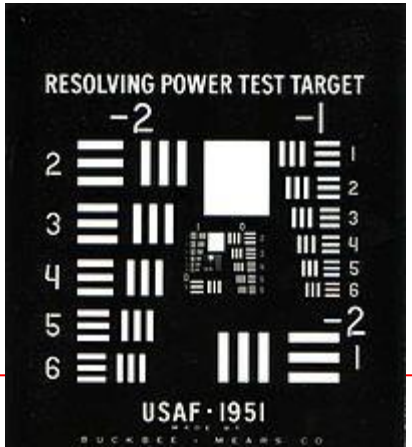
LENS.zos
Configuration 1 of 1

MTF

The most used metric for characterizing the optical system's performance is the **Modulation Transfer Function** (MTF)*. MTF is a measure of how well a lens relays contrast from object to image.

The spatial frequency (line-pair per millimeter (lp/mm)) is the standard unit of measurement for resolution. A line pair consists of one black line and one white line.





Contrast is difference between a black line and a white line.

The constrast modulation is defined as:

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

I_{max} = maximum intensity level (of bright value)

I_{min} = minimum intensity level (of dark value)

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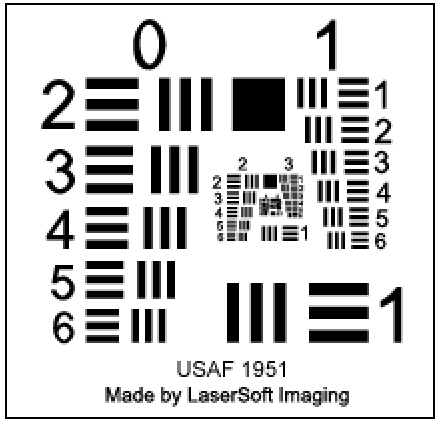
* *MTF is the magnitude of the complex optical transfer function (OTF)*

Contrast Transfer

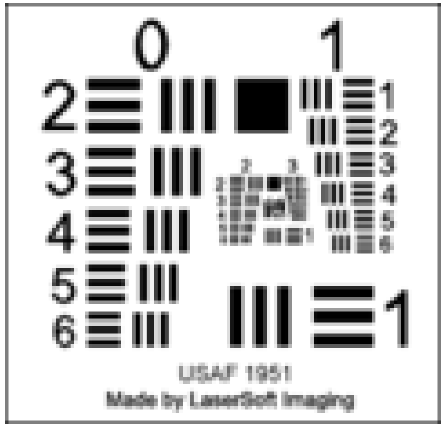
The contrast in object plane can be transferred to image plane by a transfer function called Modulation Transfer Function (MTF).

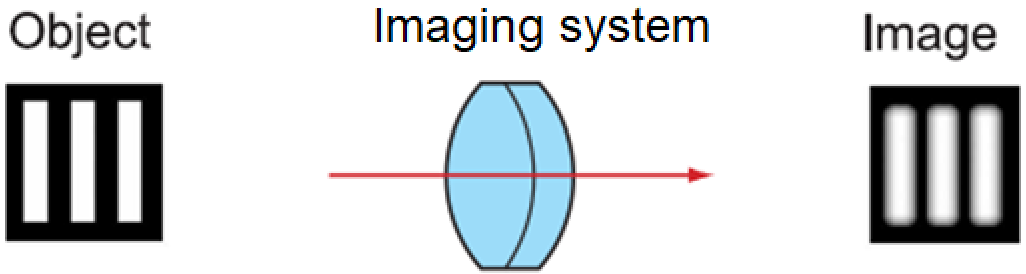
Object (M_{obj})



$$M_{img} = MTF \times M_{obj}$$

Image (M_{img})





