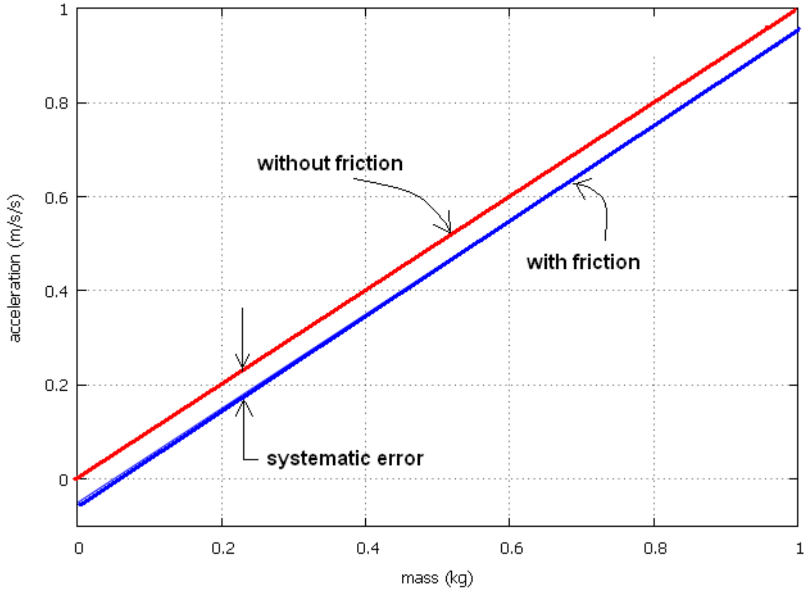
Lecture notes on
* Measurement and Error
* Least Square Fitting



Department of
Optical Engineering

University of Gaziantep

Oct 2021

PART I

Measurement and Error

System of Units

- Physics is an experimental science.
- Measurments must be expressed in units.

- There are several systems of units in use today.

1. **International System (SI) or MKS units (we'll use this)**
2. **CGS units**
3. **British Gravitational (BG)**
4. **U.S. Customary units**

e.g: Length can be measured in

meters, centimeters, yards, inches, etc.

SI Name	Symbol	Definition
Meter	**m**	***The length*** **of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.**
Kilogram	**kg**	***The mass*** **of the international prototype of the kilogram**
Second	**s**	***The duration*** **of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.**
Ampere	**A**	**The constant** ***electric current*** **which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.**
Kelvin	**K**	**The fraction 1/273.16 of the** ***thermodynamic temperature*** **of the triple point of water.**
Mole	**mol**	***The amount of substance*** **of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12 atom.**
Candela	**cd**	***The luminous intensity*** **in a given direction, of a light source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian.**

Derived SI Units

- Relying on the base units, all other units of measurement can be formed.

Quantity	Symbol	SI	Derived SI
Force	*F*	$kg.m/s^2$	Newton, N
Energy	*E*	$kg.m^2/s^2$	Joule, J
Pressure	*P*	$kg/m.s^2$	Pascal, Pa

Scaling Prefixes of SI Units

Multiplication Factor	Prefix	SI sysmbol
1,000,000,000,000,000,000,000,000 = 10^{24}	yotta	Y
1,000,000,000,000,000,000,000 = 10^{21}	zetta	Z
1,000,000,000,000,000,000 = 10^{18}	exa	E
1,000,000,000,000,000 = 10^{15}	peta	P
1,000,000,000,000 = 10^{12}	tera	T
1,000,000,000 = 10^{9}	giga	G
1,000,000 = 10^{6}	mega	M
1000 = 10^{3}	kilo	k
100 = 10^{2}	hecto	h
10 = 10^{1}	deka	da
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000,001 = 10^{-6}	micro	μ
0.000,000,001 = 10^{-9}	nano	n
0.000,000,000,001 = 10^{-12}	pico	p
0.000,000,000,000,001 = 10^{-15}	femto	f
0.000,000,000,000,000,001 = 10^{-18}	atto	a
0.000,000,000,000,000,000,001 = 10^{-21}	zepto	z
0.000,000,000,000,000,000,000,001 = 10^{-24}	yocto	y

Examples:

1 GHz = 10^{9} Hz

1 MW = 10^{6} W

1 kPa = 10^{3} Pa

1 mm = 10^{-3} m

1 μF = 10^{-6} F

Significant Figures (s.f.)

Number of digits used to express a number carries information about how precisely the number is known.

E.g: A stopwatch reading of 5.3 s (2 s.f.) is less precise than a reading of 5.32 s (3 s.f.)

The rules for significant figures:

Number	Number of s.f.	Reason	Scientific notation
504	3	in an integer all digits count (if last digit is not zero)	5.04×10^2
608 000	3	zeros at the end of an integer do not count	6.08×10^5
200	1	zeros at the end of an integer do not count	2×10^2
0.000 305	3	zeros in front do not count	3.05×10^{-4}
0.005 900	4	zeros at the end of a decimal count, those in front do not	5.900×10^{-3}

In addition, subtraction, multiplication and division, the result must have as many s.f. as the **least** precisely known number entering the calculations.

$$3.21 + 4.1 = 7.32 \approx 7.3$$

$$12.367 - 3.15 = 9.217 \approx 9.22$$

$$23 \times 578 = 13294 \approx 1.3 \times 10^4$$

$$6.244 / 1.25 = 4.9952 \approx 5.00$$

Rounding the number 542.48 = 5.4248×10^2

$5.4248 \times 10^2 \approx 5.4 \times 10^2$ **(rounded to 2 s.f.)**

$5.4248 \times 10^2 \approx 5.42 \times 10^2$ **(rounded to 3 s.f.)**

$5.4248 \times 10^2 \approx 5.425 \times 10^2$ **(rounded to 4 s.f.)**

Measurement (ölçme)

All measurements consist of three parts:

- magnitude,
- unit
- uncertainty

A measurement result of speed of light can be:

$$c = 299793 \pm 5\,\mathrm{km/h}$$

Experiment (deney)

In general, experiments are performed

- to test a theory
- to compare with other independent experiments measuring the same quantity.

- When performing experiments, we should be concern not only with the measured value but also with its accuracy.

