Lecture 20

Basic Radiometry and Photometry

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Introduction

- In optics, the electromagnetic radiation measurement is studied in two groups:

  Radiometry is the measurement of optical radiation including visible light

  Photometry is the measurement of visible light only.

- In this chapter, we will discuss these two concepts and their units in SI.

Angle

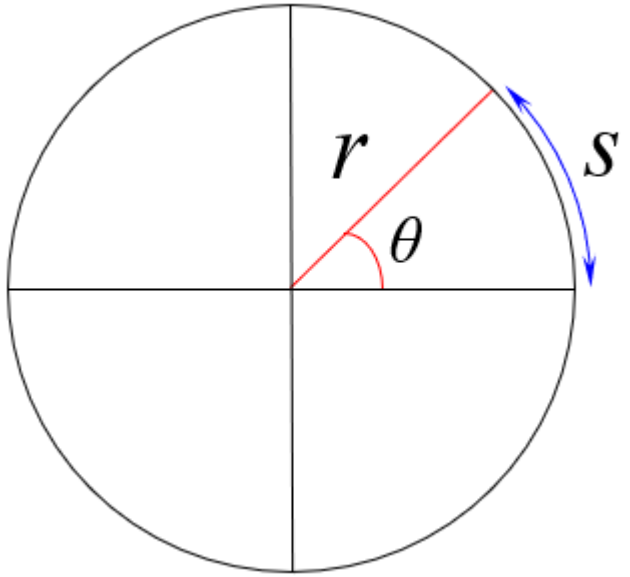
Angle in two-dimension (2D) defined as

$$\theta = k\frac{s}{r}$$

where *k* is a proportionality constant and depends on the unit of measurement that is chosen.

for radian measure *k* = 1

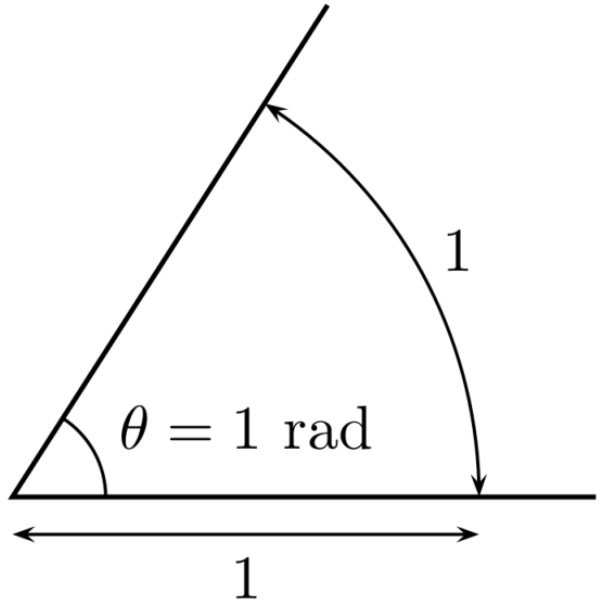
for degree measure *k* = 180/π ≈ 57.3



- Full circle is 2π radians:

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

- 1 radian defines an arc of a circle that has the same length as the circle's radius.
- 1 rad = 57.3°



Solid Angle

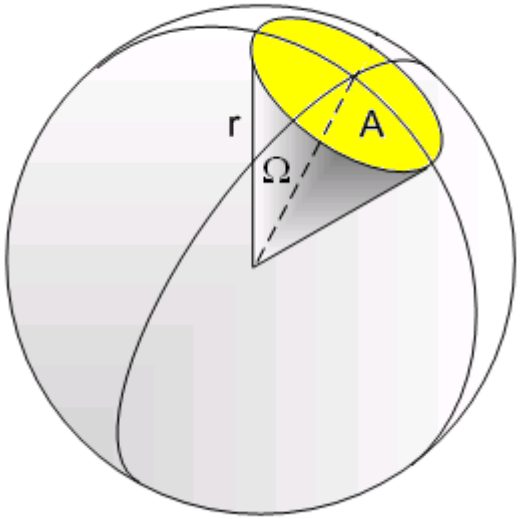
- The solid angle, Ω, is the 2D angle in 3D space that an object subtends at a point.

- Definition

$$\Omega = \frac{A}{r^2}$$

- It is a measure of how large that object appears to an observer looking from that point.

- SI unit is steradian (sr)



A : Surface area subtended from the center
r : Radius of the sphere

- The solid angle of a sphere measured from a point in its interior is 4π sr.

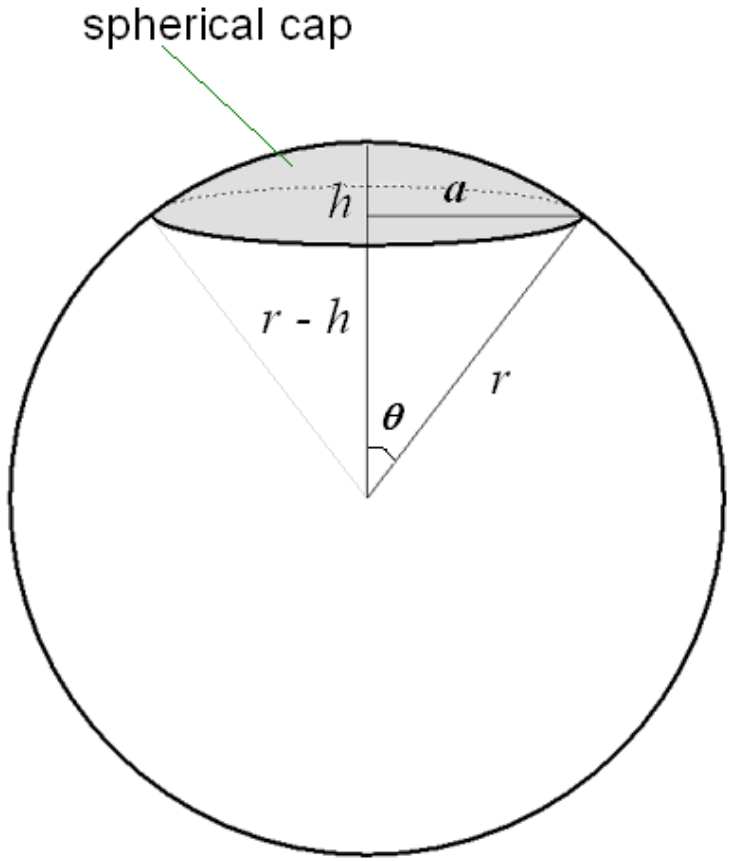
$$\Omega = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \text{ sr}$$

- Area of a spherical cap:

$$A = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$$

- Solid angle subtended:

$$\Omega = \frac{A}{r^2} = 2\pi(1 - \cos\theta)$$



Try yourself to prove these two relations!

Example

What is the solid angle of the Moon subtended from the Earth?

Distance of the Moon to the Earth is 384,400 km and the radius of the Moon of 1738 km.

SOLUTION

We can assume that the area of the moon is approximately equal to the spherical cap since the Moon-Earth distance (d) is much more grater than the radius (R) of the moon (d>>R).

$$\Omega = \frac{A}{r^2} \approx \frac{\pi R^2}{d^2} = \frac{\pi (1738)^2}{(384000)^2} = 6.4\times10^{-5}\ \text{sr}$$

Note that Moon covers about $5\text{x}10^{-4}$ *% of the sky since*

$$\frac{6.4\times10^{-5}\ \text{sr}}{4\pi\ \text{sr}} = 5.1\times10^{-6}$$

SI Base Units

The International System of Units (SI) defines seven units of measure as a basic set from which all other SI units are derived.

These SI base units and their physical quantities are:

* meter for length
* kilogram for mass
* second for time
* ampere for electric current
* kelvin for temperature
* candela for luminous intensity
* mole for the amount of substance

Name	Symbol	Definition
Meter	m	*The length* of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.
Kilogram	kg	*The mass* of the international prototype of the kilogram
Second	s	*The duration* of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom
Ampere	A	The constant *electric current* which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length
Kelvin	K	The fraction 1/273.16 of the *thermodynamic temperature* of the triple point of water
Mole	mol	*The amount of substance* of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12 atom
Candela	cd	*The luminous intensity* in a given direction, of a light source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian