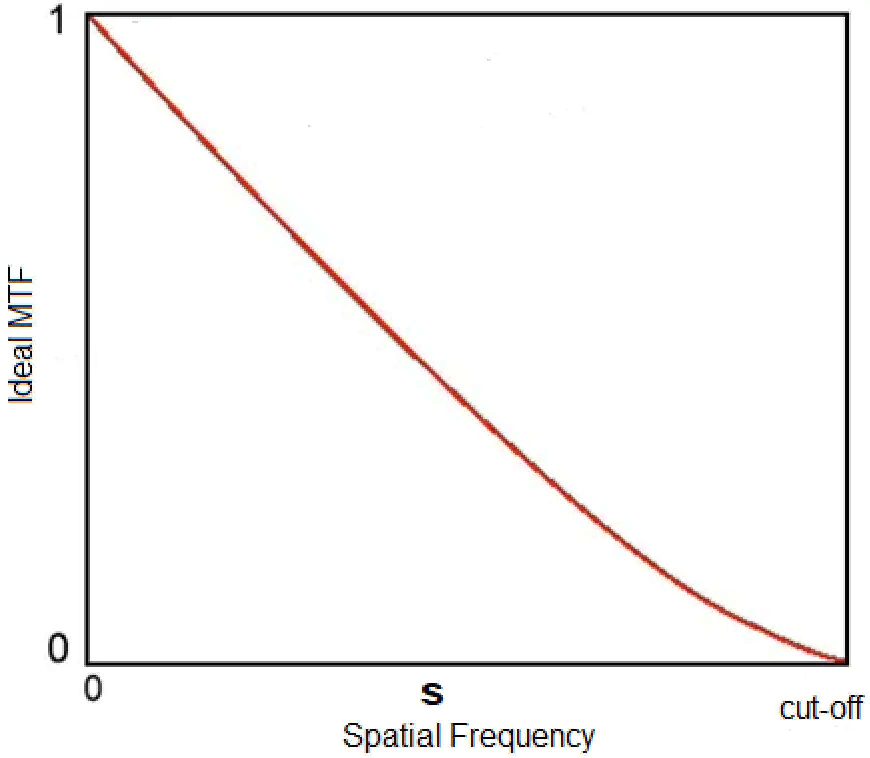
Perfect line edges before (left) and after (right) passing through a low resolution imaging lens

Diffraction-limited MTF of a Circular Lens

MTF is a function of spatial frequency (*s*):

s = number of lines within a given length.

Usually we used *lines / mm* or *line pairs / mm* unit.



The diffraction-limited incoherent MTF for a lens having a circular pupil is given by:

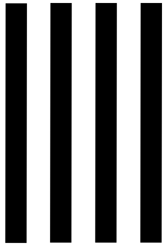
$$MTF(s) = \frac{2}{\pi}\left\{ \operatorname{acos}\left(\frac{s}{s_c}\right) - \frac{s}{s_c}\sqrt{1-\left(\frac{s}{s_c}\right)^2} \right\}$$

The cut-off resolution can be found by:

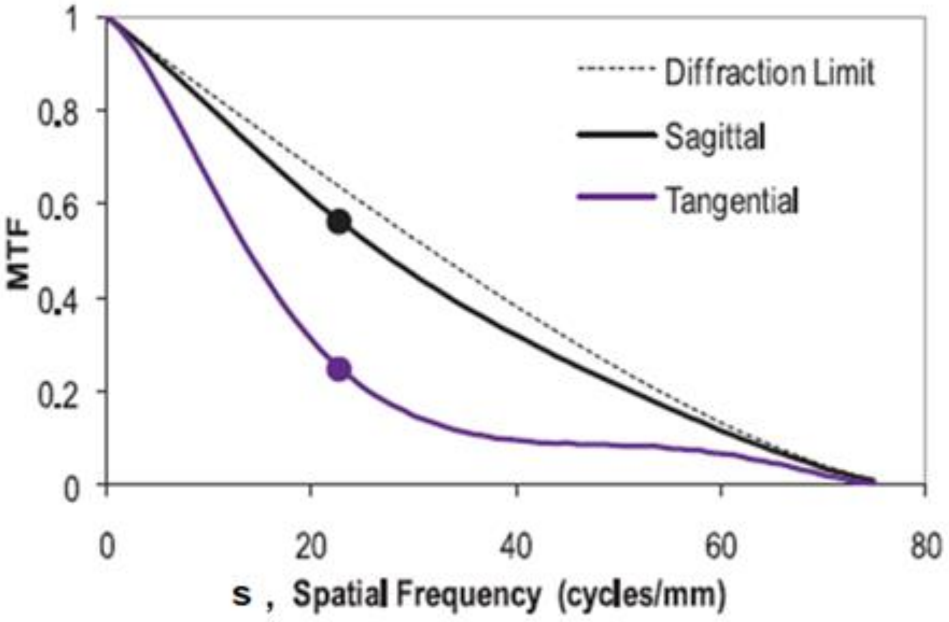
$$s_c = \frac{1}{\lambda \times (f/\#)}$$