- For example, consider a measurement of the gravitational acceleration results in:

$$g = \underbrace{9.77}_{\text{Measured value}} \pm \underbrace{0.14}_{\text{Measurement error}} \text{ m/s}^2$$

Any measured quantity has an associated error.

Error & Uncertainty

Error (=hata)

is the difference between
true value and measured value of a quantity.

In general, the true value for a physical quantity is unknown.

Uncertainty (=belirsizlik)

is the estimated error for a measurement.

The terms error and uncertainty are used instead of each other in literature.

Suppose we are asked to measure the length of a block of glass. Our experimental error depends on the method of measurement.

Method	Typical error
Cheap ruler	0.5 mm
Calipers	0.05 mm
Travelling microscope	0.005 mm
Interferometer	0.00001 mm

Sources of Errors

There are mainly two types of experimental error:

- **Systematic errors**
- **Statistical (or Random) errors**

Systematic errors biases measurements in the same direction.

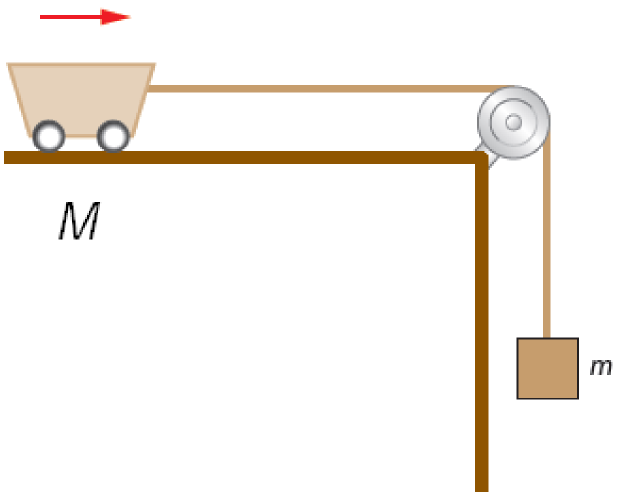
The results are too high or too low

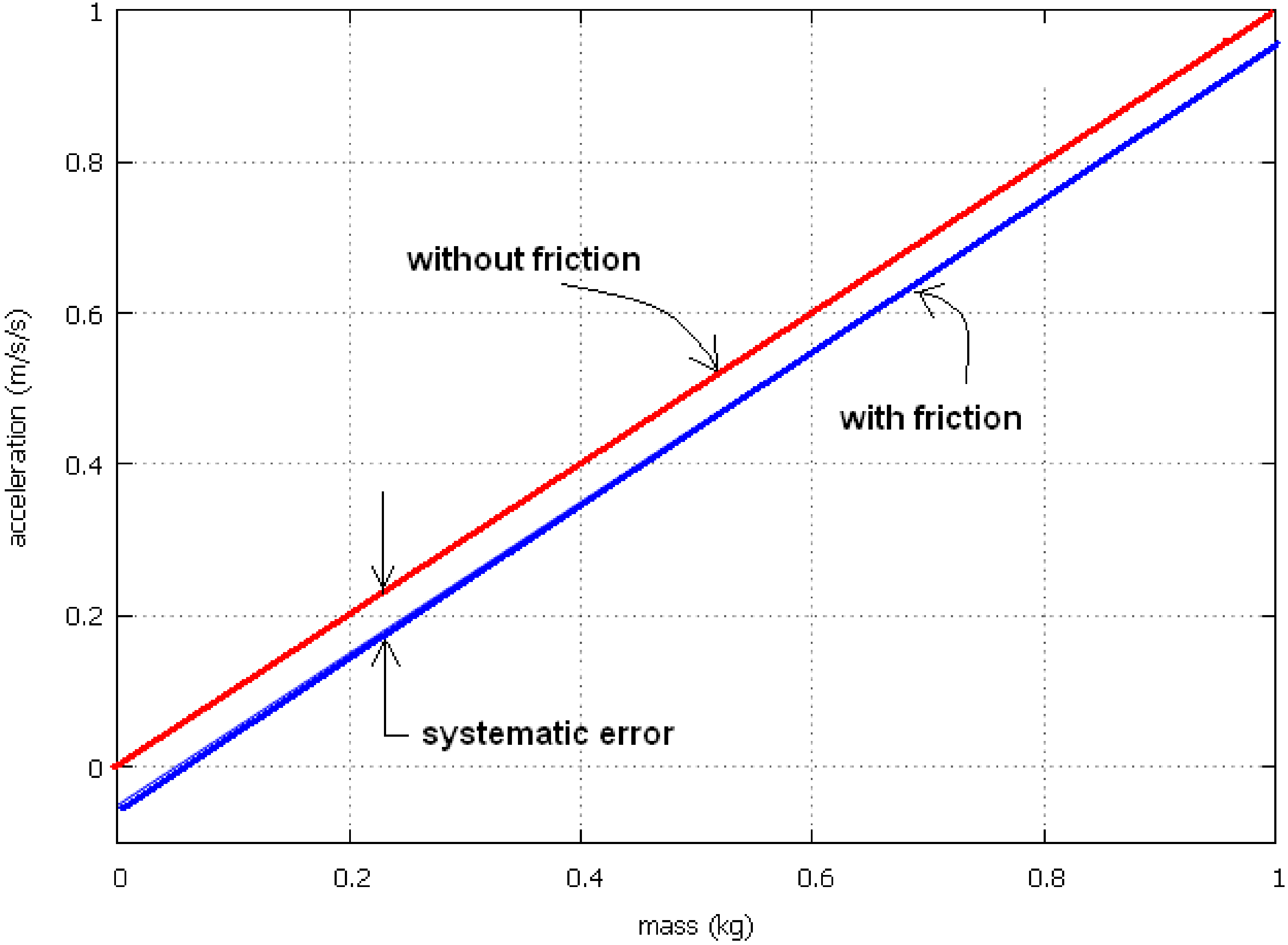
* If you use an ampermeter that shows a current 0.1 A even before connected to a circuit, every measument of current will be larger than the true value by 0.1 A.

* Suppose you want to test Newton's 2nd law.
The accelation of mass m is

$a = mg/(m+M)$ if you ignore friction

$a = mg/(m+M) - f/(m+M)$ if you dont ignore friction




without friction
with friction
systematic error
acceleration (m/s/s)
mass (kg)
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Statistical errors are random in nature.

