



Original research article

Calculating the modulation transfer function of an optical imaging system incorporating a digital camera from slanted-edge images captured under variable illumination levels: Fourier transforms application using MATLAB



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ABSTRACT

In this paper, a method for evaluating the modulation transfer function (MTF) of an optical system from the image of a slanted-edge target projected using an off-axis Newtonian collimator and recorded using a digital camera is presented. Mathematical derivation for the MTF function in terms of the line spread function employing Fourier transforms and convolution theory is provided. Furthermore, MATLAB[®] script for calculating the MTF plot from the captured slanted-edge images is given. As an illustration, MTF plots computed for slanted-edge images captured under 100, 500 and 1000 lx illumination levels are discussed.

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1. Introduction

Modulation transfer function (MTF) is the most fundamental metric for characterizing the performance of an optical imaging system [1–5]. MTF measurements are widely used for controlling the quality of optical systems and lenses in manufacturing. An MTF plot provides information on the optical system ability to truthfully transfer the spatial frequency content from the object scene to the image plane. An MTF plot is expressed in terms of spatial frequency (lines/mm or cycles/mm) on the x-axis and contrast (or modulation) on the y-axis. Optical imaging systems, which can achieve high MTF values, form high-quality images whereas optical systems with low MTF values form poor-quality images (blurred and degraded images). Several methods for measuring the MTF of an optical imaging system have been reported in literature. Some of these methods include measuring the MTF using bar targets [6], slanted-edge target [7–13], point-spread function [12,13], random targets [14] and band-limited laser speckle [15] images. Among these measurement methods, the slanted-edge and the point-spread function are the methods commonly used in industry. However, one of the major challenges of using the MTF point-spread function technique is the requirement for having the size of the point source to be smaller than the calculated point spread-function. This leads to a significant reduction in the flux reaching the image scene and consequently makes it difficult for the imager to detect the signal.

In this article, a method for measuring the MTF function for an optical system incorporating a digital camera from slanted-edge images is discussed. A MATLAB[®] script for calculating the MTF from slanted-edge images employing Fourier transforms

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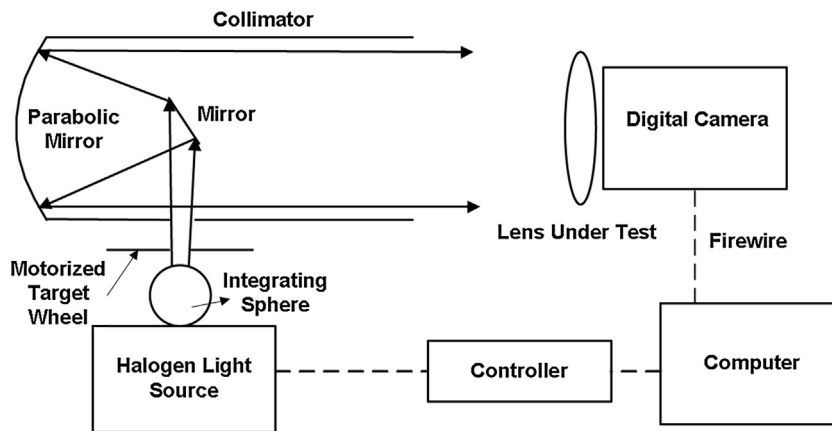


Fig. 1. Block diagram showing the experimental setup for the off-axis Newtonian all reflective collimator.

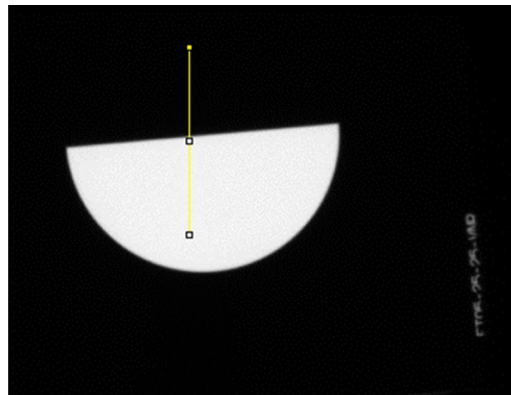


Fig. 2. Photo captured for a slanted-edge target at 500 lx illumination at a distance equivalent to approximately 700 m from the camera system. The yellow line shown in the picture represents the profile of the digitized edge function. The slanted-edge is tilted by approximately 7° from the horizontal line. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and convolution theory is presented. Finally, MTF plots computed for slanted-edge target measured under 100, 500, 1000 lx illumination conditions are discussed.