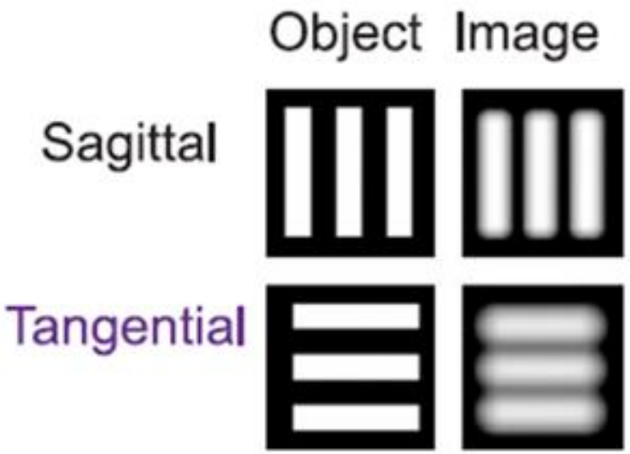
Example 2

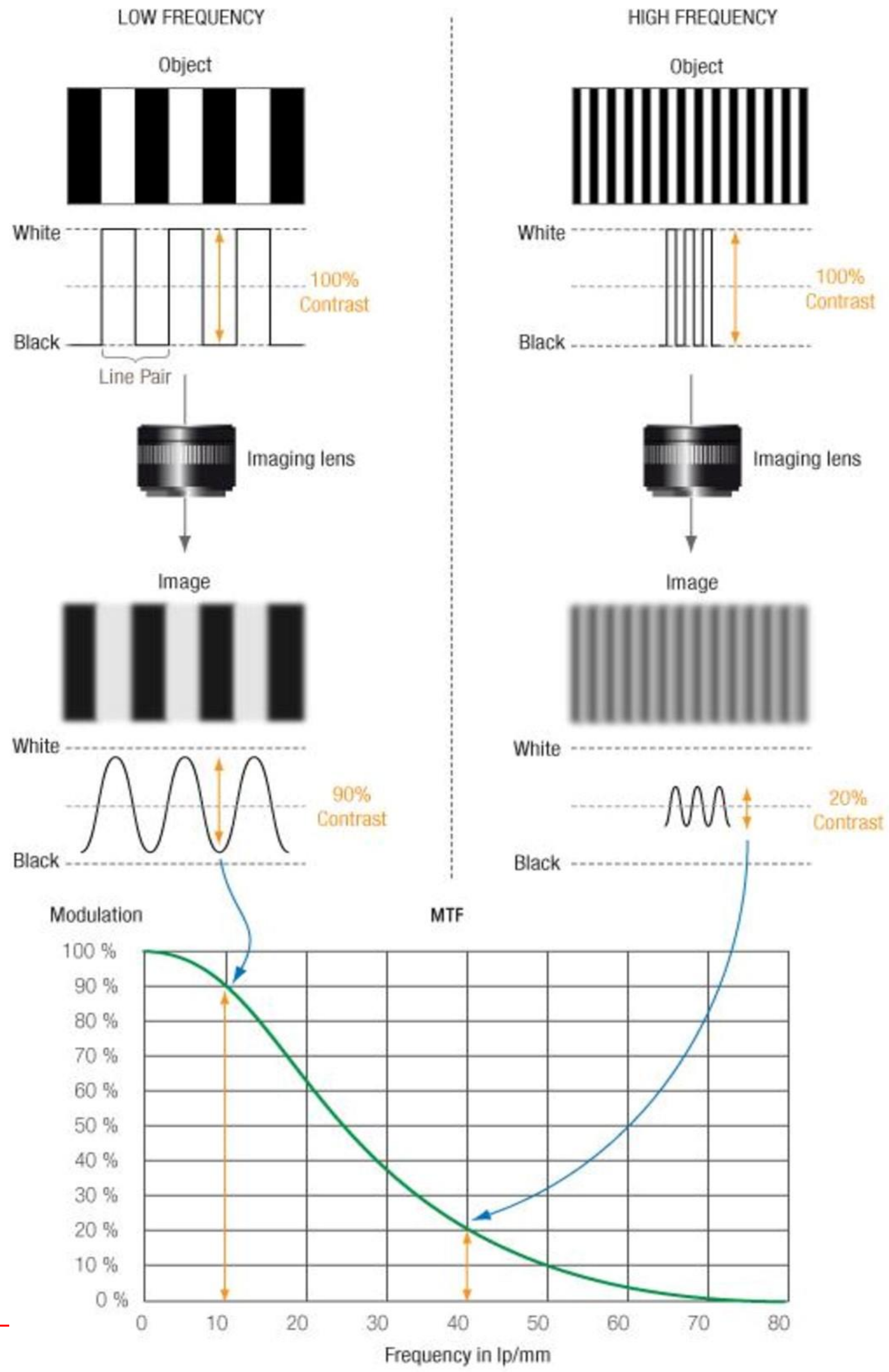
A target contains a series of black lines, each line 8 mm wide and seperated from the next line by an interval also 8 mm. What is the spatial frequency?



$$s = \frac{1\ \text{line pair}}{16\ \text{mm}} = 0.0625\ \text{mm}^{-1}$$


1
0.8
0.6
0.4
0.2
0
MTF
0
20
40
60
80
s , Spatial Frequency (cycles/mm)
Diffraction Limit
Sagittal
Tangential


