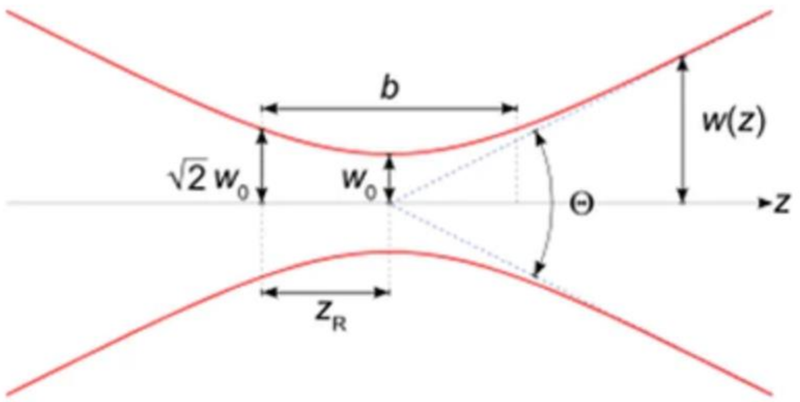
Lectures Notes on Optical Design using Zemax OpticStudio

Lecture 25

Gaussian Beam



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Content

- Gaussian Beam Definition
- Physical Optics in Zemax

References:

- Zemax Knowlegebase pages (https://support.zemax.com)
- Achen University Lectures (https://www.youtube.com/watch?v=MU4eOJw2sBQ)
- Optical System Design, R.E. Ficher et.al., 2nd Ed, McGraw-Hill Companies, (2008)

Introduction

Coherent light generated by lasers has properties different from light generated by other sources which we usually deal with in more conventional optical systems.

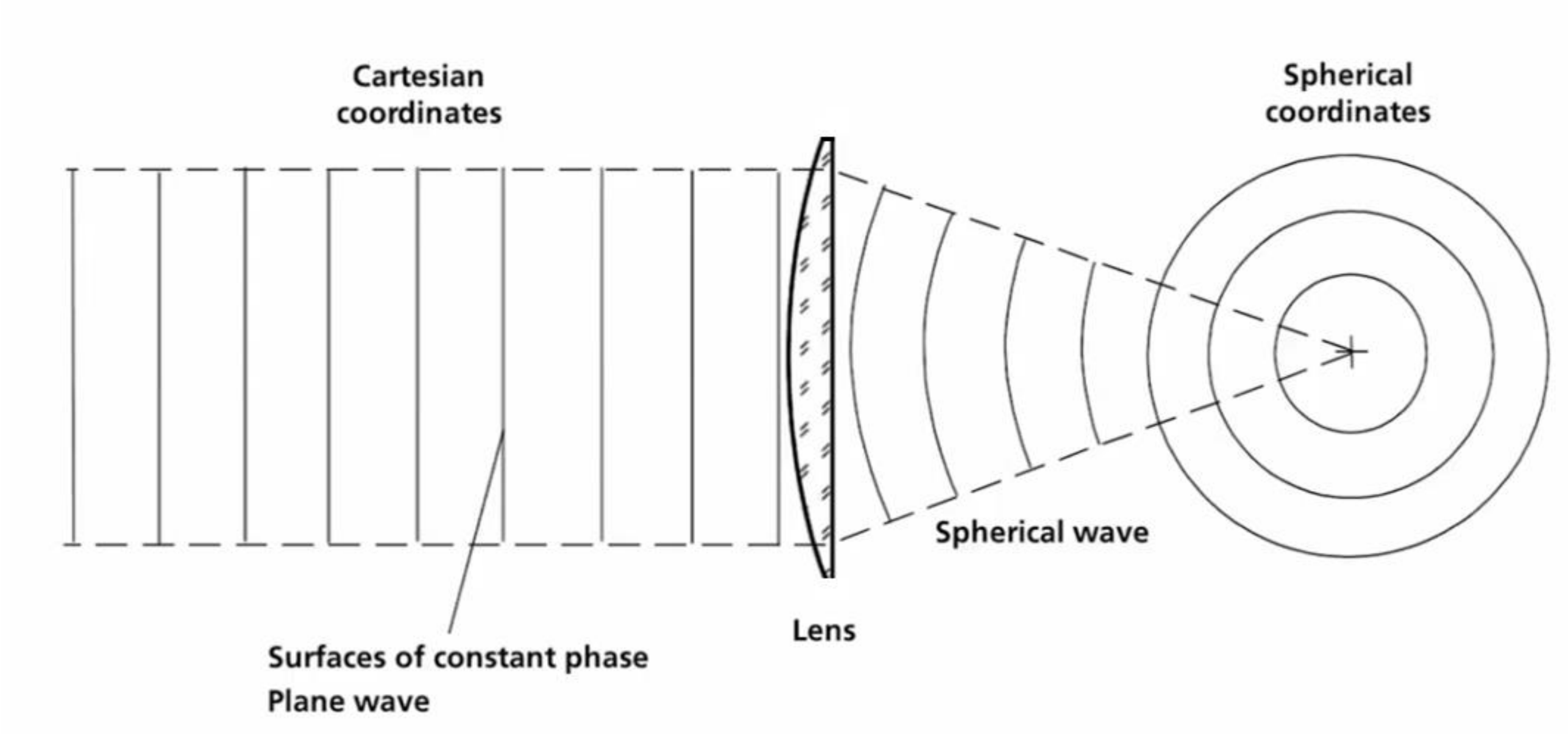
Most laser beams can be approximately described by Gaussian optics.
Gaussian optics is a type of wave optics and is very different from geometric optics.

Thereare many companies that provide laser source for the end users.
See: some laser resources

In this chapter we will investigate modelling coherent light generated by lasers.

GAUSSIAN BEAM DEFINITION

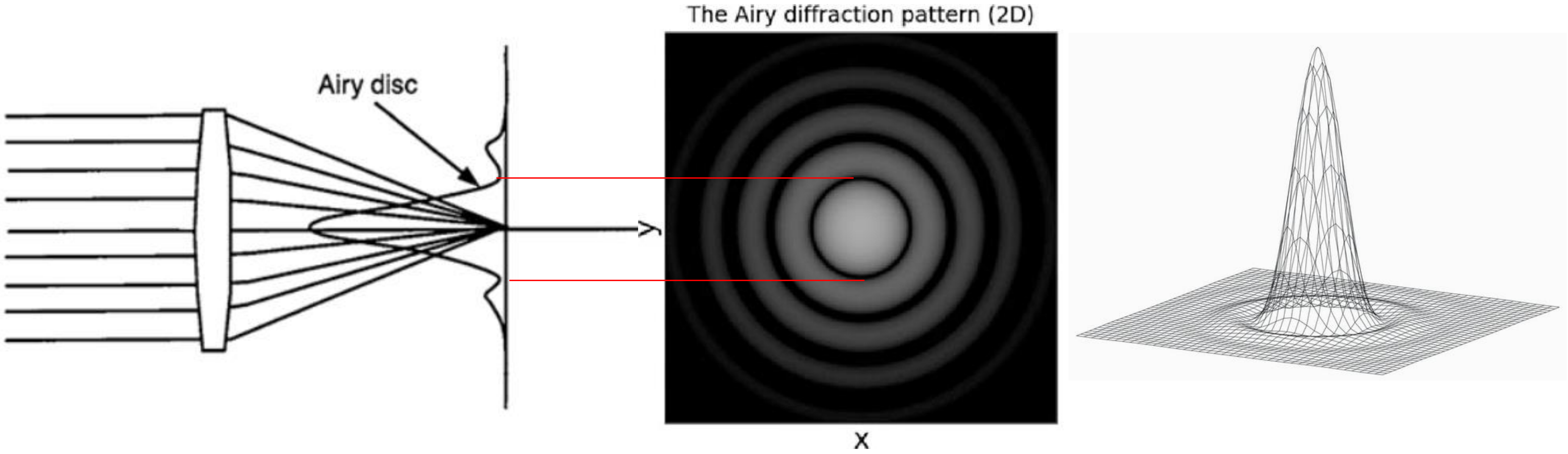
Plane Wave and Spherical Wave (Ideal Case)



See Page 4 (Wavefront and Ray) in Chapter 3 of the lecture notes.

Airy Disk

If we look through a telescope at a distant object, the light intensity across the entrance pupil and aperture stop is uniform, and this is generally known as a **top-hat** intensity profile or distribution.
if there is no aberration, a telescope objective focuses a point object into an Airy disk pattern, with the diameter determined by $D_A = 2r_A = 2.44\,\lambda\,(f/\#)$.



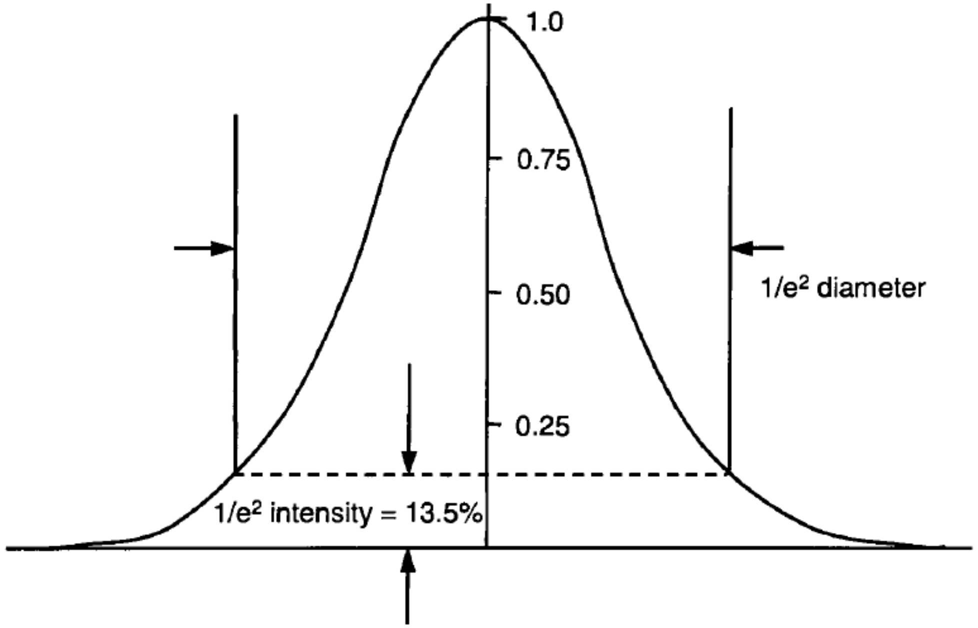
Airy disk is the smallest point to which a beam of light can be focused