Radiometry Units

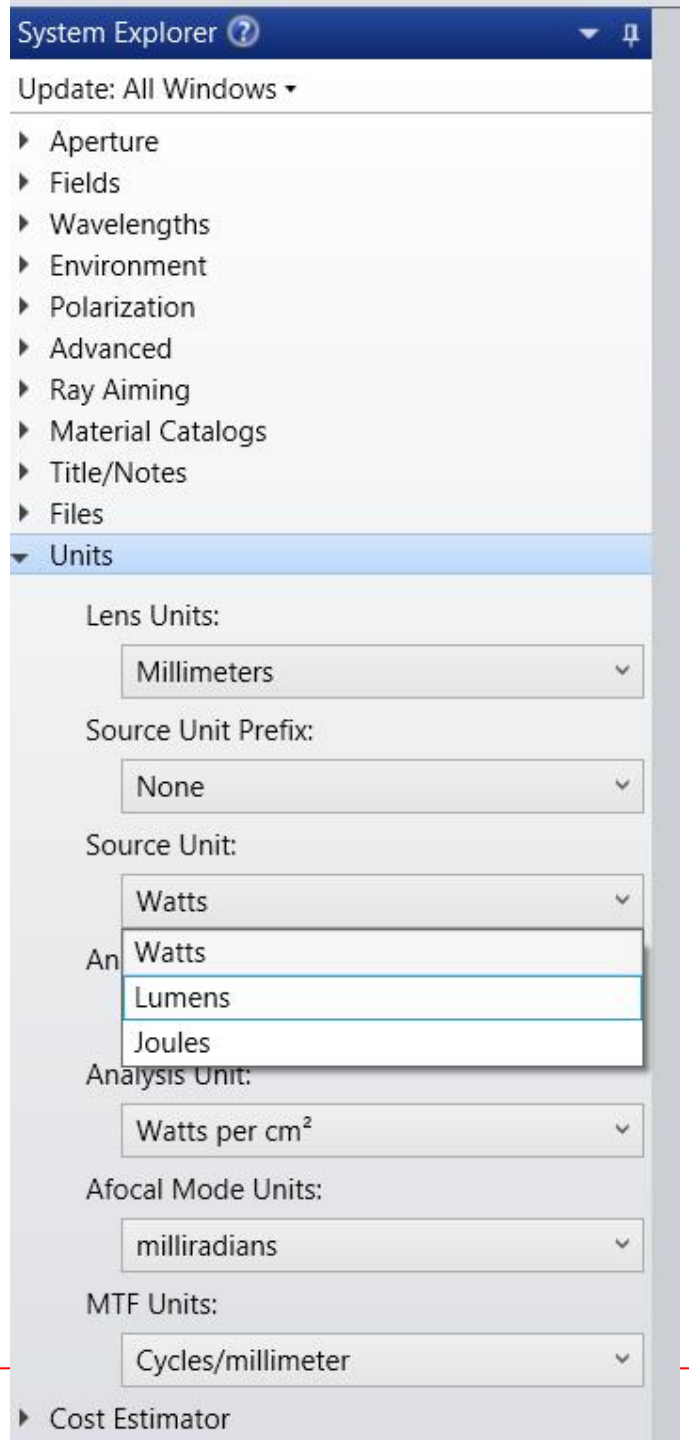
Radiometry is the field that studies the measurement of electromagnetic radiation, including *visible light*.

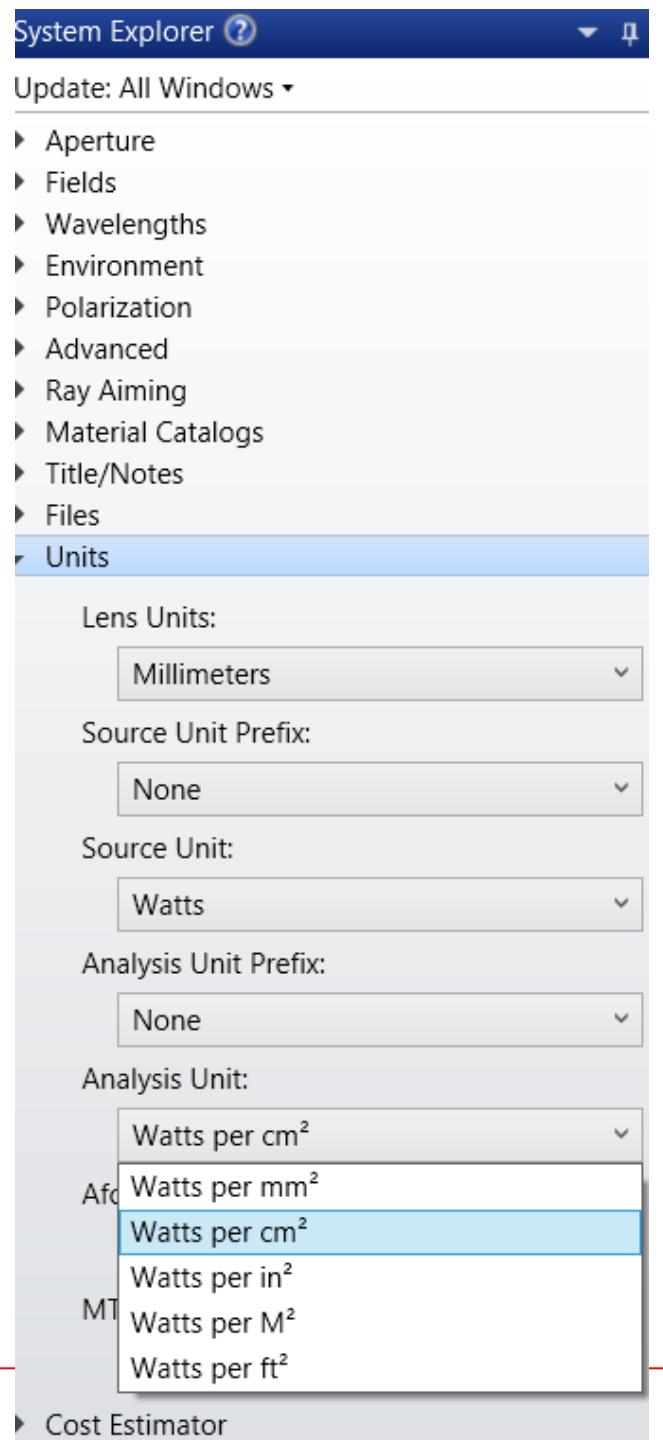
SI radiometric units are as follows:

Quantity	Symbol	SI unit	Abbr.
Radiant energy	***Q***	**Joule**	**J**
Radiant flux or Radiant power	***Φ***	**Watt**	**W**
Radiant intensity	***I***	**Watt per steradian**	**W/sr**
Irradiance	***E***	**Watt per square-meter**	**W/m^2**
Radiance	***L***	**Watt per steradian per meter-square**	**$W/sr.m^2$**

Units in Zemax

In Zemax's System Explorer Window. you can define Source Units and Analysis Units.

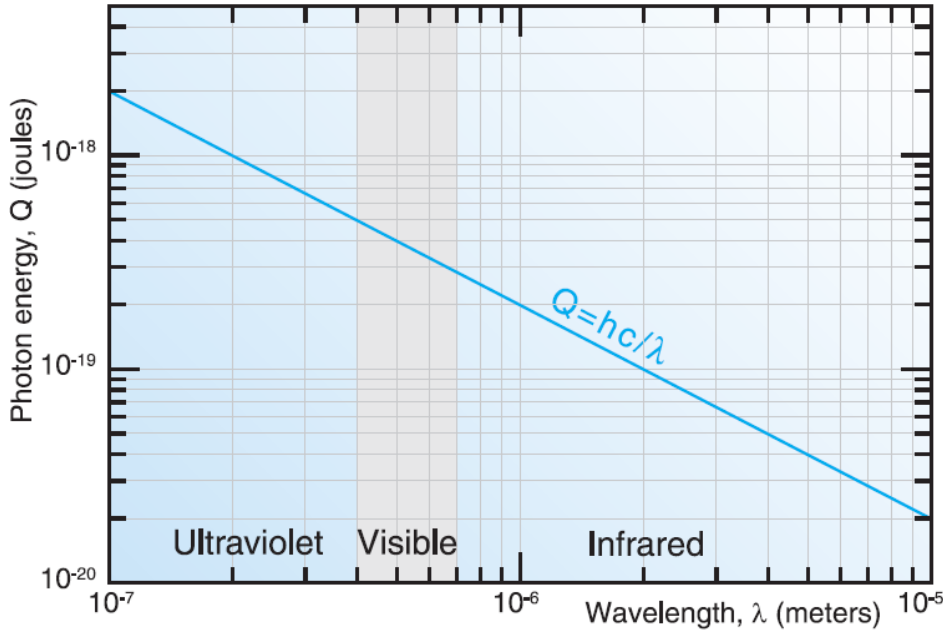




Radiant Energy

Each photon carries an energy that is described by Planck's equation:

$$E = h\nu = \frac{hc}{\lambda}$$



The number of photons (*N*) emitted from the source can be calculated by: $Q = NE$. or

$$\frac{dQ}{dt} = \frac{dN}{dt}E \quad \Rightarrow \quad \Phi = \frac{dN}{dt}\,\frac{hc}{\lambda}$$

Example:

What is the output of a 2 mW HeNe laser (632.8 nm) in photons per second?