Object
Image
Sagittal
Tangential


LOW FREQUENCY
Object
White
Black
Line Pair
100% Contrast
Imaging lens
Image
White
Black
90% Contrast
HIGH FREQUENCY
Object
White
Black
100% Contrast
Imaging lens
Image
White
Black
20% Contrast
Modulation
MTF
100 %
90 %
80 %
70 %
60 %
50 %
40 %
30 %
20 %
10 %
0 %
0
10
20
30
40
50
60
70
80
Frequency in lp/mm

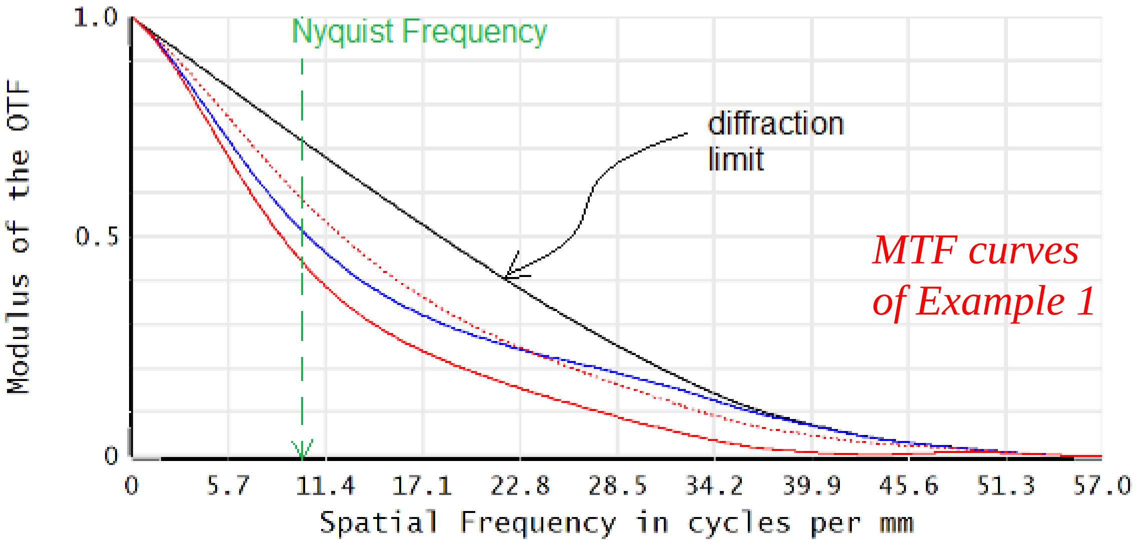
Nyquist Frequency

The Nyquist frequency (f_N) is referred to as the **sampling frequency**.
In a digital system f_N is calculated as:

$$f_N = \frac{1}{2p}$$

p = the pixel size in mm of the sensor.

Performance of the MTF is subsequently evaluated using **this value** as a reference parameter.



For the sensor in Example 1,
$p = 50\ \mu\text{m} \Rightarrow f_N = 10\ \text{lp/mm}.$
and

$$\text{cutoff resolution} = \frac{1}{\lambda(f/\#)} = 57\ \text{lp/mm}$$

Example 3

Consider photographs are taken from a high-altitude aircraft of a cruise ship. Assume that the MTF = 0.8 (typical camera lens).

Let $I_{max} = 5$ and $I_{min} = 2$.

Find M_{obj} and M_{img} so that ship's image is 0.5 mm wide in image plane.

Solution

Contrast at object plane:

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{5-2}{5+2} = 0.43 = 43\%$$

Since width of the ship is desired to be 0.5 mm, resolution is $\boldsymbol{s} = 1\ \mathrm{mm}^{-1}$ *and MTF = 0.8.*

Therefore:

$$M_{img} = MTF \times M_{obj} = 0.8 \times 0.43 = 0.34$$

This is enough for visual observations.

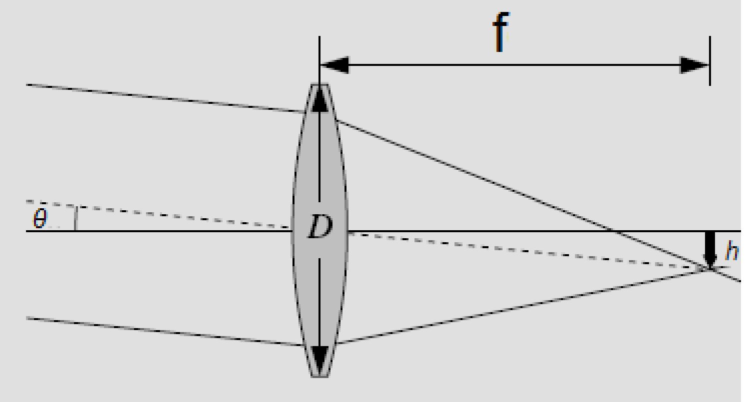
Example 4

What is the power of a lens that has an angular resolution of 0.05^o and resolves details in the focal plane that have a spatial resolution of 7 mm^{-1}?

