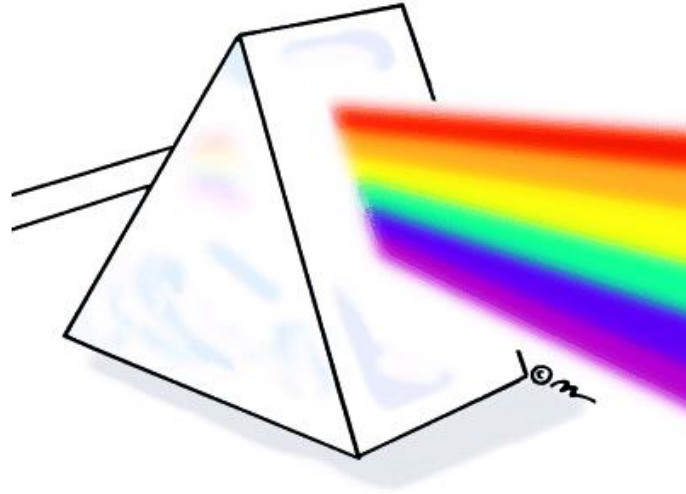


# OPAC203 PHYSICAL and MODERN OPTICS LABORATORY MANUAL



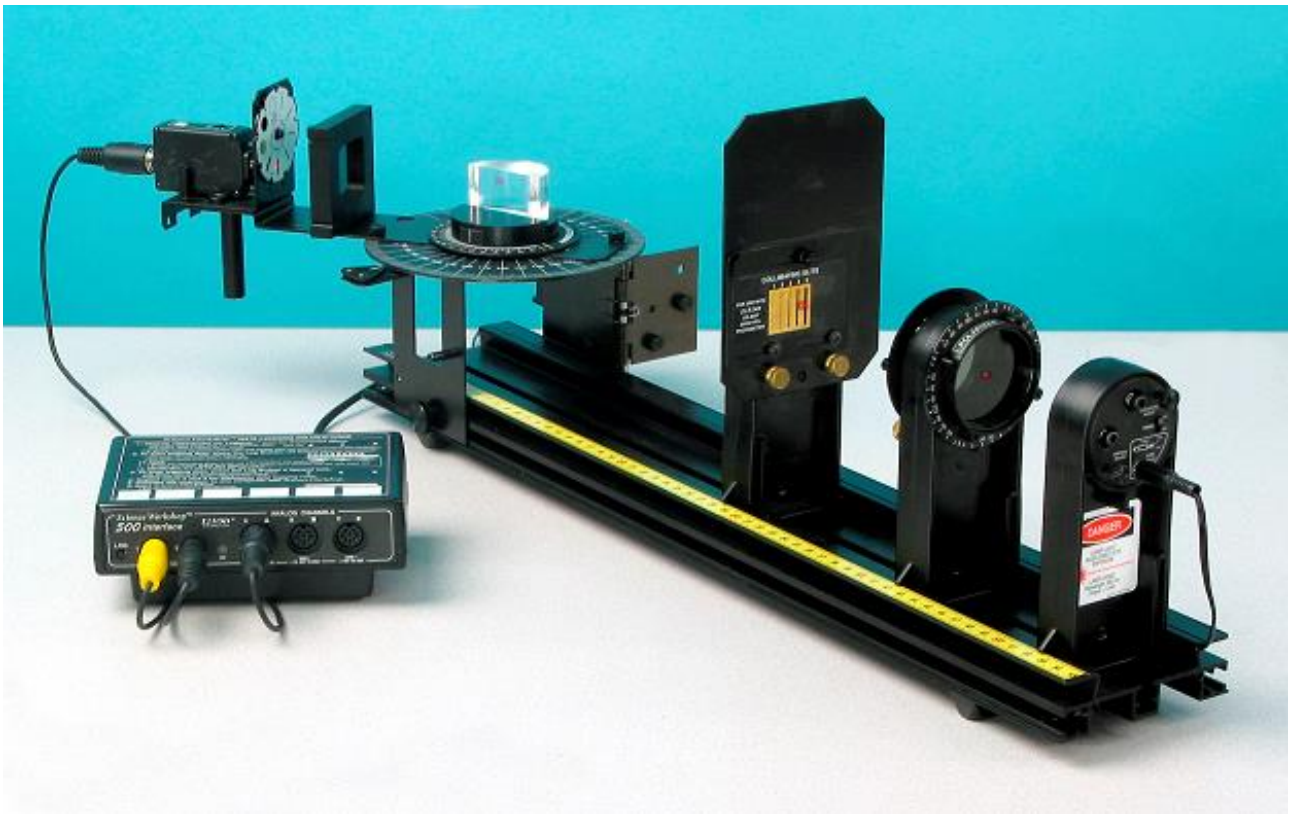
**Gaziantep University**  
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**Oct 2018**

*Some parts of this manual are extracted from [www.pasco.com](http://www.pasco.com).*



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\* Performing these experiments depend on the duration of semester.

# Preface

This is the laboratory manual for the second year lab course named Physical and Modern Optics Laboratory in Optical & Acoustical Engineering Department at Gaziantep University. These notes are compiled after experiences of 3 years in the Optics Laboratory in the department.

This manual includes 8 experiments (for one semester) mostly based on the wave optics. (2 experiments may be included later). We shall also discuss some elementary applications to a number of practical devices in the lab. It is strongly suggested that the students and Lab assistants should read the manual carefully before performing the experiments.

At the end of each experiment, student groups must prepare a report (2-3 pages) within one week and present it to the Lab Assistant. An example written report is given in Appendix.

Ahmet Bingül  
Oct, 2018

# EXPERIMENT 1

## INVERSE SQUARE LAW and LAMBERT'S COSINE LAW

### PURPOSE

You will perform two experiments. The aim of first one is to determine the light intensity of a point-like source (e.g. LED) as a function of distance. The aim of the second one is to verify the Lambert's Cosine law for point source.

### EQUIPMENT

Optics Bench, High Sensitivity Light Sensor, Rotary Motion Sensor  
LED, or Basic Optics Light Source, Data Studio or Capstone Software

### THEORY

If light spreads out in all directions, as it does from a point light source, the intensity at a certain distance from the source depends on the area over which the light is distributed. For example, for a sphere of radius  $r$ , the light from the point source will fall on a surface area of  $A = 4\pi r^2$ . The intensity is defined by the total power output of the source divided by the area over which the light is spread. That is:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{k}{r^2} \quad (1)$$

where

$I$  = Light intensity, Watt/m<sup>2</sup> (Lumen/m<sup>2</sup>)

$P$  = Light power in Watt (or Lumen)

$k$  = Numerical constant

Equation (1) is known as the *inverse square law* which states that the magnitude of the light intensity is inversely proportional to the distance from a point source. The relation between spherical geometry and the inverse-square law of radiation from a point source is illustrated in Figure 1.

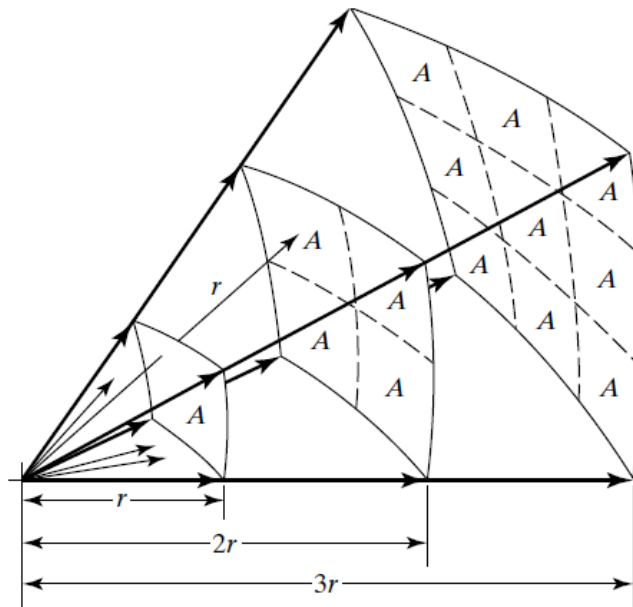
If the radiation direction makes an angle  $\theta$  with the normal of the irradiated (illuminated) surface, as in Figure 2, then the intensity is given by:

$$I = \frac{k}{r^2} \cos(\theta) \quad (2)$$

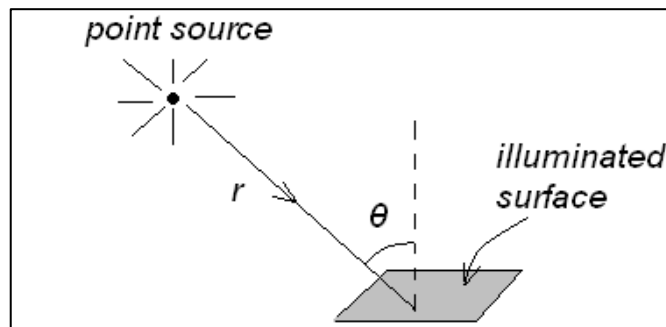
For a fixed (or constant) distance  $r$  from the point source one can write:

$$I = I_{max} \cos(\theta) \quad (3)$$

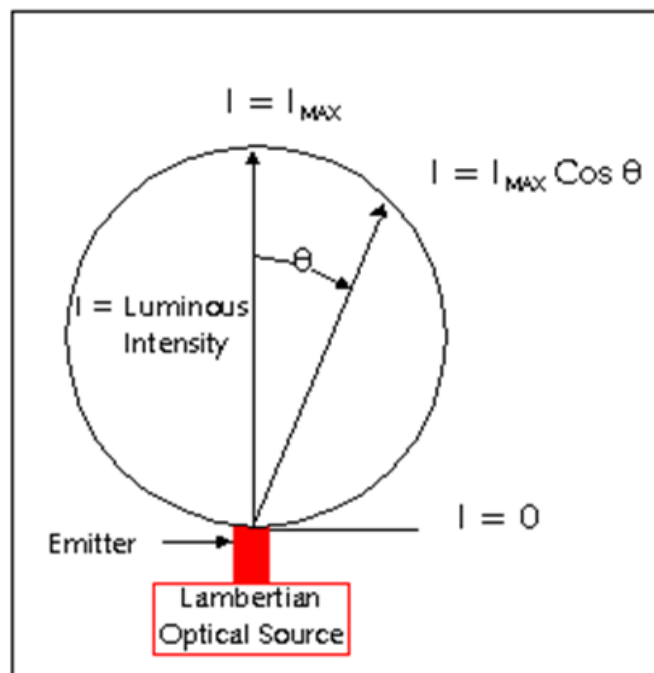
where  $I_{max} = k/r^2$ . Equation (3) is called the *Lambert's Cosine Law* which says that the intensity observed from an *ideal* point source is proportional to the  $\cos(\theta)$ , Figure 3.



**Figure 1:** Illustration of the inverse-square law. The flux leaving a point source within any solid angle is distributed over increasingly larger areas, producing an irradiance that decreases inversely with the square of the distance. It is clear that the origin of the inverse square law is a simple geometry.



**Figure 2:** Illumination of a surface



**Figure 3:** Lambert's cosine law

## PROCEDURE

### Part 1 Intensity vs Distance for a Point Source

The sensitivity of the Light Sensor must be adjusted so it does not max out and so it is not set so low that the signal is poor.

1. Set the Basic Optics Light Source (or a power LED) so the point light source is at 0.0 cm. The center of the point light source is indicated by the edge of the notch in the bottom of the light source bracket (see arrow on the bottom of the bracket). Set the front of the Light Sensor mask 10 cm from the center of the point light source. Note that you must sight down the front of the Light Sensor mask to see where it lines up with the track measuring tape (see Figure 4).

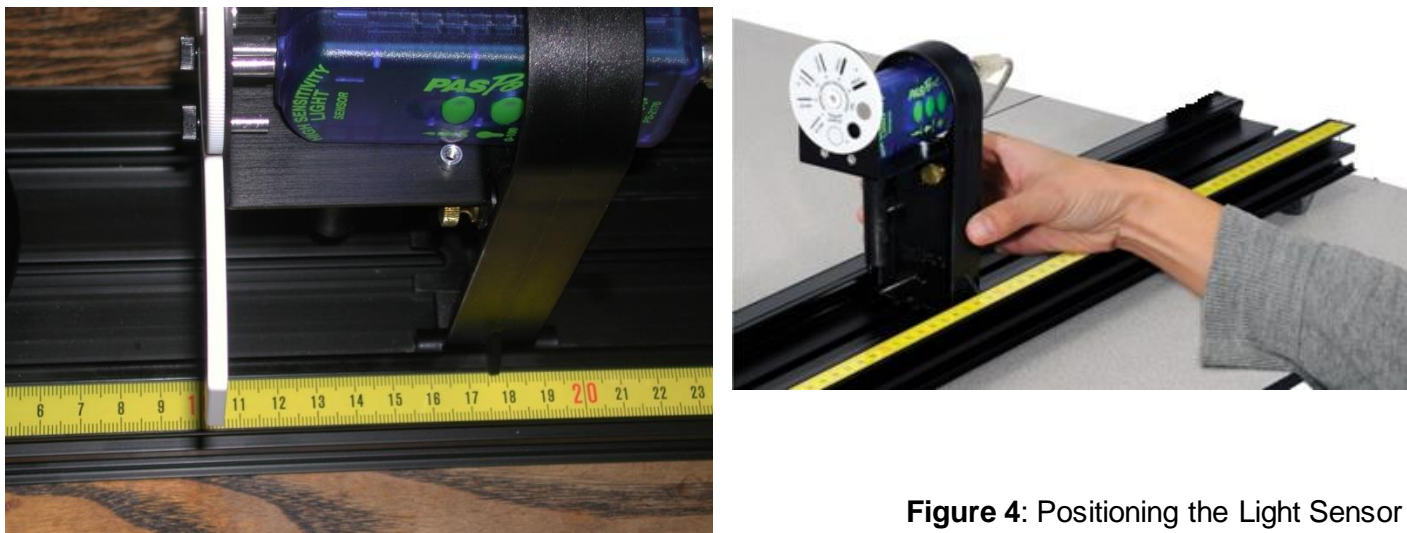


Figure 4: Positioning the Light Sensor

2. Turn off most of the room lights. We want the relative intensity levels to not vary by more than 1-2 percent as the Light Sensor is moved down the track. Start with the Light Sensor 25 cm from the center of the light source, click RECORD to collect (at least 10 seconds) data. Click STOP. Save data to a file called opac203\_lab1\_sl\_25cm.txt. Delete Last Run button.
3. Repeat the Case 1 for the sensor positions [35 cm, ... , 75 cm] from the light source. Note that you should keep your hand and body behind the screen to avoid reflecting light onto the screen.

## DATA ANALYSIS

You have a set of data files:

```
opac203_lab1_isl_25cm.txt,    opac203_lab1_isl_55cm.txt
opac203_lab1_isl_35cm.txt,    opac203_lab1_isl_65cm.txt
opac203_lab1_isl_45cm.txt,    opac203_lab1_isl_75cm.txt
```

Each file has two columns of data such that the first column is time in seconds and second one is relative light intensity in percent. To do analysis you can use one of the software: Octave, MATLAB or MS Excel.

## QUESTIONS

1. Evaluate the arithmetic mean of the relative light intensity,  $\langle I \rangle$ , and corresponding standard deviations ( $\sigma$ ) using data files and record your results to Table 1.
2. Plot the Relative Mean Intensity  $\langle I \rangle$  versus Position ( $x$ ) graph.
3. Fit the data points in the Table 1 to the functions in Table 2 to find the coefficients  $a$ ,  $b$ ,  $c$  and  $n$ .
4. Record the values of fitting parameters, their associated error and goodness of each fit in Table 2.
5. Explain the physical meaning of these fitting coefficients.
6. What is your conclusion?

**Table 1:** The experimental data

Data file name	Distance from the source x, (m)	Mean Intensity <l>	Standard Deviation $\sigma$
opac203_lab1_isl_25cm.txt	0.25		
opac203_lab1_isl_35cm.txt	0.35		
opac203_lab1_isl_45cm.txt	0.45		
opac203_lab1_isl_55cm.txt	0.55		
opac203_lab1_isl_65cm.txt	0.65		
opac203_lab1_isl_75cm.txt	0.75		

**Table 2:** Fitting procedure

Fit Equation 1	Fit Equation 2	Fit Equation 3
$I = \frac{a}{x^n}$	$I = \frac{a}{(x+b)^n}$	$I = \frac{a}{(x+b)^n} + c$
a = n =	a = b = n =	a = b = c = n =
Goodness of the fit: R2 = or Chi2/dof =	Goodness of the fit: R2 = or Chi2/dof =	Goodness of the fit: R2 = or Chi2/dof =



## Part 2 Intensity vs Angle for a point source (LED) [optional]

The setup is shown in Figure 5. Use rotary motion sensor to obtain angular measurements and High sensitive Light Sensor for the relative light intensity measurements. Turn off most of the room lights. We want the Relative Intensity levels to not vary by more than a few percent as the Light Sensor is moved down the track. Click RECORD to acquire intensity vs angle data. Save your data to a file called opac203\_lab1\_lambert.txt Click STOP. Save data to a file. Press Delete Last Run button.

