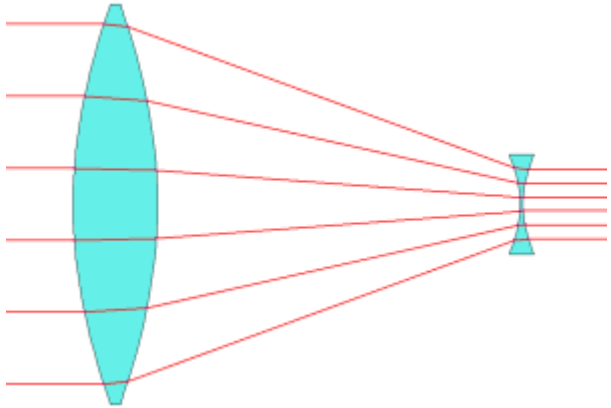
OPAC 202
Optical Design and Inst.

Chapter 12

Optical Resolution



http://www.gantep.edu.tr/~bingul/opac202

Department of

Optical & Acustical Engineering

Gaziantep University

May 2020

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Introduction

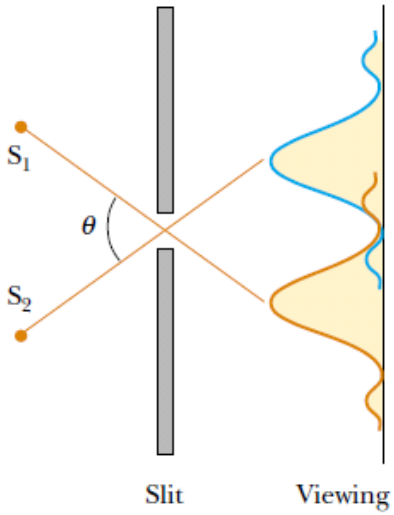
- We have seen some optical designs.
- We now turn to testing and evaluation of an optical system.
- Modern way of to test performance of an optical system is to use transfer functions.
- In this lecture, we will see optical performance tests in Zemax.

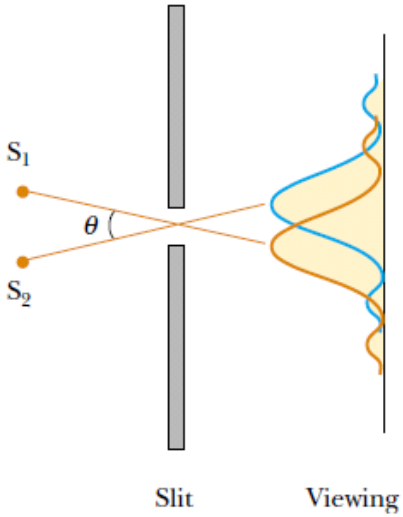
Optical Resolution

- Optical resolution describes the ability of an imaging system to resolve detail in the object that is being imaged.
- To distinguish between closely spaced objects is limited because of the wave nature of light.

- Consider two non-coherent point sources S1 and S2 (e.g. two stars). Each produce its own diffraction pattern.
  - If θ is large two distinct bright spots would be observed on the screen.
  - If θ is small each source is imaged as a bright central region.

- If the two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished and are said to be **resolved**.





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Rayleigh's Criterion

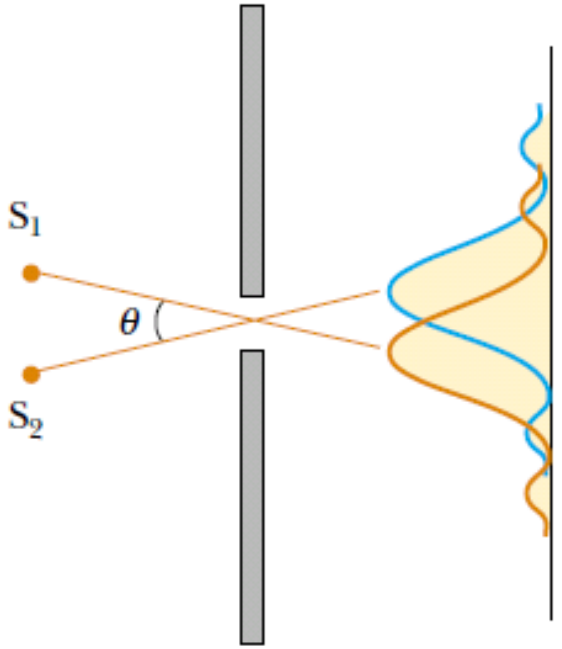
When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved.