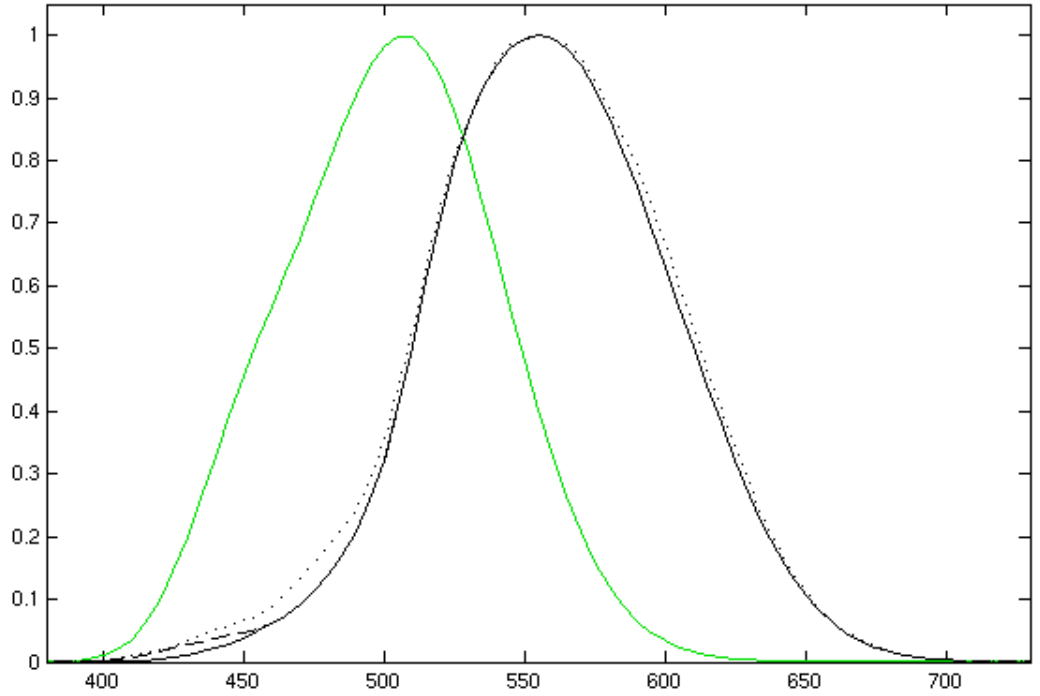
$$\frac{dN}{dt} = \frac{\Phi\lambda}{hc} = \frac{(2\times10^{-3}\ \mathrm{J/s})(632.8\times10^{-9}\ \mathrm{m})}{(6.63\times10^{-34}\ \mathrm{J.s})(3\times10^{8}\ \mathrm{m/s})} = 6.4\times10^{15}\ \mathrm{photons/s}$$

Photometry

Photometry is the science of the measurement of light, in terms of its perceived brightness to the human eye.

The human eye is not equally sensitive to all wavelengths of visible light.

Photometry attempts to account for this by weighing the measured power at each wavelength with a factor that represents how sensitive the eye is at that wavelength.



Photopic (black) and scotopic (green) luminosity functions.

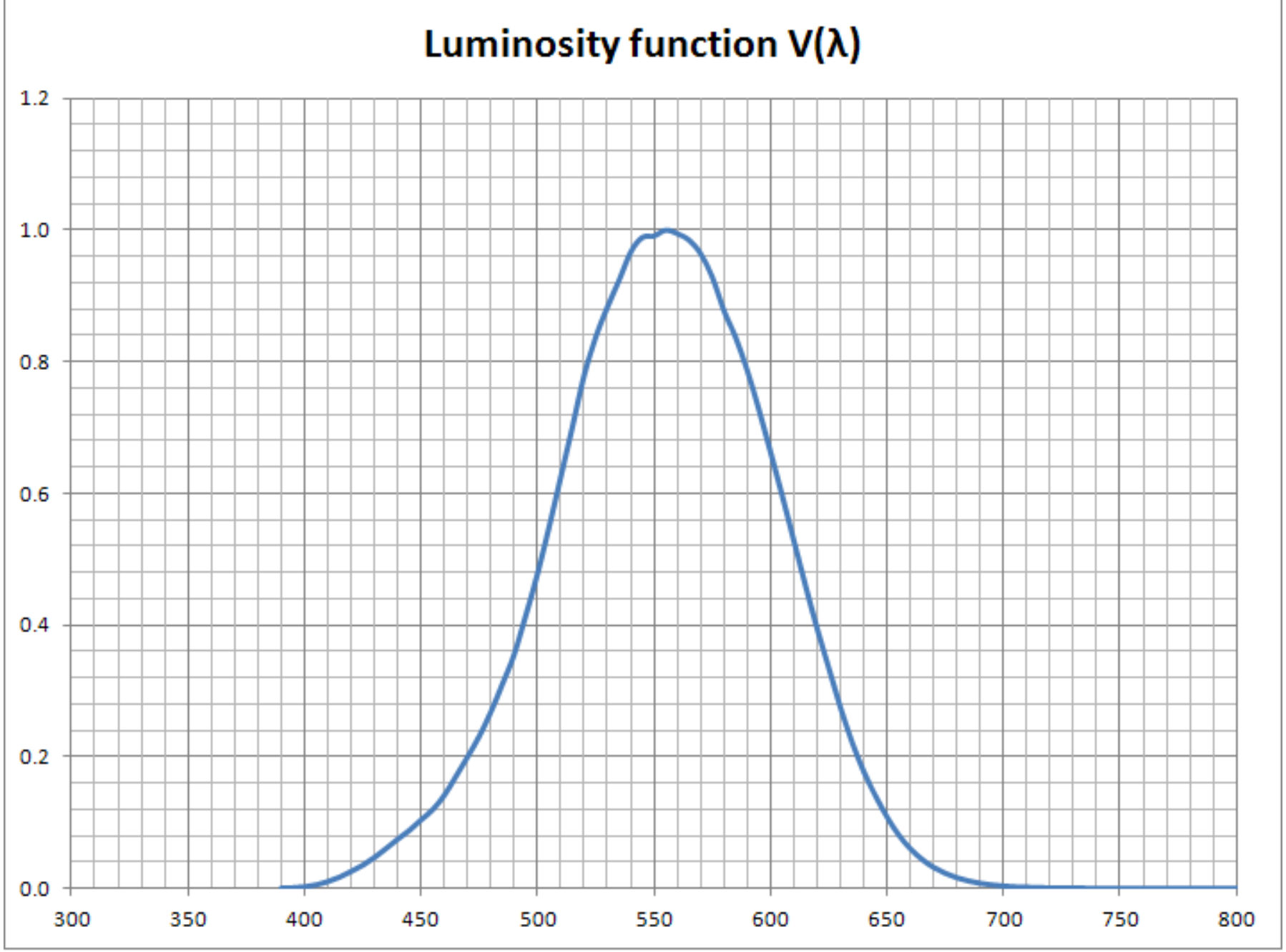
- *For everyday light levels*, the **photopic** curve best approximates the response of the human eye.
- *For low light levels*, the response of the human eye changes, and the **scotopic** curve applies.

Luminosity Function, V(λ)

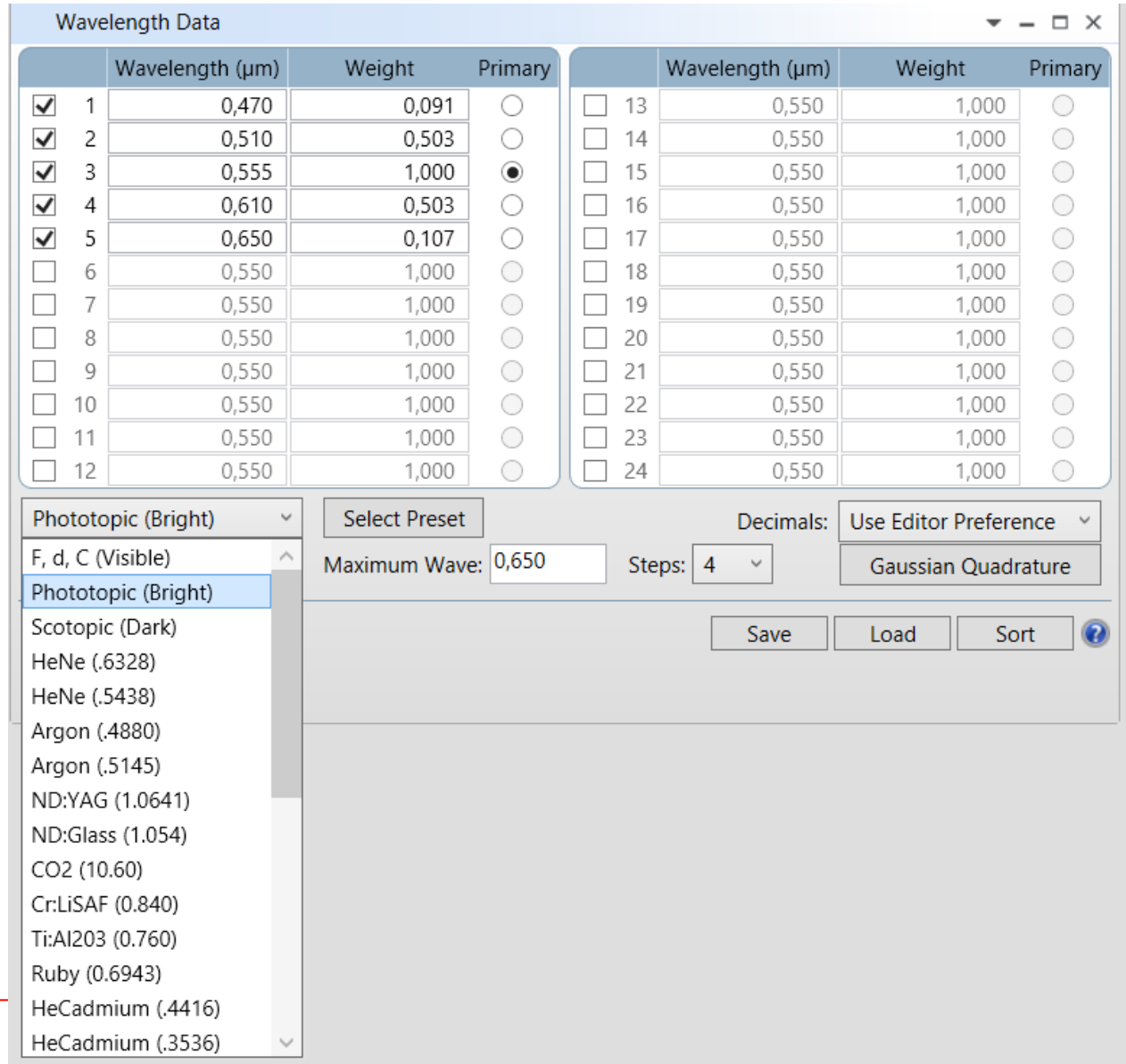
- The luminosity function (or luminous efficiency function) describes the average spectral sensitivity of human visual perception of brightness.