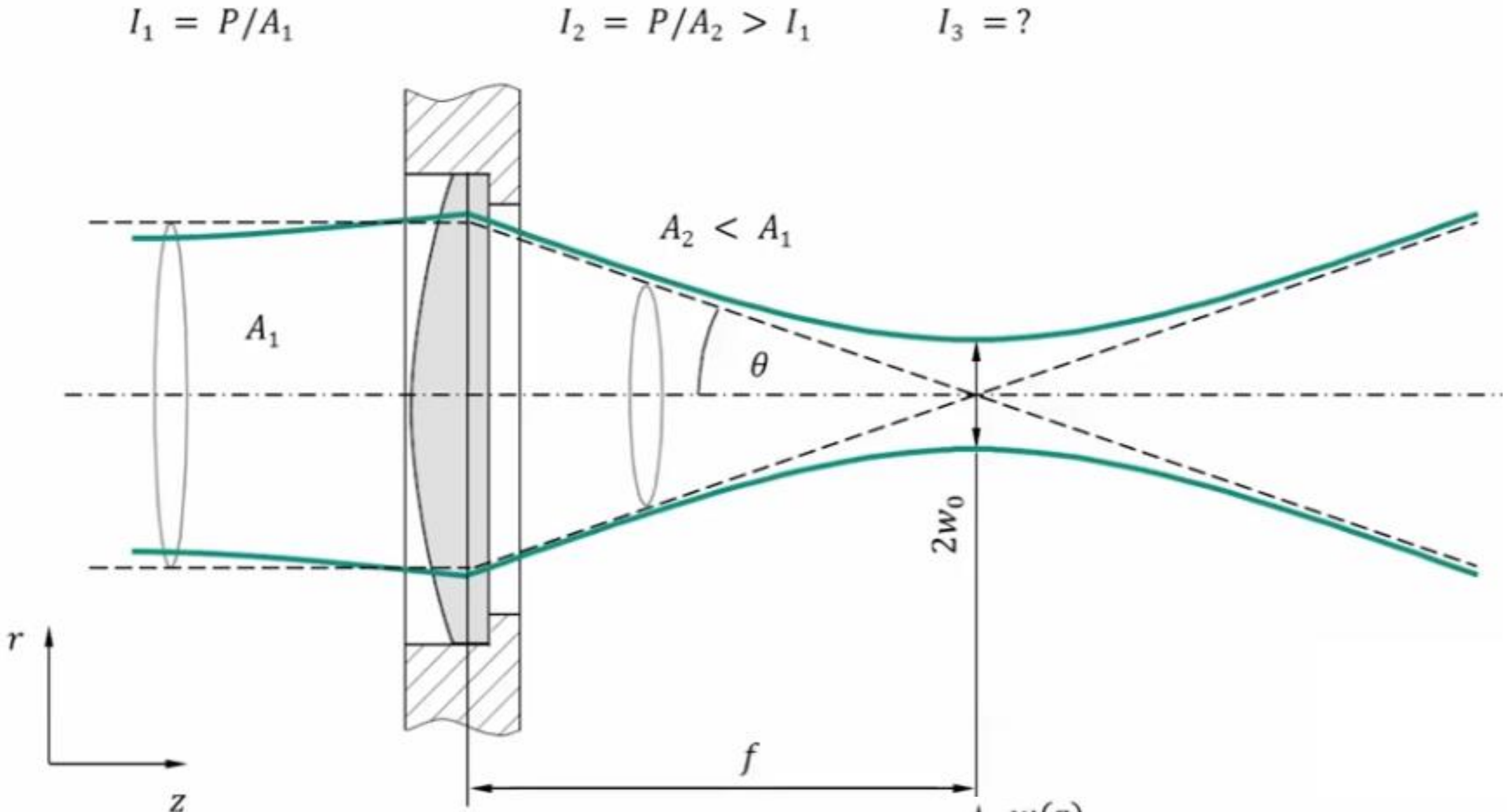
Gaussian Beam Intensity

Laser beams emitted from rotationally symmetric resonators, such as HeNe or YAG lasers with a TEM_{00} output, have an intensity distribution across the beam which is in the form of a gaussian intensity profile.

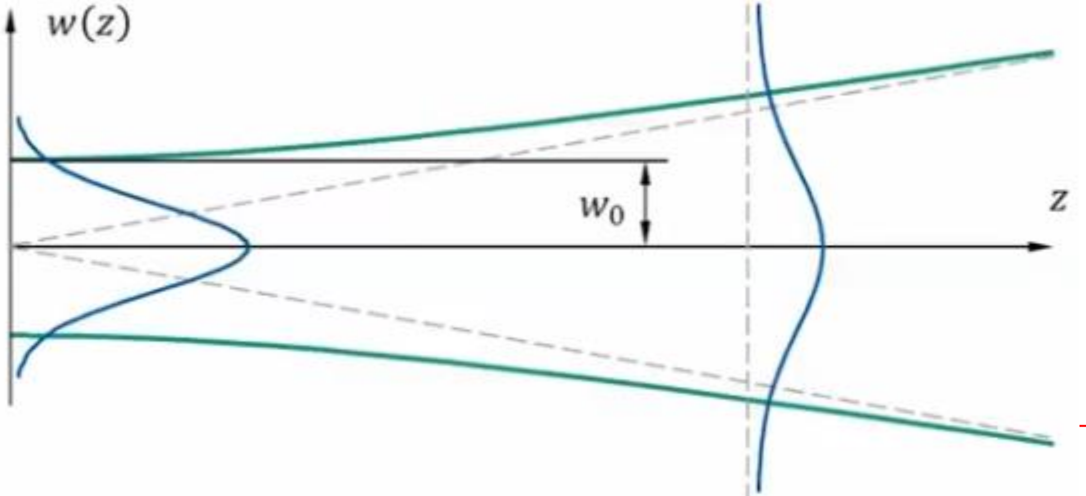
A gaussian intensity distribution in pupil space
will mathematically transform to a gaussian in image space.

The optical design of systems that facilitate laser beam propagation and focusing differs significantly from that of conventional non-laser systems, whether in the visible spectrum or another wavelength range.





P = power
A = area
I = irradiance
w_0 = waist



Light cannot be focused to an arbitrarily small spot due to its wave nature (diffraction).

Gaussian Beam

Consider an ideal Gaussian beam with waist w_0.

As shown in the schematic below

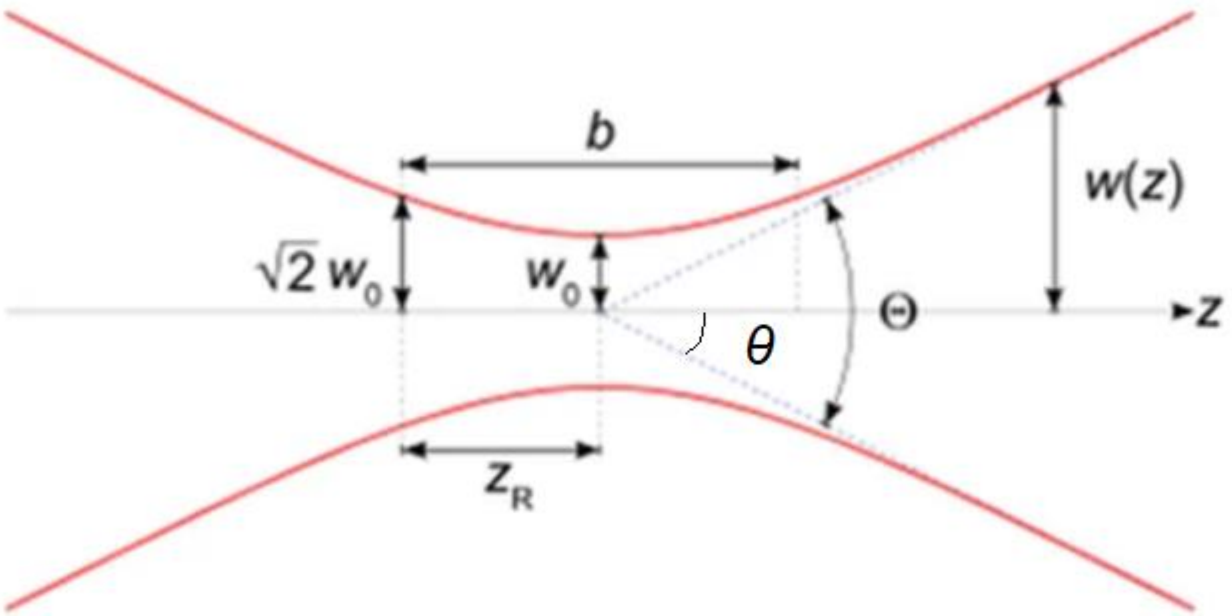
This Gaussian beam can be described

using any two of the three parameters:

wavelength : λ

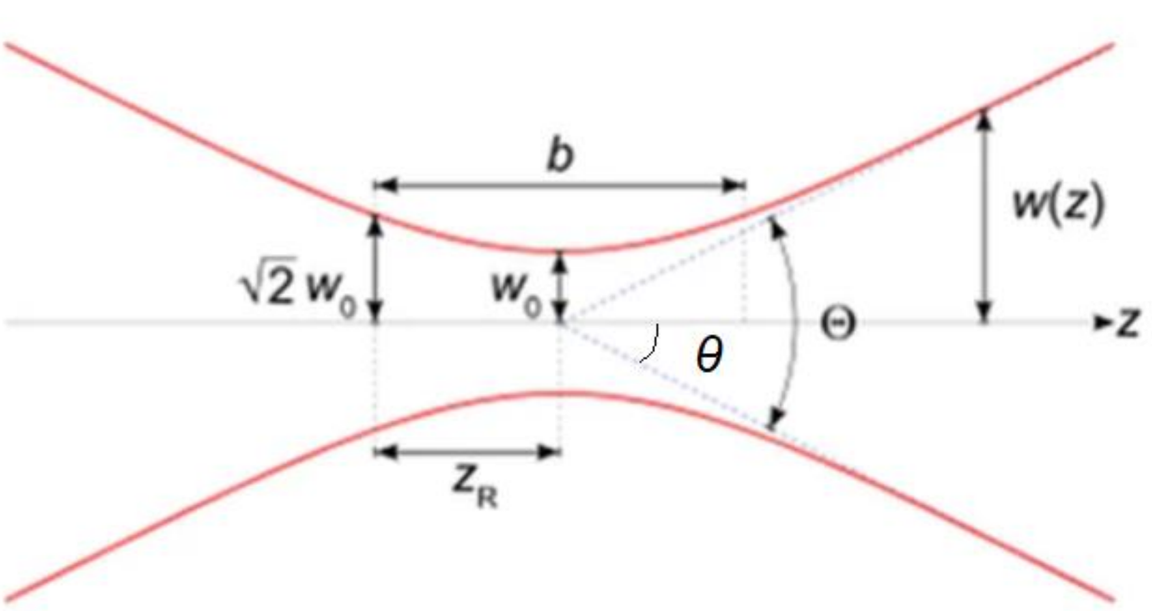
beam waist : w_0

divergence angle : $\Theta = 2\theta$



The beam size is a function of the distance from the waist. OpticStudio uses the half width to describe beam width. For a perfect gaussian shape, $1/e^2$ intensity radius of the beam as a function of z is given by

$$w(z) = w_0\left[1 + \left(\frac{z}{z_R}\right)^2\right]^{1/2}$$



For large distances the beam size expands linearly.
The divergence angle θ of the beam is given by

$$\theta = \frac{\lambda}{\pi w_0} \qquad for\ \ z \gg z_R$$

z_R is the Rayleigh Range (aka depth of focus) of the beam

$$z_R = \frac{\pi w_0^2}{\lambda}$$ (at $z = z_R$ the beam radius is $w = \sqrt{2}w_0$)