Solution

The scale of the image formed in the focal plane of a lens of focal length f can be geometrically determined. When the object is seen at the angle θ, it forms an image of height s:

$$h = f\tan\theta \approx f\,\theta$$



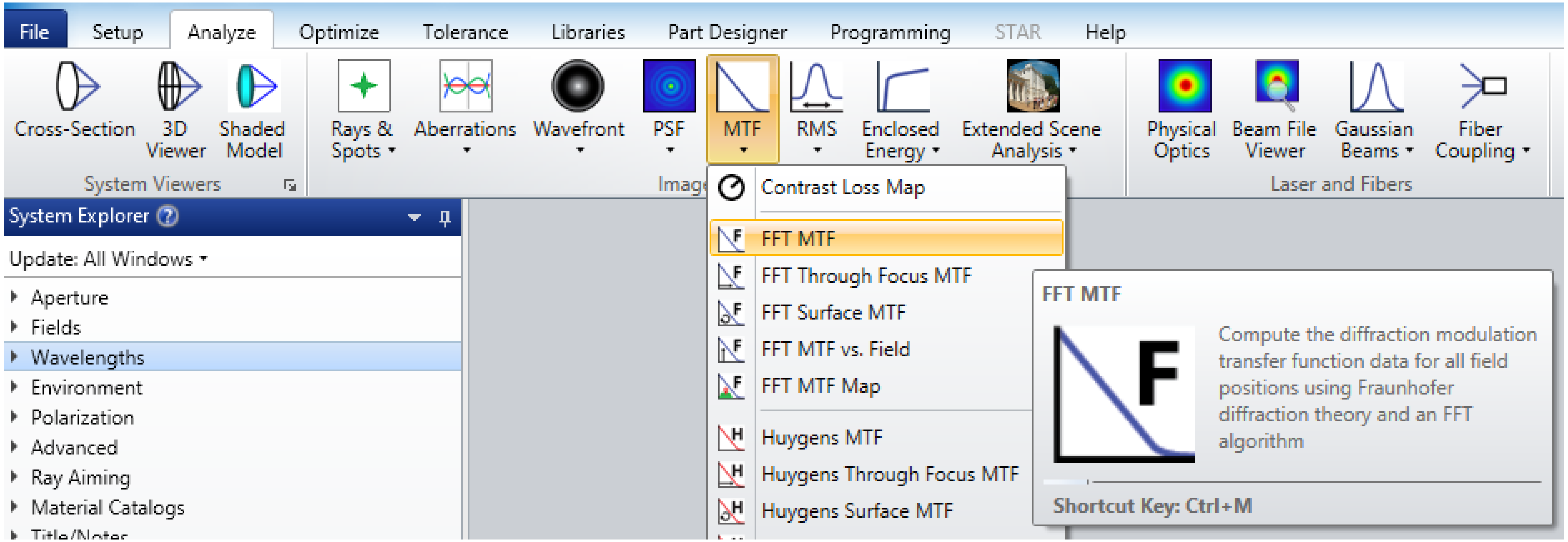
Angular res.: $\theta = 0.05^o = 8.73\;10^{-4}\;\text{rad}$

Spatial res.: $s = \dfrac{1}{h} = 7\;\text{lines/mm}$

Hence the focal length and power can be found as follows:

$$f = \frac{h}{\theta} = \frac{1}{s\theta} = 163.7\,mm \;\rightarrow P = 6.1\,D$$