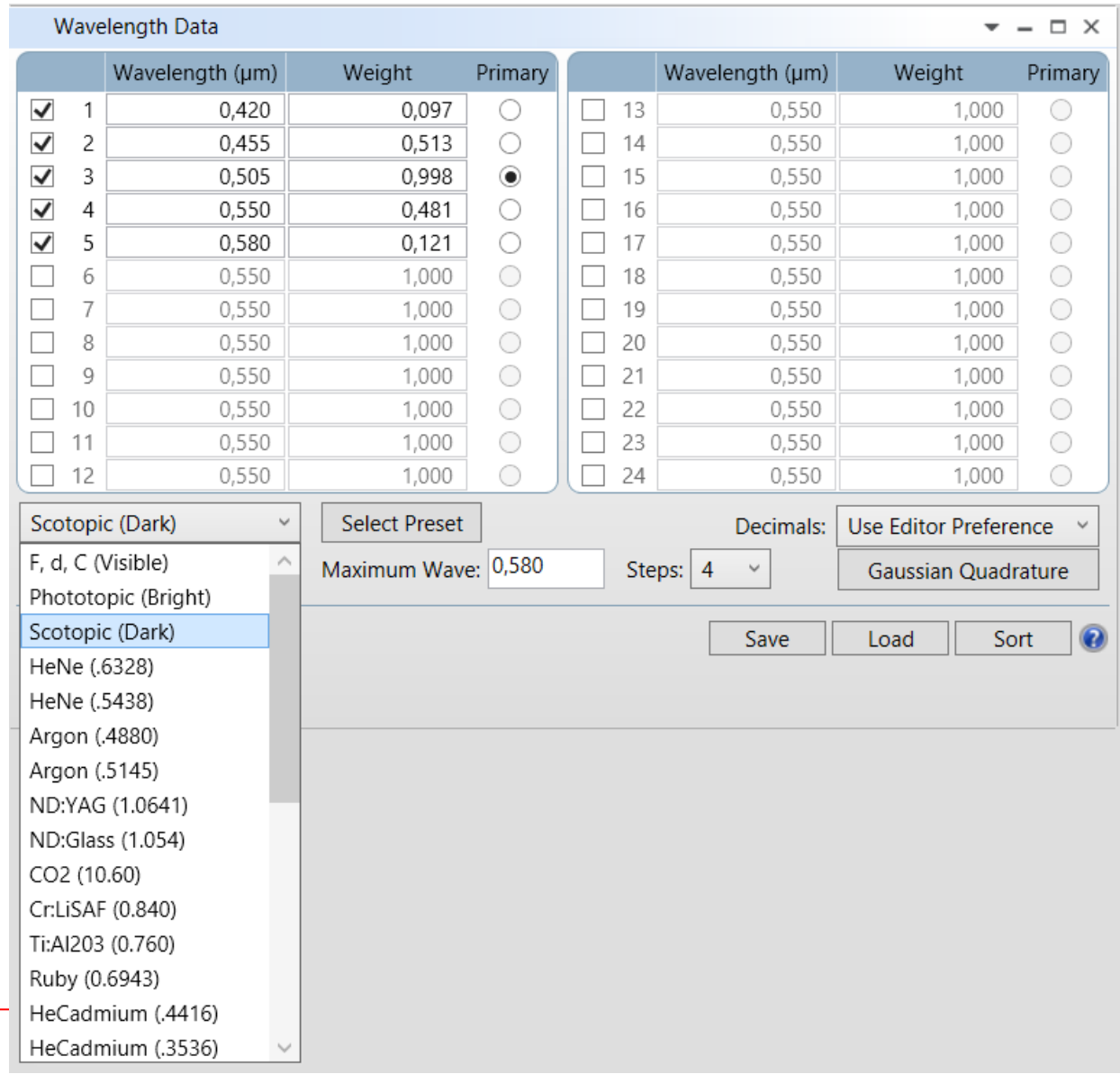
- This function is determined by International Commission on Illumination (CIE)
  It is the international authority on light, illumination, color, and color spaces.

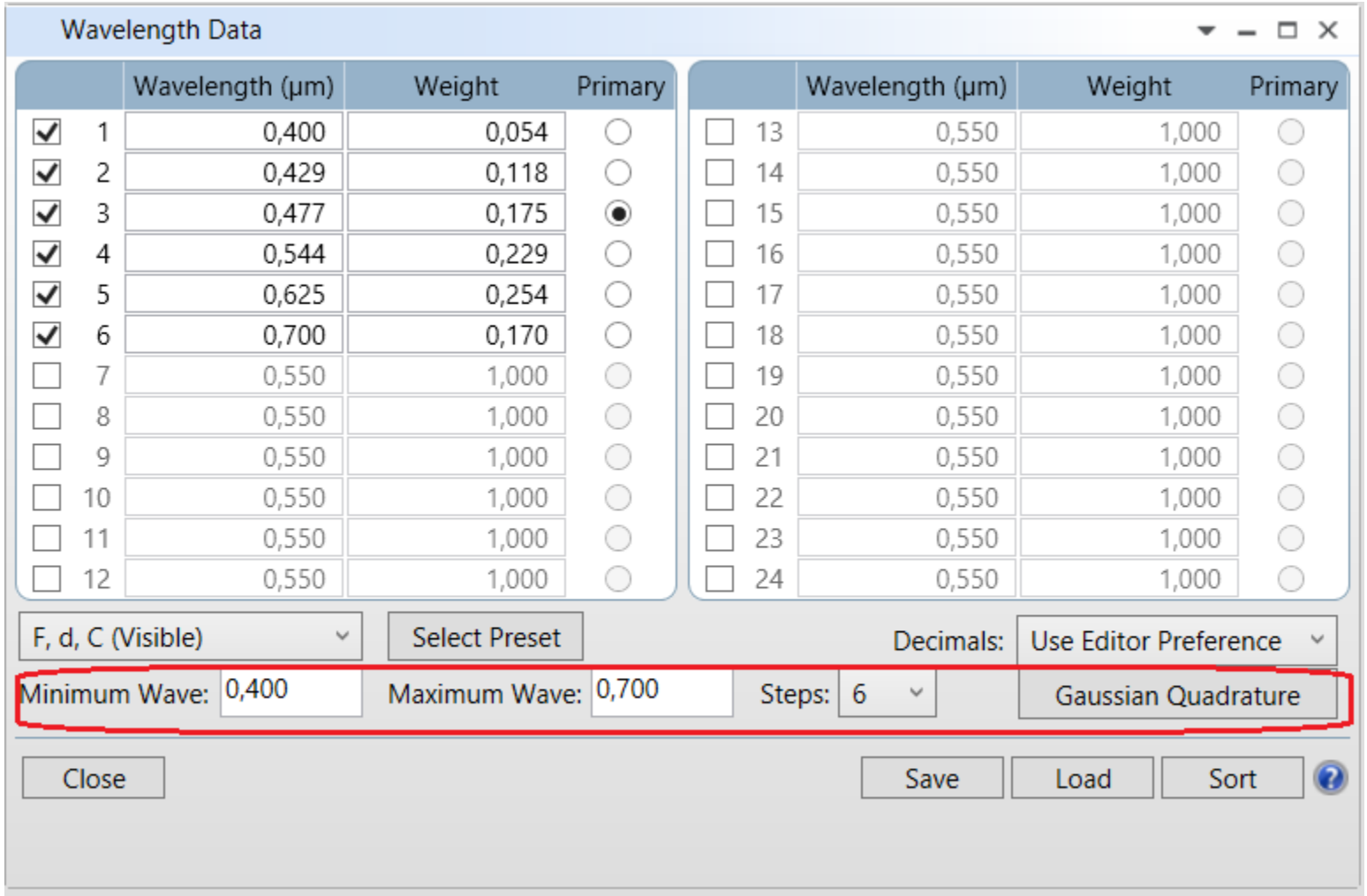
Wavelength λ (nm)	Luminosity function V(λ)
390	0.0004077
400	0.0025898
425	0.0352955
450	0.1039030
475	0.2312430
500	0.4780480
525	0.8376360
550	0.9907500
575	0.9286490
600	0.6629040
625	0.3351790
650	0.1084000
675	0.0230257
700	0.0037761
725	0.0005881
750	0.0000993
775	0.0000185
800	0.0000038
830	0.0000006