The phase (wavefront) radius of curvature of the beam is

$$R(z) = z + \frac{z_R^2}{z}$$

This means that the radius is infinite at waist location z = 0, reaches its minimum at z = z_R, and asymptotically approaches infinity as z approaches infinity.

Gaussian Beam Characterization

Transversal intensity distribution

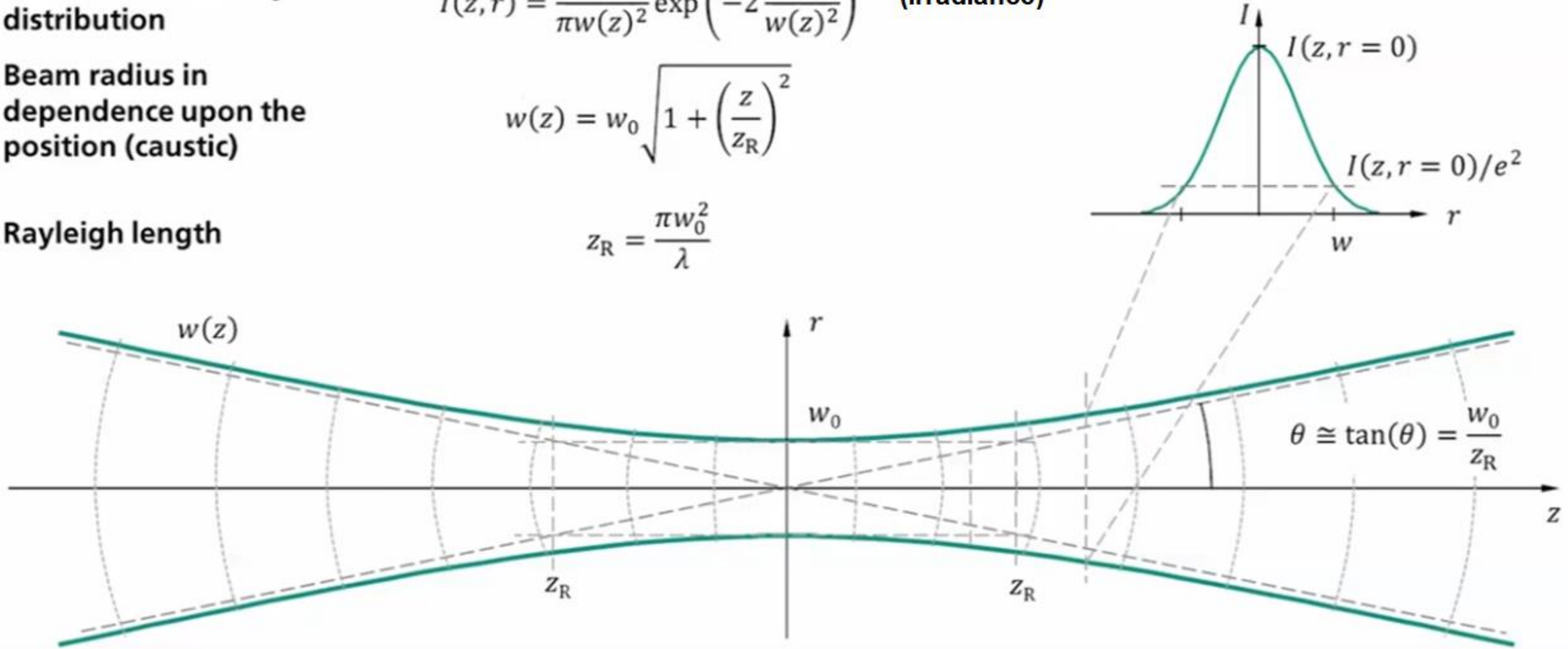
$$I(z,r) = \frac{2P}{\pi w(z)^2} \exp\left(-2\frac{r^2}{w(z)^2}\right)$$ (irradiance)

Beam radius in dependence upon the position (caustic)

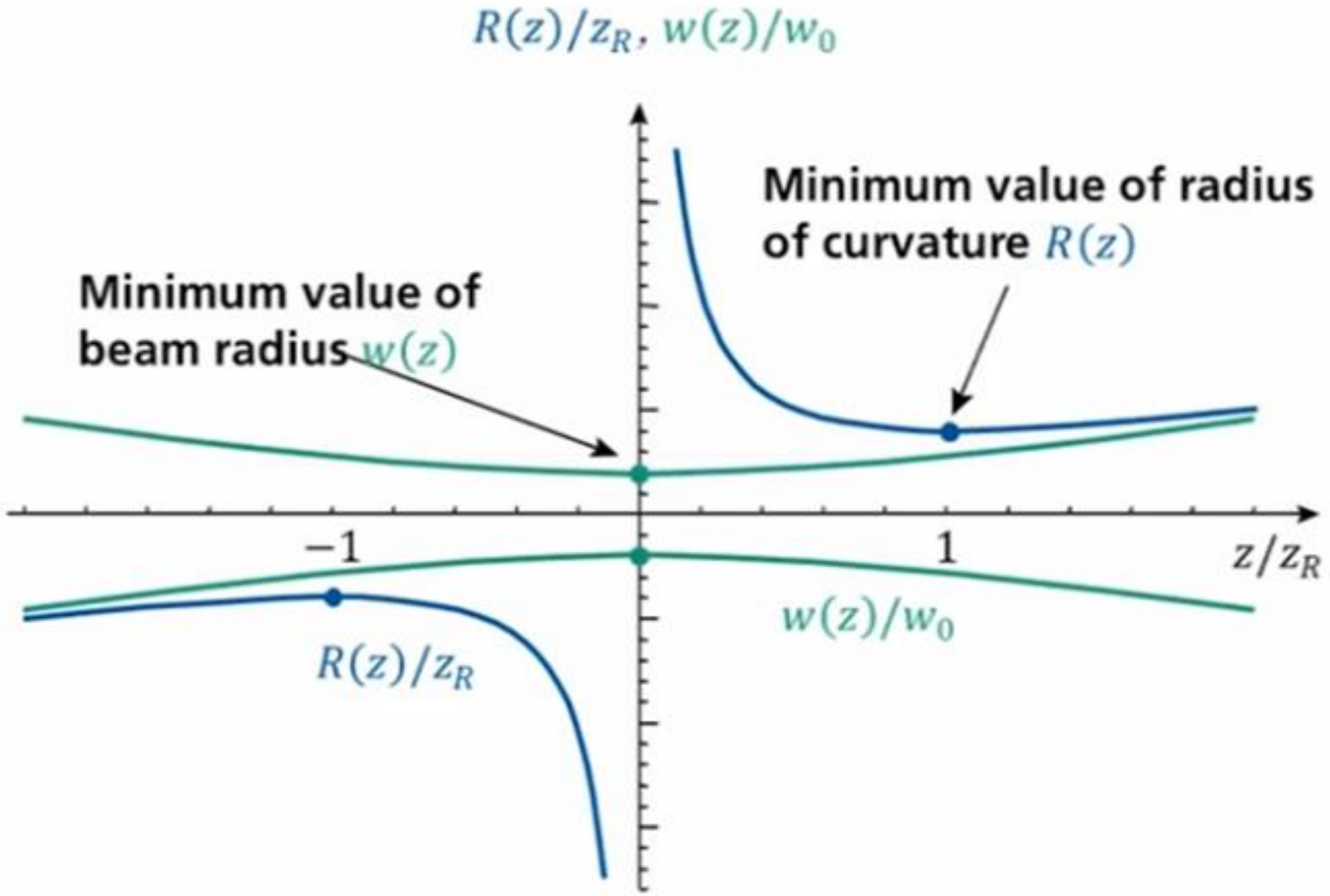
$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

Rayleigh length

$$z_R = \frac{\pi w_0^2}{\lambda}$$



Radius of Curvature R(z) and beam Radius w(z)