2. Measurement system

The MTF measurement system is shown schematically in Fig. 1. The system consists of an off-axis Newtonian collimator (clear aperture 30.5 cm; focal length = 152.4 cm; OAP surface accuracy $\sim \lambda/8$ P-V at 632 nm) equipped with a motorized target wheel mounted on a micrometer driven stage, an integrating sphere and a visible halogen lamp light source for illuminating uniformly the slanted-edge target under test. The target range can be simulated between 100 m and infinity by adjusting the micrometer stage. The digital camera (CCD; 1.45 Mpixel; pixel size: $4.65 \mu\text{m}$; 14 bit depth; $1/2''$ format) used in this test is equipped with 87 mm focal length lens and placed approximately 100 cm from the collimator exit end. A halogen lamp power supply controller connected to the halogen lamp allows adjusting the targets illumination level from 10^{-5} to 10^3 Lux.

Fig. 2 depicts a slanted-edge image captured with the digital camera system at 500 lx illumination level with the target range set to 700 m. A plot for the edge function profile (along the yellow line shown in Fig. 1) was generated from the image and saved into a data file utilizing ImageJ [16]. Each of the generated data files was read using a MATLAB® [17] script (Table 1) from which the Fourier transform was computed and plotted.

3. Theoretical background [1–13]

An edge response function (such as a slanted-edge) can be described mathematically as:

$$S(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \quad (1).$$

Table 1

MATLAB® [17] script used for calculating the slanted-edge MTF.

```

% Read slanted-edge function from Excel data sheet
values= xlsread ('example_data.xlsx');
figure (1); plot(values); legend ('Fig. 1-Slanted Edge')
xlabel ('Pixel Number'); ylabel('Gray value');dummyFig = figure(1);
% Calculate the numerical derivative for the edge function
yy= gradient(values) ;
figure (2);hold on; plot(yy); legend ('Fig. 2-Gradient-LSF')
xlabel ('Pixel Number'); ylabel ('Amplitude');dummyFig = figure(2);
% Calculate MTF
g= fft(yy);
g= abs(g); % take abs for fft(yy)
figure (3); hold on; plot(g);
legend ('Fig. 3-MTF'); xlabel ('Pixel Number'); ylabel('Amplitude');
dummyFig= figure(3);
% Smooth MTF data
gg= smooth(g) ; % smooth data
figure (4); hold on; plot(gg);
legend ('Fig. 4-MTF Smoothed'); xlabel ('Pixel Number'); ylabel ('Amplitude');
dummyFig = figure(4);
% Select the first 100 points and plot data
ggm= gg(1:100);
figure (5); hold on; plot(ggm);
legend('Fig. 5-MTF Smoothed')
xlabel ('Spatial Frequency'); ylabel('Modulation'); dummyFig = figure(5);
% Normalize MTF
ggmax= max(ggm);
ggmax= ggm/ggmax; % Normalize data
figure(6); hold on; plot(ggmax);
xlabel ('Spatial Frequency'); ylabel('Modulation');legend ('Fig. 6-MTF Normalized')
dummyFig = figure(6);
% Output results to Excel data sheet
filename = 'testdata.xlsx'; xlswrite (filename,ggmax)

```

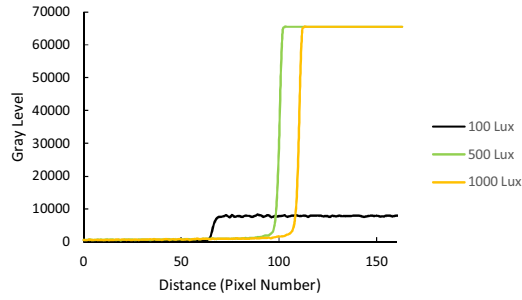


Fig. 3. Plot showing the edge functions captured under 100, 500 and 1000 lx illumination levels.

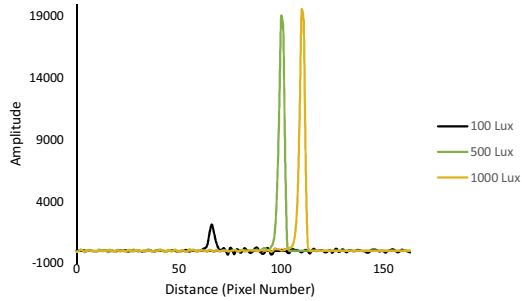


Fig. 4. Plot depicting the calculated Fourier transforms (FT) for the edge functions shown in Fig. 3.