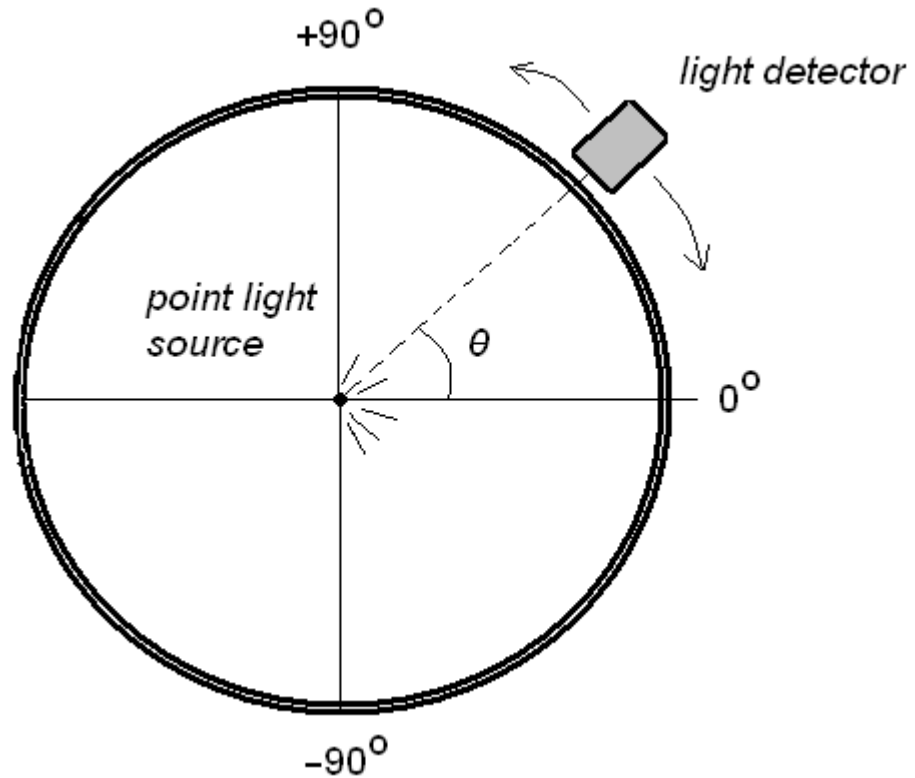


Figure 5: Setup for verification of a Lambert's Cosine Law

## QUESTIONS

1. Read the data from the text file (opac203\_lab1\_lambert.txt) produced by the PASCO software. Plot  $I - \theta$  graph in cartesian coordinate system and verify if the source obey Lambert's law Fit the data to the function  $I = a \cos(\theta) + b$  to obtain the values of the free parameters  $a$  and  $b$  and their associated errors from fitting procedure.
2. What are the physical significances of the free parameters  $a$  and  $b$ ?
3. Plot  $I - \theta$  graph in polar coordinate system and verify if the source obeys Lambert's cosine law. (For a Lambertian source, the shape will be circle in polar coordinates since  $I = \text{constant} \times \cos\theta$ ).
4. What is your overall conclusion?

## EXPERIMENT 2

### ABSORBANCE OF AN OPTICAL GLASS

#### PURPOSE

To measure reflectance, transmittance and absorbance of a transparent glass by using laser diodes.

#### EQUIPMENT

Basic Optics Bench, Red (650 nm) and Green (532 nm) Diode Laser

Glass(es) having various thickness, High Sensitivity Light Sensor, Data Studio or Capstone Software

#### THEORY

When light proceeds from one medium into another (e.g. from air to a glass), several things may happen. Some of the light radiation may be transmitted through the medium, some will be absorbed, some will be reflected at the interface between the two media or some will be scattered though the medium. Ignoring the scattering effects, the intensity  $I_0$  of the beam which is incident to the surface of the solid medium must equal the sum of the intensities of the transmitted, absorbed, and reflected beams, denoted as  $I_T$ ,  $I_A$ , and  $I_R$  respectively, Figure 1. From the conservation of energy one can write:

$$I_R + I_A + I_T = I_0 \quad (1)$$

Radiation intensity, expressed in watts per square meter, corresponds to the energy being transmitted per unit of time across a unit area that is perpendicular to the direction of propagation. These intensities are measurable as shown in Figure 2.1. Alternatively, Equation (1) can be written as:

$$T + A + R = 1 \quad (2)$$

where

$T = I_T/I_0 =$  transmissivity = transmittance

$A = I_A/I_0 =$  absorptivity

$R = I_R/I_0 =$  reflectivity

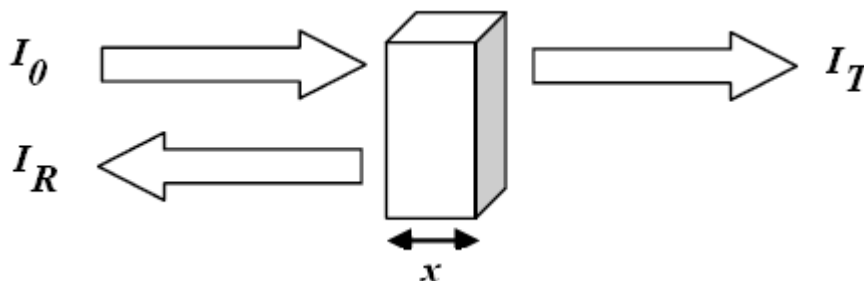


Figure 1: Measurable quantities in the experiment

Materials that are capable of transmitting light with relatively little absorption and reflection are *transparent*. Materials that are impervious to the transmission of visible light are termed *opaque*. In the lab, we will try to measure  $T$ ,  $A$ , and  $R$  for the transparent materials by using laser diodes.

## Reflectance

The reflectivity or reflectance,  $R$ , represents the fraction of the incident light that is reflected at the interface:

$$R = \frac{I_R}{I_0} \quad (3)$$

When the light is transmitted from a vacuum (or air) into solid and the light is normal (or perpendicular) to the interface then:

$$R = \left( \frac{n-1}{n+1} \right)^2 \quad (4)$$

where  $n$  is the index of refraction of the material. We will see the origin of Equation 4 later.

### Notes

- The higher the index of refraction of the solid, the greater is the reflectivity.
- Reflectivity is approximately 4-5% for typical silicate glasses.
- Reflectivity varies with wavelength.
- Reflection losses for lenses and other optical instruments are minimized significantly by coating the reflecting surface with very thin layers of dielectric materials such as magnesium fluoride ( $\text{MgF}_2$ ).

## Absorbance

Materials may be opaque or transparent to visible light. As far as we know, the light radiation is absorbed in non-metal materials by two basic mechanisms: electronic polarization and valence band-conduction band electron transitions (See lecture notes for details).

The intensity of the net absorbed radiation is dependent on the character of the medium as well as the path length. The intensity of the transmitted light decreases with distance  $x$  that the light traverses, Figure 2.

$$I_T = I_0 \exp(-\alpha x) \quad (5)$$

where

$\alpha$  = absorption coefficient (1/cm)

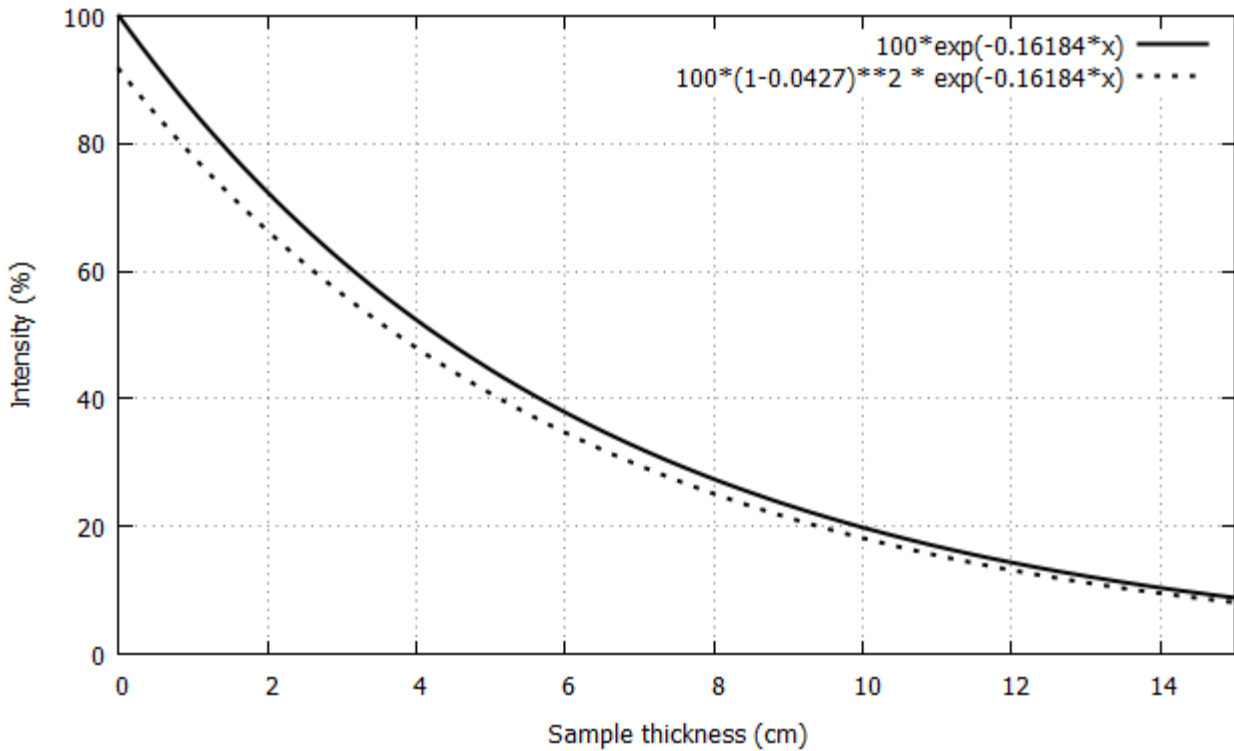
$x$  = sample thickness (cm)

Note that, Equation (5) is valid when the Fresnel reflections from the first and second surfaces are ignored. By including Fresnel reflections losses (from two surfaces at normal incidence), we have:

$$I_T = I_0(1 - R)^2 \exp(-\alpha x) \quad (6)$$

where  $R$  is the reflectivity defined in Equation (4).

The variation of intensity as a function of sample thickness ( $x$ ) for the soda lime glass at  $\lambda = 650$  nm ( $\alpha = 0.16184/\text{cm}$ ,  $n = 1.5210$  and  $R = 0.042715$ ), is demonstrated in Figure 2. The effect of including Fresnel reflections losses are also shown.



**Figure 2:** Intensity variation as a function of sample thickness for of soda lime glass at  $\lambda = 650$  nm. Solid line represents Equation 5 and dotted line represents Equation 6.

*Absorbance* is a logarithmic measure of the amount of light absorbed (at particular wavelength) as the light passes through a sample. When the Fresnel reflections are ignored it is defined as:

$$A' = \ln\left(\frac{I_0}{I_T}\right) = \ln\left(\frac{1}{T}\right) = \alpha x \quad (2.7)$$

### **Transmittance**

*Transmittance* is the fraction of incident light at a specified wavelength that passes through a sample. In other words, the transmittance of a sample is the ratio of the intensity of the light that has passed through the sample to the intensity of the light when it entered the sample as shown in Figure 1:

$$T = \frac{I_T}{I_0} \quad (8)$$

As a numerical example, consider the soda lime glass.

Let  $x = 2.0$  cm,  $\lambda = 650$  nm,  $\alpha = 0.16184/\text{cm}$  and  $n = 1.5210$  (see [refractiveindex.info](http://refractiveindex.info)) then:

Reflectance:	$R = (n-1)^2/(n+1)^2 = (1.5210-1.0000)^2 / (1.5210+1.0000)^2 = 0.0427 \approx \mathbf{4.3 \%}$
Transmittance:	$T = (1-R)^2 \exp(-\alpha x) = (1-0.0427)^2 \exp(-0.16184*2) = 0.6630 \approx \mathbf{66.3 \%}$
Absorptivity:	$A = 1 - T - R = 1 - 0.6630 - 0.0427 = 0.2943 \approx \mathbf{29.4 \%}$
Absorbance:	$A' = \ln(1/T) = \ln(1/0.6630) = \mathbf{0.4110}$

## PROCEDURE

Intensity measurements has to be done as described in inverse square lab. Data must be saved in suitable text files such as: opac203\_lab2\_sodalime\_I0.txt, opac203\_lab2\_sodalime\_IR.txt, opac203\_lab2\_sodalime\_IT.txt

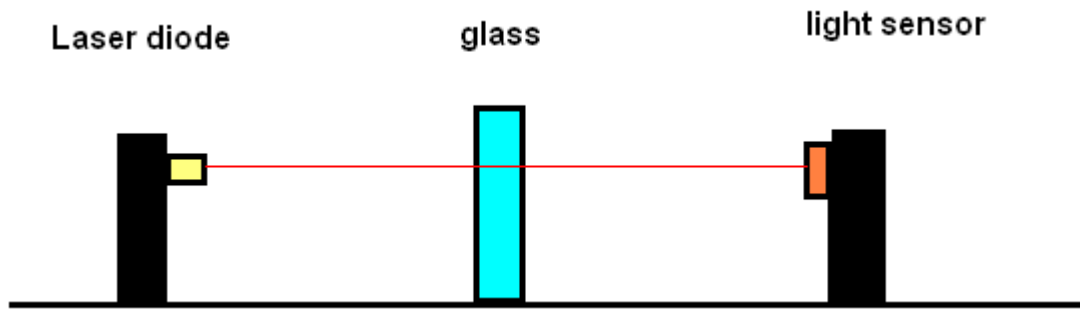


Figure 3: The setup

1. Turn on computer and red laser diode (650 nm).
2. Wait for 10 minutes until the laser diode becomes stable.
3. Measure the intensity ( $I_0$ ) the laser diode by using high sensitive light detector. (Collect at least 10 seconds data).
4. Place a slab of soda lime glass between laser diode and light sensor and measure the intensity of reflected ( $I_R$ ) and transmitted ( $I_T$ ) beams. (Again for each of them, collect at least 10 seconds data).
5. If possible, repeat above procedure for different size of glasses and the green laser (532 nm).

## QUESTIONS

1. Starting from Equation 5, derive Equation 6 by considering the effect reflection from both surfaces.
2. Explain in details the absorption mechanisms in optical region.
3. Using data files, compute mean value of  $I_0$ ,  $I_R$  and  $I_T$  and their corresponding standard deviations.
4. Evaluate the coefficients  $T$ ,  $A$ ,  $R$ ,  $\alpha$  and index of refraction ( $n$ ) and their associated errors.
5. Compare values of  $R$ ,  $\alpha$  and  $n$  values with the values in literature (e.g. refractiveindex.info) and compute the percentage differences.
6. What are the possible systematic errors in the experiments?
7. What is your overall conclusion?

## **EXPERIMENT 3 OPTICAL FILTERS**

### **PURPOSE**

To measure transmittance and absorbance of several optical filters as a function of wavelength

### **EQUIPMENT**

Some optical filters, spectro-photo-meter

### **THEORY**

Optical filters selectively transmits light having certain properties (often, a particular range of wavelengths), while blocking the remainder.

#### *Longpass filters*

A longpass (LP) filter is an optical interference or coloured glass filter that attenuates shorter wavelengths and transmits (passes) longer wavelengths over the active range of the target spectrum (ultraviolet, visible, or infrared). Longpass filters, which can have a very sharp slope (referred to as edge filters), are described by the cut-on wavelength at 50 percent of the peak transmission.

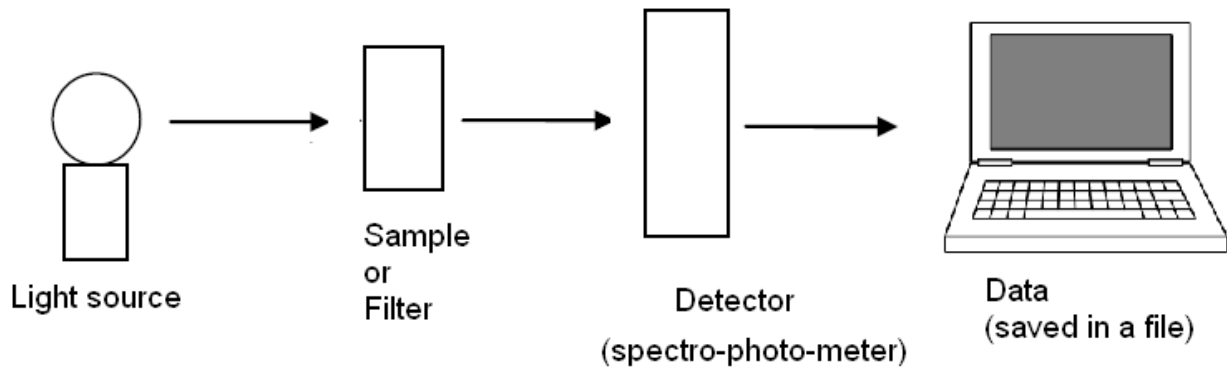
#### *Shortpass filters*

A shortpass (SP) filter is an optical interference or coloured glass filter that attenuates longer wavelengths and transmits (passes) shorter wavelengths over the active range of the target spectrum (usually the ultraviolet and visible region).

#### *Bandpass filters*

If we combine an LP filter and an SP filter we will get a Bandpass (BP) filter. These filters have usually lower transmittance values than SP and LP filters, and block all wavelengths outside of a selected interval, which can be wide or narrow, depending on the number of layers of the filter.

## PROCEDURE



**Figure 1:** The setup

### Procedure

In the laboratory, we have several optical filters with different colors. Using Thorlabs CCD Spectro-photo-meter, setup the experiment as shown in Figure 1. For each filter measure and save optical data (wavelength vs spectral light intensity) with and without filter.

### QUESTIONS

1. Explain the role of filters in optics and electronics. Compare their effects.
2. Plot Transmission spectrum ( $T$  vs  $\lambda$ ) for each filters and obtain  $\lambda_{\text{cut-on}}$  and  $\lambda_{\text{max}}$
3. Plot Absorption spectrum ( $A$  vs  $\lambda$ ) for each filters.
4. Identify which filters are longpass, shortpass and bandpass by examining your graphs.
5. By using the transmittance and absorbance values do you think there is any reflection? Explain briefly how do you decide for this.
6. For soda-lime glass, plot absorption coefficient ( $\alpha$  vs  $\lambda$ ) and

$\lambda_{\text{cut-on}}$  is cut on wavelength for the longpass / shortpass filters where transmittance is greater than 50%.

$\lambda_{\text{max}}$  is the wavelength where the transmission value is maximum for bandpass filters.

## **EXPERIMENT 4**

### **POLARIZATION OF LIGHT**

#### **PURPOSE**

To verify the Malus' Law and observe Birefringence.