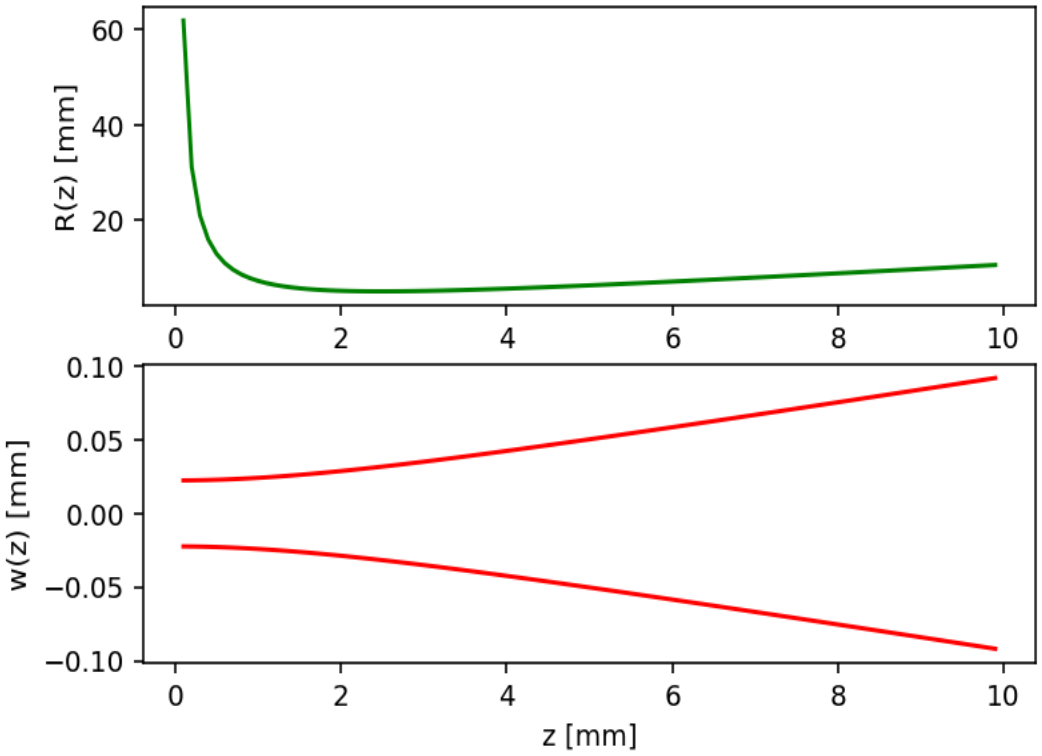
$$R(z) = z_R\left(\frac{z}{z_R} + \frac{z_R}{z}\right), \qquad \frac{R(z)}{z_R} \xrightarrow{z \gg z_R} \frac{z}{z_R}$$

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \qquad \frac{w(z)}{w_0} \xrightarrow{z \gg z_R} \frac{z}{z_R}$$

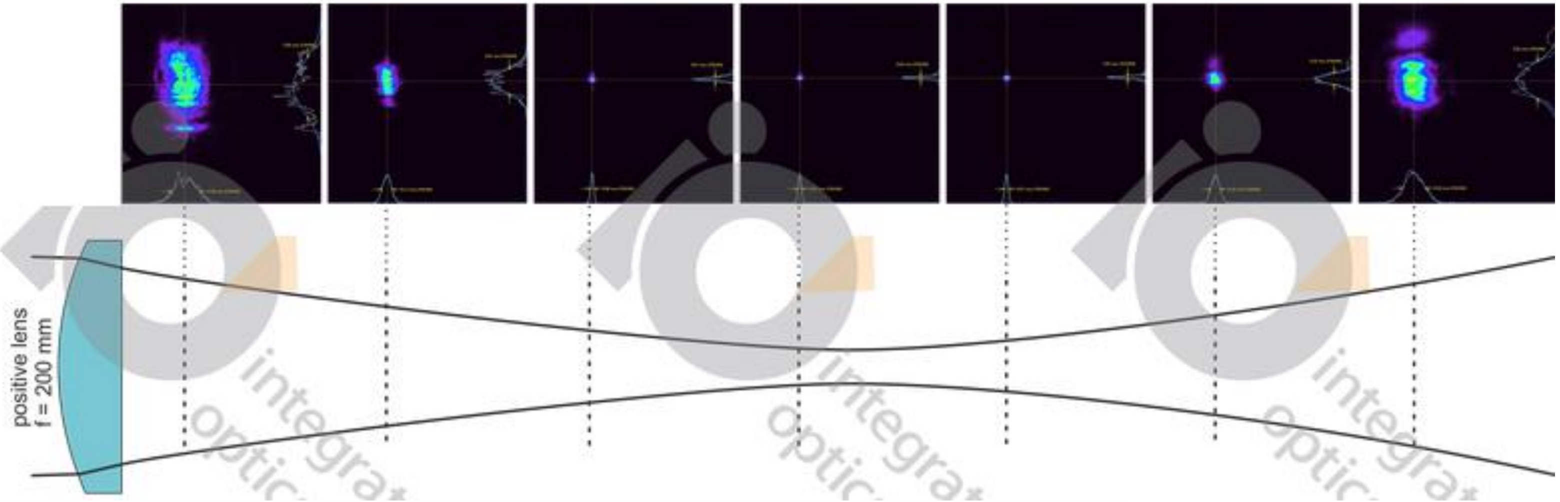
```
# gbcalc.py
# Gaussian Beam Calculator
import math
import numpy as np
import matplotlib.pyplot as plt

# *** Inputs ***
theta = 9e-3          # beam divergence (rad)
L = 0.6328 * 1e-3     # wavelength (mm)

# *** Calculations ***
w0 = L / (math.pi*theta) # beam waist
zR = math.pi*w0**2 / L   # Rayleigh range
print('w0 = ', w0,' mm')
print('zR = ', zR,' mm’)

# *** Plotting ***
z  = np.arange(0.1,10,0.1)
wz = w0*np.sqrt( 1+(z/zR)**2 )
Rz = z + zR**2/z
fig, (ax1, ax2) = plt.subplots(2)
ax1.plot(z,Rz,'g')
ax2.plot(z,wz,'r’); ax2.plot(z,-wz,'r')
ax1.set_ylabel('R(z) [mm]')
ax2.set_ylabel('w(z) [mm]'); ax2.set_xlabel('z [mm]')
```