Repeated measurements will differ from each other and from the true value by amounts which are not individually predictable.

Random errors are unbiased.

* Suppose you ask ten people to use stopwatches to measure the time it takes an athlete to run a distance of 100 m.
  You will get 10 different result (Why?).
  If you calculate average of them you will get a better estimate for the time.

* Using a digital calipper try to measure length of an object.
  10 students will get 10 diiferent results!

Measurement Errors for Some Devices (Instrumental Limitations)

- The values of experimental measurements have uncertainties due to *measurement limitations*.
- Here we will show the uncertainty for two mostly used devices in the labs.

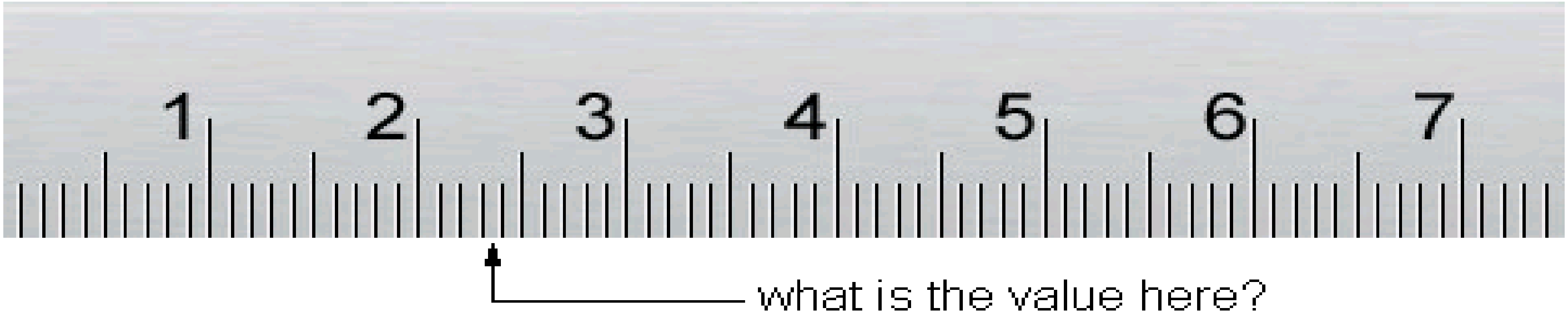
Measurement Errors for Some Devices (Instrumental Limitations)

Ruler

In Fig, the pointer indicates a value between 23 and 24 mm. With this millimeter scale one strategy is to take the center of the bin as the estimate of the value, the maximum error is then half the width of the bin. So in this case our measurement is

$$23.5 \pm 0.5 \, \mathrm{mm}$$

The value of 0.5 mm is the estimate of the random error.

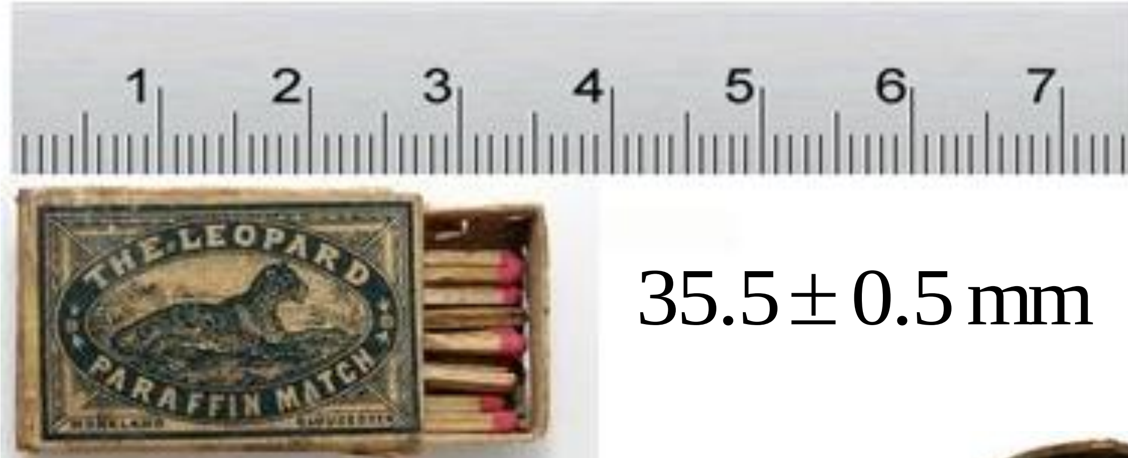


Digital Measuring Devices

All digital measuring devices has a maximum uncertainty of the order of half its last digit. For example, in Fig, for the reading from a digital voltmeter, the uncertainty is $\pm 0.01/2$ Volts.