- Minimum angular separation for slit

$$\sin\theta_{\min} = \frac{\lambda}{a}$$

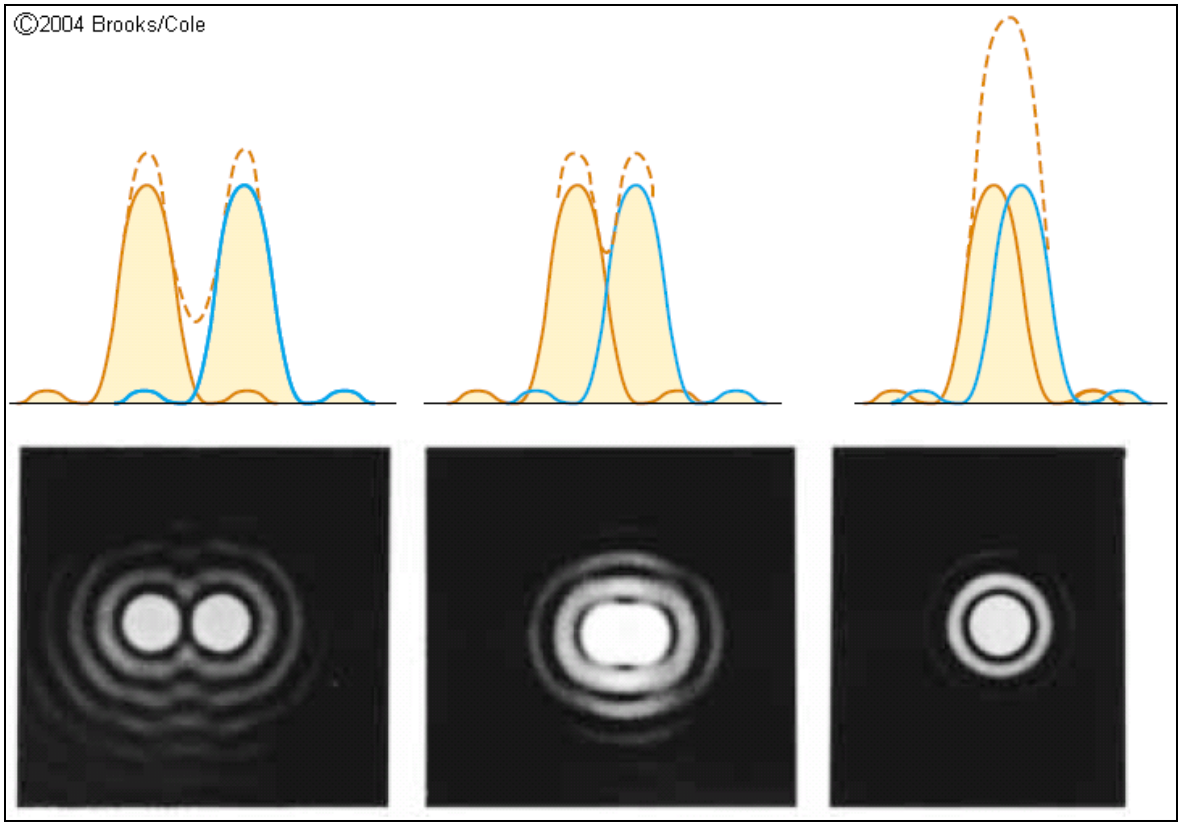
- Small angle approximation yields

$$\theta_{\min} \approx \frac{\lambda}{a}$$



- if the images are to be resolved, the angle subtended by the two sources at the slit must be greater than λ/a.

- Many optical systems use circular apertures rather than slits



- The limiting angle of resolution of the circular aperture is

$$\theta_{\min} \approx 1.22 \frac{\lambda}{D}$$

D is the diameter of the aperture (e.g. lens).

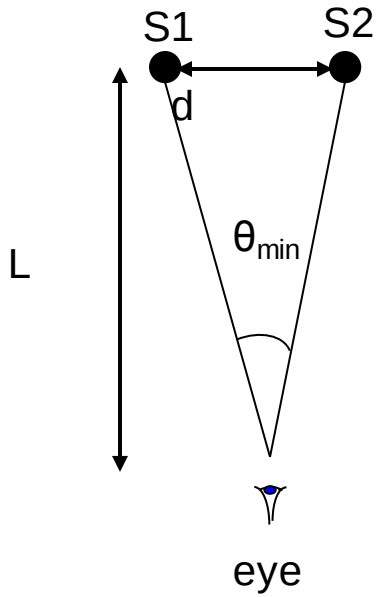
EXAMPLE

What is the limiting angle of resolution of the human eye for λ = 500 nm and pupil diameter 2 mm?

SOLUTION

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{500\times10^{-9}}{2\times10^{-3}} = 3\times10^{-4}\ \text{rad}$$

We can use this result to determine the minimum separation distance d between two point sources that the eye can distinguish if they are a distance L from the observer



$$\sin\theta_{\min} \approx \theta_{\min} = \frac{d}{L}$$

$$d \approx L\theta_{\min} = (25\ \text{cm})(3\times10^{-4}\ \text{rad}) = 8\times10^{-3}\ \text{cm}$$

This is approximately equal to the thickness of a human hair.

EXAMPLE

Light of wavelength 555 nm is used to view an object under a microscope.

The aperture of the objective has a diameter of D = 1.5 cm.

(a) What is the limiting angle of resolution?

(b) If it were possible to use visible light of any wavelength, what would be the maximum limit of resolution for this microscope?

SOLUTION

(a) $$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{555\times10^{-9}}{1.5\times10^{-3}} = 4.5\times10^{-4}\ \text{rad}$$

That is, any two points on the object subtending an angle smaller than this at the objective cannot be distinguished in the image.

(b) *We have to use the shortest wavelength available in the visible spectrum*

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{400\times10^{-9}}{1.5\times10^{-3}} = 3.3\times10^{-4}\ \text{rad}$$

EXAMPLE

The Keck telescope at Mauna Kea, Hawaii, has an effective diameter of 10 m. What is its limiting angle of resolution for 555-nm light?