Output of the program:

```
w0 = 0.02238 mm
zR = 2.48675 mm
```

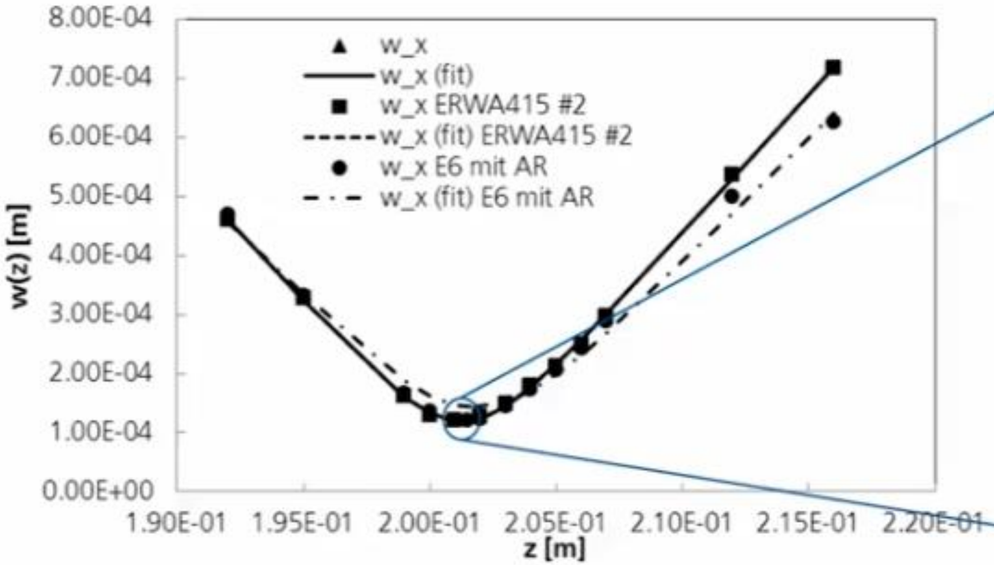


Measurement of Beam Quality

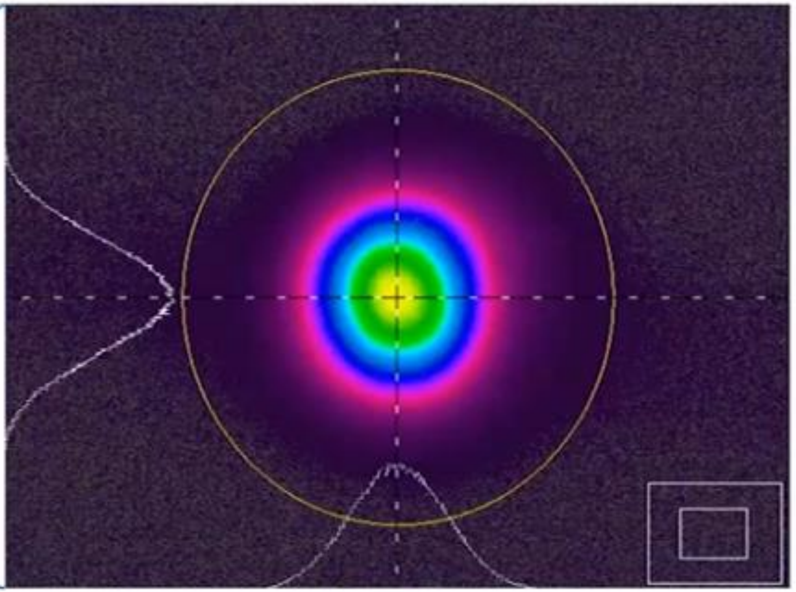


HeNe laser with wavelength $\lambda\ =\ 632.8\ \mathrm{nm}$

For real laser beams: $M^2\ >\ 1$

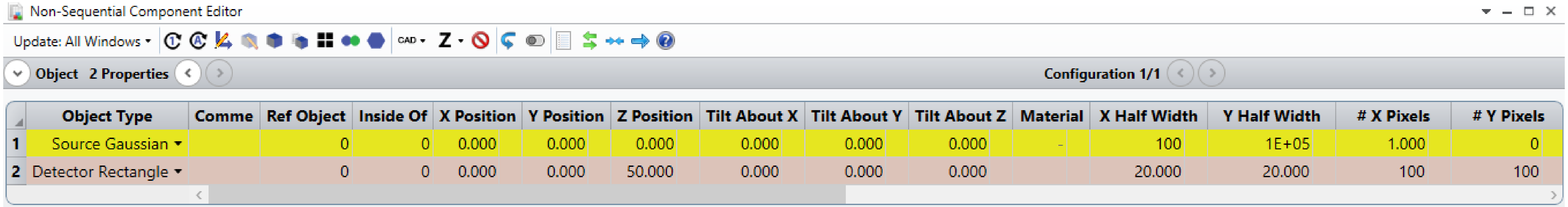


Measured caustic

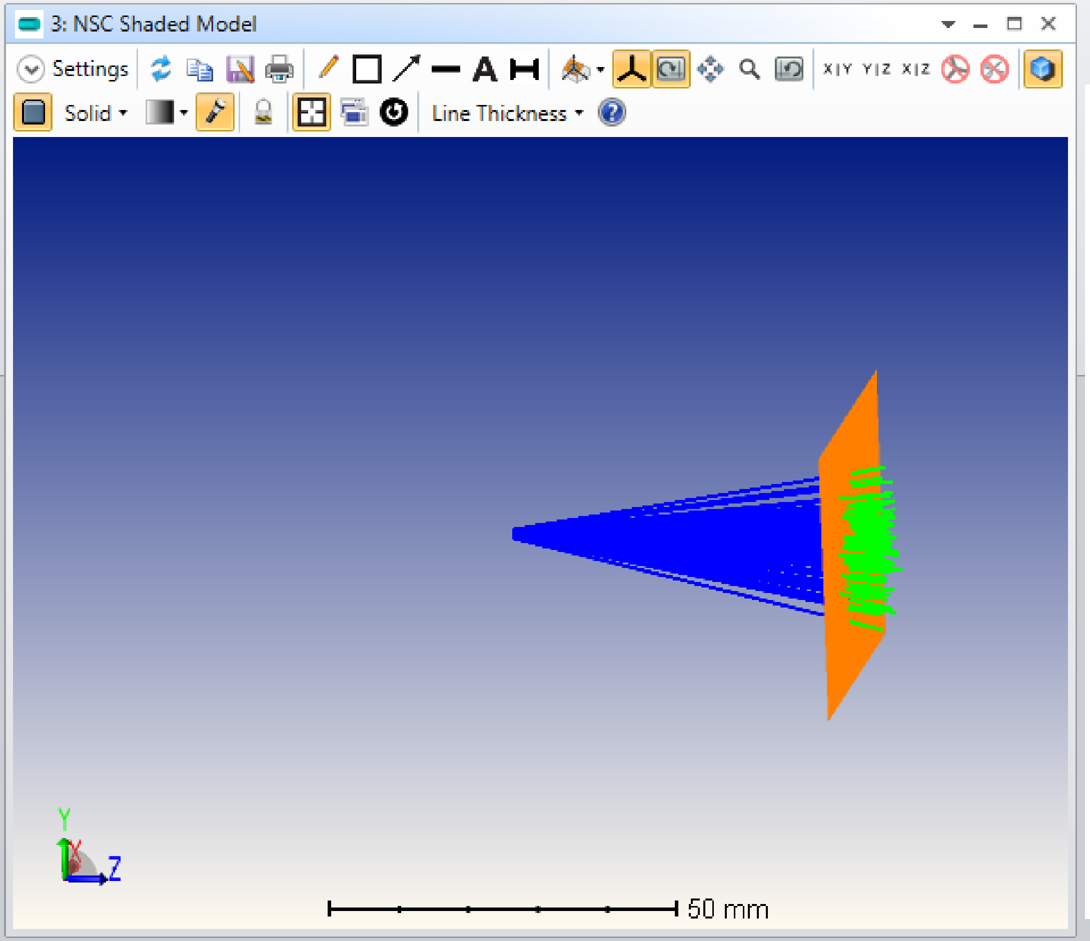


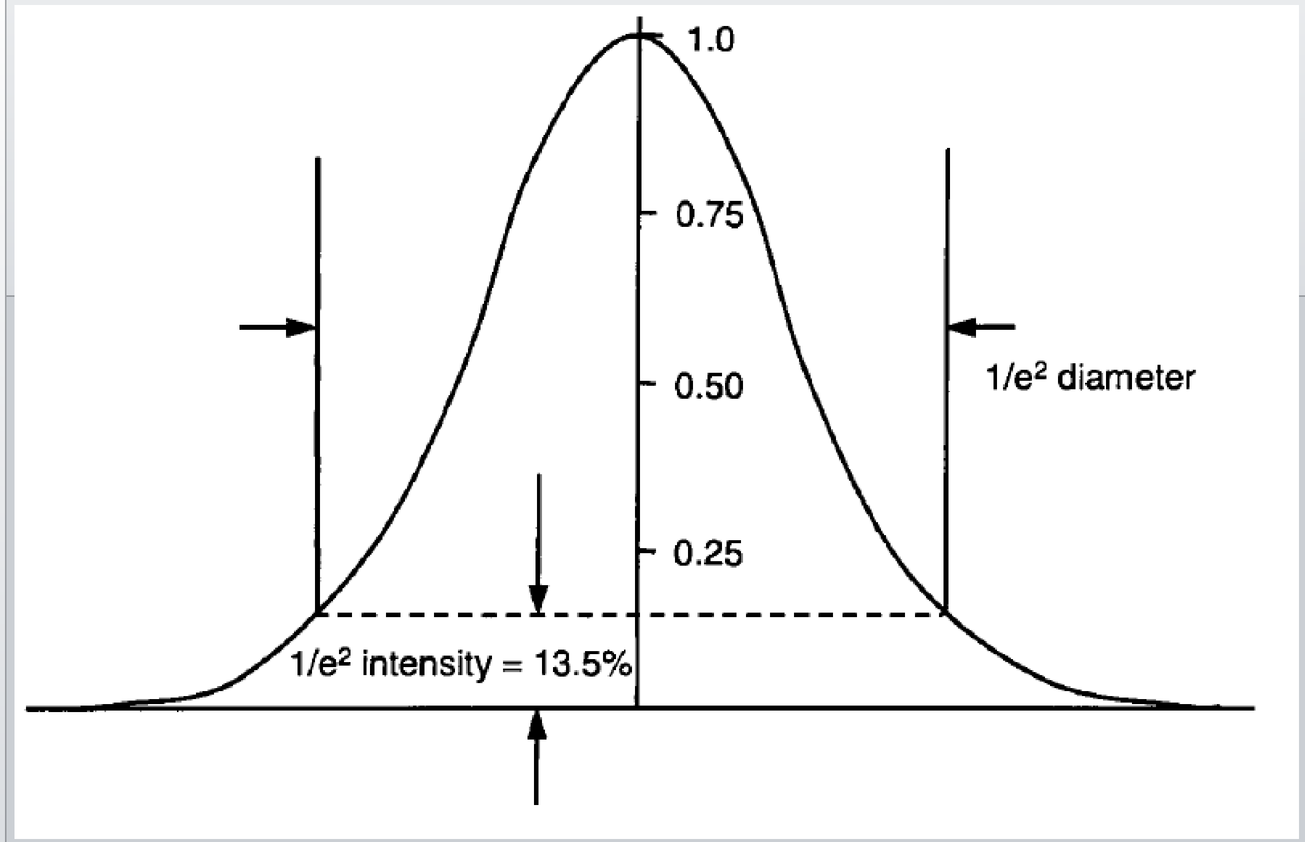
Intensity profile

Reminder: Source Gaussian in Zemax



	Object Type	Comme	Ref Object	Inside Of	X Position	Y Position	Z Position	Tilt About X	Tilt About Y	Tilt About Z	Material	X Half Width	Y Half Width	# X Pixels	# Y Pixels
1	Source Gaussian		0	0	0.000	0.000	0.000	0.000	0.000	0.000	-	100	1E+05	1.000	0
2	Detector Rectangle		0	0	0.000	0.000	50.000	0.000	0.000	0.000		20.000	20.000	100	100





M^2 Factor

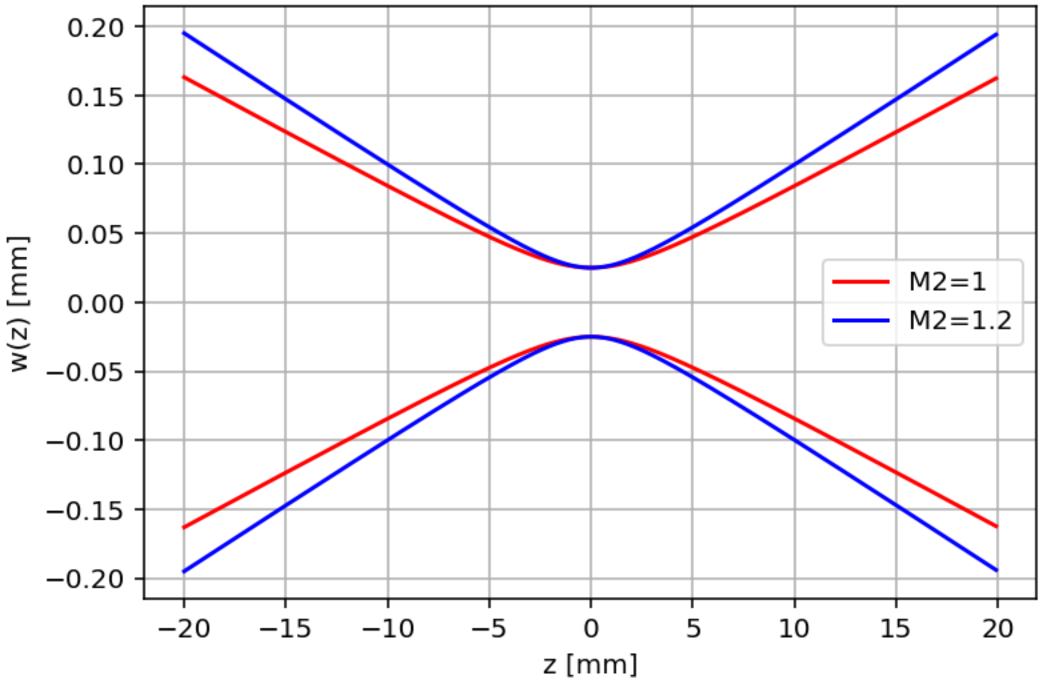
The Gaussian beam concept is so useful in photonics that a special quantity, called the M^2-factor. The M-square factor $M^2 \geq 1$ describes the deviation of a laser beam from a perfect Gaussian beam. In general, the propagation of a laser beam can be described by the following eqns:

$$w(z) = w_0\left[1+\left(\frac{M^2\lambda z}{\pi w_0^2}\right)^2\right]^{1/2} = w_0\left[1+\left(\frac{z}{z_R}\right)^2\right]^{1/2} \qquad z_R = \frac{\pi w_0^2}{M^2\lambda} \qquad I(r,z) = I_0(z)\mathrm{e}^{-2r^2/w(z)^2}$$