When the edge response function is imaged by an aberration-free (perfect) optical system (the optical system consists of a lens, CCD or CMOS imager and a display) it will not degrade the quality of the image recorded by the imager. Furthermore, when the edge function is imaged by a linear and invariant optical system the output of the optical system $O(x)$ is equal to the line spread function (LSF) convolved with the optical system impulse response $S(x)$ and can be expressed as:

$$O(x) = S(x) \times LSF(x) = \int_{-\infty}^{+\infty} LSF(\alpha) \times S(x - \alpha) d\alpha \tag{2}$$

For $(x - \alpha) < 0$ the step function $S(x) = 0$ and $S(x) = 1$ for all other values. As a result, the $ESF(x)$ can be written as

$$ESF(x) = \int_x^{\infty} 1 \cdot LSF(\alpha) d\alpha \tag{3}$$

Thus, the $ESF(x)$ derivative can be written as:

$$LSF(x) = \frac{dESF(x)}{dx} \tag{4}$$

Now, an expression for the MTF in terms of the LSF can be written as:

$$MTF(v) = \int_{-\infty}^{+\infty} LSF(x) \times e^{-i2\pi vx} dx \tag{5}$$

Here, it is worth noting that the $MTF = \|OTF\|$ where the magnitude of the OTF is the optical system transfer function. OTF is defined as the optical system normalized frequency response for a given system $F\#$ and λ . Further, the MTF is normalized to one at a spatial frequency equal to zero.

Using convolution theory and Fourier transforms the measured overall MTF for the optical system can be expressed as:

$$MTF_{opticalsystem} = MTF_{lens} \times MTF_{camera} \times MTF_{display} \tag{6}$$

4. Results and discussion

Fig. 3 illustrates the edge response profiles generated from slanted-edge target images captured under 100, 500 and 1000 lx illumination levels. The shift in the location of the edge is due to a slight shift in the spatial location of the slanted-edge target. Using the MATLAB® script outlined in the previous section, Fourier transforms were computed for the three edge response profiles (i.e., 100, 500 and 1000 lx) as depicted in Fig. 4. Clearly, the amplitude of the calculated Fourier transform increased with the increase in the illumination level. Finally, the MTF function was computed for each illumination level and plotted as shown in Fig. 5. The calculated 500 and 1000 lx MTF plots were nearly identical. However, the MTF plot computed

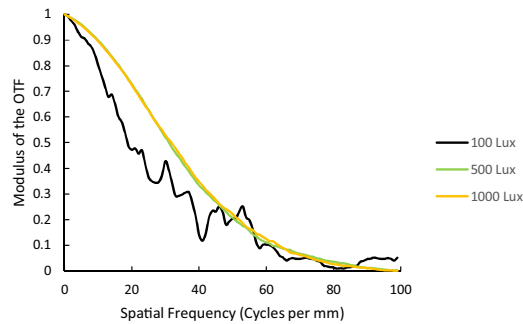


Fig. 5. Plot showing the calculated uncorrected MTF plots for the slant-edge target under 100, 500 and 1000 lx illumination levels using a digital camera equipped with 87 mm focal length lens.

for the 100 lx was lower in magnitude and had higher amplitude variations caused by the detector noise. The obtained results indicate that the MTF measurements were impacted by the illumination level and the best MTF measurements were achieved at higher illumination level. Here, it is worth noting that the computed MTF plots were uncorrected. Cunningham and Fenster in reference [18] argue that because numerical differentiation is used to calculate the line spread function from the edge spread function it is necessary to correct the magnitude of the MTF values by multiplying the uncorrected MTF plot by $1/\text{sinc}(\pi f/2f_c)$, where f_c is one-half of the sampling frequency.

5. Conclusions

This paper presented a method for calculating the MTF for an optical imaging system incorporating a digital camera from slanted-edge images under variable levels of illumination. From the obtained results the following conclusions can be drawn:

1. The amplitude of the calculated LSF increased with the increase in the illumination level.
2. The smoothness of the MTF plot was impacted by the illumination level of the slanted-edge target. The variability in the amplitude of MTF plot increased with the decrease in the illumination level.
3. At the 1000 and 500 lx illumination conditions the slanted-edge MTF plots were nearly identical.

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