SOLUTION

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = 1.22\frac{555\times10^{-9}}{10} = 6.8\times10^{-5}\ \text{rad}$$

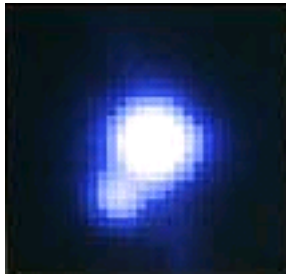
That is, any two stars that subtend an angle greater than or equal to this value are resolved (if atmospheric conditions are ideal).

The Keck telescope can never reach its diffraction limit because the limiting angle of resolution is always set by atmospheric blurring at optical wavelengths.

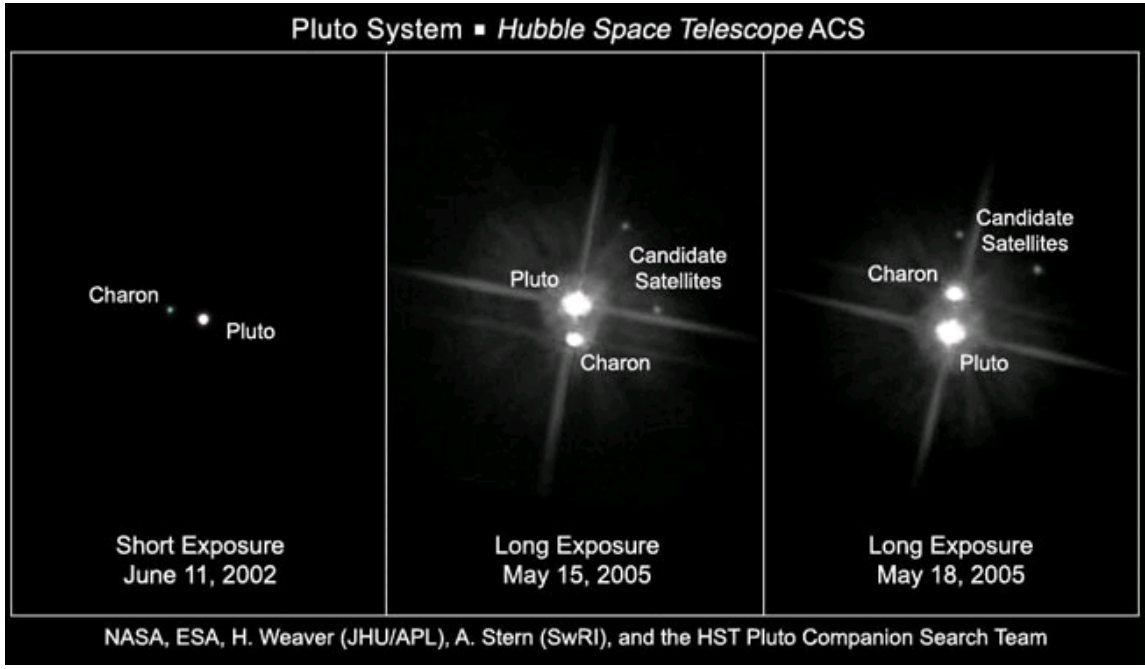
The effects of atmospheric blurring

Telescopic images of Pluto and its moon Charon

From an Earth-based telescope Atmospheric blurring (turbulence) results in Charon appearing only as a subtle bump on the edge of Pluto.



From Hubble Space Telescope (HST), photos of Pluto and Charon, are clearly resolved.



Electron microscope

- An electron microscope uses a particle **beam of electrons** to illuminate the specimen and produce a magnified image.

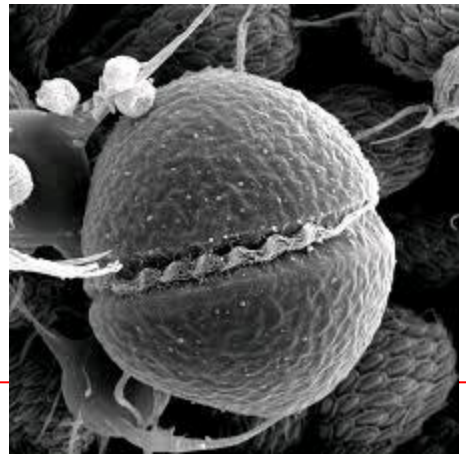
- Electron microscopes have a greater resolving power than a light-powered optical microscope, because electrons have wavelengths about 100,000 times shorter than visible light (photons),

  Resolution

  - Elecron microscope: better than 0.2 nm
  - Light microscope : limited by diffraction to about 200 nm

Head of fly

Dinoflagellate (tek hücreli)

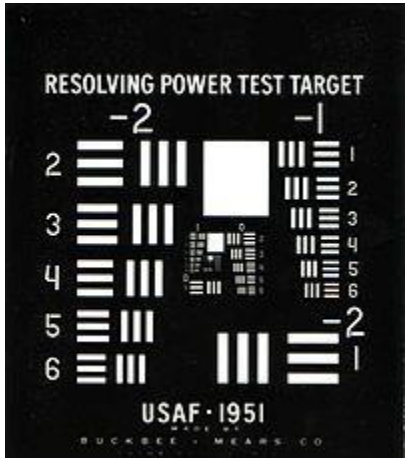


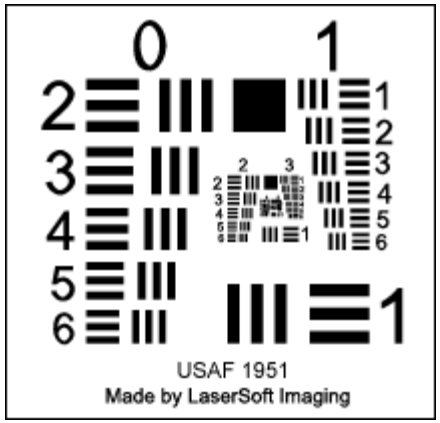
Contrast

In earlier times optical systems were often tested by using as the object a target like USAF 1951 in figure below:

[*this is defined by US Air Force MIL-STD-150A standard of 1951*]

USAF 1951 test charts:





These targets have high contrast.

[*Namely, on the right target, lines are a deep black on a white background*]

Contrast

If pattern is repetitive, such as a series of dark bars on a bright Background, the constrast modulation is defined as:

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

I_{max} = maximum intensity level (of bright value)

I_{min} = minimum intensity level (of dark value)

M = 0.0 (worse resolution)

M <= 0.2 (bad resoltion)

M >= 0.5 (good for imaging system)

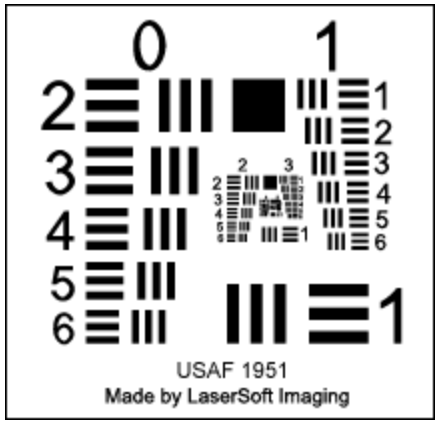
M >= 0.8 (high contrast)

M = 1.0 (perfect contrast)

Contrast Transfer

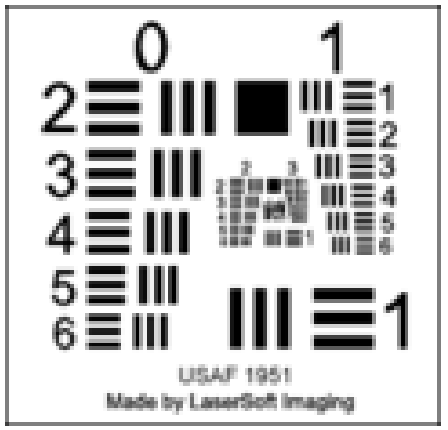
The contrast in object plane can be transferred to image plane by a transfer function called Modulation Transfer Function (MTF).

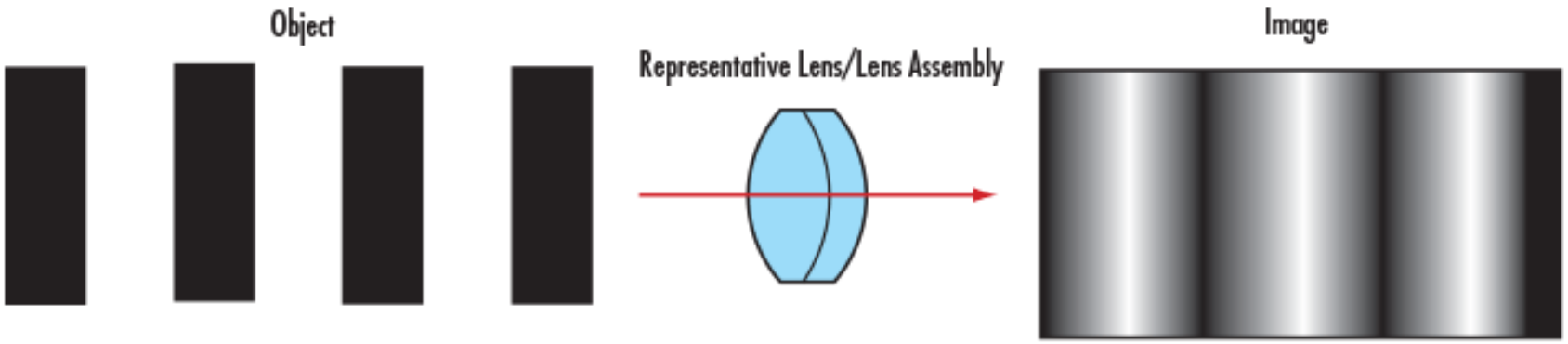
Object (M_{obj})