Luminosity function V(λ)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
300
350
400
450
500
550
600
650
700
750
800
Wavelength (nm)

Wavelength Selection in Zemax






Wavelength Data
Wavelength (µm)
Weight
Primary
1 0,400 0,054
2 0,429 0,118
3 0,477 0,175
4 0,544 0,229
5 0,625 0,254
6 0,700 0,170
7 0,550 1,000
8 0,550 1,000
9 0,550 1,000
10 0,550 1,000
11 0,550 1,000
12 0,550 1,000
13 0,550 1,000
14 0,550 1,000
15 0,550 1,000
16 0,550 1,000
17 0,550 1,000
18 0,550 1,000
19 0,550 1,000
20 0,550 1,000
21 0,550 1,000
22 0,550 1,000
23 0,550 1,000
24 0,550 1,000
F, d, C (Visible)
Select Preset
Decimals:
Use Editor Preference
Minimum Wave: 0,400
Maximum Wave: 0,700
Steps: 6
Gaussian Quadrature
Close
Save
Load
Sort

SI photometric units

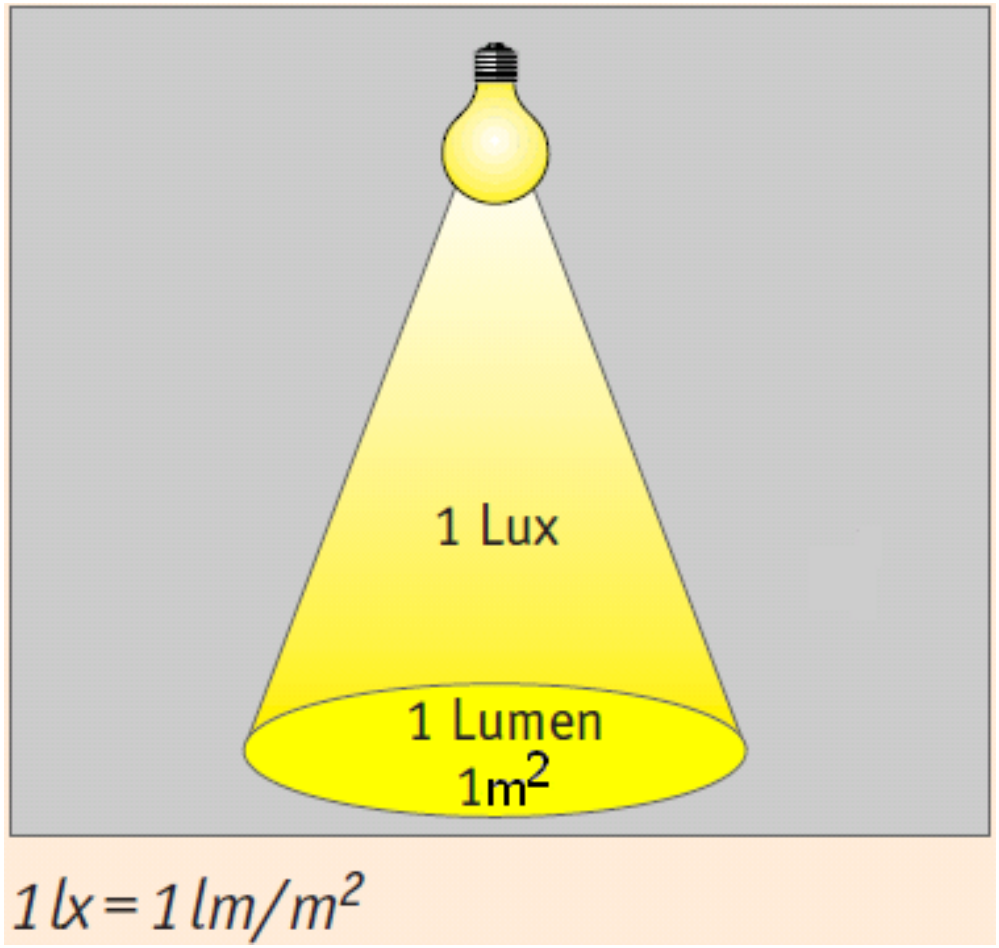
Quantity	Symbol	SI unit	Abbr.
Luminous energy	Q_v	lumen.second	lm.s
Luminous flux or Luminous power	Φ_v	lumen	lm
Luminous intensity	I_v	candela	cd = lm/sr
illuminance	E_v	lumen per meter-square	lux = lm/m^2
Luminance	L_v	lumen per steradian per meter-square	$lm/sr.m^2 = cd/m^2$

Turkish names:

Flux = Akı
Intensity = Şiddet
illuminance = Aydınlatma
Luminance = Işıldama

Typical illuminaces:

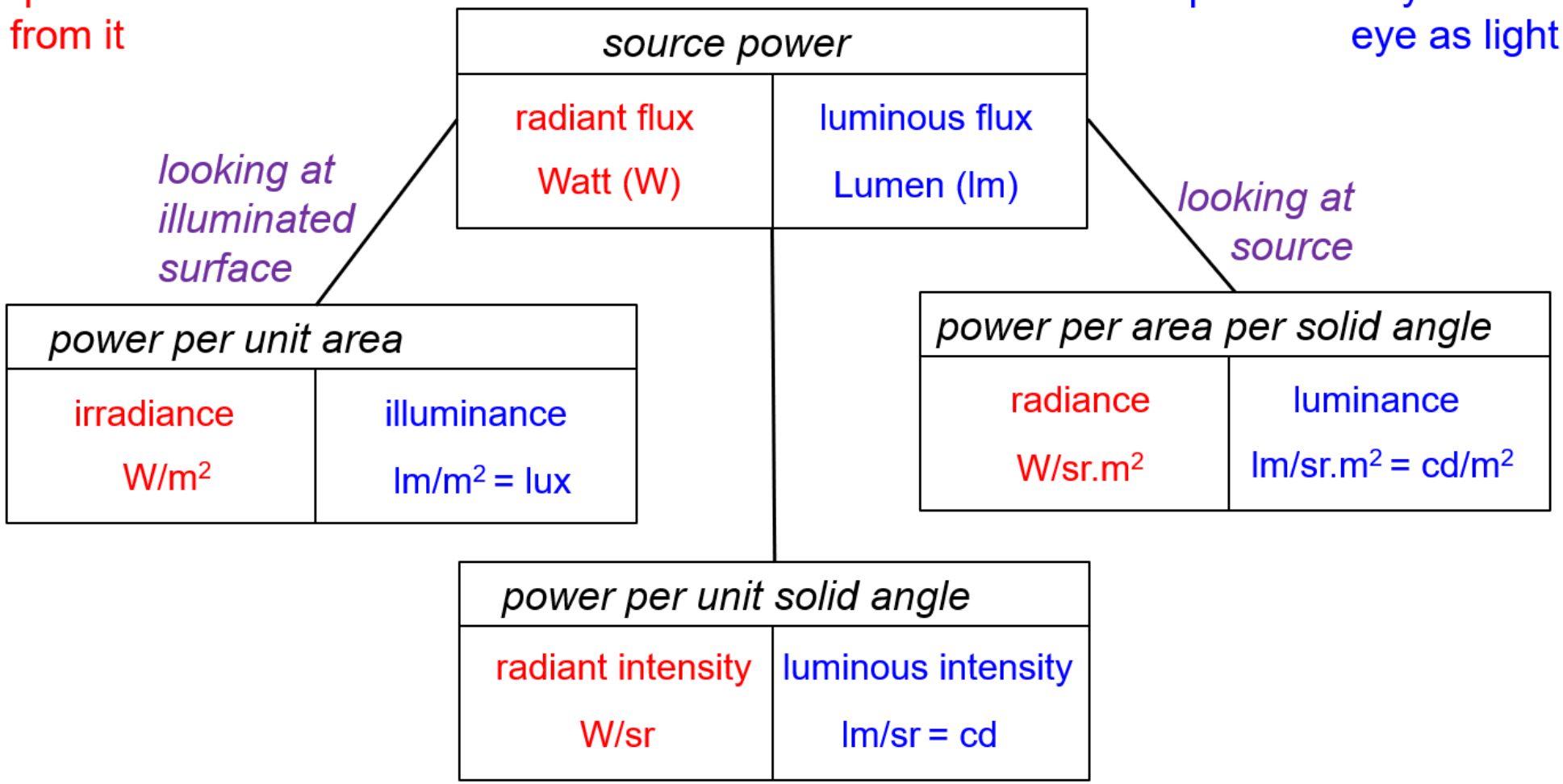
* Direct sun light 100,000 lux
* Full Moon 1 lux
* Working desk 500 lux
* Hospital corridors 20-50 lux