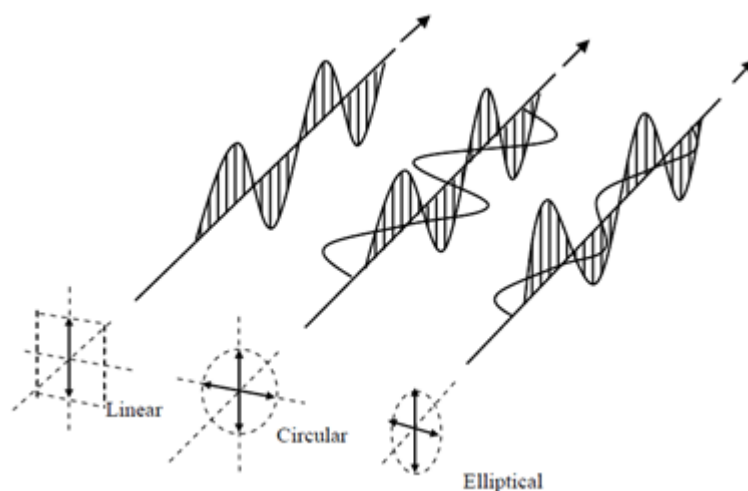
#### **EQUIPMENT**

Polarization Analyzer, Basic Optics Bench, Aperture Bracket  
Red Diode Laser, Light Sensor, Rotary Motion Sensor  
ScienceWorkshop 500 Interface, DataStudio or Capstone Software

#### **THEORY**

According to the wave model, light is a transverse electromagnetic wave. Electric and magnetic fields associated with it oscillate perpendicular to the direction of propagation. Electric field of an electromagnetic wave, in particular, can be represented by two orthogonal components. These two orthogonal components do not interfere in amplitude but are additive according to vector algebra. In an unpolarized (or randomly polarized) light there is no well-defined phase relationship between these two components. The planes of oscillations of the resultant field change randomly. Light is said to be polarized when a fixed phase and amplitude relationship is maintained between the two orthogonal field components. Resultant wave formed by the orthogonal components can have various states of polarization. Referring to Figure 1, three basic types of polarized light are:

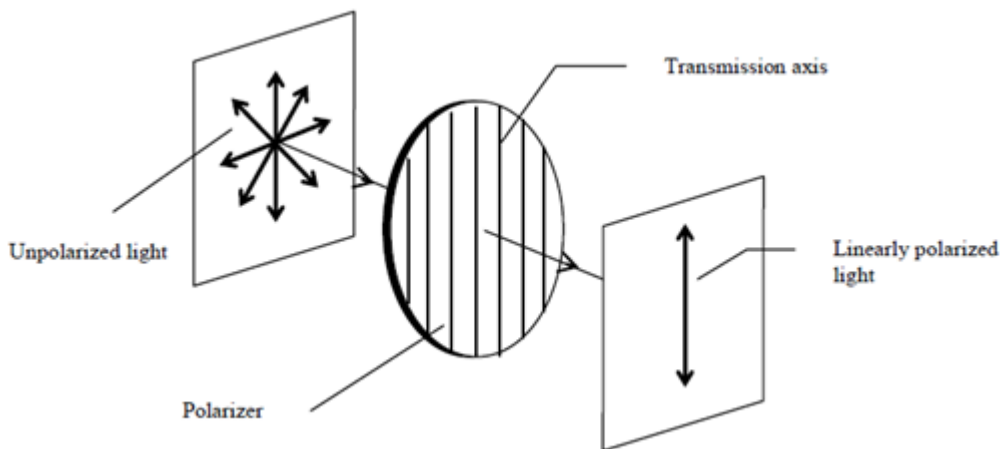
1. Linearly (or plane) polarized light. Oscillation is confined to a plane. The resultant electric field vector traces out a straight line.
2. Circularly polarized light. The resultant electric field vector traces out a circle.
3. Elliptically polarized light. The resultant electric field vector traces out an ellipse. This is the most general state of polarized light.



**Figure 1:** Linear, Circular and Elliptical polarization



Processes of preferential absorption in a dichroic material, reflection and transmission at oblique incident, double refraction in a birefringent material, and scattering by particles can be used for producing polarization. An ideal polarizer transmits only light whose electric field is parallel to the transmission axis of the polarizer and blocks light with the orthogonal field, Figure 2.



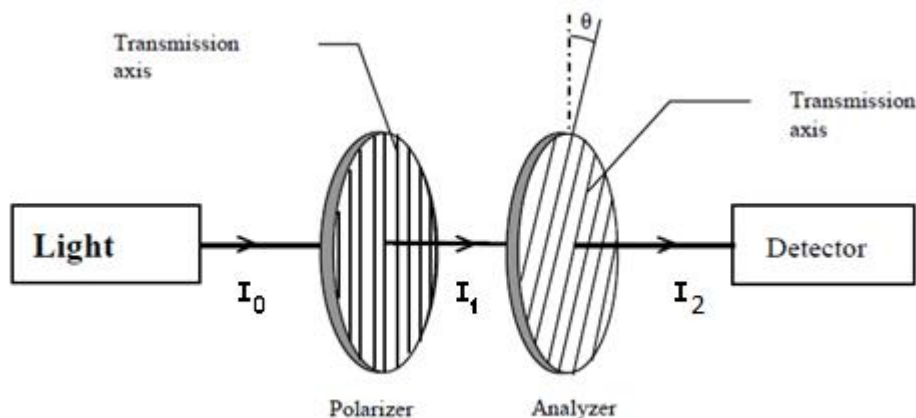
**Figure 2:** Linear Polarization

Real polarizers are not perfect and transmit light with minimum intensity when polarizer axis is perpendicular to the polarization of purely linearly polarized light. The maximum transmitted intensity occurs when the polarizer axis is parallel to the incident polarization direction.

In present experiment **Malus' law** is verified. Suppose, as illustrated in Figure 3, we have two linear polarizers made of dichroic material. The first is called polarizer and produces plane polarized light by strongly absorbing the component of the incident electric field perpendicular to its axis. The second is called the analyzer.

Let the angle between the axis of the polarizer and analyzer is  $\theta$ . It is easily shown that if the intensity of the unpolarized light incident on the analyzer is  $I_1$ , then the intensity,  $I_2$ , out of the polarized light leaving the analyzer is given by:

$$I_2 = I_1 \cos^2(\theta) \tag{1}$$



**Figure 3:** Verification of Malus' Law

Consider now, the unpolarized light passes through 3 polarizers as in Figure 4. The first and last polarizers are oriented at  $90^\circ$  with respect each other. The second polarizer has its polarization axis rotated an angle  $\theta$  from the first polarizer. Then, the intensity after passing through the third polarizer is:

$$I_3 = \frac{I_1}{4} \sin^2(2\theta) \tag{2}$$

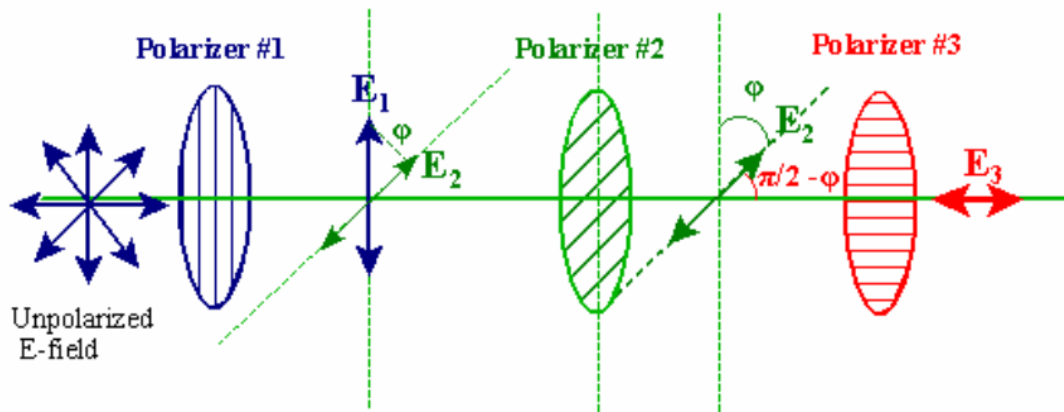


Figure 4: Three polarizers

**Birefringent Materials** [Extracted from *Hyperphysics and Wikipedia*]

[Dear lab assistant, If you have time, please show students the Birefringence effect and stress distribution on transparent materials at the end of lab.]

Crystalline materials may have different indices of refraction associated with different crystallographic directions. A common situation with mineral crystals is that there are two distinct indices of refraction, and they are called birefringent materials. If the y- and z- directions are equivalent in terms of the crystalline forces, then the x-axis is unique and is called the optic axis of the material. The propagation of light along the optic axis would be independent of its polarization; it's electric field is everywhere perpendicular to the optic axis and it is called the ordinary- or o-wave. The light wave with E-field parallel to the optic axis is called the extraordinary- or e-wave, Figure 5. Birefringent materials are used widely in optics to produce polarizing prisms and retarder plates such as the quarter-wave plate. Putting a birefringent material between crossed polarizers can give rise to interference colors, Figure 6.

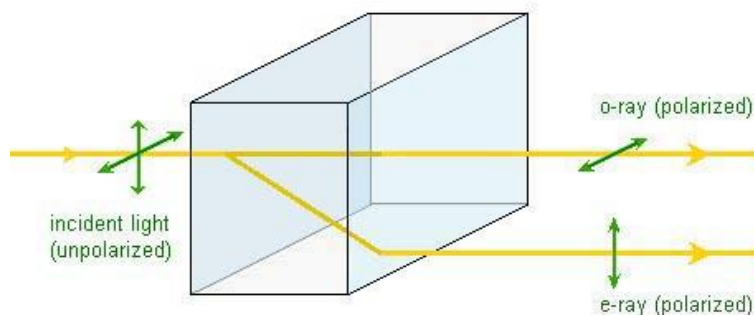
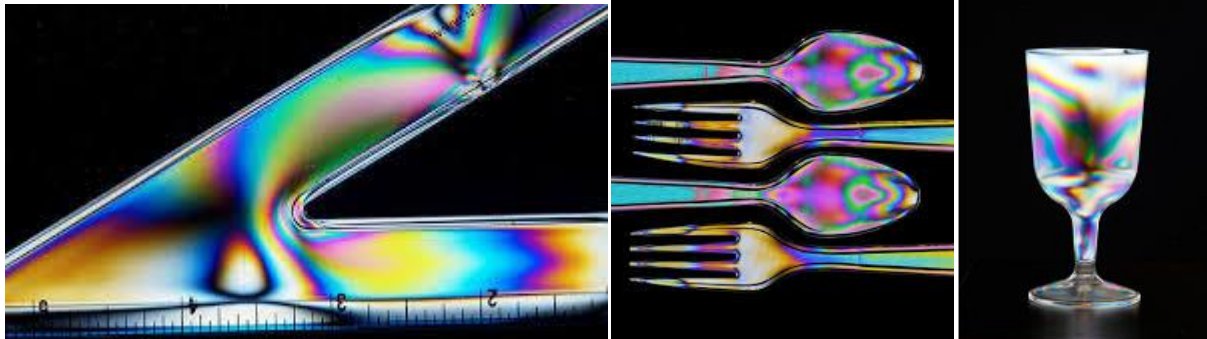


Figure 5: Birefringence

A widely used birefringent material is calcite. Its birefringence is extremely large, with indices of refraction for the o- and e-rays of 1.6584 and 1.4864 respectively.

Isotropic solids do not exhibit birefringence. However, when they are under mechanical stress, birefringence results. The stress can be applied externally or is “frozen in” after a birefringent plastic ware is cooled after it is manufactured using injection molding, Figure 6. When such a sample is placed between two crossed polarizers, colour patterns can be observed, because polarization of a light ray is rotated after passing through a birefringent material and the amount of rotation is dependent on wavelength. The experimental method called photoelasticity used for analyzing stress distribution in solids is based on the same principle.



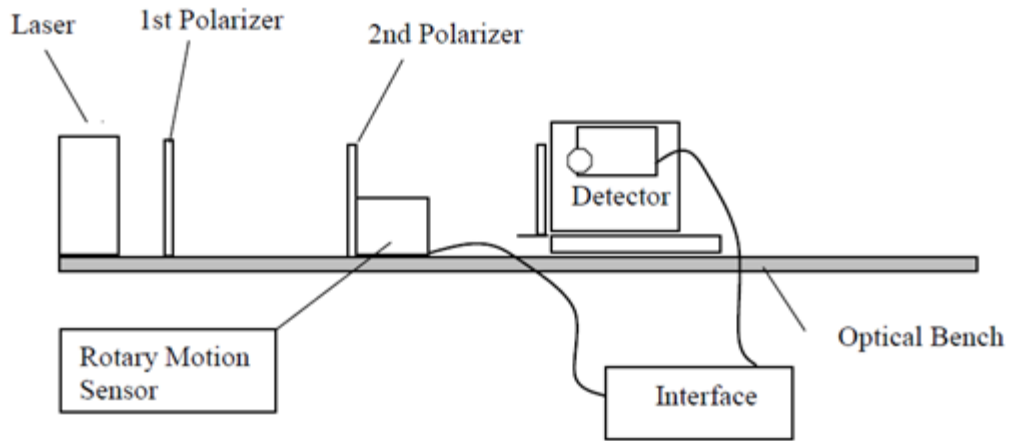
**Figure 6:** Birefringence effect can be used to see mechanical stress distributions

## PROCEDURE

### Part 1 *Two Polarizers to verify Malus' Law*

Laser light (peak wavelength = 650 nm) is passed through two polarizers. As the second polarizer (the analyzer) is rotated by hand, the relative light intensity is recorded as a function of the angle between the axes of polarization of the two polarizers. The angle is obtained using a Rotary Motion Sensor that is coupled to the polarizer with a drive belt. The plot of light intensity versus angle can be fitted to the square of the cosine of the angle.

1. Turn on the laser. Adjust the position of the both polarizers so that their transmission axes become parallel.
2. Turn on the interface. Before starting to take data, open the graph icon from the main menu.
3. Then start taking data by pushing the play button once on the keyboard of the interface.
4. Next slowly rotate the 2nd Polarizer up to at least  $400^\circ$   
(i.e. polarizer which is connected to the rotary motion sensor.)
5. In order to stop the measurement, push again the play button once.
6. Go back to the main menu by pushing the home button on the interface and choose the files icon. In this sub menu you will see your measurement data as untitled file. You should rename and save it. Then plug in your flash disk to the interface and send your saved file to it.
7. Go back to the main menu by pushing the home button on the interface and choose the files icon. In this sub menu you will see your measurement data as untitled file. You should rename and save it. Then plug in your flash disk to the interface and send your saved file to it.
8. Turn off the laser and the interface.



**Figure 7:** The setup for two polarizers



**Figure 8:** PASCO equipment for two polarizers

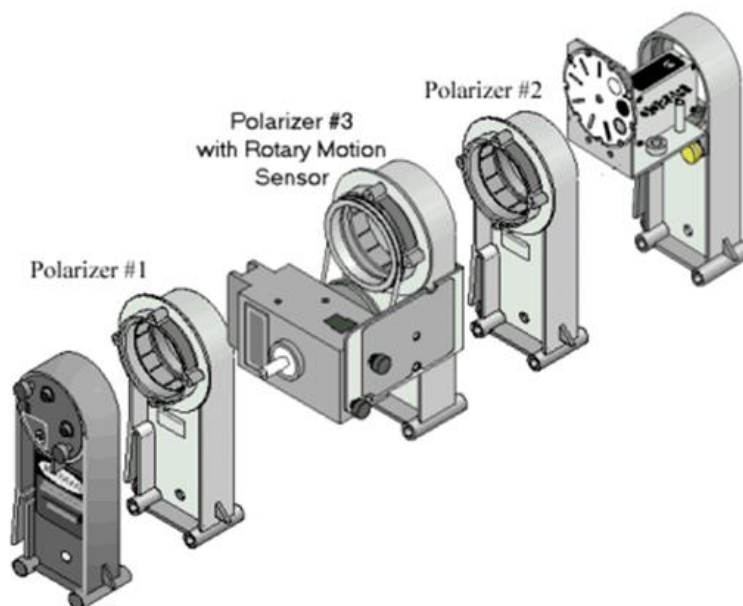
## QUESTIONS

1. Give two example usage of polarizers in our daily life.
2. Derive Equation 1.
3. Which type of polarization do you observe in the experiment(s)? (i.e. Linear, circular or elliptical polarization)
4. Why isotropic solids do not exhibit birefringence?
5. Plot a graph with measured intensities  $I(\theta)$  as ordinates and the angle  $\theta$  between the polarizers as abscissa. Fit your data to the non-linear function  $I(\theta) = A \cos^n(\theta + \varphi) + B$  to obtain the fit parameters  $A$ ,  $B$ ,  $\varphi$  and  $n$  and their associated errors. What are the physical meanings of the constants  $A$ ,  $B$ ,  $\varphi$ , and  $n$ ? Record your results to Table 1.
6. What is your overall conclusion?

**Table 1:** Fit parameters for two polarizers

Fit parameter	Physical Meaning	value +- error (from fit)
$A$		
$n$		
$\varphi$		
$B$		

## Part 2 Three Polarizers [Optional]



1. Now repeat the experiment with 3 polarizers. Place one polarizer on the track and rotate it until the transmitted light is a maximum.
2. Then place a second polarizer on the track and rotate it until the light transmitted through both polarizers is a minimum.
3. Then place a third polarizer on the track between the first and second polarizers. Rotate it until the light transmitted through all three polarizers is a maximum.
4. Press Start and record the Intensity vs. angle for 400 degrees as you rotate the third polarizer that has the Rotary Motion Sensor.
5. Select your data from 2 polarizers and from 3 polarizers. What two things are different for the Intensity vs. Angle graph for 3 polarizers compared to 2 polarizers?

### QUESTIONS

1. For 3 polarizers, what is the angle between the middle polarizer and the first polarizer to get the minimum transmission through all 3 polarizers?
2. Plot a graph with measured intensities  $I(\theta)$  as ordinates and the angle  $\theta$  between the polarizers as abscissa. Fit your data to the non-linear function  $I(\theta) = A \sin^n(2\theta + \varphi) + B$  to obtain the fit parameters  $A$ ,  $B$ ,  $\varphi$  and  $n$  and their associated errors. What are the physical meanings of the constants  $A$ ,  $B$ ,  $\varphi$ , and  $n$ ? Record your results to Table 2.
3. What is your overall conclusion?