Thus, assuming the voltmeter is calibrated accurately, the measured voltage is

$$9.160 \pm 0.005\,\mathrm{V}$$



35.5 ± 0.5 mm

12.880 ± 0.005 V

10.55 ± 0.25 m/s

$71 \pm 1\ ^{o}C$

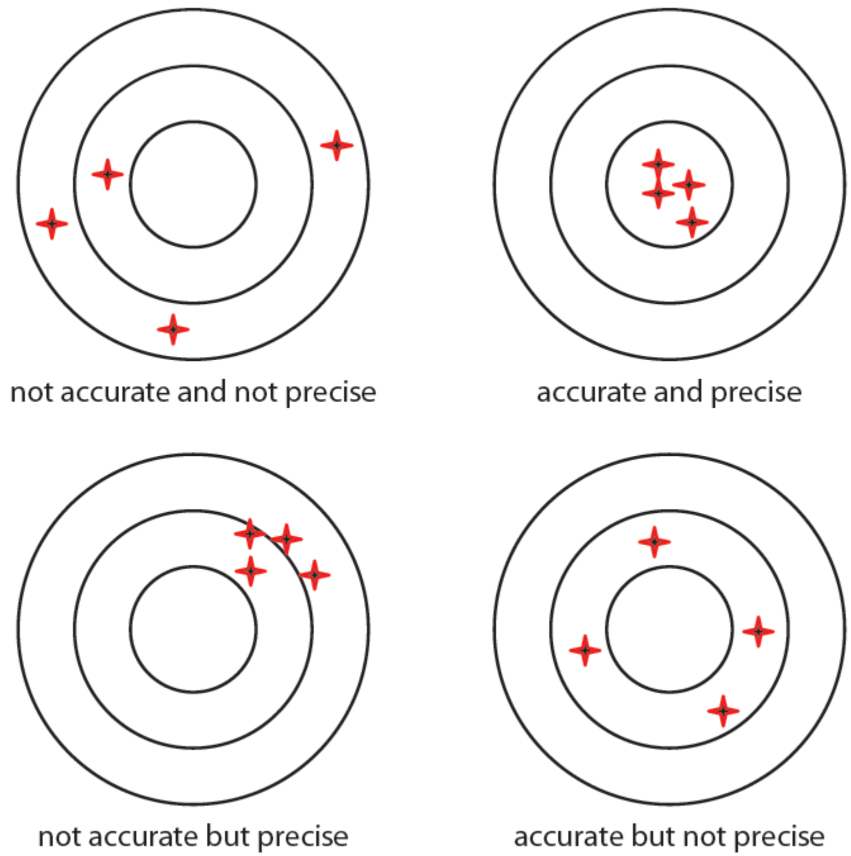
Accuracy & Precision
(Doğruluk & Kesinlik)

A measurement is said to be accurate if the systematic error is small.

The measured value is close to accepted value.

A measurement is said to be precise if the statistical error is small.

Individual measurement results are close to each other



Percentage Error

Percentage Error measures the accuracy of a measurement by the difference between
a measured or experimental value E and
a true or accepted value A.

Percentage error is calculated by:

$$PE = \frac{|E - A|}{A} \times 100\,\%$$

PART II

Basic Statistics

Mean and Standard Deviation

For a set of *N* measurements $\{x_1, x_2, x_3, ..., x_N\}$:

The arithmetic mean: $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

The standard deviation: $\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$

The standard error: $\sigma_E = \frac{\sigma}{\sqrt{N}}$

- Note that the square of the standard deviation is known as variance.

$$Variance \equiv \sigma^2$$

- Final result obtained from this kind of measurement should be reported in the form of:

$$\bar{x} \pm \sigma_E$$

Example

Consider the data of 10 different measurements for the mass density in g/cm^3 of a liquid.

d = { 1.10, 1.12, 1.09, 1.09, 1.07, 1.14, 1.11, 1.16, 1.07, 1.08}

The mean, standard deviation, variance and standard error of the measurement are as follows:

$$\bar{x} = (1.10 + 1.12 + 1.09 + 1.09 + 1.07 + 1.14 + 1.11 + 1.16 + 1.07 + 1.08)/10 = 1.103\ \mathrm{g/cm^3}$$

$$\sigma = \sqrt{[(1.10 - 1.103)^2 + (1.12 - 1.103)^2 + \cdots + (1.08 - 1.103)^2]/9} = 0.030\ \mathrm{g/cm^3}$$

$$\sigma^2 = 0.0009\ (\mathrm{g/cm^3})^2$$

$$\sigma_E = 0.03/\sqrt{10} = 0.009\ \mathrm{g/cm^3}$$

The result of the measurement is reported as $d = 1.103 \pm 0.009$ g/cm^3 or 1.103(9) g/cm^3.

Error Propagation (=Hata Birikimi)

In a typical experiment, one is seldom interested in taking data of a single quantity. More often, the data are processed through multiplication, addition or other functional manipulation to get final result. Experimental measurements have uncertainties due to measurement limitations which propagate to the combination of variables in a function.

In statistical data analysis, the propagation of error (or propagation of uncertainty) is the effect of measurement uncertainties (or errors) on the uncertainty of a function based on them.

For a function of one variable, f(x), if x has a measurment error σ_x, then the associated error σ_f is computed from:

$$\sigma_f = \left| \frac{\partial f}{\partial x} \right| \sigma_x$$

Specific cases:

$$\sigma(x^2) = 2x\sigma_x \quad \text{or} \quad \frac{\sigma(x^2)}{x^2} = 2\frac{\sigma_x}{x}$$

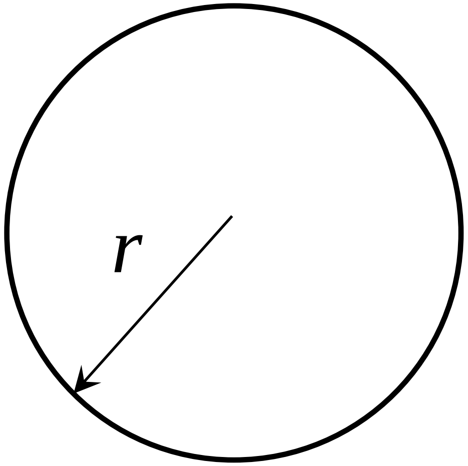
$$\sigma(x^n) = nx^{n-1}\sigma_x \quad \text{or} \quad \frac{\sigma(x^n)}{x^n} = n\frac{\sigma_x}{x}$$

$$\sigma(\sin x) = \cos x\, \sigma_x$$

$$\sigma(\ln x) = \frac{1}{x}\sigma_x$$

Example

The radius of a circle is measured as



$$r = 2.5 \pm 0.3\,\mathrm{cm}$$

Calculate the area of the circle and its uncertainty.

Answer: $A = 19.6 \pm 4.7\,\mathrm{cm}^2$

In general, if x_1, x_2, …, x_n are <u>independent variables</u> having associated errors (standard deviations)

$$\sigma_1, \sigma_2, \ldots, \sigma_n$$

then the standard deviation for any quantity of the form

$$f = f\,(x_1, x_2, \ldots, x_n)$$

derived from these errors can be calculated from:

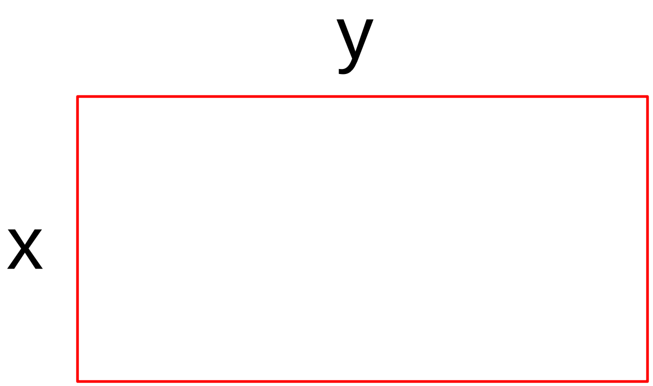
$$\sigma_f^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_2^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma_n^2$$

Errors of simple functions after application of Eqn. (5).