Unit Comparison

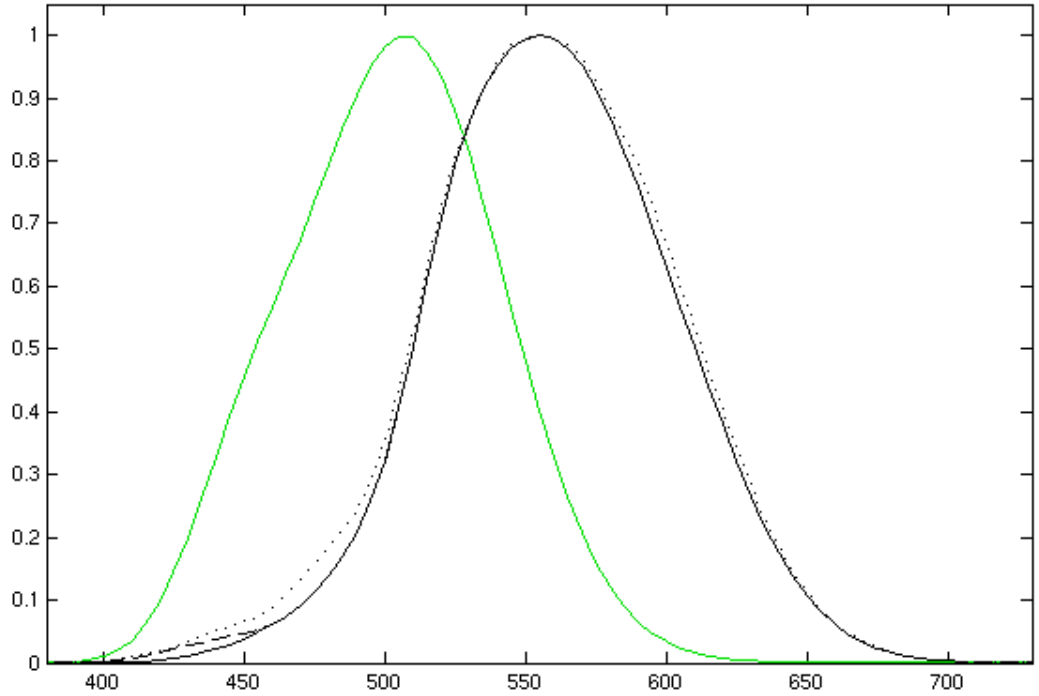
Radiometry measures the entire radiant power and quantities derived from it

Photometry measures that part of radiant power perceived by human eye as light



Radiometry and Photometry Conversion

The radiant power at each wavelength is weighted by a luminosity function V(λ) that models human brightness sensitivity.



Conversion from Watt to lumen:

For Mono-chromatic light:

$$\Phi_v = (683\ \mathbf{lm/W})\ \Phi V(\lambda)$$

For Poly-chromatic light:

$$\Phi_v = (683\ \mathbf{lm/W}) \int_0^{\infty} \Phi(\lambda)\ V(\lambda) d\lambda$$

Table 1: Basic Radiometric and Photometric units

Quantity	**Radiometric unit**	**Photometric unit**	**Conversion**
Power	*P* (in Watts, W)	Φ (in lumens, lm)	$\Phi = 683P(\lambda)V(\lambda)$
Intensity	*I* (in W/m^2)	*E* (in lm/m^2 = lux)	$E = 683I(\lambda)V(\lambda)$
Here V(λ) is a function of wavelength, λ. Its form can be found in many text books.			

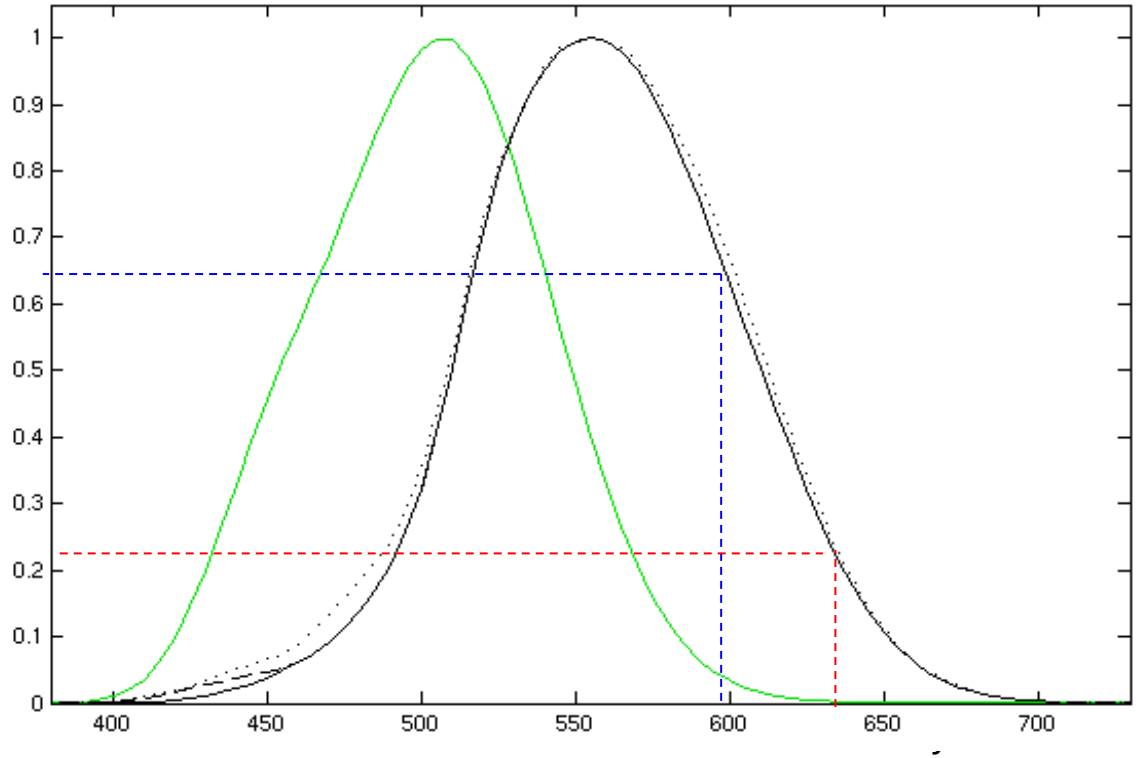
Example

Compare brightness of two 5 mW laser pointers at 635 nm and 600 nm.

* at λ =600 nm, V(λ) = 0.650 ➔ $\Phi_v = (683\ \frac{\mathrm{lm}}{\mathrm{W}})(0.005\ \mathrm{W})(0.65) = 2.22\ \mathrm{lm}$

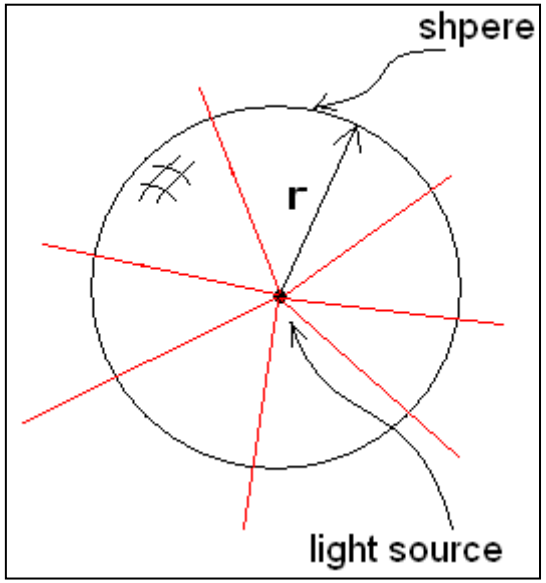
* at λ = 635 nm, V(λ) = 0.217 ➔ $\Phi_v = (683\ \frac{\mathrm{lm}}{\mathrm{W}})(0.005\ \mathrm{W})(0.217) = 0.74\ \mathrm{lm}$

The shorter wavelength (600 nm) laser pointer will create a spot that is almost 3 times as bright as the longer wavelength (635 nm) laser assuming the same beam diameter.



Inverse Square Law

If light spreads out in all directions, as it does from a point light source, the intensity at a certain distance from the source depends on the area over which the light is distributed.



For example, for a sphere of radius *r*, the light from the point source will fall on a surface area of $A = 4\pi r^2$. The irradiance is defined by the total power output of the source (Φ) divided by the area over which the light is spread.

$$E = \frac{\Phi}{A} = \frac{\Phi}{4\pi r^2} = \frac{I}{r^2}$$

E = Irradiance in Watt/m^2	(or Illuminance in lux ≡ lumen/m^2)
Φ = Light power in Watt	(or Luminous power in lumen)
I = Radiant Intensity in Watt/sr	(or Luminous Intensity in cd)