**Table 2:** Fit parameters for three polarizers

Fit parameter	Physical Meaning	value +- error (from fit)
$A$		
$n$		
$\varphi$		
$B$		

## EXPERIMENT 5 OPTICAL ACTIVITY

### PURPOSE

To measure optical activity manifested by a solution of sugar molecules in water and to observe the relationship between optical activity and the sugar concentration in a solution.

### EQUIPMENT

Polarizers, Laser diode, Optical Bench, Optical sensor, Several solution of sugar molecules in water

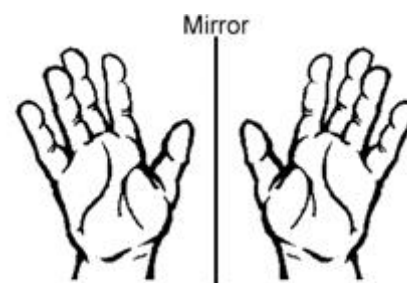
### THEORY

Optical Activity is the rotation of the polarization plane of a beam of linearly polarized light while it is passing through a medium. The mediums having that kind of ability are called *optically active* e.g. sugar molecules and amino acids.

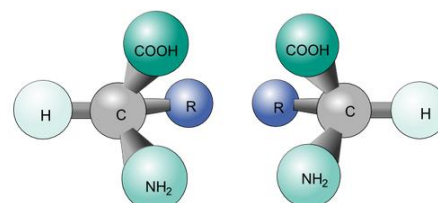
Certain compounds, mostly organic rotate the plane of polarized light. The phenomenon is called optical rotation. Materials with these properties are said to have optical activity and consists of chiral molecules (those containing asymmetric carbon atoms).

Chirality is the property of an object of being non-superimposable on its mirror image. Chiral centers that have opposite configurations rotate polarized light the same number of degrees, but in opposite directions (enantiomers). Rotation of plane polarized light counterclockwise is *levorotation*, and rotation of plane polarized light clockwise is *dextrorotation*. Racemic mixtures (equal parts of two enantiomers) will have no net rotation because the equal but opposite rotations cancel each other.

A very simple illustration of an optically active material is showed in Figure 1.

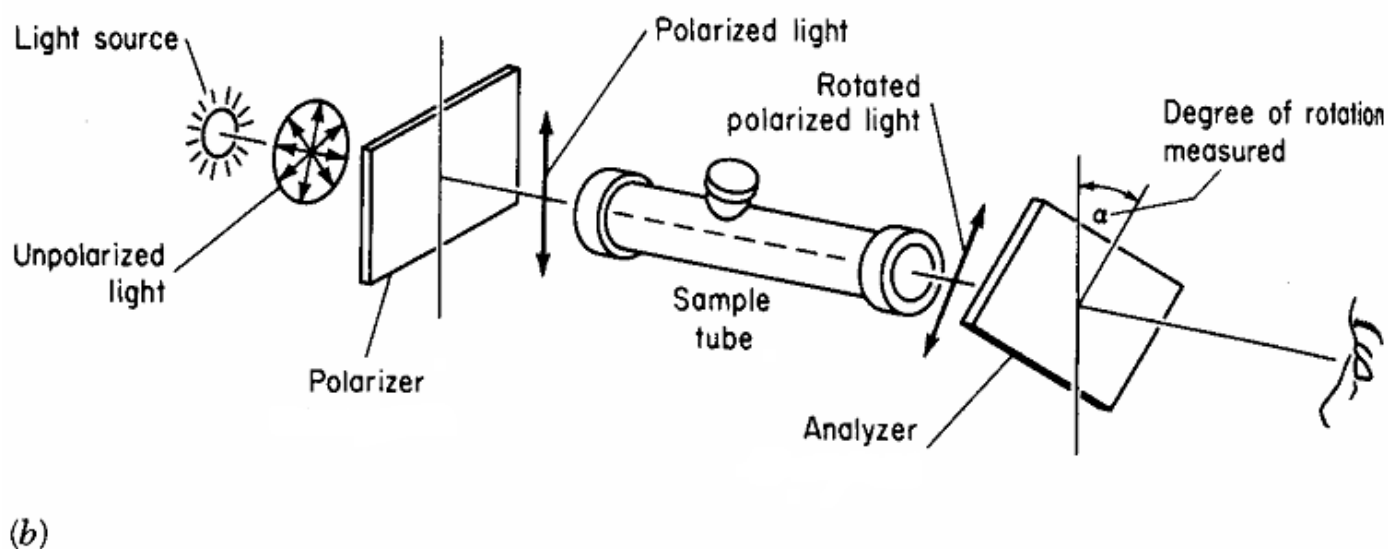
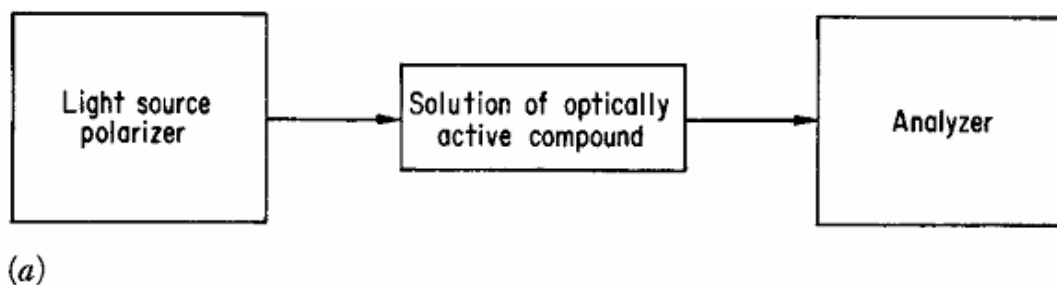


The mirror image of a chiral substance cannot be superimposed on the original image. Hands are chiral, as are sugars and amino acids.



**Figure 1:** Chiral Molecules

Optical activity is the ability of a chiral molecule to rotate the plane of plane-polarized light, measured using a *polarimeter*. Figure 2 shows a principle of a polarimeter set up and its main components together with their function. Unpolarized light from the light source is first polarized. This polarized light passes through a sample cell. If an optical active substance is in a sample tube, the plane of the polarized light waves is rotated. The rotation is noticed by looking through the analyser as a change in intensity of illumination. To reach the same illumination as was without an optical active sample the analyser must be turned around for an angle.



**Figure 2:** Principle of a polarimeter set up

The degree to which a substance rotates light may be used to determine:

- i) the identity of the substance,
- ii) the enantiomeric purity of the substance or
- iii) the concentration of a known substance in a solution.

Experiments show that the measured angle of rotation depends upon many variables:

- the type or nature of sample (example: sugar solution)
- concentration of the optical active components
- the length of the sample tube
- the wavelength of the light source
- temperature of the sample

As a summary, first a plane polarized beam of light is sent towards the tube which has a solution of sugar/water molecules. Then, the amount of the observed rotation  $\alpha$  depends on the path length  $L$  (length of the tube), the concentration of the solution  $C$ , and the intrinsic ability of the molecules that indicates the interaction with the polarized light, which is called specific rotation  $[\alpha]$  or rotation power.

$$[\alpha]_{\lambda}^T = \frac{\alpha}{cL} \quad (1)$$

Specific rotation is determined at a specified temperature  $T$  (usually 20 °C) and a wavelength ( $\lambda$ ) of light source (usually sodium lamp with its D line at 589 nm). In practical measurements readings are taken at

different units:  $\alpha$  in  $^{\circ}$  (deg),  $c$  in g/mL (or g/cm<sup>3</sup>),  $L$  in dm and so  $[\alpha]_{589}^{20^{\circ}}$   $\alpha$  is usually tabulated in  $^{\circ}$  cm<sup>3</sup>/g dm. Following table shows the specific rotation of different substances.

Substance in a solution H <sub>2</sub> O solvent	Specific rotation $[\alpha]_{589}^{20^{\circ}}$ ( $^{\circ}$ cm <sup>3</sup> /g dm)
sucrose	+ 66.54
glucose	+ 52.74
fructose	- 93.78
maltose	+ 137.5
lactose	+ 55.3
dextrose	+ 194.8

The following experimental data shows that  $\alpha$  is inversely proportional to wavelength  $\lambda$ .

Light source	Wavelength	Specific rotation $[\alpha]$ ( $^{\circ}$ cm <sup>3</sup> /g dm)
Mercury (green)	546 nm	+ 78.4178
Sodium (yellow)	589 nm	+ 66.5885
HeNe Laser (red)	632 nm	+ 57.2144
Near Infrared (NIR)	883 nm	+ 28.5462

The following table shows  $\alpha$  calculated for some temperatures of a sucrose solution at some concentration:

Temperature $^{\circ}$ C	Rotation of a sucrose solution $\alpha$ ( $^{\circ}$ angle degree)
20	40.000
21	39.981
25	39.906

Notice the decrease of the rotation of sucrose solution with rising temperature. Also the effect of temperature is relatively small.

Polarimetry is a powerful tool to determine the active purity of raw materials such as vitamins, steroids, and antibiotics, since for most chiral compounds, only one isomer (enantiomer) has biological activity. From the pharmacological point of view, a worst scenario is one which the undesired enantiomer causes serious toxicity. The most common application is for sugar content. Methods for determination of sugar in chocolate, wines, jams, jellies, and flour are well documented, as well as lactose in milk products.

## PROCEDURE

We have several tubes whose lengths are 1 dm. Each of them contains different amount (concentration) of glucose. You can reference Figure 2. Optical data (light intensity) can be obtained by a photo-diode.

1. Place your reference tube (containing only water) on the holder and make sure that the polarizer and analyzer are perpendicular by keeping the first polarizer fixed and rotating the analyzer until you obtain minimum transmission.
2. Note the position where you obtained minimum position.
3. Insert one of the tube to holder.
4. Again rotate the polarizer until you get minimum transmission and record this position.
5. Follow same procedure with the rest of the solution tubes.
6. Record your results to Table 3.



**Table 3:** Glucose concentration ( $c$ ) vs observed angle of rotation ( $\alpha$ ) data

Wavelength of the light source used in exp.	$\lambda =$
Glucose Concentration $c$ (g/mL)	Observed angle of rotation $\alpha$ (degree)
0.21	
0.27	
0.32	
0.40	

## QUESTIONS

1. Why some molecules are optically active?
2. According to your measurements, is the solution dextrorotatory or laevorotatory?
3. What is a polarimeter? Give two example applications in real life.
4. What is the SI unit of specific angle of rotation  $[\alpha]$ ?
5. Plot the angle of rotation versus the concentration graph.
6. Plot your data [concentration ( $c$ ) vs angle of rotation ( $\alpha$ )].
7. By fitting your data to linear function, extract the value of the specific rotation  $[\alpha]$  of the glucose molecules for the wavelength used in the experiment and corresponding uncertainty. Compare your result with the value in the literature.

## EXPERIMENT 6 BREWSTER'S ANGLE MEASUREMENT

### PURPOSE

To measure Brewster's Angle

### EQUIPMENT

Brewster Angle Measurement Kit (Light Sensor, Laser, PMMA Block, Polarizers, ...)

### THEORY

Polarization is a property of certain types of waves that describes the orientation of their oscillations.

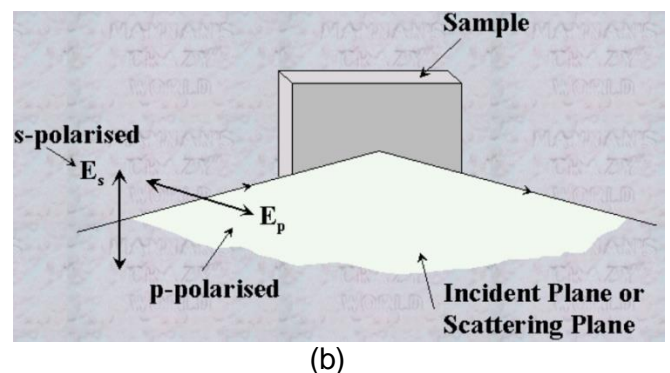
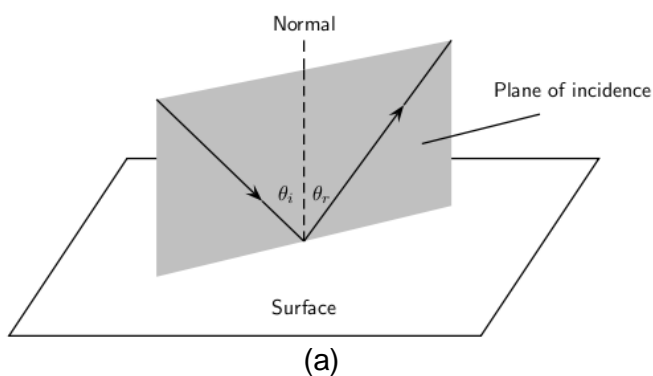
- Only transverse waves have polarization effect (e.g electromagnetic waves, gravitational waves).
- Longitudinal waves do not have polarization since the direction of vibration and direction of propagation are the same (e.g. sound waves in air)
- To explain polarization effect for electromagnetic waves, it is sufficient to consider only the electric field component.

Polarization can be obtained from an unpolarized (= Randomly polarized) beam

- by selective absorption (see previous experiment)
- by reflection (this experiment)
- by scattering

### *S-Wave and P-Wave*

- The component of the electric field parallel to plane of incidence is termed p-like (parallel) and
- the component perpendicular to this plane is termed s-like (from senkrecht, German for perpendicular).
- Polarized light with its electric field along the plane of incidence is thus denoted *p-polarized*, while light whose electric field is normal to the plane of incidence is called *s-polarized*, Figure 1.



**Figure 1:** (a) The plane of incident ray reflected from a surface. (b) s-wave and p-wave.

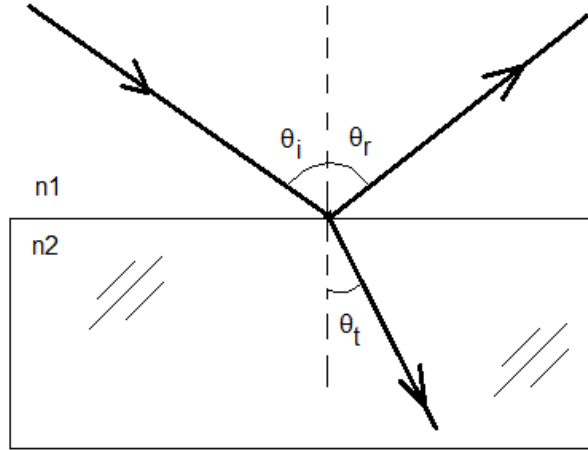
## Fresnel Equations

Consider a light ray falling on a surface as in Figure 2. We have law reflection:

$$\theta_i = \theta_r$$

and law of refraction:

$$n_1 \sin\theta_i = n_2 \sin\theta_t \quad (1)$$



**Figure 2:** When light moves from a medium of a given refractive index  $n_1$ , into a second medium with refractive index  $n_2$ , both reflection and refraction of the light may occur.

What about the intensities of reflected light and refracted light? The Fresnel Equations describe what fraction of the light is reflected and what fraction is refracted (ie, transmitted). They also describe the phase shift of the reflected light. These equations can be extracted from the equations of the electromagnetic theory. The results are as follows:

The reflectance for s-polarized light is:

$$R_s = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2 = \left| \frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2}} \right|^2 \quad (2)$$

while reflectance for p-polarized light is:

$$R_p = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2 = \left| \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2} - n_2 \cos\theta_i}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2} + n_2 \cos\theta_i} \right|^2 \quad (3)$$

Transmission coefficients can be found from the conservation of energy:

$$T_s = 1 - R_s \quad (4)$$

$$T_p = 1 - R_p \quad (5)$$

If the incident light is unpolarized (mixture of s-wave and p-wave), the reflection coefficient is:

$$R = \frac{R_s + R_p}{2} \quad (6)$$

There are number of consequences of Fresnel equations. We will discuss some of them here.