MTF Analysis in Zemax

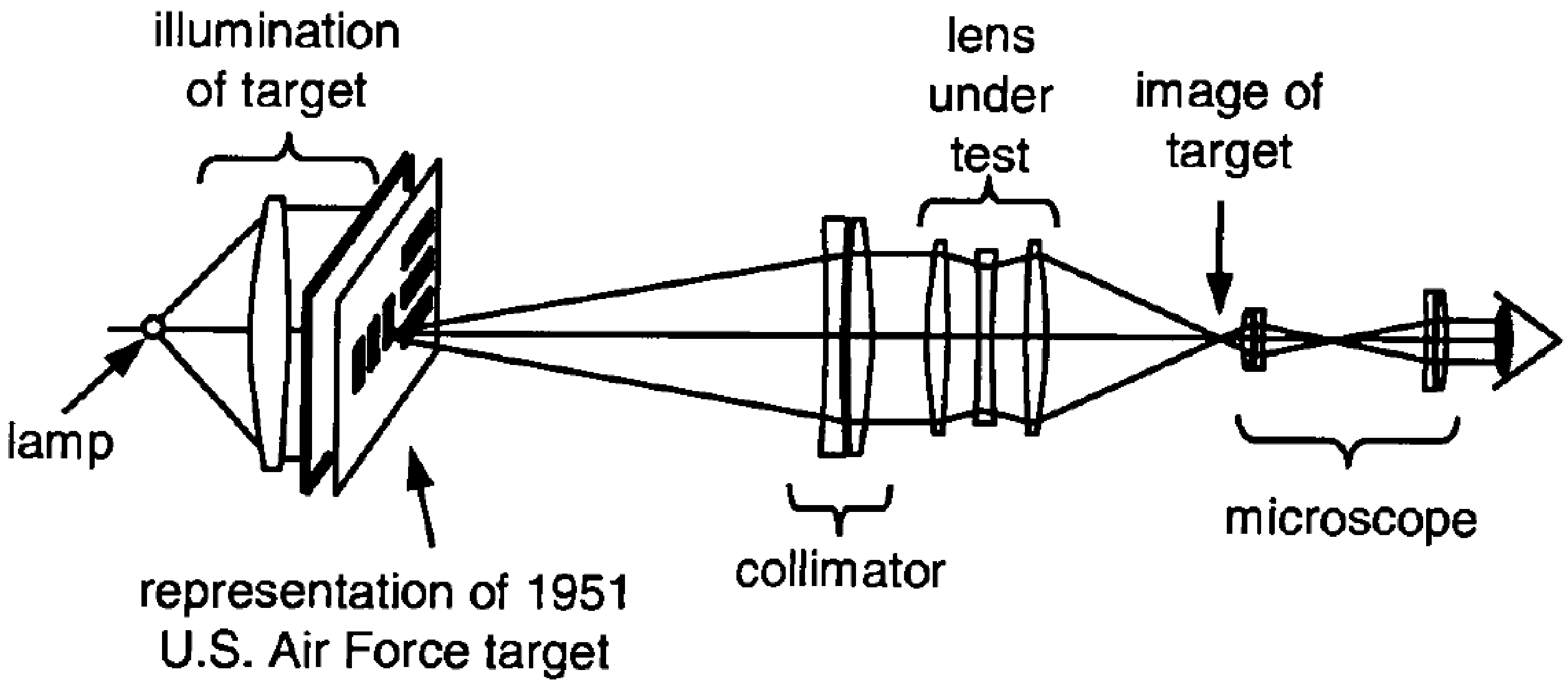


FFT MTF

Compute the diffraction modulation transfer function data for all field positions using Fraunhofer diffraction theory and an FFT algorithm