- For a perfect Gaussian laser beam, $M^2 = 1$
- Most gas lasers have $M^2 \approx 1$
- Most solid-state lasers have an $M^2 = [1.1, 1.5]$
- Some lasers, such as laser diode piles and high-power YAG lasers, can have an M^2 value over 10.
- M-square can be measured using the relation:

$$M^2 = \frac{\pi w_0 \theta}{\lambda}$$

```
# m2.py
# Gaussian Beam Comparator
import math
import numpy as np
import matplotlib.pyplot as plt

# Inputs
L  = 0.6328 * 1e-3  # wavelength (mm)
M2 = 1.2            # M-square factor
w0 = 0.025          # waist
# Calculations
zR1 = math.pi*w0**2 / (L)
zR2 = math.pi*w0**2 / (L*M2)
# plotting
z  = np.arange(-20,20,0.1)
wz1 = w0*np.sqrt( 1+(z/zR1)**2 )
wz2 = w0*np.sqrt( 1+(z/zR2)**2 )
plt.plot(z,wz1,'r')
plt.plot(z,wz2,'b')
plt.plot(z,-wz1,'r')
plt.plot(z,-wz2,'b')
plt.xlabel('z [mm]')
plt.ylabel('w(z) [mm]')
plt.legend(["M2=1", "M2=1.2"], loc="best")
plt.grid(True)
```



Example 1

Consider He-Ne laser beam at 633 nm with a spot size of 1 mm. For a Gaussian beam ($M^2 = 1$) what is the divergence of the beam? What are the Rayleigh range and the beam width at 25 m?

$$2\theta = \frac{4\lambda}{\pi(2w_o)} = \frac{4(633 \times 10^{-9}\ \mathrm{m})}{\pi(1 \times 10^{-3}\ \mathrm{m})} = 8.06 \times 10^{-4}\,\mathrm{rad} = 0.046^\circ$$

$$z_o = \frac{\pi w_o^2}{\lambda} = \frac{\pi\left[(1 \times 10^{-3}\ \mathrm{m})/2\right]^2}{(633 \times 10^{-9}\ \mathrm{m})} = 1.24\ \mathrm{m}$$

$$\begin{aligned} 2w &= 2w_o\left[1 + (z/z_o)^2\right]^{1/2} = (1 \times 10^{-3}\,\mathrm{m})\left\{1 + \left[(25\ \mathrm{m})/(1.24\ \mathrm{m})\right]^2\right\}^{1/2} \\ &= 0.0202\ \mathrm{m} \quad \text{or} \quad 20\ \mathrm{mm}. \end{aligned}$$

What if $M^2 = 2$?

Exercise

Consider a 5 mW He-Ne laser that is operating at 633 nm, and has a spot size of 1 mm.

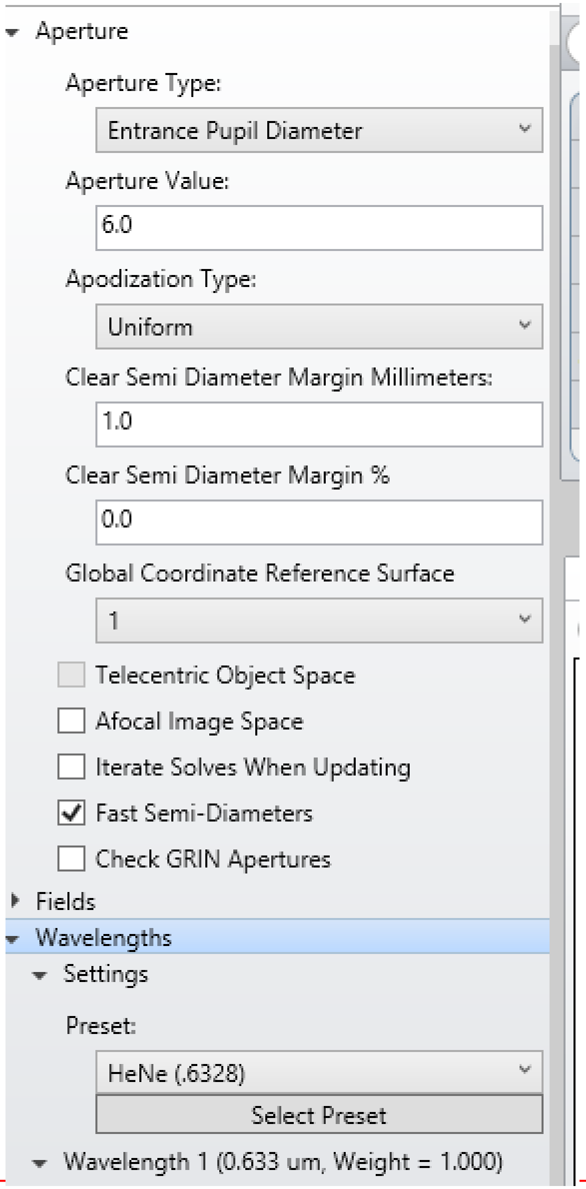
Find

(a) the maximum irradiance of the beam [Ans: 1.27 W/cm^2]

(b) the axial (maximum) irradiance at 25 m from the laser [Ans: 3.13 mW/cm^2].