**\*\*\* Fresnel Equations, Case 1:  $n_1 < n_2$**

Figure 3a shows reflection factors ( $R_s$  and  $R_p$ ) as a function of incident angle  $\theta_i$  for  $n_1=1$  and  $n_2=1.5$ . At certain value  $\theta_i$ , the intensity of the reflected p-polarized light is zero while reflected s-polarized light is always increasing. This specific angle of incidence is known as the Brewster's angle and its value can be evaluated as follows. Consider Equation 3. If  $R_p = 0$  then

$$n_1 \cos\theta_t = n_2 \cos\theta_i$$

or

$$n_1/n_2 = \cos\theta_i / \cos\theta_t$$

On the other hand, from Snell's law we have (Equation 1):

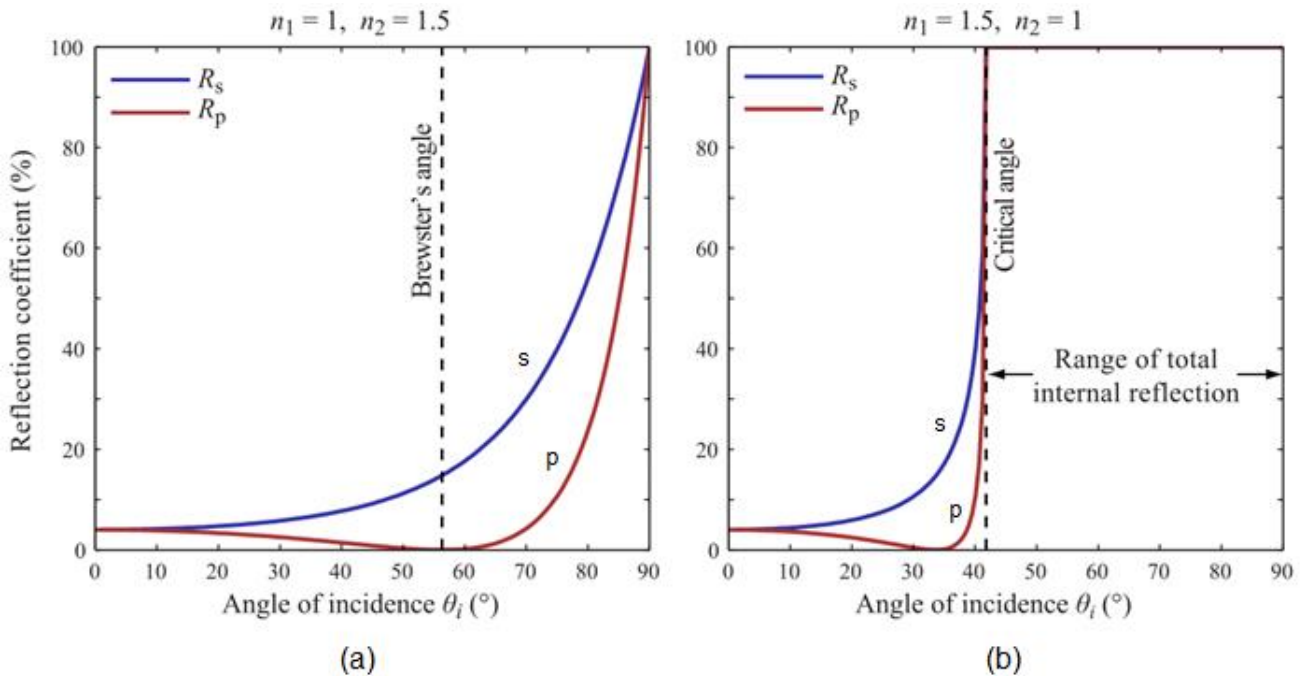
$$n_1/n_2 = \sin\theta_t / \sin\theta_i$$

Equating last two equations yields:

$$\cos\theta_i / \cos\theta_t = \sin\theta_t / \sin\theta_i$$

or

$$\sin\theta_i \cos\theta_i = \sin\theta_t \cos\theta_t$$



**Figure 3:**  $R_s$  and  $R_p$  values as a function of incident angle (a) for  $n_1=1.0$  and  $n_2=1.5$ , and (b) for for  $n_1=1.5$  and  $n_2=1.0$

Since  $\sin(2\beta) = 2 \sin\beta \cos\beta$ , we can write

$$\sin(2\theta_i) = \sin(2\theta_t) \quad (7)$$

Equation (7) has two solutions. First  $\theta_i = \theta_t$  which is not acceptable physically for our case (Why?). Second solution is,  $2\theta_i + 2\theta_t = 180^\circ$  or

$$\theta_i + \theta_t = 90^\circ \quad (8)$$

You can try.  $\theta_i = 30^\circ$  and  $\theta_t = 60^\circ$  then  $\sin(2 \times 30^\circ) = \sin(2 \times 60^\circ) = 0.866$ . This solution is acceptable.

This incidence angle satisfying Equation 8 is the known as Brewster's angle,  $\theta_B$ , Figure 4. Applying Snell's law when  $\theta_i = \theta_B$ , results in

$$n_1 \sin(\theta_B) = n_2 \sin(\theta_t) \quad (9)$$

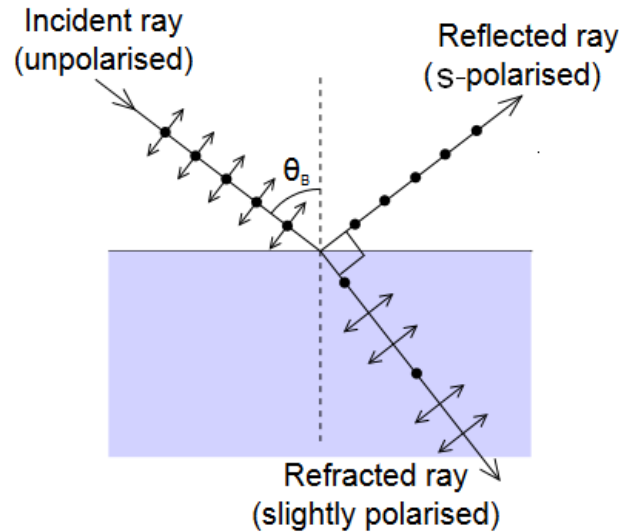
Since  $\theta_B + \theta_t = 90$ , we can write  $\sin(\theta_t) = \sin(90 - \theta_B) = \cos(\theta_B)$ . Substituting for  $\sin(\theta_t)$  in Equation 9 gives:

$$n_1 \sin(\theta_B) = n_2 \cos(\theta_B)$$

Therefore

$$\tan \theta_B = \frac{n_2}{n_1} \quad (10)$$

For example,  $n_1=1$  and  $n_2=1.5$  then  $\theta_B = \text{atan}(1.5/1.0) = 56.3^\circ$ .



**Figure 4.** When  $\theta_i = \theta_B$ , light with a p-polarization is perfectly transmitted through a transparent dielectric surface, with no reflection. Only s-wave is reflected.

### \*\*\* Fresnel Equations, Case 2: $n_1 > n_2$

When  $n_1 > n_2$ , all of s-wave and p-wave reflect from the surface if angle of incidence is greater than the critical angle defined by  $\sin\theta_c = n_2 / n_1$ . This phenomena is called the total internal reflection as shown in Figure 3b. Mathematically speaking,  $R_s = R_p = 1$  if  $n_1 > n_2$  and  $\theta_i > \theta_c$ .

**\*\*\* Fresnel Equations, Case 3:  $\theta_i = 0$**

If the angle of incidence is zero then transmitted angle is also zero. Therefore, substituting  $\theta_i = \theta_t = 0$  in Equations (2) and (3) gives the reflection coefficients:

$$R_s = R_p = R = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2 \tag{11}$$

The physical predictions of the Fresnel Equations are summarized in Table 1.

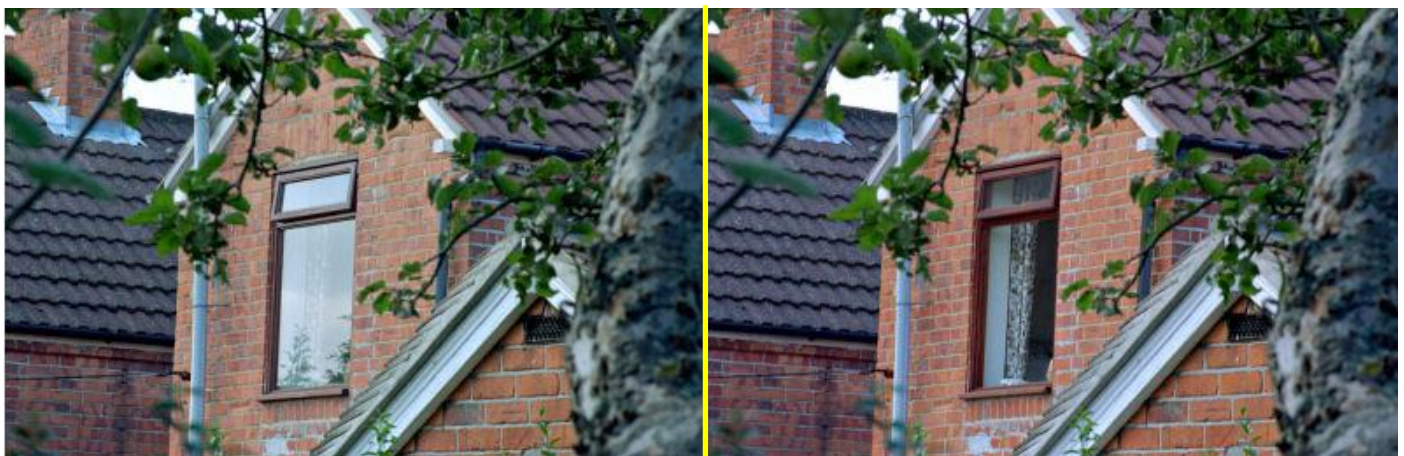
**Table 1:** Summary of consequences of Fresnel Equations

Case	Physical prediction	Equation
Case 1: $n_1 < n_2$	When $\theta_i = \theta_B$ , light with a p-polarization is perfectly transmitted through a transparent dielectric surface, with no reflection. Only s-wave is reflected. This specific angle is called the Brewster angle.	$\tan \theta_B = \frac{n_2}{n_1}$
Case 2: $n_1 > n_2$	Both reflectances are 1 if the angle of incidence is greater than the critical angle	$R_s = R_p = 1$ if $n_1 > n_2$ and $\theta_i > \theta_c$ .
Case 3: $\theta_i = 0$	Reflectances are the same when light falls perpendicular onto surface	$R_s = R_p = R = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2$

***Application of Brewster Angle [Extracted from Wikipedia]***

Polarized sunglasses use the principle of Brewster's angle to reduce glare from the sun reflecting off horizontal surfaces such as water or road. In a large range of angles around Brewster's angle, the reflection of p-polarized light is lower than s-polarized light. Thus, if the sun is low in the sky, reflected light is mostly s-polarized. Polarizing sunglasses use a polarizing material such as Polaroid sheets to block horizontally-polarized light, preferentially blocking reflections from horizontal surfaces.

Photographers use the same principle to remove reflections from water so that they can photograph objects beneath the surface. In this case, the polarizing filter camera attachment can be rotated to be at the correct angle (see Figure 5).



**Figure 5:** Photographs taken of a window with a camera polarizer filter rotated to two different angles. In the picture at left, the polarizer is aligned with the polarization angle of the window reflection. In the picture at right, the polarizer has been rotated 90° eliminating the heavily polarized reflected sunlight.

The following Experimental Accessory is used to study the polarization of reflected light and to determine the Brewster's angle.

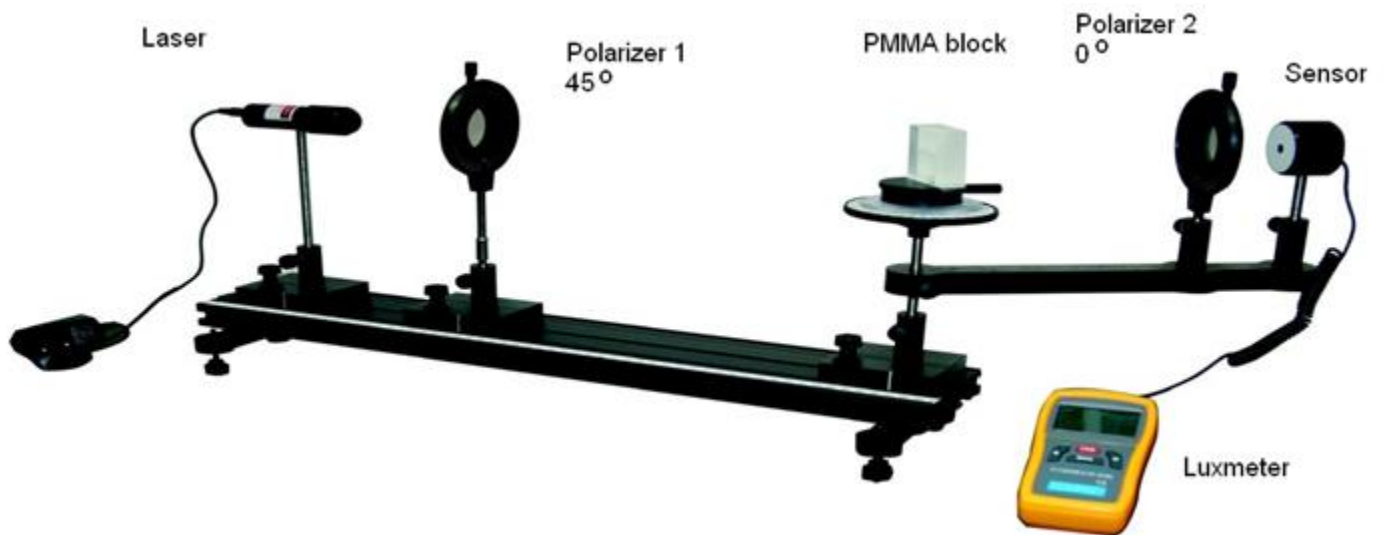
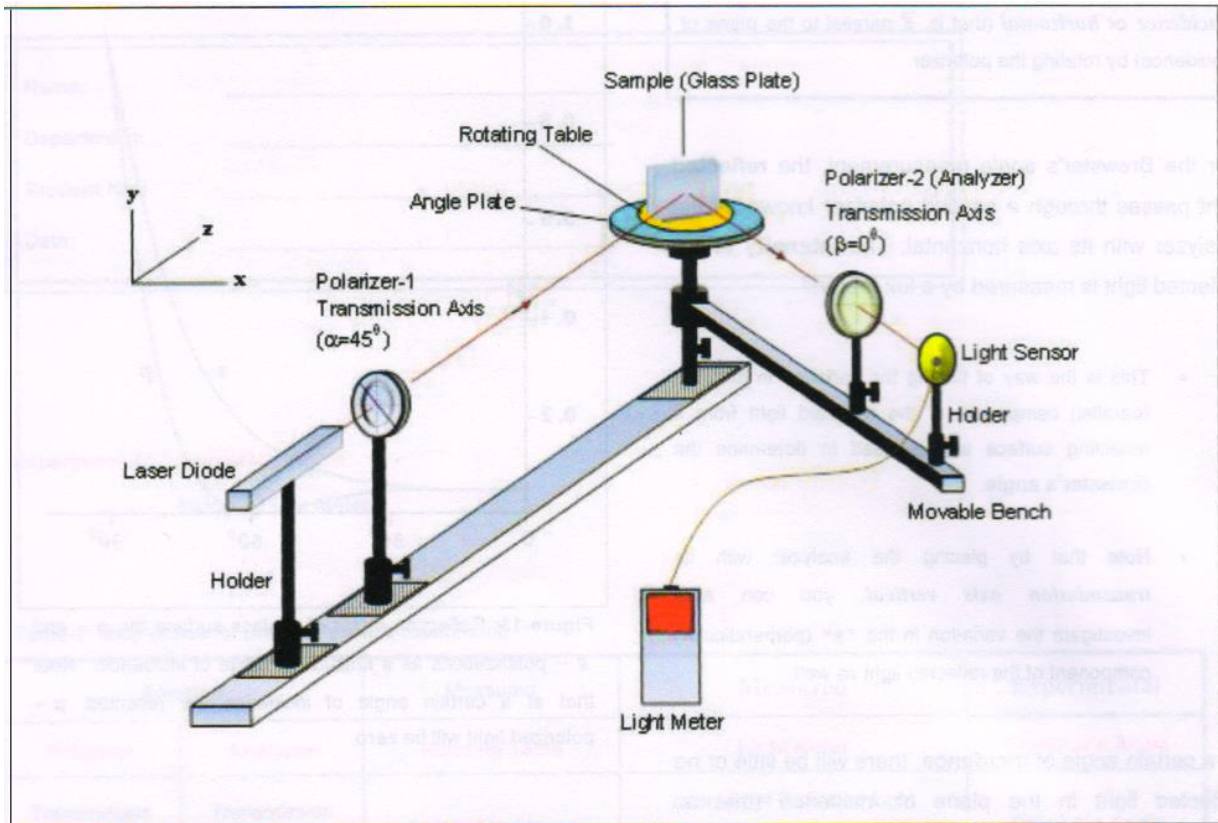


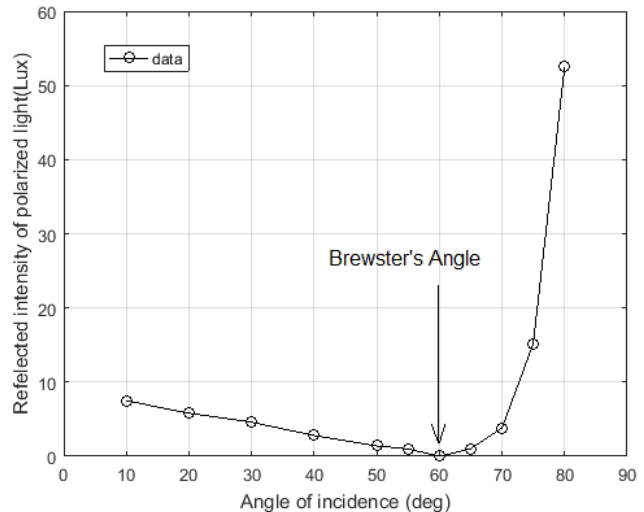
Figure 6: Experimental Kit for the Brewster Angle Measurement

## PROCEDURE

1. Setup the experimental apparatus as shown in Figure 6.
2. Turn on the laser diode and wait for 10 minutes.
3. Setup the first polarizer 45 degrees and second one zero degree.
4. By changing the angle of incidence, measure using the light sensor intensity of polarized light reflected from the PMMA block.
5. Report your result to Table 2.

**Table 2:** The experimental data

Angle of incidence $\theta_i$ (deg)	Intensity or illumination on sensor
10	
20	
30	
40	
45	
50	
55	
60	
65	
70	
80	



**Figure 7:** Example plot of data obtained from a typical experiment. Brewster angle is around  $60^\circ$ .

## QUESTIONS

1. Give an example application indicating the importance of the Brewster's Angle in our daily life.
2. Explain why the transmission axis of the first polarizer is  $45^\circ$ .
3. What is the range Brewster's angles in degrees for the water-air interface if white light (400 nm-700 nm) falls on the boundary?
6. Plot  $\theta$  vs Intensity graph. Fit the data in the Table 2 to Equation 3 ( $R_p$ ) for  $n_1 = 1$  and  $n_2 = n$  to obtain the index of refraction of the PMMA block and its associated error. (An example plot of data obtained from a typical Brewster angle experiment is shown in Figure 7. If you did not apply fitting procedure, you may obtain that the Brewster Angle is about  $60^\circ$  which is not correct value.)
4. In fact  $n = 1.5$ . Compare your results with the actual value and compute the percentage error.
5. What is your overall conclusion?



## **EXPERIMENT 7**

### **SINGLE SLIT AND DOUBLE SLIT DIFFRACTION**

#### **PURPOSE**

- To observe diffraction and interference effects of light passing through various apertures.
- To use the diffraction patterns produced by single/double slit apertures and to calculate the wavelength of the light source(s) which is used.