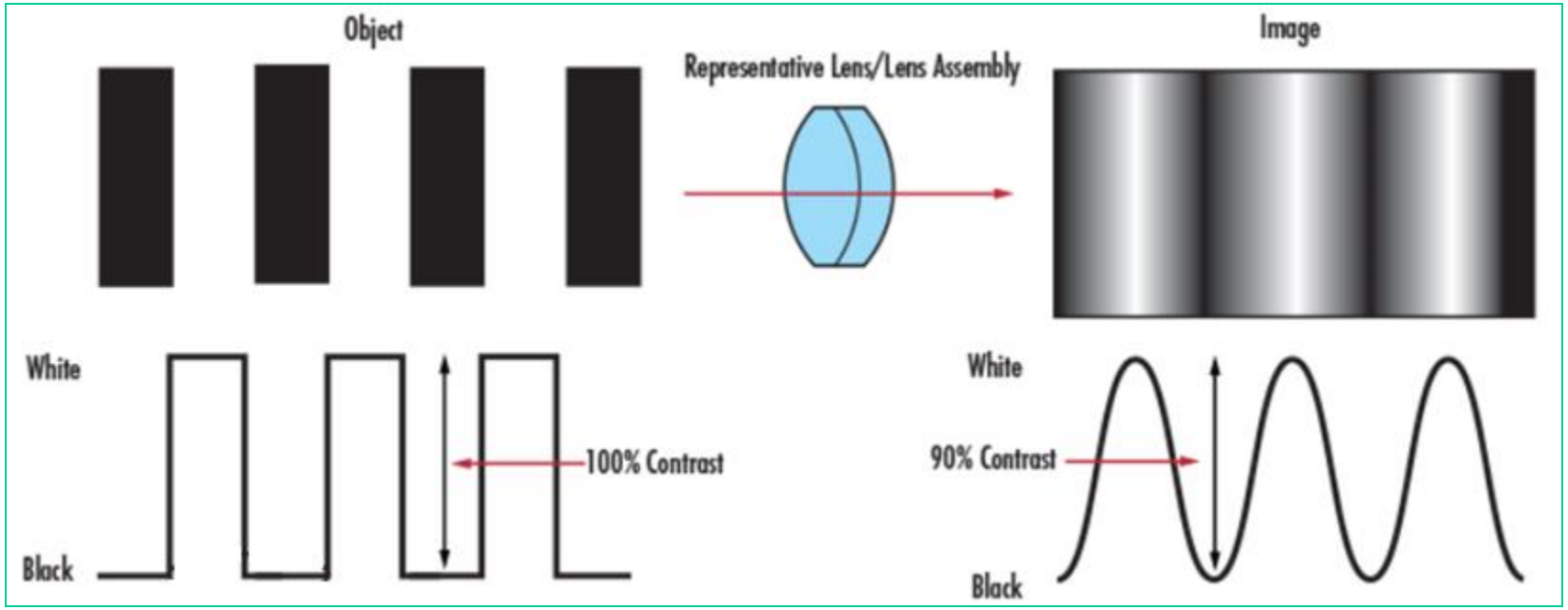
$$M_{img} = MTF \times M_{obj}$$

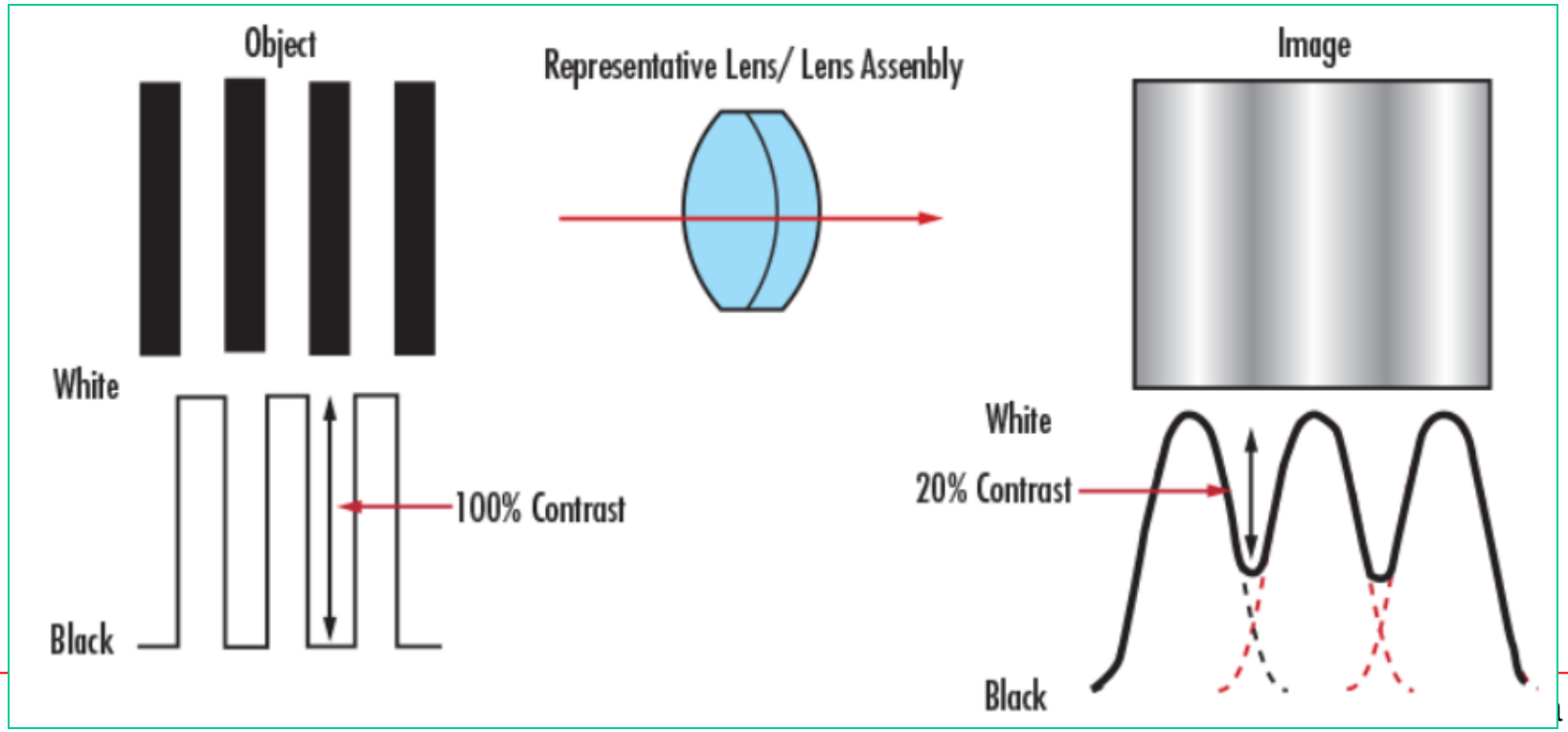
Image (M_{img})