Function	Derivative(s)	Variance	Standard deviation
$f = kx$; $k \in \mathrm{R}$	$\frac{\partial f}{\partial x} = k$	$\sigma_f^2 = k^2 \sigma_x^2$	$\sigma_f = k\sigma_x$
$f = x + y$	$\frac{\partial f}{\partial x} = 1$ and $\frac{\partial f}{\partial y} = 1$	$\sigma_f^2 = \sigma_x^2 + \sigma_y^2$	$\sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2}$
$f = x - y$	$\frac{\partial f}{\partial x} = 1$ and $\frac{\partial f}{\partial y} = -1$	$\sigma_f^2 = \sigma_x^2 + \sigma_y^2$	$\sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2}$
$f = x\,y$	$\frac{\partial f}{\partial x} = y$ and $\frac{\partial f}{\partial y} = x$	$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$	$\sigma_f = f\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$
$f = x / y$	$\frac{\partial f}{\partial x} = \frac{1}{y}$ and $\frac{\partial f}{\partial y} = -\frac{x}{y^2}$	$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$	$\sigma_f = f\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$

Example

What is the area and associated error for the rectangle if



$$x = 1.0 \pm 0.1\,\mathrm{m}$$

$$y = 2.0 \pm 0.2\,\mathrm{m}$$

Answer: $A = 2.00 \pm 0.28\,\mathrm{m}^2$

Example

Suppose we wish to calculate the average speed (displacement/time) of an object. Assume the displacement is measured as $x = 22.2 \pm 0.5\,\mathrm{cm}$ during the time interval $t = 9.0 \pm 0.1\,\mathrm{s}$. What is the speed of the object and its uncertainty?

Answer: $v = 2.467(62)\,\mathrm{cm/s}$

PART III

Least Square Fitting

Introduction

- Data is often given for discrete values along a continuum.

The distance required to stop an automobile is a function of its speed. The following data is collected to get this relationship:

- You may require estimates at points between discrete values.

v(km/h)	d(m)
24	4.8
32	6.0
40	10.2
48	12.0
64	18.0
80	27.0

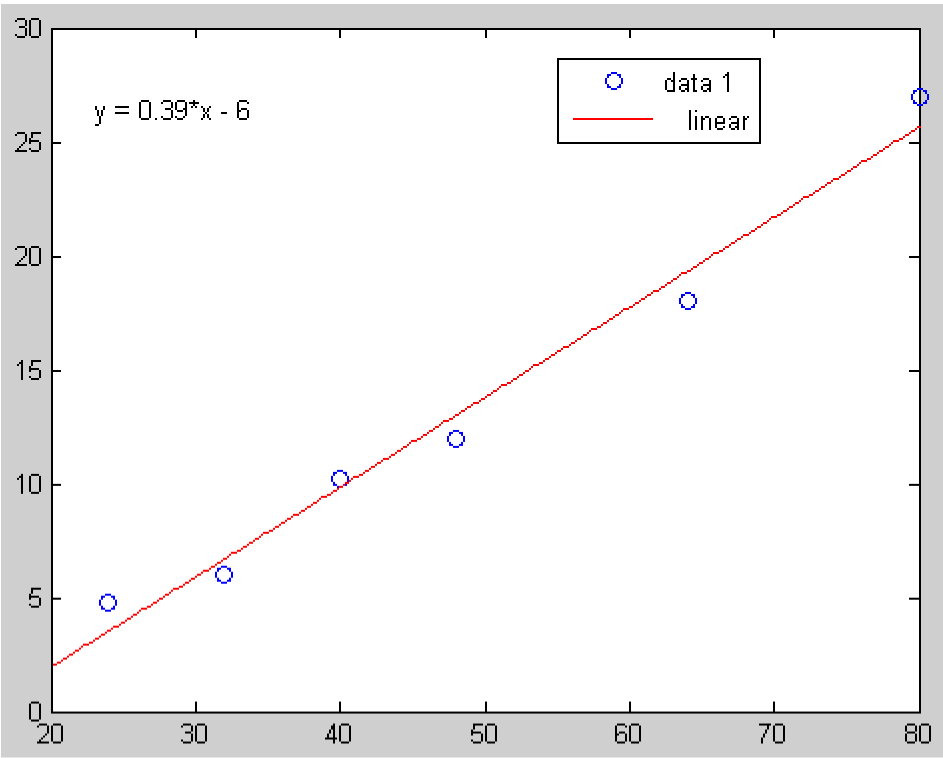
- In this section we will consider how to obtain values between the given experimental points using **Least square fitting method.**