#### **EQUIPMENT**

Single and double slit accessory  
Diode Laser sources (650 nm, 532 nm)  
Rotary motion sensor  
High sensitive light sensor  
Linear Translator

#### **THEORY**

Certain optical phenomena may be accurately described by the hypothesis that *light is an electromagnetic wave*. These phenomena include diffraction and interference. Although it is customary to discuss interference and diffraction of light as separate topics, they are essentially similar. Both are special cases of superposition of waves. Roughly speaking waves emanating from more than one very small source may exhibit interference. Diffraction is interference of a wave with itself. According to Huygen's Principle waves propagate such that each point reached by a wave-front acts as a new wave source. Interference between secondary waves emitted from different parts of the wave-front can cause the bending of light around objects and cause intensity fluctuations much like the interference pattern from separate sources.

For the interference is to be observed, the light has to be coherent, i.e.: the phase of the light wave is well defined at all times. For incoherent light, the interference is hard to observe because it is washed out by the very rapid phase jumps of the light.

Relative sizes and distances also affect the result of interference experiment. For instance when light propagates through and around objects whose dimensions are much greater than the wavelength, the wave effects are negligible so that its behavior can be adequately described by ray optics. Consider also a situation where a point light source illuminates a slit. If both the light source and observation point are very far from the slit relative to its width, it is a good approximation to assume that incident and diffracted waves are plane. This condition is called **Fraunhofer** diffraction or far-field diffraction. Using this approximation, it is easy to compute the distance a wave must travel to reach the observation point from each part of the slit and then to use the principle of superposition to deduce the observable intensity. The same method is used if the light source, the observer, or both are close to the slit, but the geometry harder to work out. This situation is called **Fresnel** diffraction or near-field diffraction.

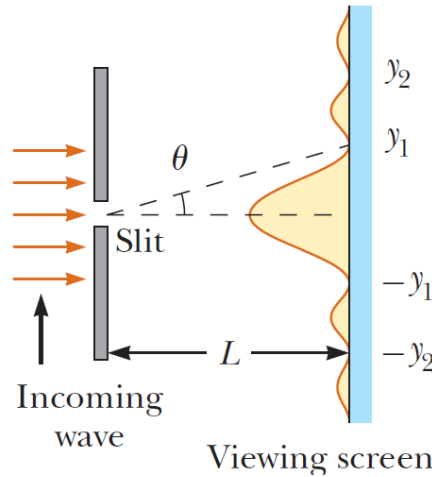
In this experiment, under far field condition, diffraction and interference of light when it propagates through single and double slits will be explored. From the resulting diffraction patterns the wavelength of the incident light will be determined. We will directly give equations that governs the diffraction effects. For more theoretical background please see your lecture notes.

### Single Slit Diffraction

Intensity distribution ( $I_s$ ) of the diffraction pattern produced by a single slit of width  $a$ , illuminated by plane wave of wavelength  $\lambda$  is given by:

$$I_s = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad (1)$$

where  $\alpha = \pi a \sin(\theta)/\lambda$  and  $I_0$  is the intensity at the middle of central maximum, and  $\theta$  is the observation angle (Figure 1 and Fig 2).



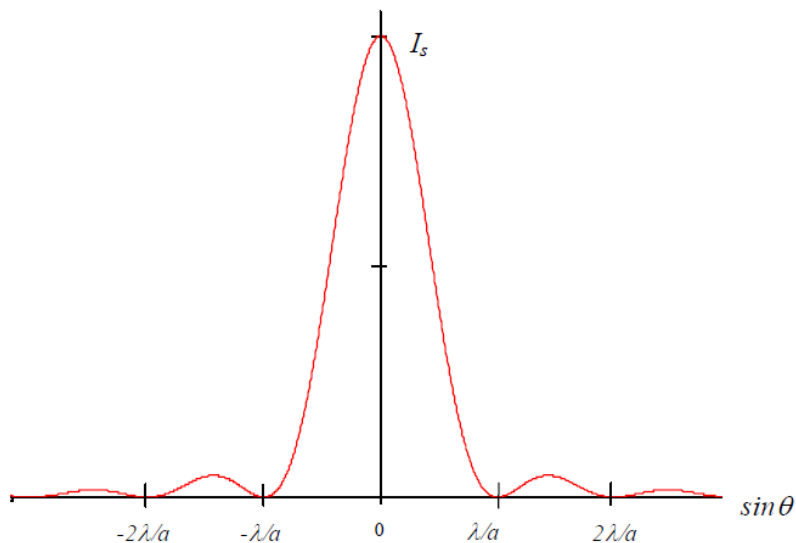
**Figure 1:** Single slit diffraction.  $L$  is the distance between slit and viewing screen.

The general condition for destructive interference (dark pattern) is:

$$\sin \theta = \frac{m\lambda}{a} \quad (2)$$

where  $m = \dots -2, -1, 0, 1, 2, 3 \dots$ . If  $L \gg a$  then  $\sin(\theta) \approx \tan(\theta) = y / L$ . Hence we have:

$$\frac{y}{L} \approx \frac{m\lambda}{a} \quad (3)$$



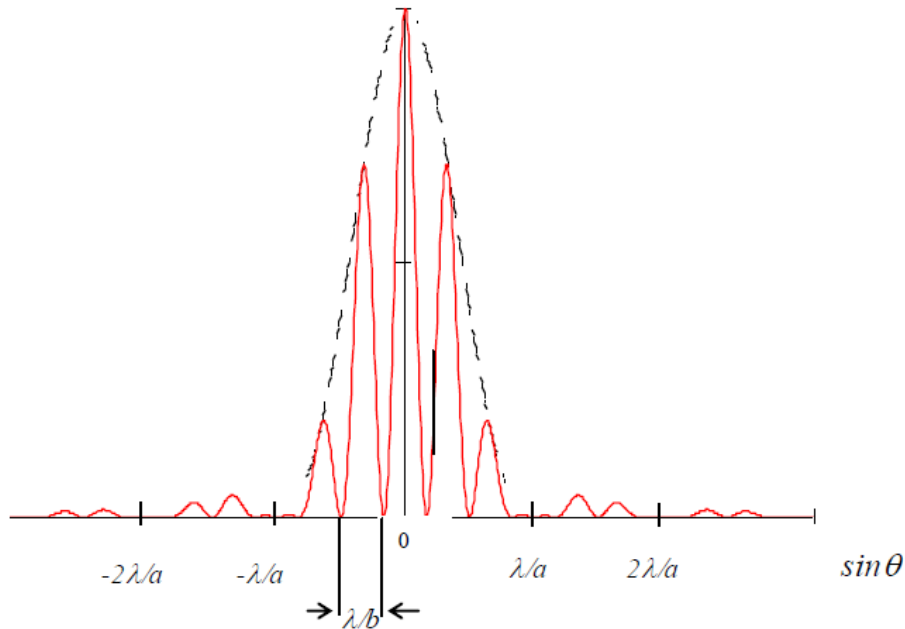
**Figure 2:** Intensity vs  $\sin \theta$  distribution in a single slit diffraction

### Double Slit Diffraction

Intensity distribution ( $I_d$ ) produced by a double slit of separation  $b$  and of slit widths  $a$  is given by:

$$I_d = 4I_s \cos^2 \beta = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \beta \quad (4)$$

where  $\alpha = \pi a \sin(\theta)/\lambda$  and  $\beta = \pi b \sin(\theta)/\lambda$ . An example plot of Equation 4 is shown in Figure 3.



**Figure 3:** Intensity pattern of a double slit diffraction & interference

If the distance between slits is much larger than that of the slit separation (i.e.  $L \gg b$ ) then the position ( $y$ ) for the maxima (bright) fringes can be evaluated from:

$$\frac{y}{L} \approx \frac{m\lambda}{b} \quad (5)$$

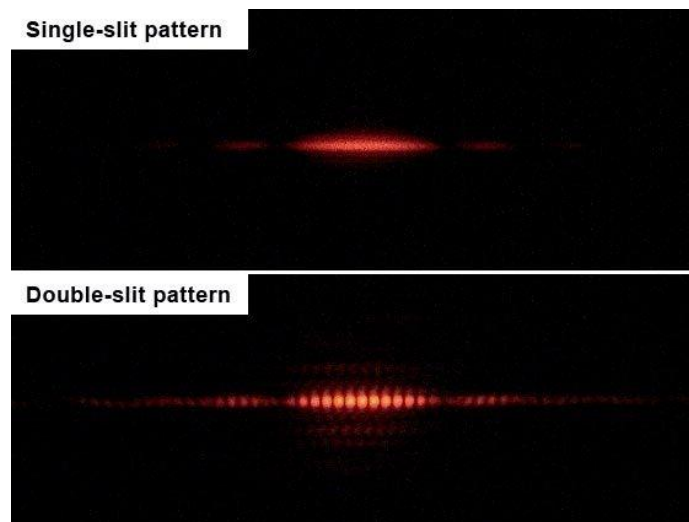
Also, the difference between the neighbor bright peaks is given by:

$$\Delta y \approx \frac{\lambda L}{b} \quad (6)$$

### Observed Distributions

In the single-slit pattern shown at the top of Figure 5, most of the intensity is contained in the central region. However, due to the wave nature of light and the finite width of the slit, there is diffraction, giving rise to faint side lobes on both sides of the main central lobe.

In the double-slit pattern shown at the bottom Figure 5, the pattern has a similar shape, but it is divided into many smaller interference fringes due to constructive and destructive interference between the waves passing through one slit and those passing through the other.



**Figure 5** Single slit and double slit interference pattern on the screen for the same wavelength.

## PROCEDURE

*Dear lab assistant, you will use PASCO optical system (linear translator, rotary motion sensor and high sensitivity light sensor) in these experiments. Please set up the equipment before the lab starts. You may need high position resolution. So increase the data acquisition frequency to 100-200 Hz or more.*

1. Turn off most of the room lights. We want the relative intensity levels to not vary by more than 1 percent as the Light Sensor is moved on the linear translator.
2. Turn on computer and red laser diode (650 nm).
3. Wait for 10 minutes until the laser diode becomes stable.
6. Set the distance between slit(s) and the light sensor at least  $L = 1.0$  m.
7. Measure the intensity  $I$  as a function of distance  $y$  obtained by linear translator.
8. Acquire the single slit and double slit diffraction data electronically using PASCO system
9. If possible, repeat above procedure for different size of glasses and the green laser (532 nm).

Take a note:

Distance:  $L =$

Single slit width:  $a =$

Double slit separation:  $b =$

## QUESTIONS

1. Fit your single slit data to Equation 1 to get wavelength and its associated error.  
*Hint let  $I_0$  and  $a$  as free parameters in the fit.*
2. For laser sources, compute percentage difference between your observations and expected values.
3. Using your double slit data, evaluate the difference ( $\Delta y$ ) between neighbor peak positions and then calculate wavelength using Equation (6).
4. Compute the average wavelength and corresponding standard deviation.
5. Again, for laser sources compute percentage difference between your observations and expected values.
6. Compare your findings in case 1 and 4.
7. What are the possible systematic errors in the experiments?
8. What is your overall conclusion?

## EXPERIMENT 8 DIFFRACTION GRATING SPECTROMETER

### PURPOSE

- To Familiar with the diffraction grating and the grating spectrometer
- To observe and measure the discrete wavelengths emitted from gas discharge lamps

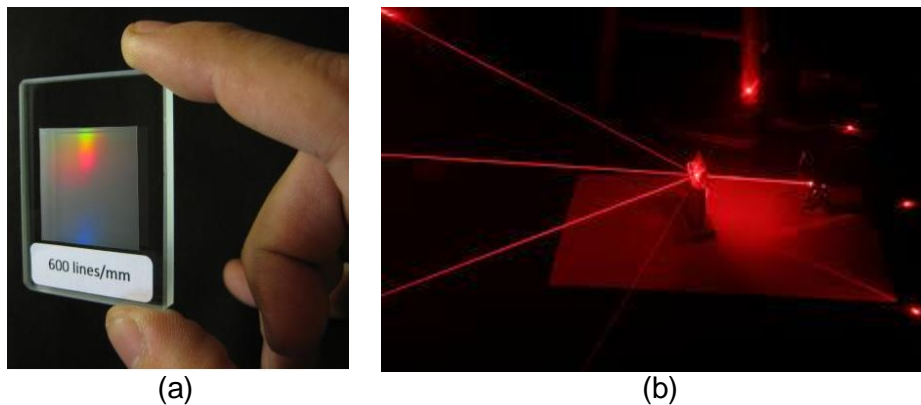
### EQUIPMENT

- Diffraction grating (80, 100, 300, 600 lines/cm)
- Discharge lamps (Helium, Mercury, etc )

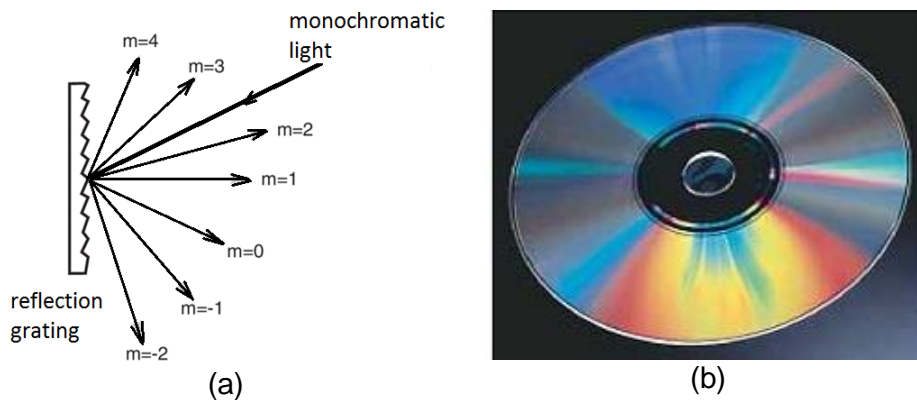
### THEORY

#### *The diffraction Grating*

A large number of equally spaced parallel slits is called a diffraction grating. Gratings containing 10,000 lines (or slits) per centimeter are common, and are very useful for precise measurements of wavelengths. Diffraction grating containing slits is called a *transmission grating* which we will use in our experiments, Figure 1. Another type of diffraction grating is the *reflection grating*, made by ruling fine lines on a metallic or glass surface from which light is reflected and analyzed, Figure 2. The analyses of both gratings are basically the same.



**Figure 1** (a) Transmission Grating which we will use in the lab. (b) Grating in use with red laser.



**Figure 2** (a) The reflection grating (b) A CD (illuminated with white light) is a diffraction grating

### Grating Analysis

The analysis of a diffraction grating is very similar to that of Young's double-slit experiment. We assume parallel rays of light are incident on the grating as shown in Figure. 3. If  $d$  is the distance between slits, then  $\Delta l = d \sin\theta$ , and

$$\sin \theta = \frac{m\lambda}{d} \quad (1)$$

is the criterion to have a brightness maximum  $m = 0, 1, 2, 3$ , etc. is called the order of the pattern. The pattern with  $m = 0$  is called zero order, that with  $m = \pm 1$ , first order, etc. Note that if the grating has  $N$  slits per cm then the distance between slits is  $d = 1/N$  cm.

The intensity distribution for wavelength  $\lambda$  is given by:

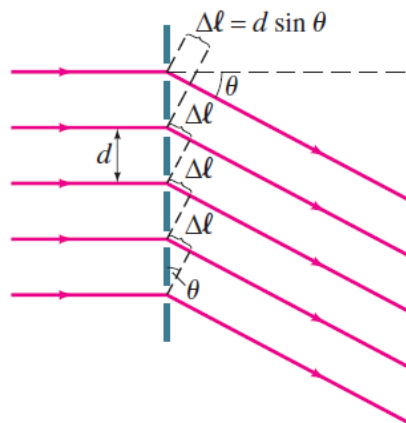
$$I = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin N\alpha}{\sin \alpha} \right)^2 \quad (2)$$

Here

$I_0$  = intensity in the  $\theta = 0$  direction emitted by any one of the slits.

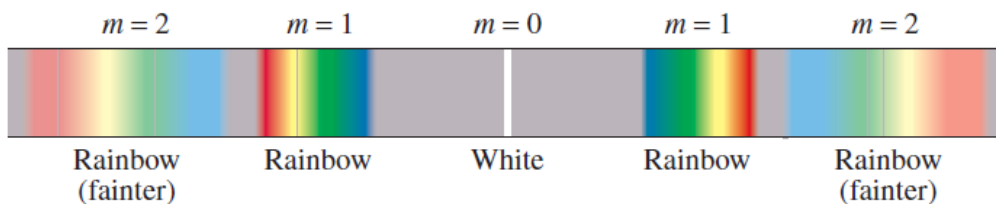
$\beta = \pi a \sin(\theta)/\lambda$

$\alpha = \pi d \sin(\theta)/\lambda$  (where  $a$  is the slit width and  $d$  is the slip separation).



**Figure 3:** Diffraction grating.  $\Delta l$  is the path difference.

Since  $d$  is grating constant and  $m$  is constant for a given order, the angle  $\theta$  changes with wavelength. For each order ( $m = \pm 1, \pm 2, \dots$ ) different wavelengths are separated out and observed as distinct spectrum. If white light strikes a grating, the central maximum ( $m = 0$ ) will be a sharp white line. But for all other orders, there will be a distinct spectrum of colors spread out over a certain angular width Figure 4. Because a diffraction grating spreads out light into its component wavelengths, the resulting pattern is called a *spectrum*.



**Figure 4:** Spectra produced by white light. The second order will normally be dimmer than the first order.













**Gas Discharge Lamps [Extracted from Wikipedia]**

Gas-discharge lamps are a family of artificial light sources that generate light by sending an electric discharge through an ionized gas, a plasma. Typically, such lamps use a noble gas (argon , neon , krypton , and xenon) or a mixture of these gases. Some include additional substances, like mercury, sodium, and metal halides, which are vaporized during startup to become part of the gas mixture.

In operation, some of the electrons are forced to leave the atoms of the gas near the anode by the electric field applied between the two electrodes, leaving these atoms positively ionized. The electrons ejected from these atoms flow onto the anode, while the cations thus formed are accelerated by the electric field and flow towards the cathode. Typically, after traveling a very short distance, the ions collide with neutral gas atoms, which transfer their electrons to the ions. The atoms, having lost an electron during the collisions, ionize and speed toward the cathode while the ions, having gained an electron during the collisions, return to a lower energy state while releasing energy in the form of photons . Light of a characteristic frequency is thus emitted. In this way, electrons are relayed through the gas from the anode to the cathode. The color of the light produced depends on the emission spectra of the atoms making up the gas, as well as the pressure of the gas, current density , and other variables.