PHYSICAL OPTICS
IN ZEMAX

Example2: Paraxial (Abberation Free) Gaussian Beam Propagation



Aperture

Aperture Type: Entrance Pupil Diameter

Aperture Value: 6.0

Apodization Type: Uniform

Clear Semi Diameter Margin Millimeters: 1.0

Clear Semi Diameter Margin %: 0.0

Global Coordinate Reference Surface: 1

☐ Telecentric Object Space

☐ Afocal Image Space

☐ Iterate Solves When Updating

☑ Fast Semi-Diameters

☐ Check GRIN Apertures

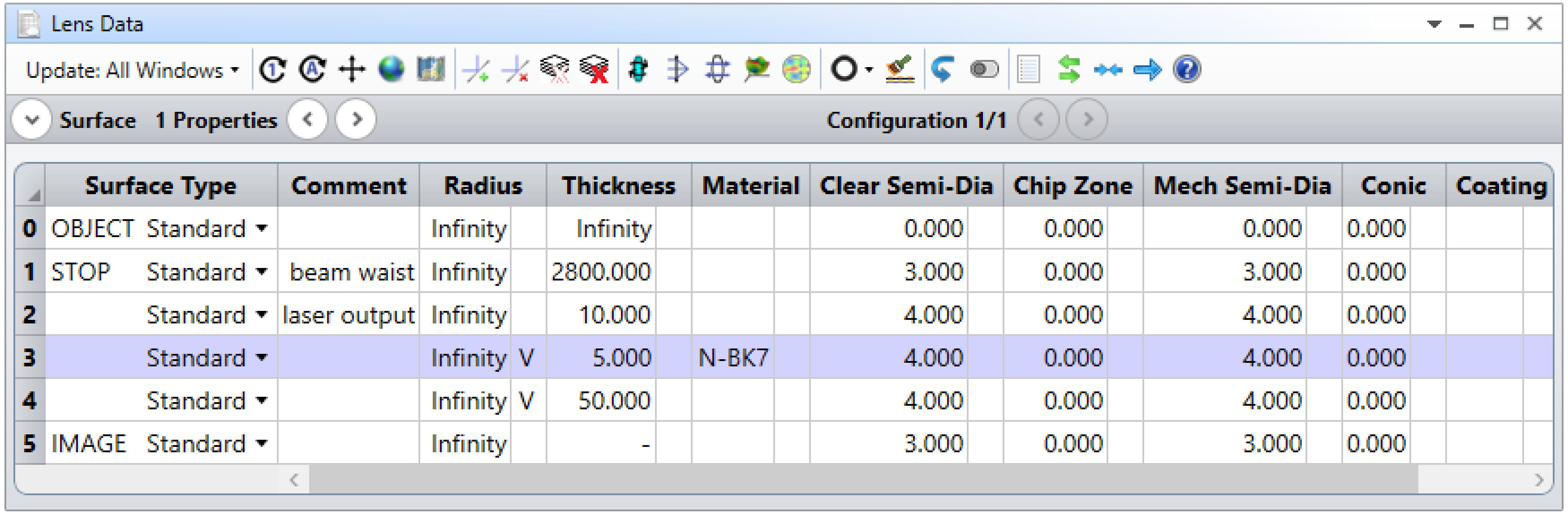
Fields

Wavelengths

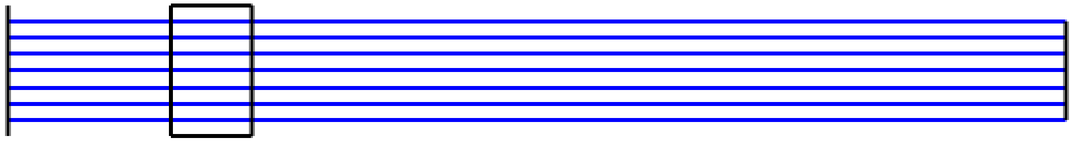
Settings

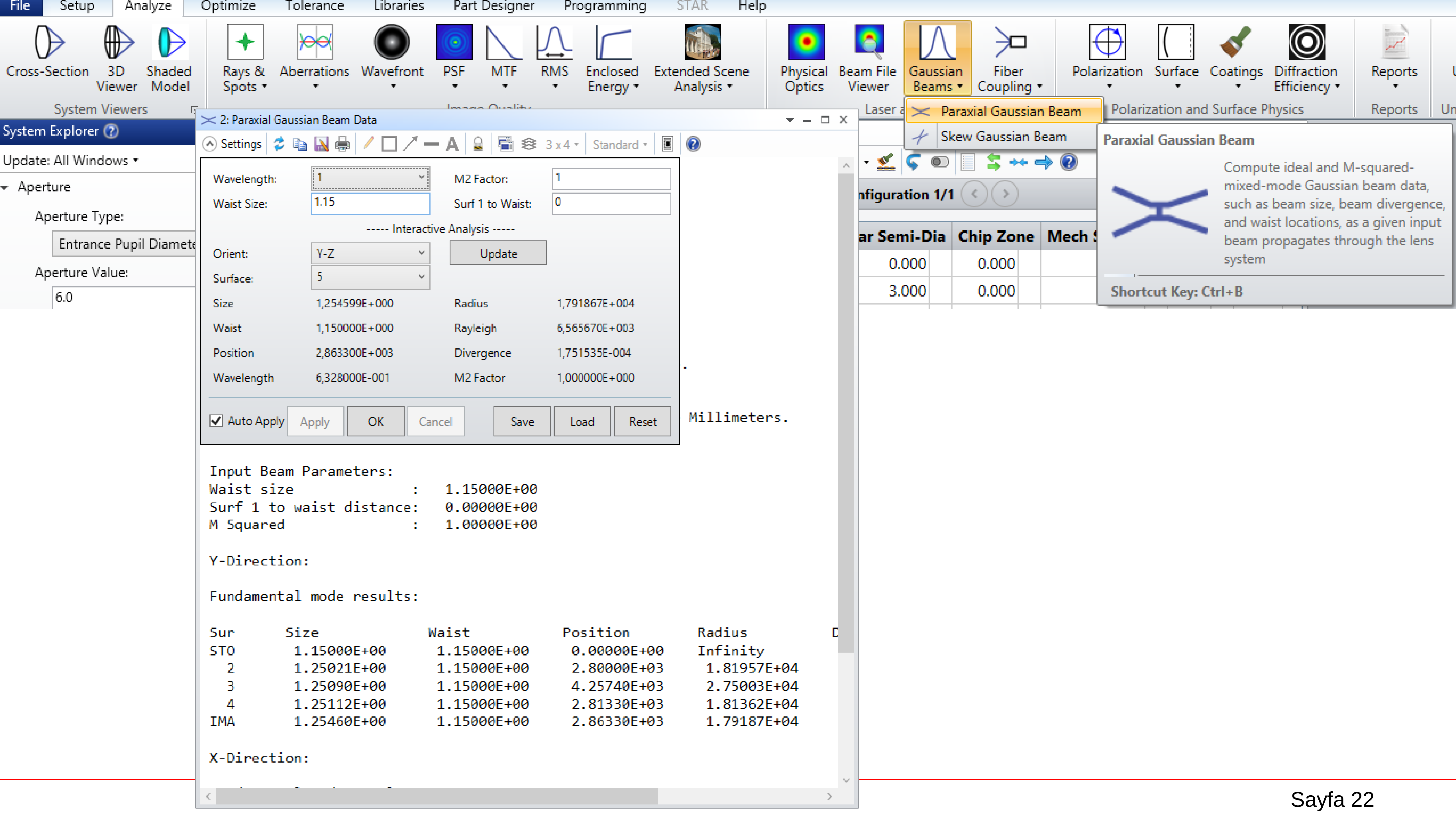
Preset: HeNe (.6328)

Wavelength 1 (0.633 um, Weight = 1.000)



	Surface Type		Comment	Radius	Thickness	Material	Clear Semi-Dia	Chip Zone	Mech Semi-Dia	Conic	Coating
0	OBJECT	Standard		Infinity	Infinity		0.000	0.000	0.000	0.000	
1	STOP	Standard	beam waist	Infinity	2800.000		3.000	0.000	3.000	0.000	
2		Standard	laser output	Infinity	10.000		4.000	0.000	4.000	0.000	
3		Standard		Infinity V	5.000	N-BK7	4.000	0.000	4.000	0.000	
4		Standard		Infinity V	50.000		4.000	0.000	4.000	0.000	
5	IMAGE	Standard		Infinity	-		3.000	0.000	3.000	0.000	





Paraxial Gaussian Beam

Compute ideal and M-squared-mixed-mode Gaussian beam data, such as beam size, beam divergence, and waist locations, as a given input beam propagates through the lens system

Shortcut Key: Ctrl+B

Wavelength: 1
M2 Factor: 1
Waist Size: 1.15
Surf 1 to Waist: 0

----- Interactive Analysis -----

Orient: Y-Z
Surface: 5

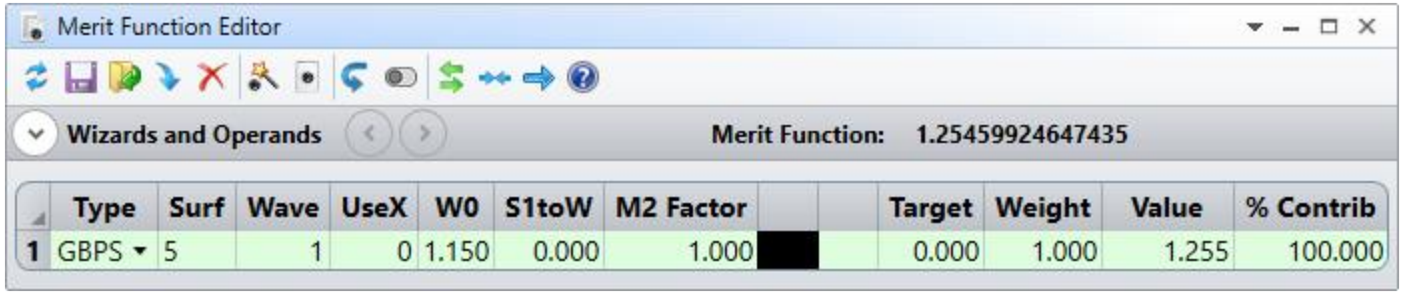
Size	1,254599E+000	Radius	1,791867E+004
Waist	1,150000E+000	Rayleigh	6,565670E+003
Position	2,863300E+003	Divergence	1,751535E-004
Wavelength	6,328000E-001	M2 Factor	1,000000E+000

☑ Auto Apply

```
Millimeters.

Input Beam Parameters:
Waist size               :   1.15000E+00
Surf 1 to waist distance:   0.00000E+00
M Squared                :   1.00000E+00

Y-Direction:

Fundamental mode results:

Sur       Size            Waist           Position        Radius          D
STO        1.15000E+00     1.15000E+00     0.00000E+00    Infinity
  2        1.25021E+00     1.15000E+00     2.80000E+03     1.81957E+04
  3        1.25090E+00     1.15000E+00     4.25740E+03     2.75003E+04
  4        1.25112E+00     1.15000E+00     2.81330E+03     1.81362E+04
IMA        1.25460E+00     1.15000E+00     2.86330E+03     1.79187E+04

X-Direction:
```

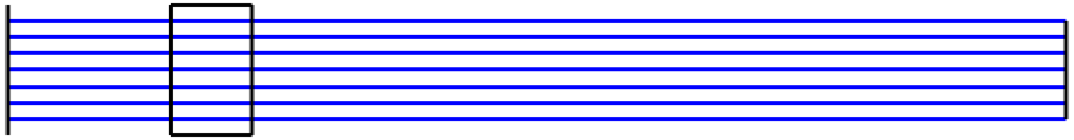
Optimize the lens to get minimum spot size.

Before optimization:

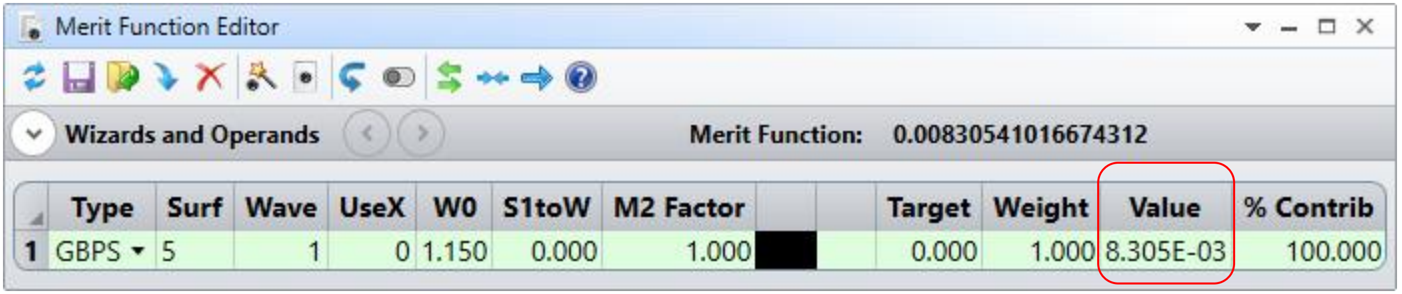


Merit Function: 1.25459924647435

	Type	Surf	Wave	UseX	W0	S1toW	M2 Factor			Target	Weight	Value	% Contrib
1	GBPS ▾	5	1	0	1.150	0.000	1.000	■		0.000	1.000	1.255	100.000

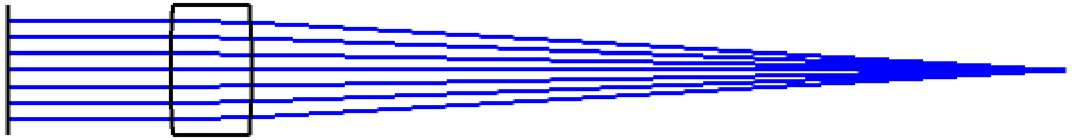


After optimization:



Merit Function: 0.00830541016674312

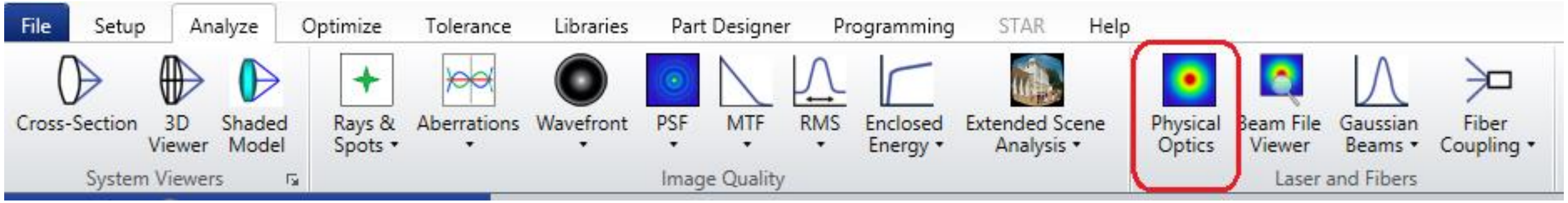
	Type	Surf	Wave	UseX	W0	S1toW	M2 Factor			Target	Weight	Value	% Contrib
1	GBPS ▾	5	1	0	1.150	0.000	1.000	■		0.000	1.000	8.305E-03	100.000