Shortcut Key: Ctrl+M

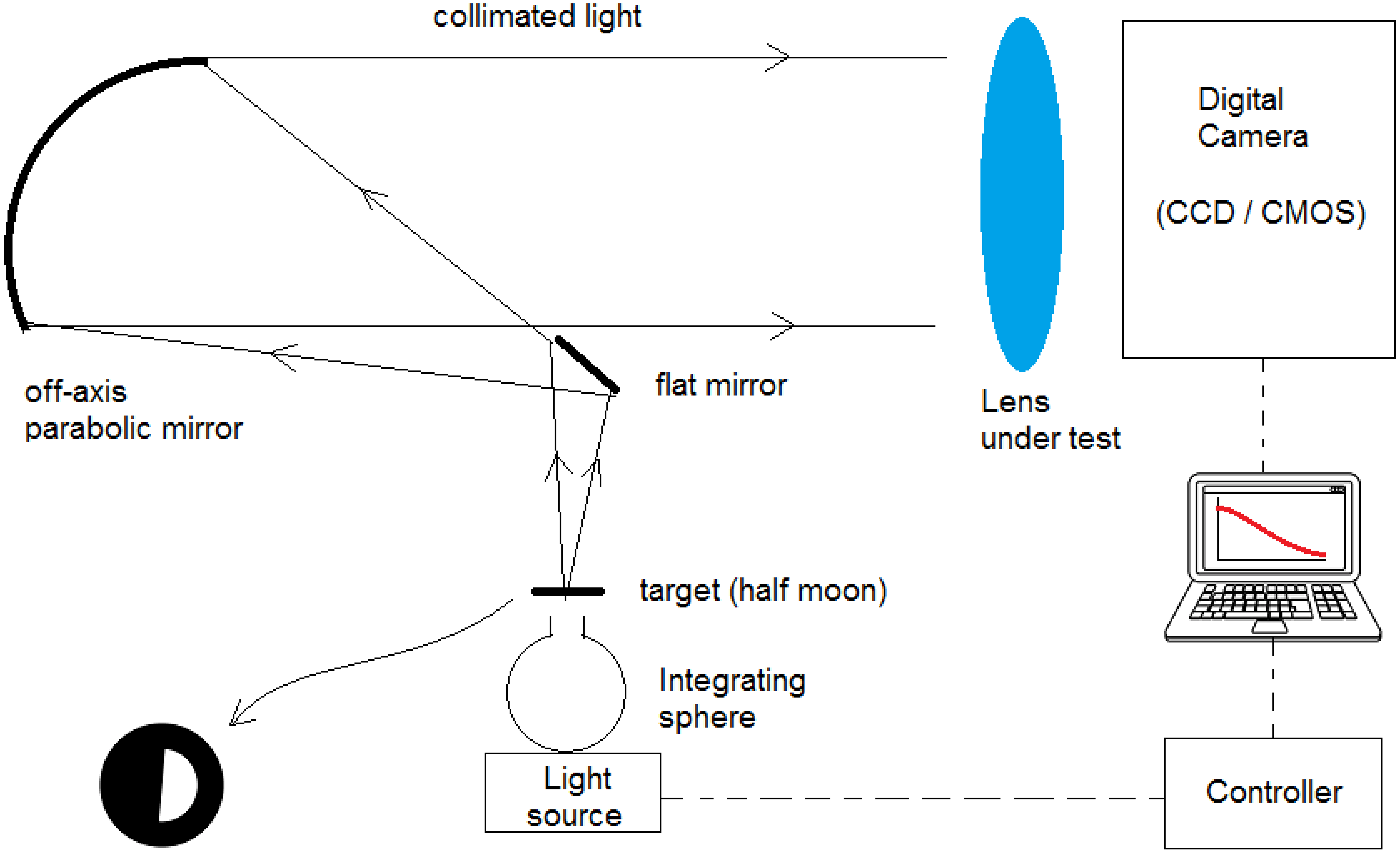
Measuring MTF via USAF Target



Using the Air Force Target to Test a Camera Lens

https://harvestimaging.com/blog/?p=1294

Measuring MTF via Slanted Edge

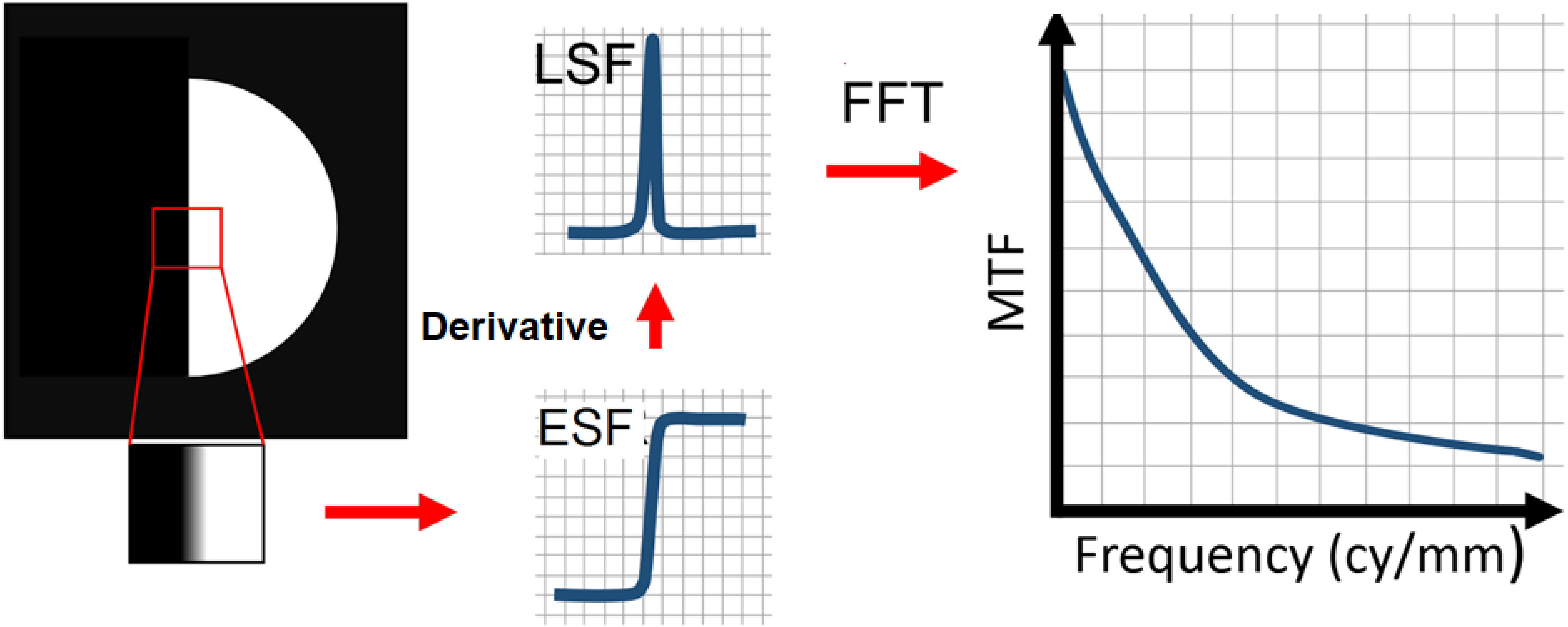


See related scientific paper from web page.

Measuring MTF via Slanted Edge

MTF is calculated from ESF and LSF using FFT.

- **Edge Spread Function** (ESF) is measured using a half-Moon target which is tilted a few degrees. This target gives differences between blackbody and background in a single image. SFR measurement readings is a step-function whose derivative is **Line Spread Function** (LSF).
- LSF is a function of the angle of view which describes the sharpness of the camera.
- MTF comes as a result of the **Fast Fourier Transform** of the LSF.



Evaluating MTF using MATLAB

You may use the following functions implemented in **image processing tool**:

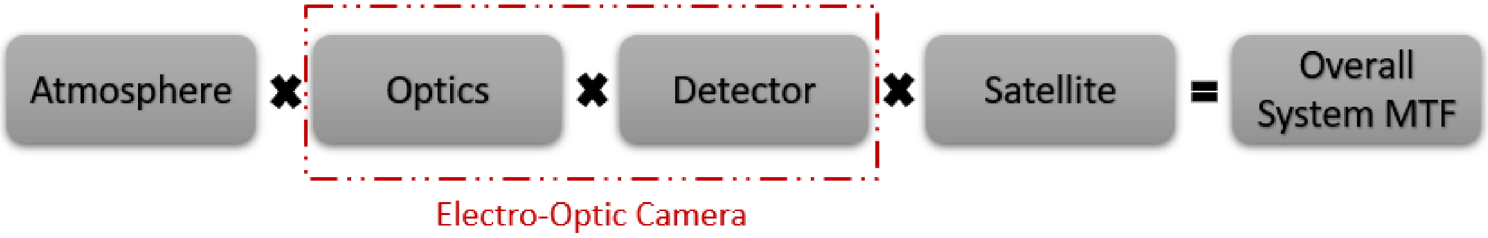
`imread()`	open an image file
`imshow()`	display an image file