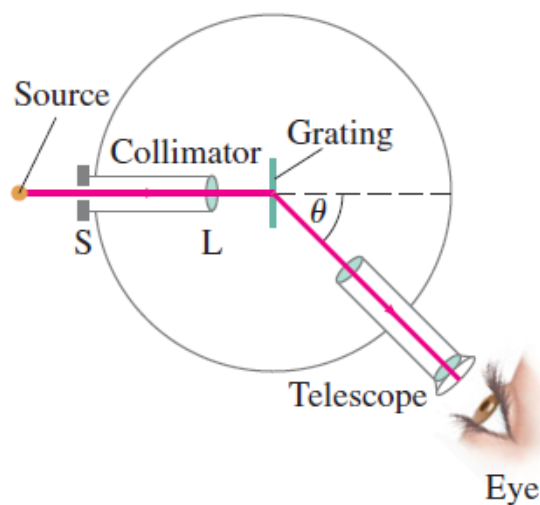
Each gas, depending on its atomic structure emits certain wavelengths, its emission spectrum, which determines the color of the light from the lamp. Some examples are given in Table 1.

**Table 1:** Gas discharge lamp and its spectra

Gas	Spectrum	Image
Helium		
Neon		
Argon		
Hydrogen		
Oxygen		
Mercury		

## Grating Spectrometer

A spectrometer or spectroscope, Figure 5, is a device to measure wavelengths accurately using a diffraction grating (or a prism) to separate different wavelengths of light.



**Figure 5:** The diffraction spectrometer

Light from a source passes through a narrow slit S in the “collimator.” The slit is at the focal point of the lens L, so parallel light falls on the grating. The movable telescope can bring the rays to a focus. Nothing will be seen in the viewing telescope unless it is positioned at an angle that corresponds to a diffraction peak (first order is usually used) of a wavelength emitted by the source. The angle can be measured to very high accuracy, so the wavelength can be determined to high accuracy using Equation 1:

$$\lambda = \frac{d}{m} \sin \theta \quad (3)$$

### *Use of Spectrometers in our Daily Life*

An important use of a spectrometer is for the identification of atoms or molecules. As described in the previous section, when a gas is heated or an electric current is passed through it, the gas emits a characteristic line spectrum. That is, only certain discrete wavelengths of light are emitted, and these are different for different elements and compounds. Sun, produces a continuous spectrum including a wide range of wavelengths. Figure 6 shows the Sun’s “continuous spectrum,” which contains a number of dark lines called absorption lines.

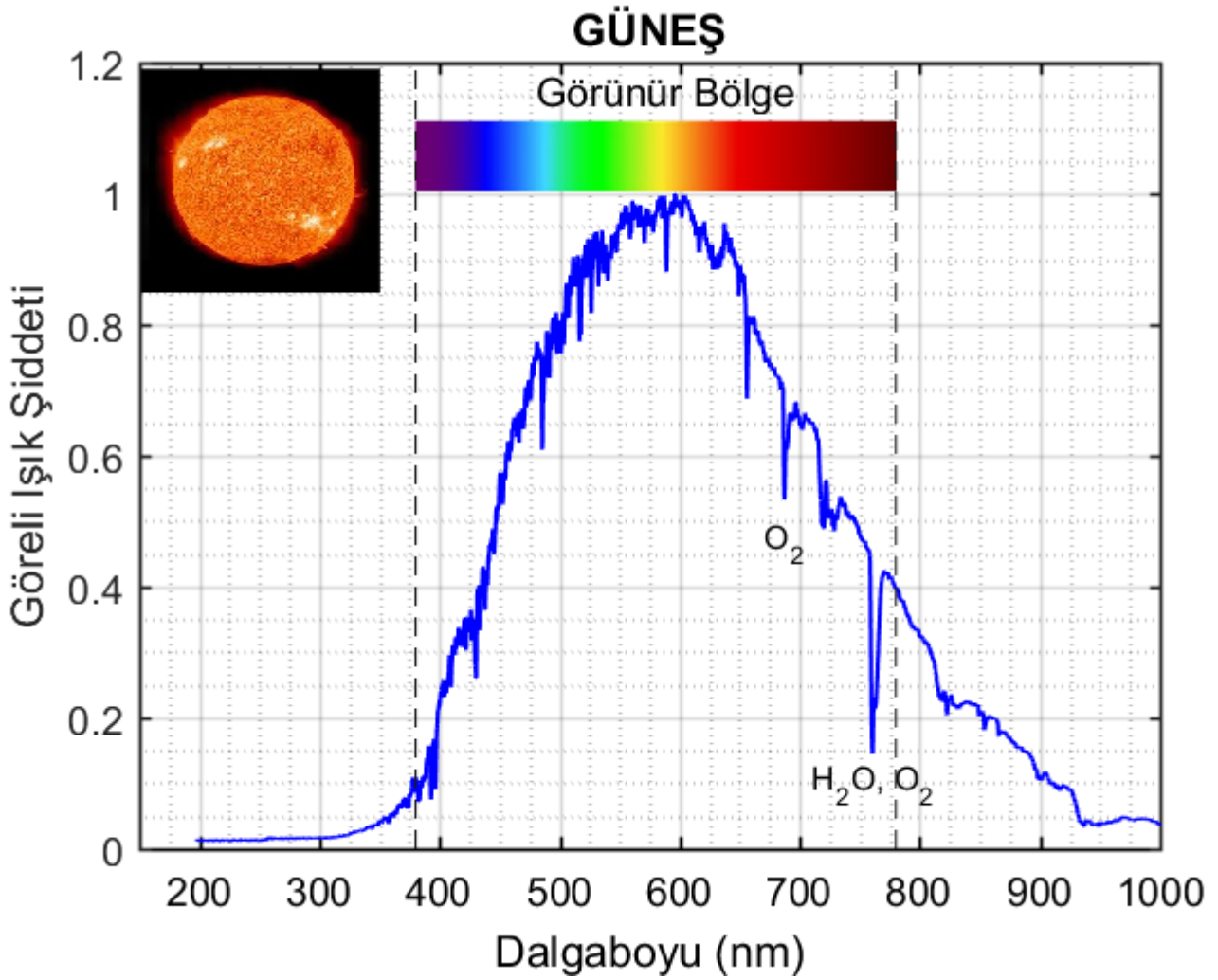
Spectroscopy is useful for determining the presence of certain types of molecules in laboratory specimens where chemical analysis would be difficult. For example, biological DNA and different types of protein absorb light in particular regions of the spectrum (such as in the UV). The material to be examined, which is often in solution, is placed in a monochromatic light beam whose wavelength is selected by the placement angle of a diffraction grating or prism. The amount of absorption, as compared to a standard solution without the specimen, can reveal not only the presence of a particular type of molecule, but also its concentration.

Light emission and absorption also occur outside the visible part of the spectrum, such as in the UV and IR regions. Glass absorbs light in these regions, so reflection gratings and mirrors (in place of lenses) are used. Special types of film or sensors are used for detection.





(a)



**Figure 6:** (a) Solar absorption spectrum. (b) Intensity distribution of radiation from Sun.

### A numerical example: Hydrogen Spectrum

Consider light emitted by hot hydrogen gas is observed with a spectrometer using a diffraction grating having 10000 lines/cm ( $d = 1/10000 = 1 \times 10^{-4}$  cm = 1000 nm). The spectral lines nearest to the center ( $0^\circ$ ) are a violet line at  $24.2^\circ$ , a blue line at  $25.7^\circ$ , a blue-green line at  $29.1^\circ$ , and a red line at  $41.0^\circ$  from the center. What are the wavelengths of these spectral lines of hydrogen?

From Equation 3: observed wavelength is proportional to  $\sin(\theta)$ ,  $\lambda = (d/m) \sin(\theta)$

$$\lambda_1 = (1000 \text{ nm}/1) \sin(24.2) = 410 \text{ nm}$$

$$\lambda_2 = (1000 \text{ nm}/2) \sin(25.7) = 434 \text{ nm}$$

$$\lambda_3 = (1000 \text{ nm}/3) \sin(29.1) = 486 \text{ nm}$$

$$\lambda_4 = (1000 \text{ nm}/4) \sin(41.0) = 656 \text{ nm}$$

*Note that: In an unknown mixture of gases, these four spectral lines need to be seen to identify that the mixture contains hydrogen.*

## PROCEDURE

*Referring to Figure 5.*

1. Place a discharge lamp adjacent to the collimator and switch it on. It will take about 10 minutes before the intensity of the light becomes constant. Looking through the collimator you should see a small rectangle of light coming from the discharge lamp. The rectangle is formed by the slit on the end of the collimator.
2. It is necessary to adjust the spectrometer so that the collimator produces parallel light and the telescope receives parallel light from the grating.
3. Fix the mount for the grating to the spectrometer table. Place the grating into the mount. The table height should be set so that the full width of the collimated beam falls on the grating, without obstruction by the table or grating mount.
4. Rotate the table of the spectrometer until the grating plane is perpendicular to the axis of the collimator and with the ruled surface facing the telescope.
5. Look through the telescope. You should see a sharp image of the slit in the center of the field of view. This is the central maximum ( $m = 0$ ). Qualitatively scan the spectrum by rotating the telescope in either direction from the on-axis alignment with the collimator, in order to determine how many orders are visible.
6. Realign the telescope arm of the spectrometer with the collimator. Looking through the telescope you should again see the image of the slit. If you did not observe the zero order exactly at zero ( $\theta = 0^\circ$ ), then you record this reading and make all subsequent measurements relative to this value.
7. Look through the telescope while swinging it around slowly, until a series of coloured spectral lines are seen again. Record your values in Table 2.
8. Repeat the same series of measurements on the opposite side of the zero position and fill your values in Table 2.
9. Repeat steps 1-8 for a different discharge lamp and Fill Table 3.

**Table 1:** Spectrum lines to be used. The wavelength values are in nm.

Helium	Mercury	Sodium
447.1 Blue-Violet	404.6 Violet	404.6 Violet
471.3 Blue	407.8 Violet	466.8 Blue
501.7 Green	435.8 Blue	498.2 Blue-Green
587.5 Yellow	491.6 Blue-Green	515.3 Blue-Green
667.8 Red	546.1 Green	588.9 Yellow
706.5 Red	577.0 Orange	615.4 Red

**Table 2:** Observed angle vs wavelength data

Gas discharge lamb name =				Grating spacing (d) =		
Color	$\theta$ in deg (cw rotation)	$\theta$ in deg (ccw rotation)	$\theta$ in deg (average)	$\lambda$ (calculated)	$\lambda$ (literature)	% Error

**Table 3:** Observed angle vs wavelength data

Gas discharge lamb name =				Grating spacing (d) =		
Color	$\theta$ in deg (cw rotation)	$\theta$ in deg (ccw rotation)	$\theta$ in deg (average)	$\lambda$ (calculated)	$\lambda$ (literature)	% Error

**QUESTIONS**

1. What is the advantage of using the reflection grating in some optical devices?
2. The bright maxima are much sharper and narrower for a grating compared to a double slit. Why?
3. White light strikes a grating containing  $N$  lines/cm. For which value of  $N$ , blue color of the third-order spectrum overlaps the red of the second order?
4. What is the origin of absorption lines of Sun?
5. Fill Table 2 and Table 3 for two different gas discharge lamb.
6. What is your overall conclusion?

## **A P P E N D I C E S**

**Appendix A: Experimental Errors**

**Appendix B: Basic MATLAB Tutorial**

**Appendix C: Sample Written Report**

# Appendix A: Experimental Errors

## A1. Introduction

In general, experiments are performed to test a theory and to compare with other independent experiments measuring the same quantity. When performing experiments, we should not consider that the job is over once we obtain a numerical value for the quantity we are trying to measure. We should be concerned not only with the measured value but also with its *accuracy*. The accuracy is derived from an *experimental error* for the measured quantity. For example, consider a measurement of the gravitational acceleration due to the gravity is obtained as

$$g = 9.77 \pm 0.14 \text{ m/s}^2$$

We will mention about what we mean by the error (or accuracy)  $\pm 0.14$ .

## A2. Definitions

In this section, we will give some fundamental terms used in the measurement systems.

### Error

The error is the difference between true value and measured value of a quantity.

### Uncertainty

The uncertainty is the estimated error for a measurement. Since a true value for a physical quantity may be unknown, it is sometimes not possible to determine the true error of the measurement. So, this term is more sensible for the error computations although the terms error and uncertainty are used instead of each other in literature.

### Percentage Error

Percentage error measures the accuracy of a measurement by the difference between a measured or experimental value  $E$  and a true or accepted value  $A$ . Percentage error is calculated by

$$\frac{|E - A|}{A} \times 100\% \quad (\text{A.1})$$

### Percentage Difference

Percentage difference measures precision of two measurements by the difference between the measured or experimental values  $E_1$  and  $E_2$  expressed as a fraction the average of the two values. The equation to use to calculate the percent difference is:

$$\frac{|E_1 - E_2|}{(E_1 + E_2)/2} \times 100\% \quad (\text{A.2})$$

### Mean and Standard Deviation

For a set of  $n$  measurements  $\{x_1, x_2, \dots, x_n\}$ , the arithmetic mean, is defined by:

$$\langle x \rangle = \bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (\text{A.3})$$

The Standard deviation is a measure that is used to quantify the amount of variation or dispersion of the measured values. It is defined as:

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} \quad (\text{A.4})$$

Note that the square of the standard deviation is known as *variance*.

The standard error of the mean is given by:

$$\sigma_E = \frac{\sigma}{\sqrt{n}} \quad (\text{A.5})$$

Final result obtained from this kind of repeated measurement should be reported in the form of

$$\bar{x} \pm \sigma_E$$

**EXAMPLE A1:** Consider the data of 10 different measurements for the mass density in  $\text{g/cm}^3$  of a liquid.

$$d = \{1.10, 1.12, 1.09, 1.09, 1.07, 1.14, 1.11, 1.16, 1.07, 1.08\}$$

The mean, standard deviation, variance and standard error of the measurement are as follows:

$$\bar{x} = (1.10 + 1.12 + 1.09 + 1.09 + 1.07 + 1.14 + 1.11 + 1.16 + 1.07 + 1.08)/10 = 1.103 \text{ g/cm}^3$$

$$\sigma = \sqrt{[(1.10 - 1.103)^2 + (1.12 - 1.103)^2 + \dots + (1.08 - 1.103)^2]}/9 = 0.030 \text{ g/cm}^3$$

$$\sigma^2 = (0.03 \text{ g/cm}^3)^2 = 0.0009 (\text{g/cm}^3)^2$$

$$\sigma_E = 0.030/\sqrt{10} = 0.009 \text{ g/cm}^3$$

The result of the measurement is reported as  $d = 1.103 \pm 0.009 \text{ g/cm}^3$  or  $1.103(9) \text{ g/cm}^3$ .

### A3. Measurement Errors for Some Devices (Instrumental Limitations)

The values of experimental measurements have uncertainties due to measurement limitations. Here we will show the uncertainty for two mostly used devices in the labs.

#### Ruler

In Fig A1, the pointer indicates a value between 23 and 24 mm. With this millimeter scale one strategy is to take the center of the bin as the estimate of the value, the maximum error is then half the width of the bin. So in this case our measurement is  $23.5 \pm 0.5 \text{ mm}$ . The value of 0.5 mm is the estimate of the random error.



**Fig A1:** Standard ruler has a maximum uncertainty of  $\pm 0.5 \text{ mm}$ . So, the value and its error is  $23.5 \pm 0.5 \text{ mm}$ .

#### Digital Measuring Devices

All digital measuring devices has a maximum uncertainty of the order of half its last digit. For example, in Fig 5, for the reading from a digital voltmeter, the uncertainty is  $\pm 0.01/2 \text{ Volts}$ . Thus, assuming the voltmeter is calibrated accurately, the value is  $12.88 \pm 0.005 \text{ V}$ .



**Fig A2:** Digital voltmeter having a maximum uncertainty of  $\pm 0.005 \text{ V}$  for this reading. So the value and its error is  $12.880 \pm 0.005 \text{ V}$ .

### A4. Error Propagation

In a typical experiment, one is seldom interested in taking data of a single quantity. More often, the data are processed through multiplication, addition or other functional manipulation to get final result. Experimental measurements have uncertainties due to measurement limitations which propagate to the combination of variables in a function.

In statistical data analysis, the propagation of error (or propagation of uncertainty) is the effect of measurement uncertainties (or errors) on the uncertainty of a function based on them.

For a function of one variable,  $f(x)$ , if  $x$  has a measurement error  $\sigma_x$ , then the associated error is computed from:

$$\sigma_f = \left| \frac{\partial f}{\partial x} \right| \sigma_x \quad (\text{A.6})$$

**EXAMPLE A2:** The radius of a circle is measured as  $r = 2.5 \pm 0.3$  cm.

Calculate the area of the circle and its uncertainty.

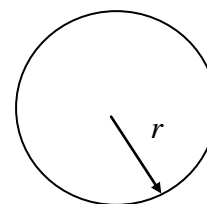
The area of the circle is:  $A = \pi r^2 = \pi(2.5)^2 = 19.6 \text{ cm}^2$

Its derivative with respect to  $r$  is:

$$\frac{\partial A}{\partial r} = 2\pi r = 2\pi(2.5) = 15.7 \text{ cm}$$

and the uncertainty in the area is:

$$\sigma_A = \frac{\partial A}{\partial r} \sigma_x = (15.7)(0.3) = 4.7 \text{ cm}^2$$



Therefore the area and its uncertainty is  $A = 19.6 \pm 4.7 \text{ cm}^2$ .

In general, if  $x, y, z, \dots$  are independent variables having associated errors (standard deviations)  $\sigma_x, \sigma_y, \sigma_z, \dots$  then the error for any quantity of the form  $f = f(x, y, z, \dots)$  derived from these errors can be calculated from:

$$\sigma_f^2 = \left( \frac{\partial f}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial f}{\partial y} \right)^2 \sigma_y^2 + \left( \frac{\partial f}{\partial z} \right)^2 \sigma_z^2 + \dots \quad (\text{A.7})$$

Equation (A.7) is known as the *error propagation formula* used for uncorrelated (independent) measurements.