Perfect line edges before (left) and after (right) passing through a low resolution imaging lens.


Object
Representative Lens/Lens Assembly
Image
White
100% Contrast
Black
White
90% Contrast
Black


Object
Representative Lens/ Lens Assembly
Image
White
100% Contrast
Black
White
20% Contrast
Black

MTF

MTF is a function of spatial frequency (*R*).

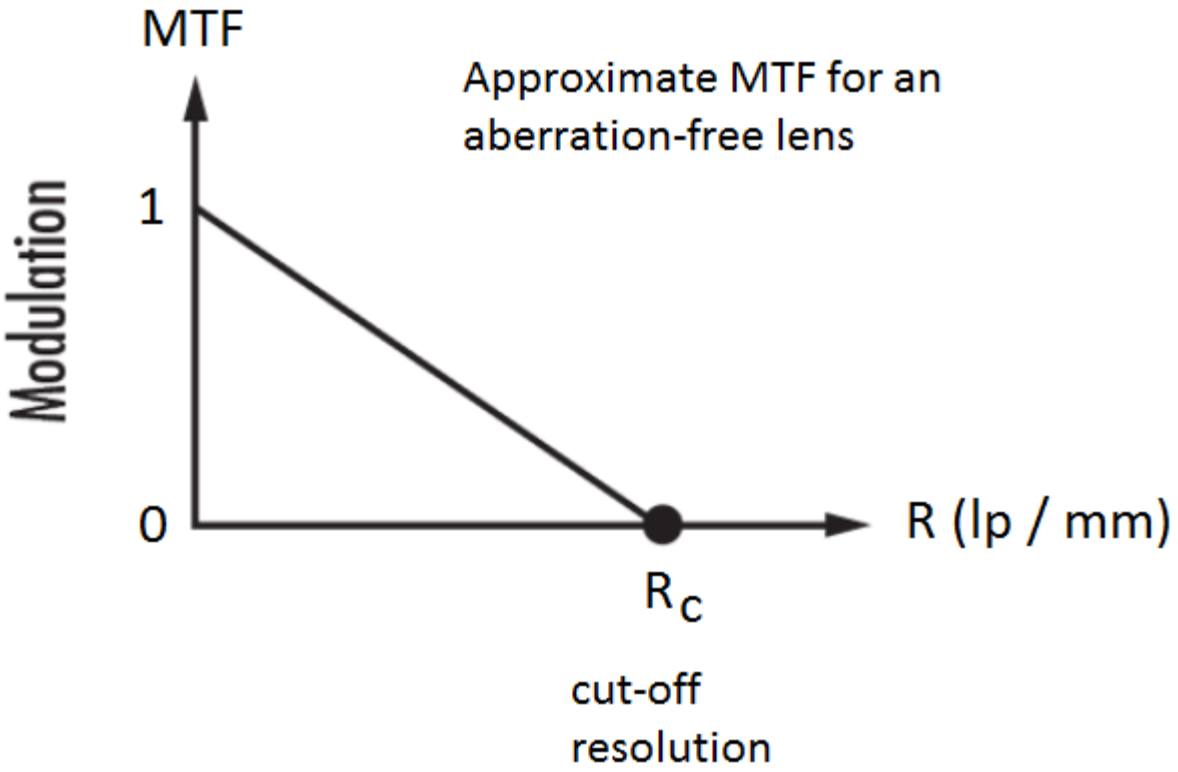
$$\text{MTF} = f(R)$$

R = number of lines within a given length.

Usually we used *lines / mm* or *line pairs / mm* unit.

The cut-off resolution can be found by:

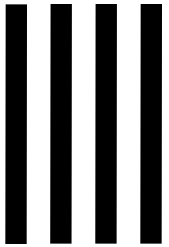
$$R_c = \frac{1}{\lambda \times (f/\#)}$$



EXAMPLE

A target contains a series of black lines, each line 8 mm wide and seperated from the next line by an interval also 8 mm. What is the spatial frequency?