`rgb2gray()`	convert rgb image to grayscale
`imrotate()`	rotate an image
`imcrop()`	crop an image
`fft()`	Fast Fourier Transform function

System MTF

Each factor influencing performance should be reviewed to determine the MTF budget of the optical system.

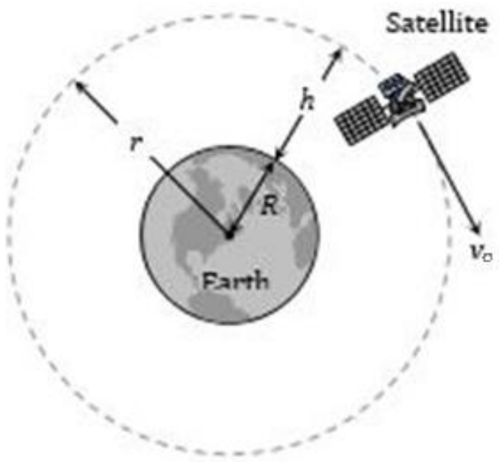
$$MTF_{system} = MTF_{atmosphere} * MTF_{optics} * MTF_{detector} * \cdots$$

For example, MTF operation for satellite system.



Atmosphere ✖ Optics ✖ Detector ✖ Satellite = Overall System MTF

Electro-Optic Camera



Signal-to-Noise Ratio (SNR)

- SNR measures the radiometric performance of the captured image.
- SNR is computed for the light path from the source to the imager.
- SNR is one important performance evaluation parameter for
    - space-borne electro-optic imagers
    - Thermal Camera systems.

- SNR is the ratio of the imaging signal to the total noise:

$$SNR = \frac{S_{total}}{\sigma_{total}}$$

See paper 'Signal-to-noise ratio model in Python for high-resolution space-borne electro-optic imagers' at: *https://doi.org/10.1117/1.JRS.17.014508*