**EXAMPLE A3:** Suppose we wish to calculate the average speed (displacement/time) of an object. Assume the displacement is measured as  $x = 22.2 \pm 0.5$  cm during the time interval  $t = 9.0 \pm 0.1$  s. Then,

$$v = \frac{x}{t} = \frac{22.2}{9.0} = 2.47 \text{ cm/s}$$

The derivatives are:

$$\frac{\partial v}{\partial x} = \frac{1}{t} = 0.11 \text{ and } \frac{\partial v}{\partial t} = -\frac{x}{t^2} = -0.27$$

Finally the error in the speed is

$$\sigma_v = \sqrt{0.11^2 \times 0.50^2 + 0.27^2 \times 0.10^2} = 0.06 \text{ cm/s}$$

Therefore, the reported result would be  $v = 2.47 \pm 0.06 \text{ cm/s}$  or  $v = 2.47(6) \text{ cm/s}$ .

**Exercise:** Refractive index,  $n$ , of a glass is calculated by using Snell's law:  $n = n_{\text{air}} \sin(\theta_1) / \sin(\theta_2)$  where measured values and their estimated errors are given by  $n_{\text{air}} = 1$ ,  $\theta_1 = 61(1)$  degrees and  $\theta_2 = 36(1)$  degrees. Calculate the refractive index of the glass and its uncertainty. [Answer:  $n = 1.488(24)$ ].

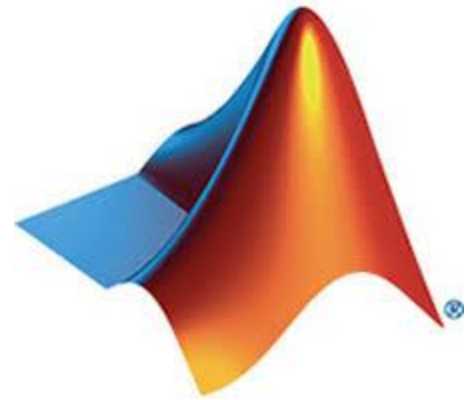
# Appendix B: Basic MATLAB Tutorial

<http://www.mathworks.com/products/matlab>

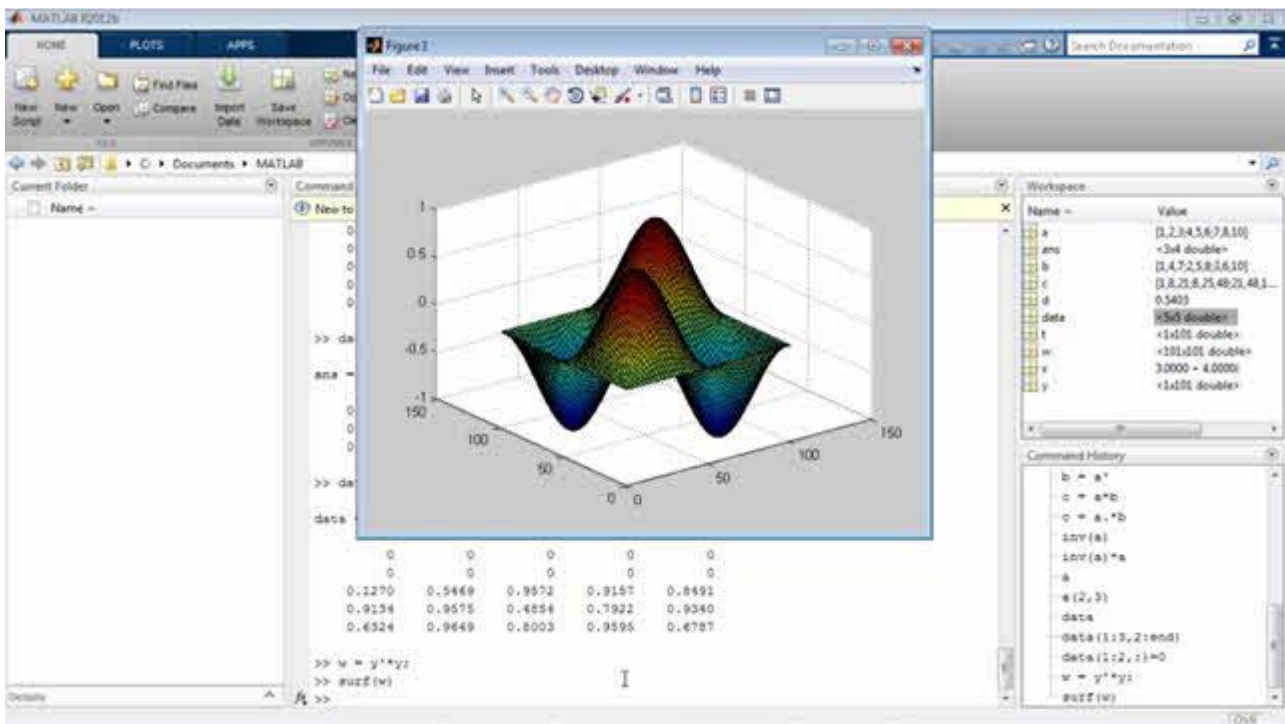
<http://www1.gantep.edu.tr/~bingul/ep375>

This is a basic tutorial for the Matlab program which is a high-performance language for technical computing for platforms:

- Linux,
- MS Windows
- Mac



Sep 2017



MATLAB integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.

Typical uses include

- Math and computation
- Algorithm development
- Data acquisition Modeling,
- simulation, and prototyping
- Data analysis,
- Scientific and engineering graphics
- Curve fitting
- etc



## Basic Commands

- \* `help command`      get help for a command
- \* `clear`                    clears all the memory (workspace)
- \* `clear x`                    removes variable `x` from the memory
- \* `whos`                        lists all the variables (and details) on the workspace
- \* Semicolon (;) at the end will suppress the output
- \* Command history: upper & lower arrows, also command name guess
- \* If you don't use a variable name, your calculation is labeled and assigned by `ans` variable.

Variables	Arrays	Matrices	Complex numbers
<pre>&gt;&gt; x = 4; &gt;&gt; y = sqrt(x) + 2 y = 4</pre>	<pre>&gt;&gt; A = [5 -3 4 2]; &gt;&gt; B = [0 3 9 4];  &gt;&gt; A.*B 0 -9 36 8  &gt;&gt; A./B ans = inf -1 0.4444 0.5  &gt;&gt; A.^B ans = 1 -27 262144 16</pre>	<pre>&gt;&gt; B=[1 2 3;0 5 6;7 8 9] B = 1 2 3 0 5 6 7 8 9  &gt;&gt; det(B) ans = -24  &gt;&gt; inv(B) ans = 0.1250 -0.2500 0.1250 -1.7500 0.5000 0.2500 1.4583 -0.2500 -0.2083</pre>	<pre>&gt;&gt; z = 3 + 4i; &gt;&gt; abs(z) ans = 5</pre>

## Some Intrinsic Functions

<u>Function</u>	<u>Description</u>
<code>abs(x)</code>	$ x $
<code>sin(x)</code>	sine of $x$ (x is in radian)
<code>cos(x)</code>	cosine of $x$
<code>tan(x)</code>	tangent of $x$
<code>sind(x)</code>	sine of $x$ (x is in degrees)
<code>cosd(x)</code>	cosine of $x$
<code>tand(x)</code>	tangent of $x$
<code>asin(x)</code>	angle in radian from $\sin(x)$
<code>acos(x)</code>	angle in radian from $\cos(x)$
<code>atan(x)</code>	angle in radian from $\tan(x)$
<code>log(x)</code>	$\ln(x)$
<code>log10(x)</code>	$\log_{10}(x)$
<code>exp(x)</code>	$e^x$
<code>mod(x,y)</code>	$x$ modulo $y$ (mod(12,5) = 2)

Solutions of linear algebraic equations	Roots
$\begin{aligned} x + y + z &= 6 \\ -2x + y &= 0 \\ 3x + 2y + z &= 10 \end{aligned}$ <pre>&gt;&gt; A = [1 1 1; -2 1 0; 3 2 1]; &gt;&gt; b = [6 0 10]'; &gt;&gt; x = inv(A) * b x = 1 2 3</pre>	<p>Find the roots of <math>x^5 + 2x^4 - 5x^3 + x + 3 = 0</math></p> <pre>&gt;&gt; c = [1 2 -5 0 1 3]; &gt;&gt; roots(c) ans = -3.4473 1.1730 + 0.3902i 1.1730 - 0.3902i -0.4494 + 0.6062i -0.4494 - 0.6062i</pre> <p>Find the root of <math>\sin(x)+2-x=0</math></p> <pre>&gt;&gt; solve('sin(x)+2-x=0') ans = 2.5542</pre>

## Curve Fitting

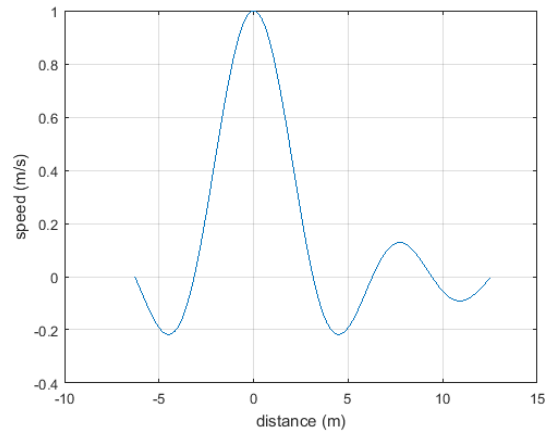
```
>> cftool
```

### Basic Statistics

```
>> x = [1 2 2.5 3 3.1];  
>> sum(x)  
ans = 11.6000  
  
>> mean(x)  
ans = 2.3200  
  
>> std(x)  
ans = 0.8585  
  
>> max(x)  
ans = 3.1000  
  
>> min(x)  
ans = 1  
  
>> length(x)  
ans = 5
```

### Plotting

```
>> x = -2*pi:0.1:4*pi;  
>> y = sin(x)./x;  
>> plot(x, y)  
>> xlabel('distance (m)')  
>> ylabel('speed (m/s)')  
>> grid on
```



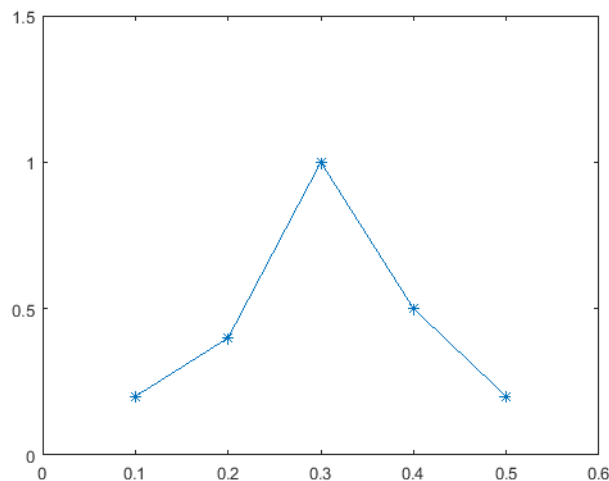
## Reading Data file

Consider you have two column data file called `data.txt`. Let its content be:

```
0.1 0.2  
0.2 0.4  
0.3 1.0  
0.4 0.5  
0.5 0.2
```

In MATLAB command line you can read and plot data as follows:

```
>> [x y] = textread('data.txt');  
>> plot(x, y, '*')  
>> axis([0.0 1.0 0.0 1.5])
```



## Appendix C: Sample Written Report

In this section, we will present you a sample report (starting from the next page), obtained in the experiment named “Snell’s Law of Refraction” (Experiment 4 of OPAC103).

- Try to obey the rules as in this sample.
- Bring your reports to lab research assistant on time (weekly).
- Include to your report the data sheet(s) signed by the lab assistant in the lab.



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Department of Optical and Acoustical Engineering  
OPAC 103 Geometric Optics Laboratory

LAB REPORT FOR EXPERIMENT 4  
SNELL'S LAW OF REFRACTION

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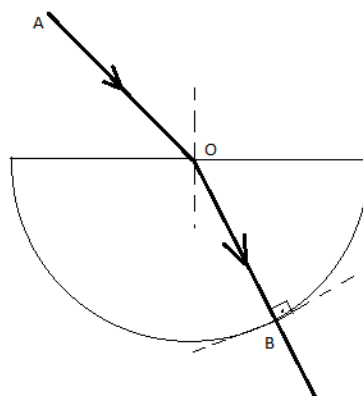
06 Oct 2017

The data we have obtained in the experiment is as follows:

Angle of incidence $\theta_1$ (in degrees)	Angle of refraction $\theta_2$ (in degrees)	$\sin(\theta_1)$	$\sin(\theta_2)$	index of refraction $n$
5	3	0.087156	0.052336	1.6653
10	7	0.173648	0.121869	1.4249
15	10	0.258819	0.173648	1.4905
20	13	0.342020	0.224951	1.5204
25	16	0.422618	0.275637	1.5332
30	19	0.500000	0.325568	1.5358
35	22	0.573576	0.374607	1.5311
40	25	0.642788	0.422618	1.5210
45	28	0.707107	0.469472	1.5062
50	31	0.766044	0.515038	1.4874
55	33	0.819152	0.544639	1.5040
60	35	0.866025	0.573576	1.5099
65	37	0.906308	0.601815	1.5060
70	39	0.939693	0.629320	1.4932
Average, $\langle n \rangle =$				<b>1.5163</b>
Standard Deviation, $\sigma =$				<b>0.0511</b>
Standard Error, $\sigma_E =$				<b>0.0137</b>

### Answers of Questions:

1. The D-shaped material has an advantage in this experiment. First, the light enters from its flat surface and refracts according to Snell's law, Equation 1 in the lab sheet. Then the light travels inside the material and leaves it from the circular side. If the light enters nearly the center of the D-shape as in Figure 1, then it hits the circular face at right angle (perpendicular). As a result, the light leaves the material without refracting. Therefore, we can only apply Snell's law on the first surface.



**Figure 1:** Light follows the path AOB. At point O it refracts. At point B, it will leave the D-Shape without refracting since Snell's law does not work for the light rays normally entering or leaving the surface.

2. From the table, the mean value of the data and its standard error is:

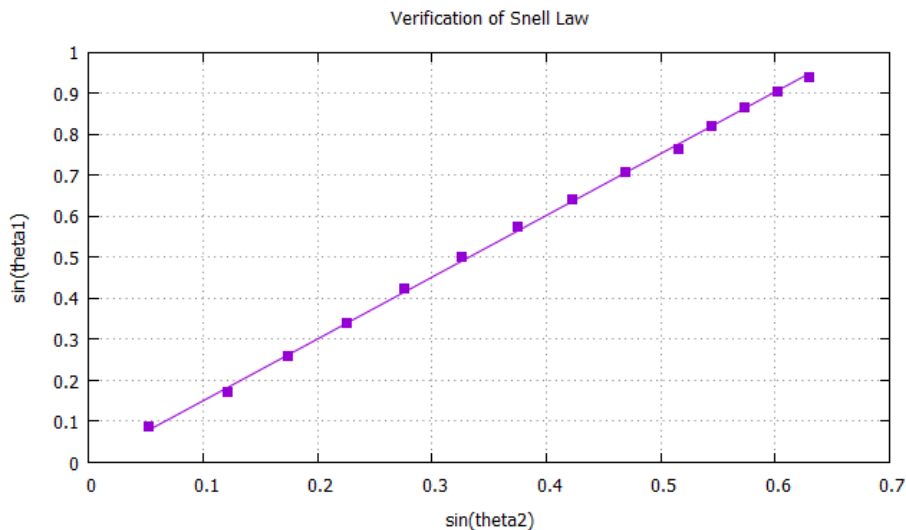
$$n = 1.5163 \pm 0.0137$$

Or the interval that encloses the true index of refraction (with 68% confidence level) is:

$$n = [1.5026, 1.5300].$$

3. Alternatively, we can extract the index of refraction using linear fitting. Figure 2 shows the result of fitting procedure applied for our data where  $\sin(\theta_2)$  values are plotted in x-axis and  $\sin(\theta_1)$  values are plotted in y-axis. According to Snell's law  $\sin(\theta_2) = n \sin(\theta_1)$ , that is,  $\sin(\theta_2)$  is linearly proportional to  $\sin(\theta_1)$  and the proportionality constant (or the slope) is  $n$ , the index of refraction. Therefore, the slope of the line obtained from fitting procedure gives the index of refraction. The extracted value using gnuplot program is:

$$n = 1.50665 \pm 0.004418.$$



**Figure 2:** Linear fitting result using gnuplot. The slope of the best fit line gives the index of refraction

The gnuplot script (called `fitter.txt`) used to extract  $n$  is given below. The file `data.txt` contains the values of  $\sin(\theta_2)$  vs  $\sin(\theta_1)$  values in the form of two columns, respectively. The fitting result is also given.

<pre># fitter.txt reset set grid unset key set title 'Verification of Snell Law' set xlabel 'sin(theta2)' set ylabel 'sin(theta1)' f(x) = n*x fit f(x) 'data.txt' via n plot 'data.txt' pt 5 ps 1, f(x) lc 1</pre>	<pre>Final set of parameters          Asymptotic Standard Error ===== n                               = 1.50665          +/- 0.004418      (0.2932%)</pre>
--	--

#### 4. Conclusion

In this experiment, Snell's law of refraction is validated by using two different methods, the arithmetic mean and linear fitting method. The aim of the experiment is to measure the index of refraction of a material by collecting several pairs of the angle of incidence and angle of refraction data. The results of the method are  $n = 1.5163 \pm 0.0137$  and  $n = 1.50665 \pm 0.004418$ , respectively. The actual value of the index of refraction is  $n = 1.5$ . Hence, the percentage errors are as follows:

$$\begin{aligned} \text{P.E. (mean)} &= 100\% \times (1.51630 - 1.5000) / 1.5000 = 1.1\% \\ \text{P.E. (fit)} &= 100\% \times (1.50665 - 1.5000) / 1.5000 = 0.4\% \end{aligned}$$

The results of both methods are quite consistent with the expected value. However, the fitting method exhibits a better performance since its associated error and its percentage error is much smaller.