SOLUTION



$$R = \frac{1 \text{ line pair}}{16 \text{ mm}} = 0.0625 \text{ mm}^{-1}$$

MTF and OTF

Contrast in image can be calculated from:

$$M_{img} = MTF \times M_{obj}$$

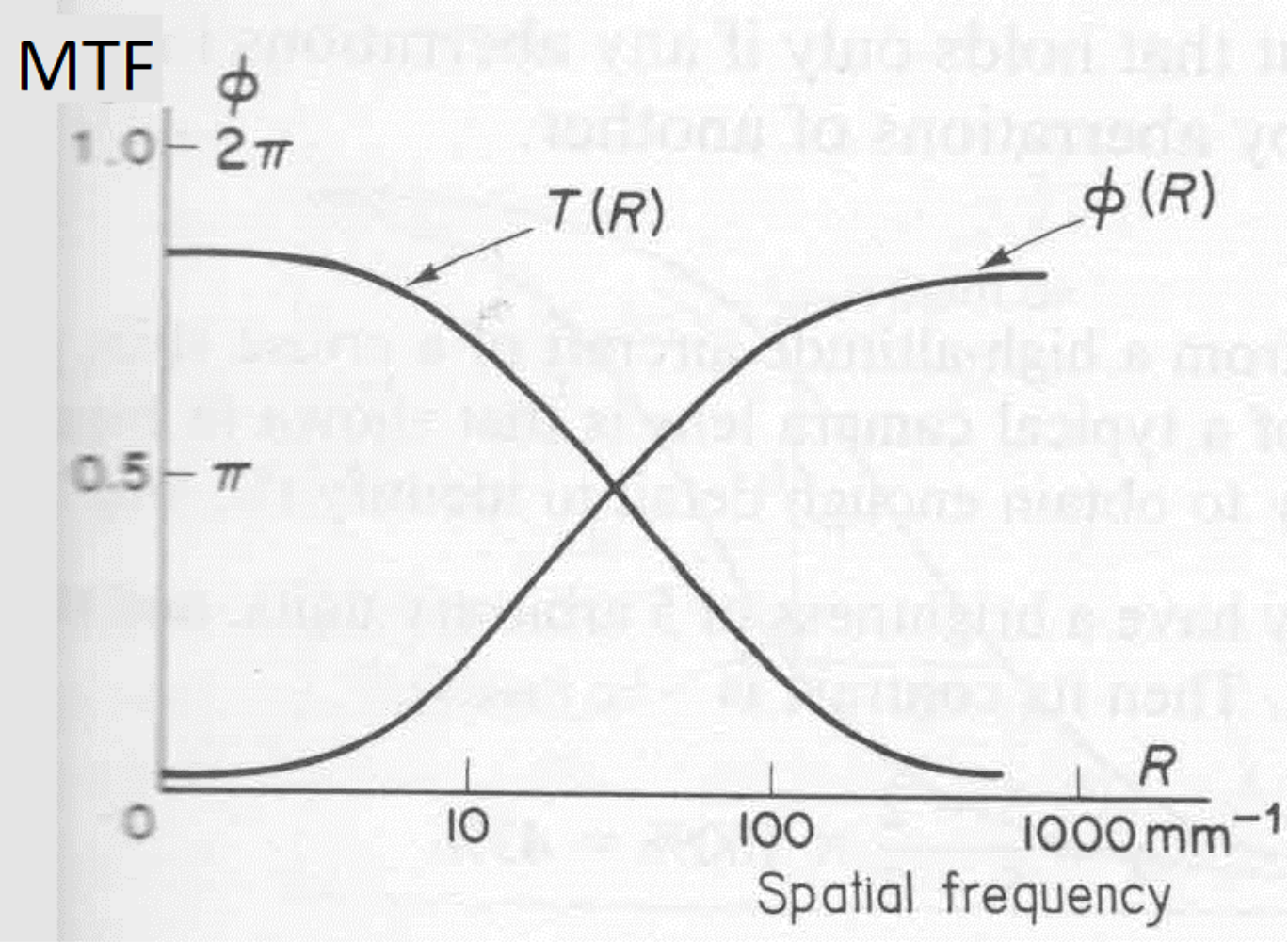
Due to aberrations such as coma and distortion,
or due to turbulent atmosphere, some image detail may change with respect to their nominal positions. That is called change of phase. Phase shift (Φ) is a also function of R, $\Phi = f(R)$.
Hence, in addition to MTF we have also PTF.

We define Optical Transfer Function as follows:

$$OTF = MTF \times \exp(i\phi)$$

Modulus of OTF = MTF (since |OTFxOTF*| = MTF)

MTF and OTF



An example MTF and phase transfer factor for a camera as a function of spatial-frequency, *R*.

EXAMPLE

Consider photographs are taken from a high-altitude aircraft of a cruise ship. Assume that the MTF = 0.8 (typical camera lens).

Let I_{max} = 5 and I_{min} = 2.

Find M_{obj} and M_{img} so that ship's image is 0.5 mm wide in image plane.

SOLUTION

Constrast at object plane:

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{5-2}{5+2} = 0.43 = 43\%$$

Since width of the ship is desired to be 0.5 mm, resolution is $R = 1\,\mathrm{mm}^{-1}$ *and MTF = 0.8. Therefore:*

$$M_{img} = MTF \times M_{obj} = 0.8 \times 0.43 = 0.34$$

This is enough for visual observations.

EXAMPLE

What is the power of a lens that has an angular resolution of 0.05^o and resolves details in the focal plane that have a spatial resolution of $7\ mm^{-1}$?