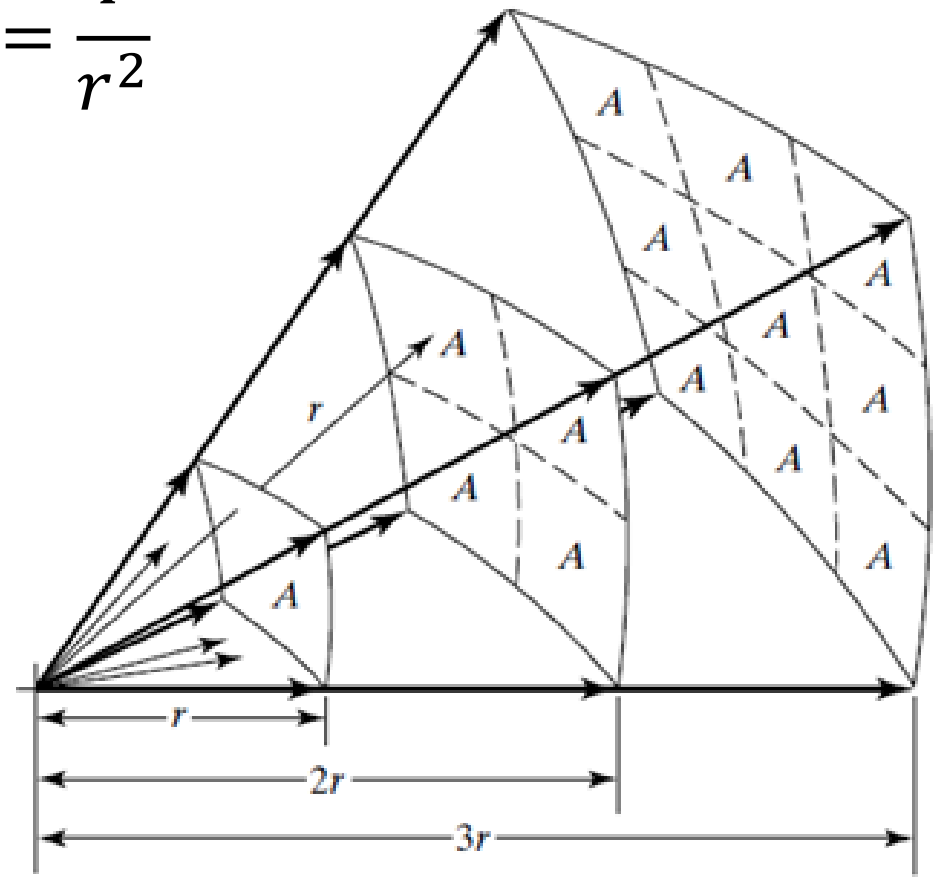
This Equation is known as the *inverse square law* which states that the magnitude of the light irradiance (brightness or intensity) on a surface is inversely proportional to square of the distance from a point source.

$$E = \frac{I}{r^2}$$

The light power is distributed on the surface of area A at a distance r from the source.

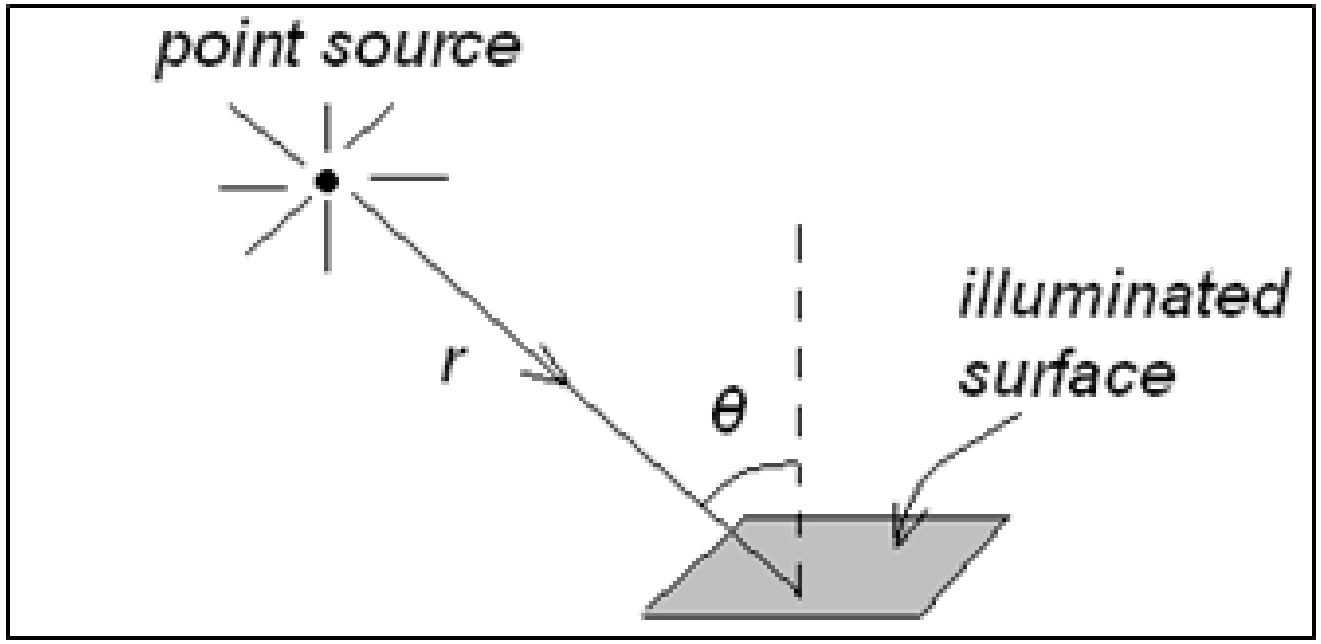
Same power is distributed on area 4A at a distance 2r.

Same power is distributed on area 9A at a distance 3r



If the radiation direction makes an angle θ with the normal of the irradiated (illuminated) surface, as in Figure, then the irradiance (intensity) is given by:

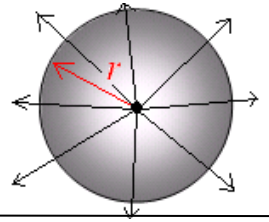
$$E = \frac{I}{r^2}\cos\theta$$



Radiometric & Photometric Equations

Radiative flux of point source:

$$\Phi = 4\pi I$$

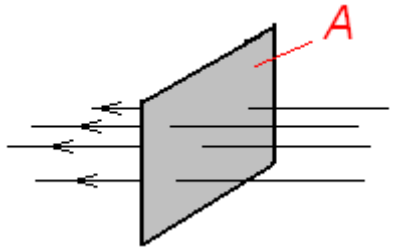


Luminous flux of point source:

$$\Phi_v = 4\pi I_v$$

Irradiance on area *A*:

$$E = \frac{\Phi}{A}$$

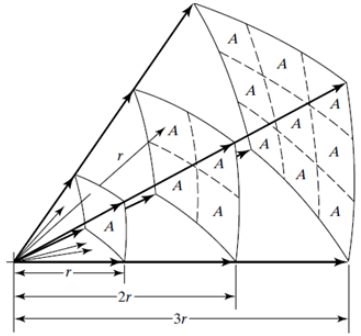


illuminance on area *A*:

$$E_v = \frac{\Phi_v}{A}$$

Irradiance of a point source of intensity *I*

$$E = \frac{\Phi}{A} = \frac{4\pi I}{4\pi r^2} = \frac{I}{r^2}$$

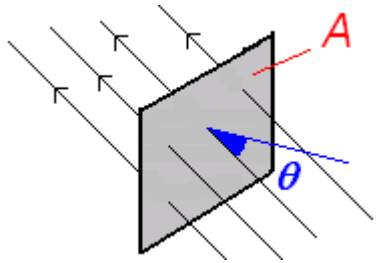


illuminance of a point source of intensity I_v

$$E_v = \frac{\Phi_v}{A} = \frac{4\pi I_v}{4\pi r^2} = \frac{I_v}{r^2}$$

If radiation direction makes an angle θ with the normal of irradiated surface

$$E = \frac{I}{r^2}\cos\theta$$



If radiation direction makes an angle θ with the normal of illuminated surface

$$E_v = \frac{I_v}{r^2}\cos\theta$$

Photometer / Luxmeter

- Photometer is an instrument for measuring **light intensity level.**
- Most of the, photometers are used to measure iluminance (E_v) or irradiance (E).
- Measuring E_v is important in *illumination Engineering.*
- Most photometers detect the light with photoresistors, photodiodes or photomultipliers

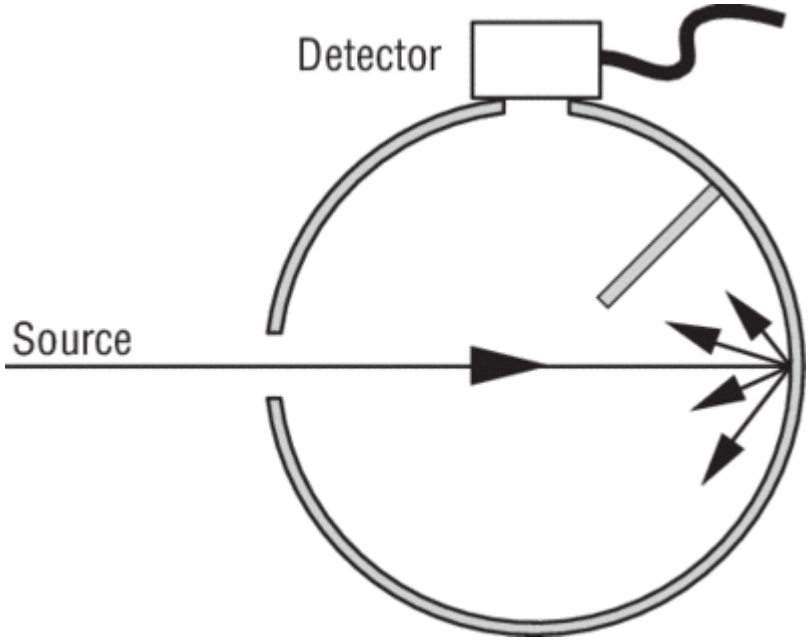
Goniophotometer

- Goniophotometer is a device used for measurement of the light emitted from an object at different angles.