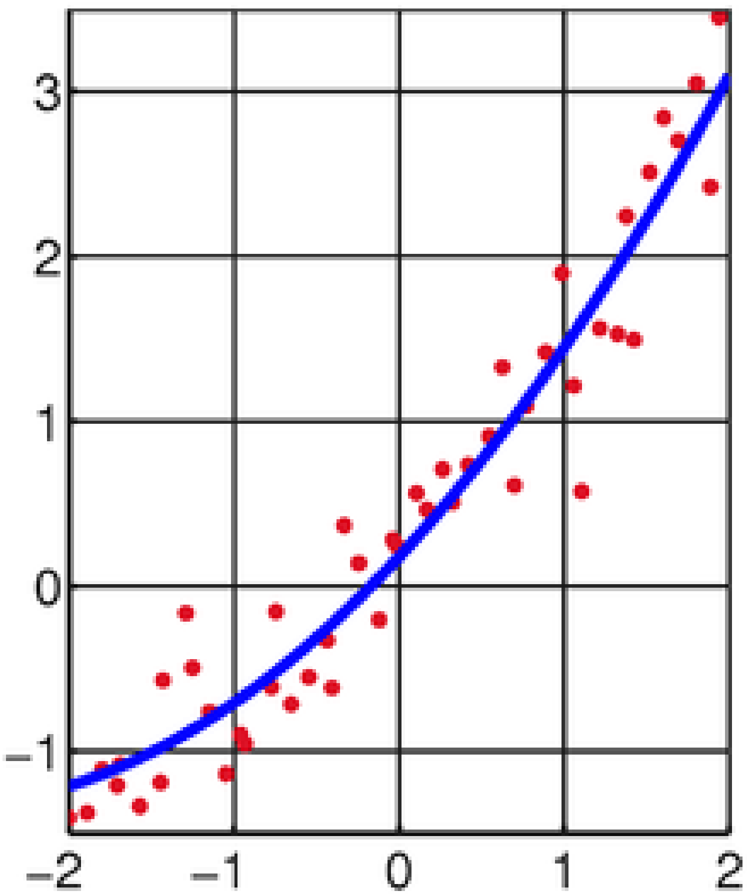
Least Square Fitting Method

- The method of least squares is a standard approach to the approximate solution of over-determined systems, i.e. sets of equations in which there are more equations than unknowns.

- "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in solving every single equation.

- Least squares problems fall into two categories: **linear** or **non-linear** least squares.





- The most important application is in data fitting.
- The best fit in the least-squares sense minimizes the sum of squared residuals defined by:

residual = data – fit_model_function

$$r_i = y_i - \hat{y}_i$$

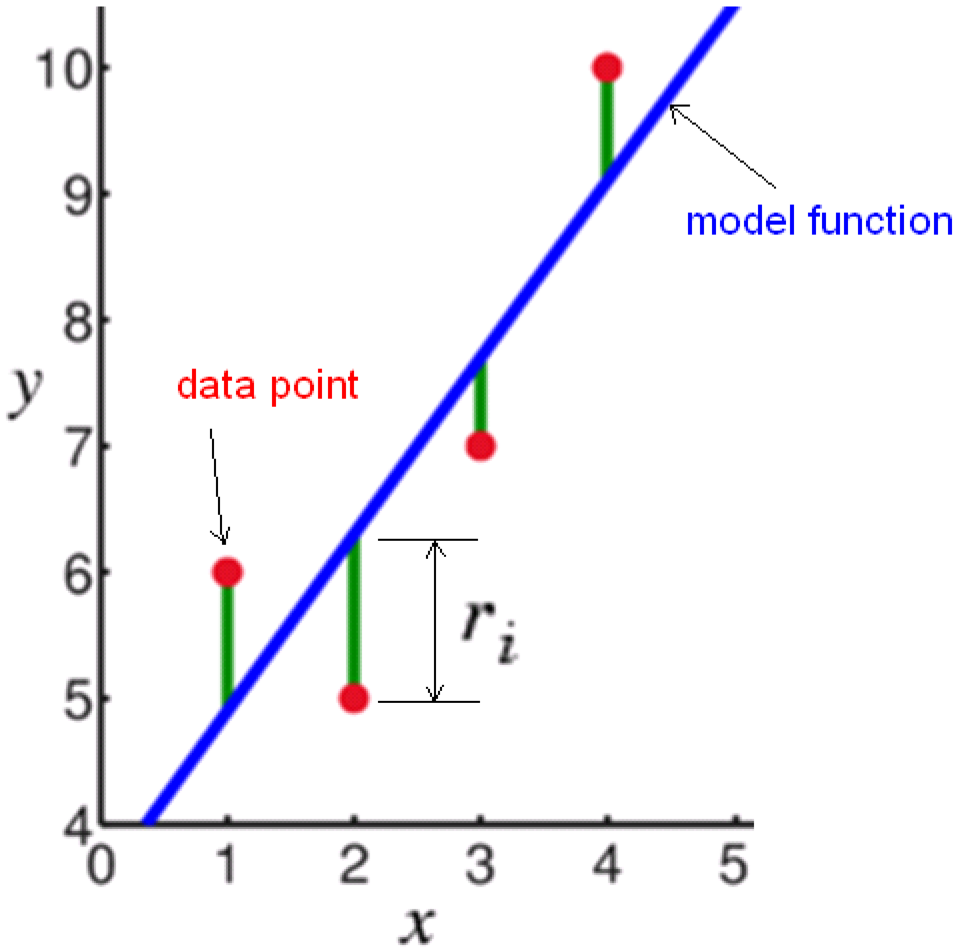
The summed square of residuals is given by

$$S = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Experimental data

x	y
x_1	y_1
x_2	y_2
.	.
.	.
.	.
x_n	y_n

where ***n*** is the number of data points included in the fit and ***S*** is the sum of squares error estimate.



Experimental data

x	y
x_1	y_1
x_2	y_2
.	.
.	.
.	.
x_n	y_n

$$r_i = y_i - \hat{y}_i$$

Linear Least Square Method (two parameters)

- Consider we want to fit the a data (x_i, y_i) to a function **y = ax + b** then the square sum of the residuals is:

$$S = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

- To minimize *S*, we should solve the following equations simultaneously:

$$\frac{\partial S}{\partial a} = 0 \qquad \frac{\partial S}{\partial b} = 0$$

- Solutions are:

$$a = \frac{n\sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)^2} \qquad b = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

Goodness of the Fit:

$$r^2 = \frac{S_t - S}{S_t}$$

where

$$S = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

$$S_t = \sum_{i=1}^{n} (y_i - y_m)^2$$

$$y_m = \frac{\sum_{i=1}^{n} y_i}{n}$$ (mean value of y)

For a good fit

$$S \to 0$$

$$r^2 \to 1$$

Weighted Linear Least Square Method

- Weighted least squares regression minimizes the error estimate:

$$S = \sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2$$

where w_i are the weights which determine how much each response value influences the final parameter estimates.

- If you know the variances of your data, then the weights are given by:

$$w_i = 1/\sigma_i^2$$

Example: *Linear Fit*

The distance required to stop an automobile is a function of its speed. The following data is collected to get this relationship:

v(km/h)	d(m)
24	4.8
32	6.0
40	10.2
48	12.0
64	18.0
80	27.0

Fit the data to a linear function and compute the goodness of the fit.