SOLUTION

$$\theta_{\min} = 0.05^o = 0.00087266\ \text{rad}$$

.

$$f = \frac{1}{\theta_{\min} R} = \frac{1}{(0.00087266)(7)} = 163.70\ \text{mm} = 0.1637\ \text{m}$$

$$P = \frac{1}{f} = 6.1\ \text{D}$$

EXAMPLE

Consider a Cassegrain Telescope is used take photos (in visible region) from a satellite which is at 600 km altitude. We want to resolve 3 m on Earth surface using this telescope. Assume that λ = 500 nm.

(a) What is the angular resolution of the telescope?

(b) What is the minimum diameter of the primary mirror?

(c) What is focal length required to resolve *R* = 100 lp / mm in image plane?

(d) What is the cut-off resolution?

(e) What is the value of MTF at *R* = 50 lp / mm?

SOLUTION

(a) $\theta_{\min} = (3/600000) = 5.0\times10^{-6}$ rad

(b) $\theta_{\min} = 1.22\lambda / D = (1.22)(0.5\times10^{-6}) / D = 5.0\times10^{-6}$ rad $\rightarrow$ $D = 12.2$ cm

(c) $f = 1/(\theta_{\min} R) = 2.0$ m

(d) $R_c = 1/(\lambda[f/\#]) = 122$ mm^{-1} *(e) MTF ~ 0.4 (use zemax)*

MTF in Zemax : Analyze > MTF

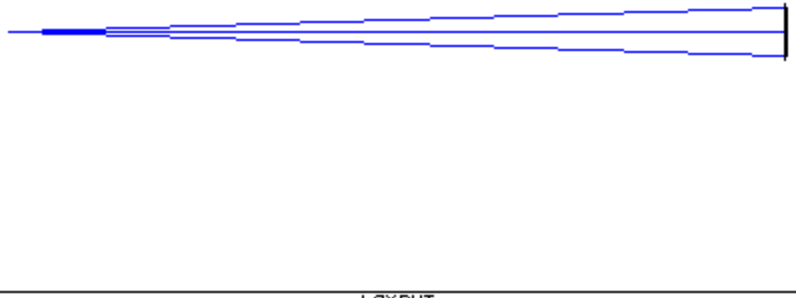
Lens Data Editor

Edit Solves View Help

Surf:Type		Comment	Radius	Thickness		Glass	Semi-Diameter	Conic	Par 0(u
OBJ	Standard		Infinity	Infinity			0.000	0.000	
STO	Standard		Infinity	0.000			61.000	0.000	
2	Standard		-4000.000	-2000.000	M	MIRROR	61.000	-1.000	
IMA	Standard		Infinity	-			7.105E-015	0.000	

1: Layout Z = -353.57489, Y = -70.465629

Update Settings Print Window Text Zoom



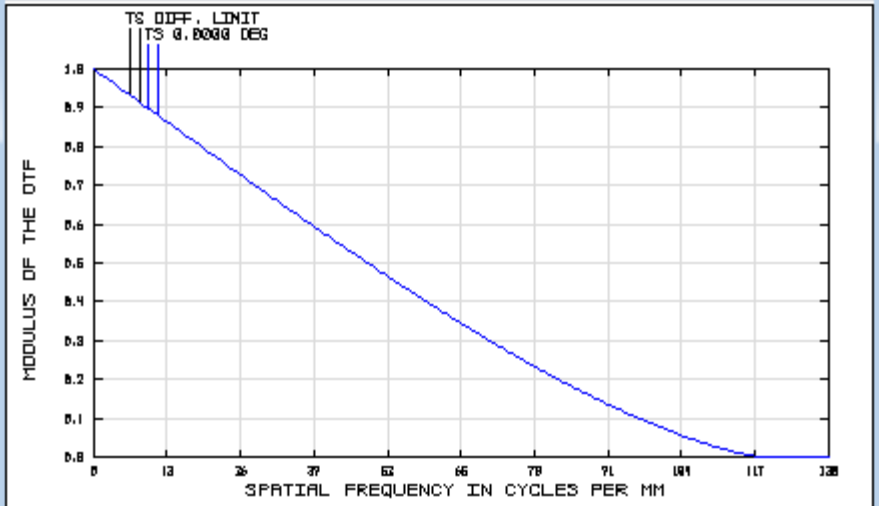
LAYOUT

FRI MAY 3 2019
TOTAL AXIAL LENGTH: 2000.00000 MM

LENS.ZMX
CONFIGURATION 1 OF 1

2: FFT MTF Freq = 83.36, MTF = 0.6139

Update Settings Print Window Text Zoom



POLYCHROMATIC DIFFRACTION MTF

FRI MAY 3 2019
DATA FOR 0.5000 TO 0.5000 µm.
SURFACE: IMAGE

LENS.ZMX
CONFIGURATION 1 OF 1

Measuring MTF using MATLAB

You need to use functions implemented in image processing tool:

imread()	open an image file
imshow()	display an image file
size()	size of an image
imresize()	resize an image
rgb2gray()	convert rgb image to grayscale
im2bw()	convert an image to BlackWhite
imhist()	histogram of the image
histeq()	histogram equilization
imwrite()	save image
imcomplement()	comlement of an image
imadd()	add a value to each pixel
imrotate()	rotate an image
imcrop()	crop an image
edge()	edge detection for an image
bwarea()	return area (number of pixels) for a given region