- A goniophotometer can be used for various applications:
  - Measurement of luminous flux of a light source
  - Measurement of luminous intensity distribution from a source much smaller than the size of the goniophotometer.

Integrating Sphere

- An integrating sphere collects electromagnetic radiation from a source completely external to the optical device, usually for flux measurement or optical attenuation.



- Radiation introduced into an integrating sphere strikes the reflective walls and undergoes multiple diffuse reflections. After numerous reflections, the radiation is dispersed highly uniformly at the sphere walls. The resulting integrated radiation level is directly proportional to the initial radiation level and may be measured easily using a detector.

A Large Integrating Sphere

Exercises

1. What is the luminous intensity of a light bulb at 590 nm having the luminous flux of 270 lm?

2. For reading a book one needs a 50 lux illuminance. A bulb of 100 cd is used for illumination. Calculate the required perpenrdicular distance of the lamb to the book.

3. Calculate irradiance and illuminace of a 10 mW laser pointer at 620 nm with beam size of 4 mm^2.

4. A laser pointer produces a 2 mW. The beam enters the eye and is focused by the cornea and lens to a spot on the retina 16 μm in diameter. Find the irradiance on the retina, assuming that all of the 2 mW of power is focused on the retina.

5. Irradiance on Earth surface due to Sun is about 1000 W/m^2 and diameter is about 32’.

An f/4 thin lens with focal length 100 mm is used to get image of the sun on a paper on Earth.

(a) Calculate the total power of the sun?

(b) Find the diameter of the image of Sun on the paper.

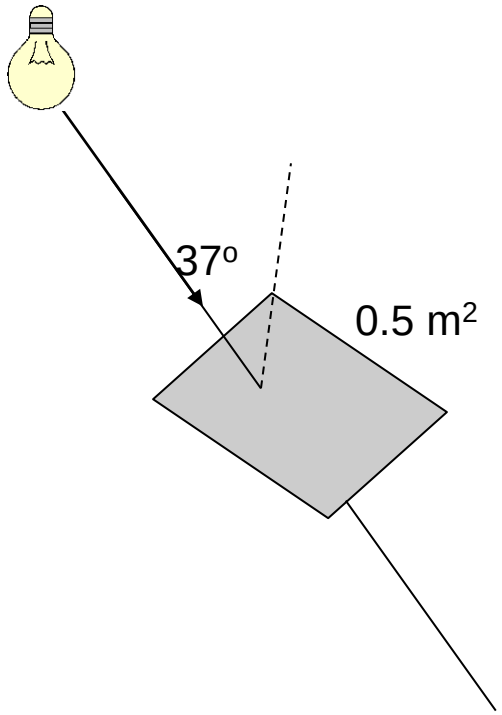
(c) Find the irradiance of the of the image of Sun on the paper.

The light rays emerging from a point source of intensity 100 cd fall on a planar surface whose area is 0.5 m^2 at distance 1 m from the source. The rays make an angle of 37^o with the normal of a planar surface.

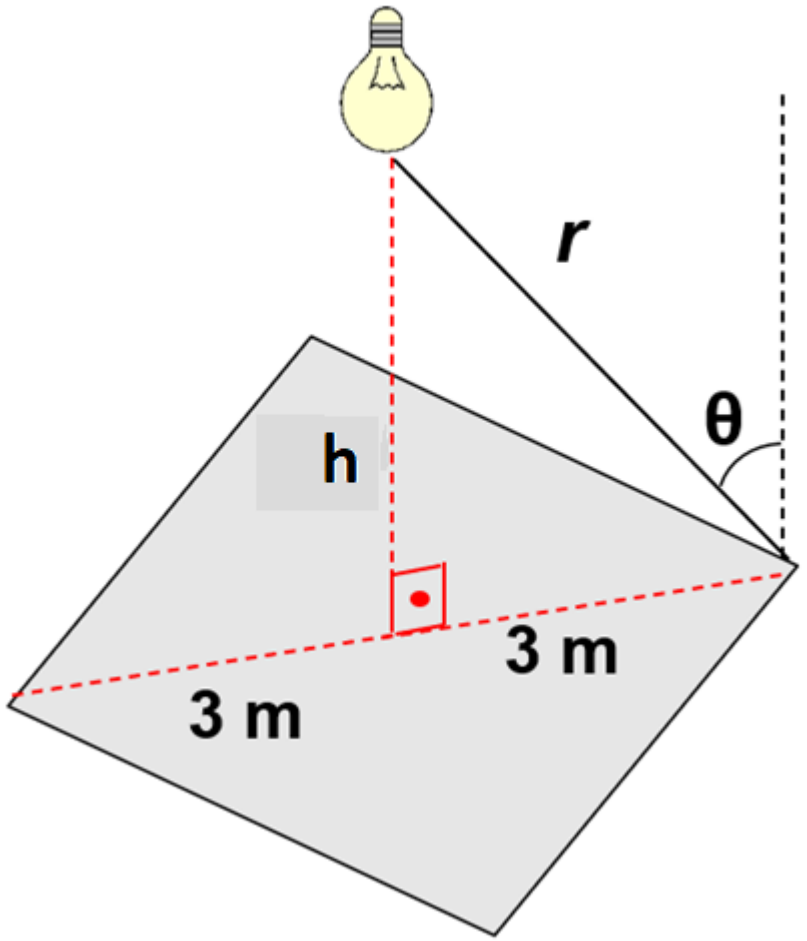
(a) Find the total flux of the source.

(b) Find the illuminance on the surface.

(c) Find the flux on the surface.



A 1000 cd-bulb is hanged at a height of h = 4 m from the center of the floor of a room having square shape with diagonal length of 6 m as shown in figure. Calculate the illuminance of the bulb at any corner of the floor.



A 1000 cd bulb is hanged at height *h* from the center of the floor of a room having square shape with diagonal length of 6 m as shown in figure. For which value of the height *h* from the ground the illuminace at the bottom corner of the floor is maximum?

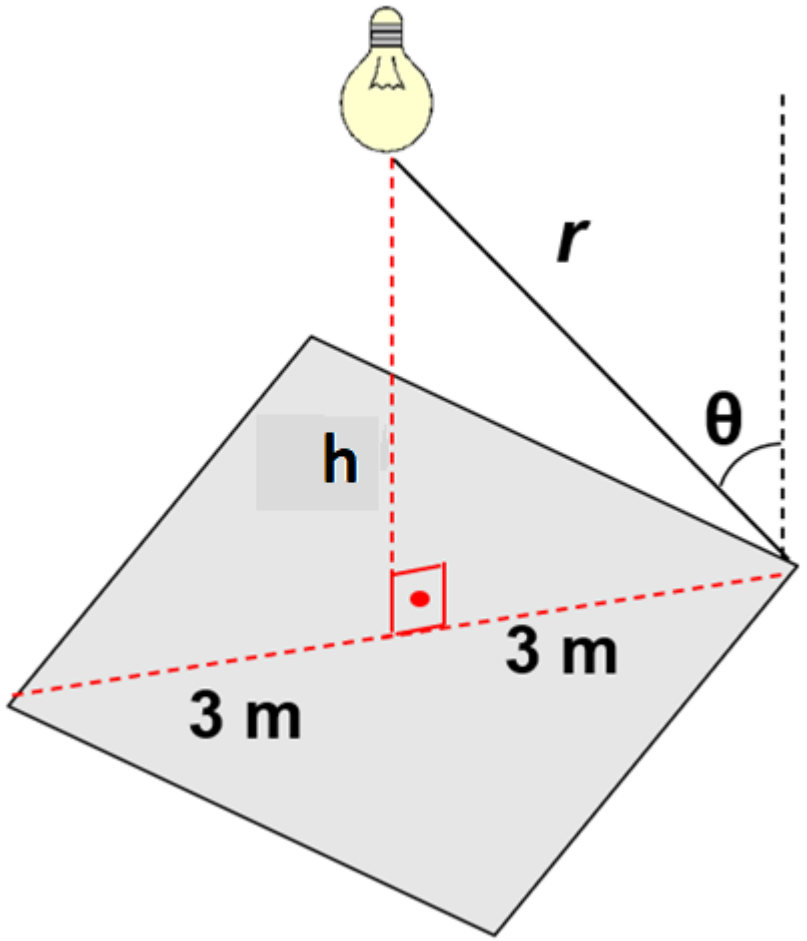


Table gives data on measured radiant power spectrum of a light source at various wavelengths measured in nm. Convert each radiant power given in watts to lumens and compute total luminous flux of the source.

λ(nm)	Φ(mW)	Φ_v(lm)
400	0.19	
450	0.52	
500	2.25	
550	1.69	
575	1.33	
700	0.15	

Quiz

1. What is the difference between the radiometry and photometry?

2. The solid angle of a half-sphere measured from a point in its center is
   (a) 4π sr (b) 2π sr (c) π sr (d) π/2 sr

3. The solid angle subtended at the center of a cube by one of its faces is
   (a) 2π sr (a) π/2 sr (c) π/3 sr (d) 2π/3 sr

4. What is the SI unit of
   (a) radiant intensity (b) radiant flux (c) radiance (d) irradiance
   (e) luminous intensity (f) luminous flux (g) luminance (h) illuminance?