Example: *Weighted Linear Fit*

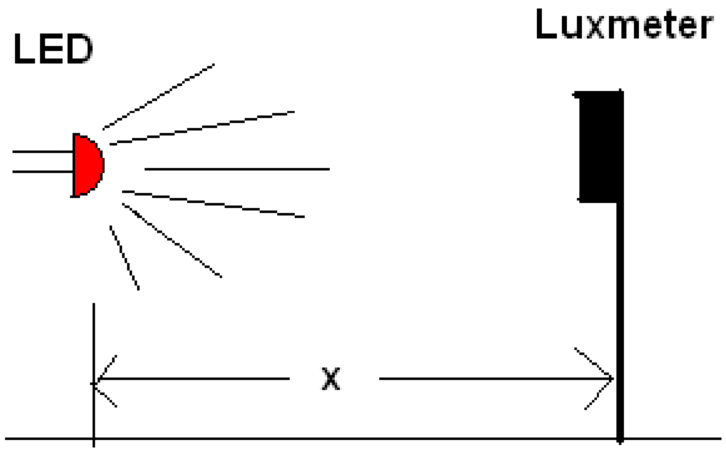
The distance required to stop an automobile is a function of its speed. The following data is collected to get this relationship:

v(km/h)	d(m)
24	4.8 +- 0.3
32	6.0 +- 0.4
40	10.2 +- 1.0
48	12.0 +- 1.1
64	18.0 +- 1.4
80	27.0 +- 1.5

The **+- value** represents the measurement error (one standard deviation).
Fit the data to a linear function and compute the goodness of the fit.

Example: Non-*Linear Fit*

Illumination (E) vs distance (x) from an LED data is collected.



x(cm)	E(lux)
10	1008
20	285
30	133
40	79
50	50

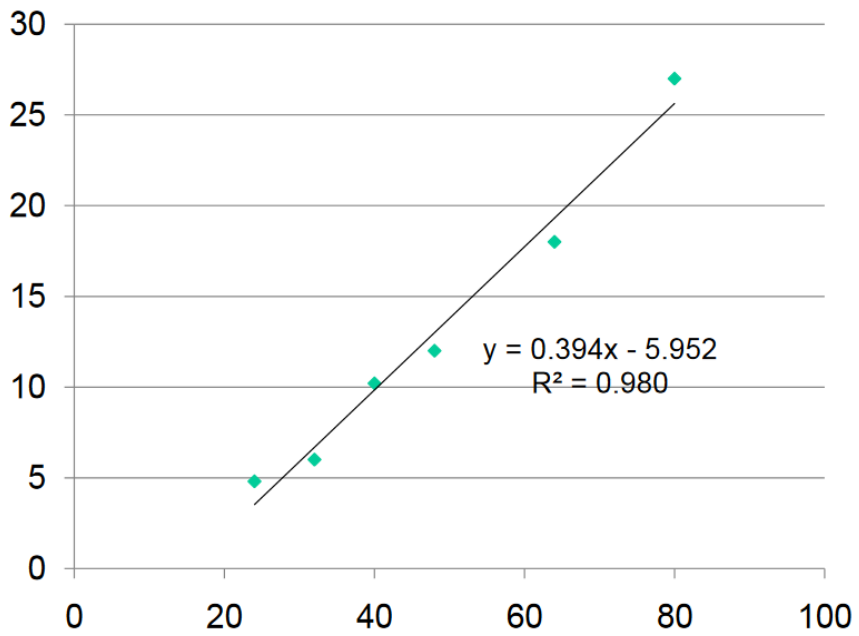
(a) Fit the data to the function $E = A / x^n$

where A and n are free parameters.

(b) By linearizing the system, find the fit parameters again.

MATLAB Curve Fitting Tool

Curve Fitting can be performed in MS Excel



Curve Fitting can also be performed in Matlab

```
>> cftool
```

Questions

1. What is a random error? Give an example for it.
2. What is a systematic error? Give an example for it.
3. How can we reduce random and systematic errors in an experiment?
4. How does the limited accuracy of the measuring apparatus result in a random error?
5. How do uncontrolled changes in the environment result in a systematic error?
6. What is the difference between error and uncertainty?
7. What is meant by accuracy and precision?
8. Why instrument calibration is necessary?
9. What is the difference between percentage error and percentage difference?

10. Two measurements of body temperature before and after a drug is administered: 37.2 ^oC and 37.8 ^oC. Is temperature rise significant for errors (a) 0.01 ^oC and (b) 0.5 ^oC.

11. For a measurement of gravitational acceleration g = 9.811 m/s^2, calculate the percentage error with respect to best measured value (having very small uncertainty) of 9.80665 m/s^2.

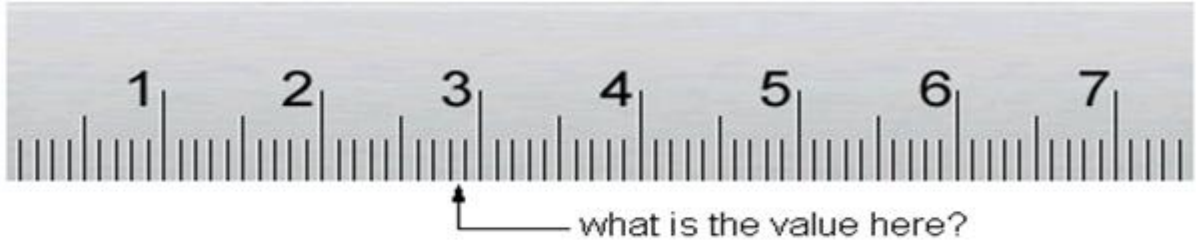
12. Two measurement of gravitational acceleration are given by:

$$g_1 = 9.77 \pm 0.14\,\mathrm{m/s}^2 \quad \text{and} \quad g_2 = 9.82 \pm 0.10\,\mathrm{m/s}^2$$

Which one is the better measurement?