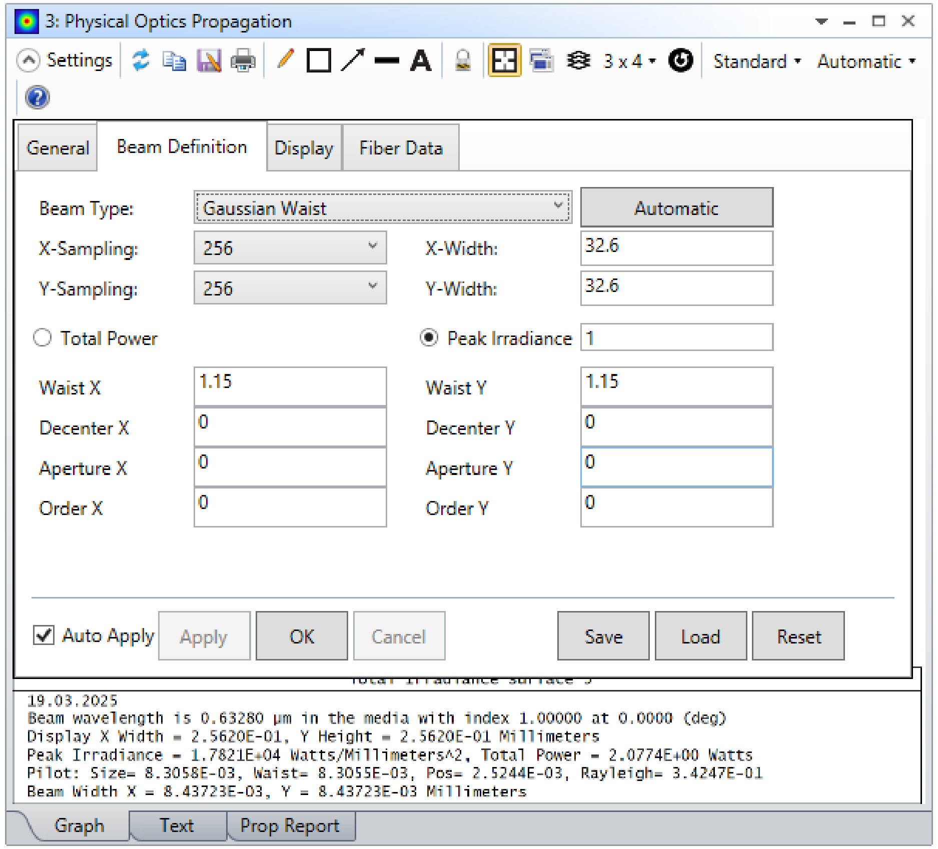


Physical Optics

We examine

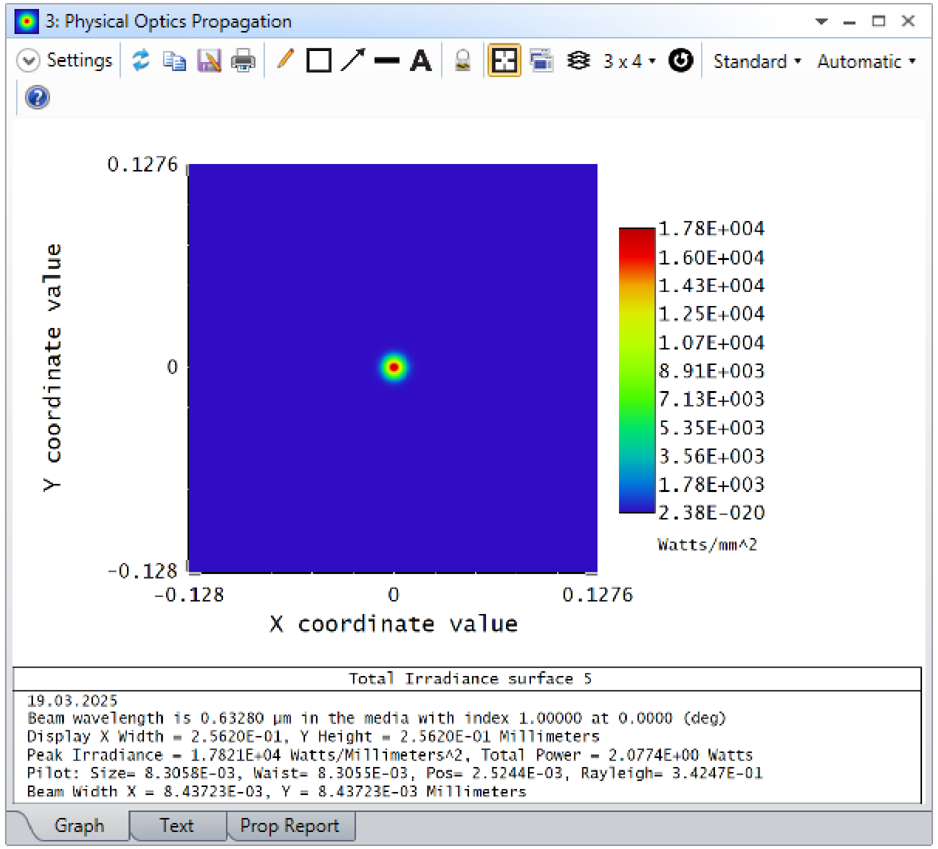
the same example …





Beam Type: Gaussian Waist
X-Sampling: 256
Y-Sampling: 256
X-Width: 32.6
Y-Width: 32.6
○ Total Power
● Peak Irradiance 1
Waist X 1.15
Waist Y 1.15
Decenter X 0
Decenter Y 0
Aperture X 0
Aperture Y 0
Order X 0
Order Y 0
☑ Auto Apply

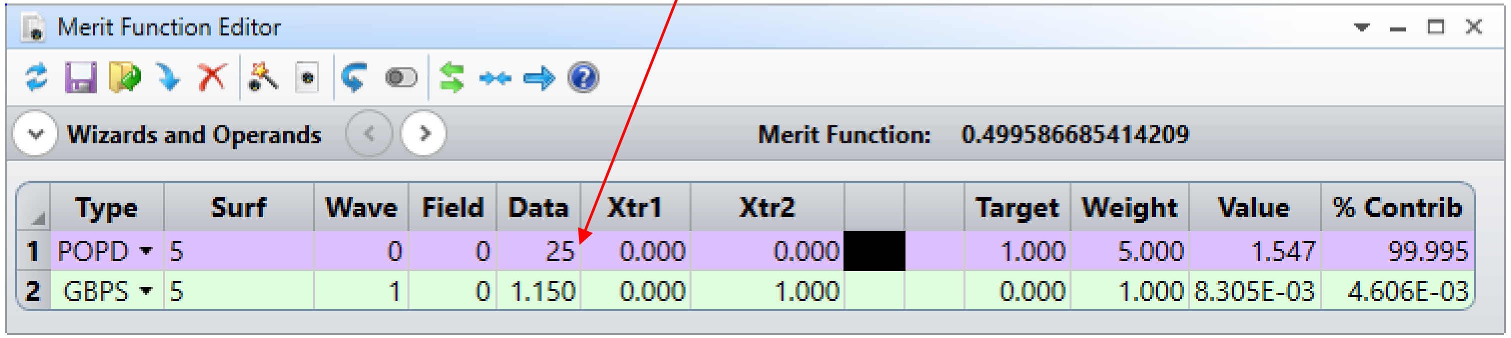
```
Total Irradiance surface 5
19.03.2025
Beam wavelength is 0.63280 µm in the media with index 1.00000 at 0.0000 (deg)
Display X Width = 2.5620E-01, Y Height = 2.5620E-01 Millimeters
Peak Irradiance = 1.7821E+04 Watts/Millimeters^2, Total Power = 2.0774E+00 Watts
Pilot: Size= 8.3058E-03, Waist= 8.3055E-03, Pos= 2.5244E-03, Rayleigh= 3.4247E-01
Beam Width X = 8.43723E-03, Y = 8.43723E-03 Millimeters
```



```
Total Irradiance surface 5
19.03.2025
Beam wavelength is 0.63280 µm in the media with index 1.00000 at 0.0000 (deg)
Display X Width = 2.5620E-01, Y Height = 2.5620E-01 Millimeters
Peak Irradiance = 1.7821E+04 Watts/Millimeters^2, Total Power = 2.0774E+00 Watts
Pilot: Size= 8.3058E-03, Waist= 8.3055E-03, Pos= 2.5244E-03, Rayleigh= 3.4247E-01
Beam Width X = 8.43723E-03, Y = 8.43723E-03 Millimeters
```

Re-optimize to obtain minimum M^2 Value.

Before optimization

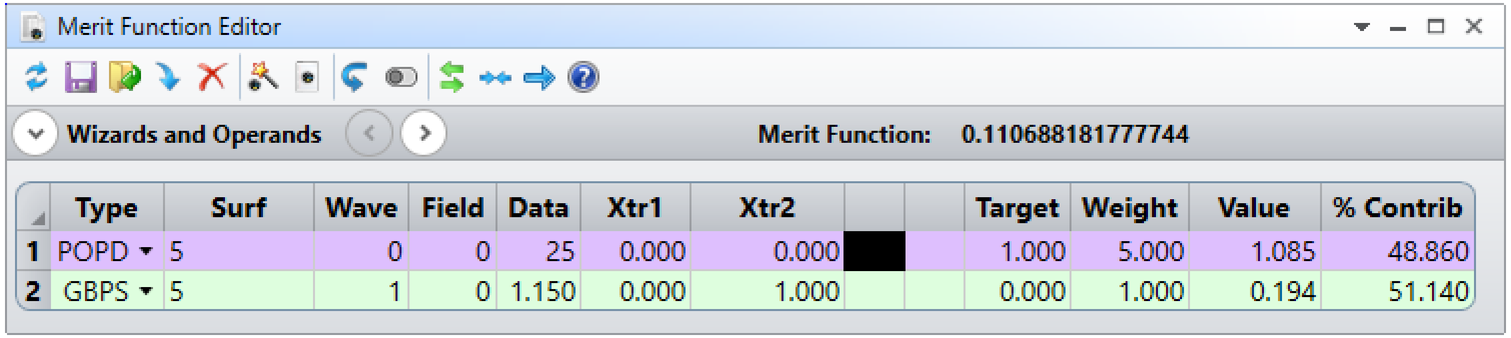


to get value of M^2

Merit Function: 0.499586685414209

	Type	Surf	Wave	Field	Data	Xtr1	Xtr2			Target	Weight	Value	% Contrib
1	POPD ▾	5	0	0	25	0.000	0.000	■		1.000	5.000	1.547	99.995
2	GBPS ▾	5	1	0	1.150	0.000	1.000			0.000	1.000	8.305E-03	4.606E-03

After optimization



Merit Function: 0.110688181777744

	Type	Surf	Wave	Field	Data	Xtr1	Xtr2			Target	Weight	Value	% Contrib
1	POPD ▾	5	0	0	25	0.000	0.000	■		1.000	5.000	1.085	48.860
2	GBPS ▾	5	1	0	1.150	0.000	1.000			0.000	1.000	0.194	51.140

Example 3: Zemax Examples

Investigate the examples at:

C:\<Zemax>\Samples\Non-sequential\Coherence Interference and Diffraction