13. What is the value of reading given below?

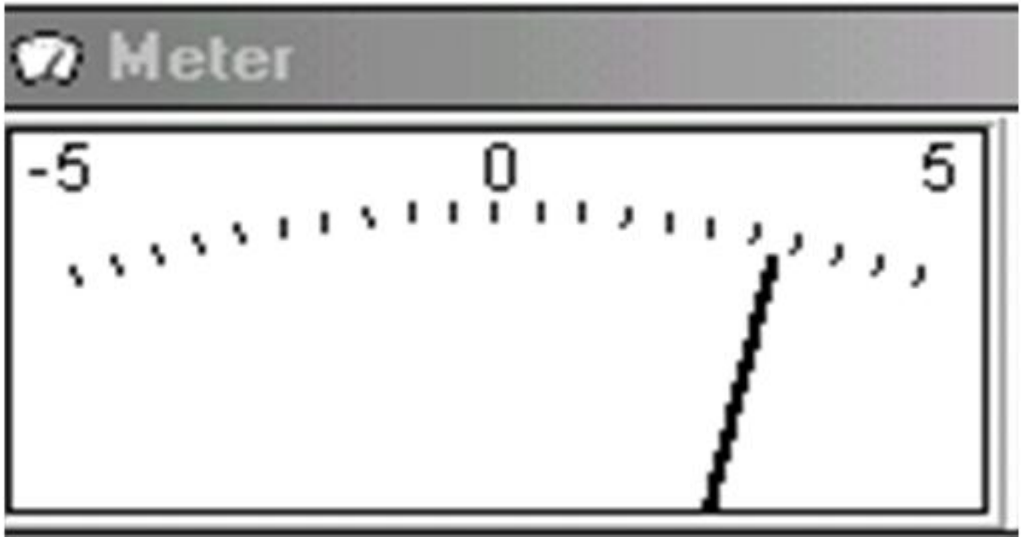
(a) 28.5 +- 0.5 mm
(b) 28.0 +- 0.5 mm
(c) 29.0 +- 0.5 mm
(d) 27.5 +- 0.5 mm



14. What is the value of reading given below?

(a) 12.58 +- 0.05 g
(b) 12.58 +- 0.01 g
(c) 12.580 +- 0.005 g
(d) 12.580 +- 0.010 g

16. What is the value of reading given below?



17. By measuring yourself with 10 different rulers, you obtain the estimates of your height.

165.6 ± 0.3 cm
165.1 ± 0.4 cm
166.4 ± 1.0 cm
166.1 ± 0.8 cm
165.5 ± 0.5 cm
165.5 ± 0.4 cm
165.9 ± 0.6 cm
165.5 ± 0.2 cm
166.0 ± 0.7 cm
164.9 ± 0.4 cm

(a) Which of these is the best estimate of your height?

(b) Use Eqn. (1), (2) and (3), without considering errors, calculate the mean height and standard error of the data.

(c) Use Eqn (6), calculate the height and its standard deviation by combining these 10 measurements.

18. Determine the distance between the points A and B.

$$A(0.0 \pm 0.2\,\text{cm},\ 0.0 \pm 0.3\,\text{cm})$$

$$B(3.0 \pm 0.3\,\text{cm},\ 4.0 \pm 0.2\,\text{cm})$$

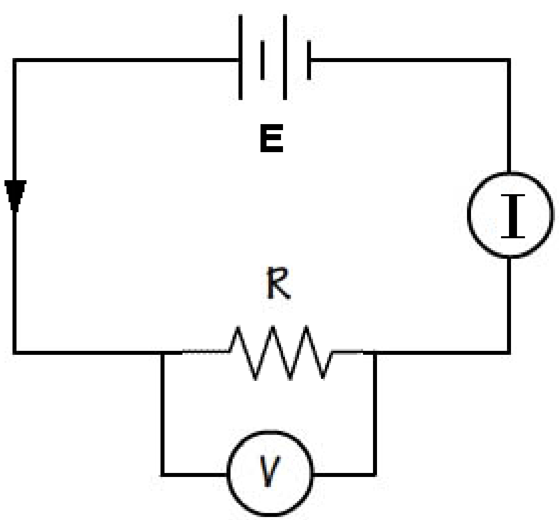
19. Suppose that $x = 2.0 \pm 0.2$ $y = 3.0 \pm 0.6$ $z = 4.52 \pm 0.02$
Find w = x + y - z and its uncertainty.

20. A resistance *R* is connected in a circuit as shown in figure. Calculate power in the resistor and its uncertainty, if

$$R = 10\,\Omega \pm 1\%$$

$$V = 100\,\text{V} \pm 1\%$$

$$I = 10\,\text{A} \pm 1\%$$



21. The resistance of a certain size of copper wire is given as

$$R = R_0[1+\alpha(T-20)]$$

where

R_0 is the resistance at 20 ^{o}C,

alpha is the temperature coefficient of resistance and

T is the resistance of the wire.

Calculate the resistance (R) of the wire and its uncertainty for:

$$R_0 = 6.00 \pm 0.02 \ \Omega$$

$$\alpha = (4.00 \pm 0.04)\times 10^{-3} \ {}^{o}C^{-1}$$

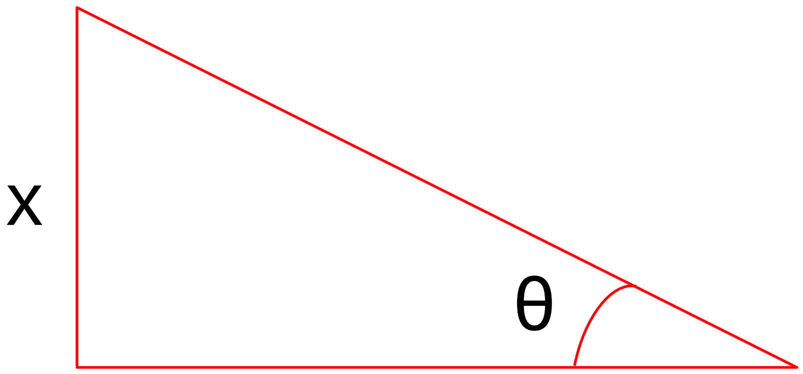
$$T = 30 \pm 1 \ {}^{o}C$$

22. Two resistors R_1 and R_2 are to be connected in series and parallel. The values of the resistances are

$$R_1 = 100.0 \pm 0.3\,\Omega, \quad R_2 = 50.0 \pm 0.1\,\Omega$$

Calculate the value and uncertainty in the combined resistance for both series and parallel arrangements.

23. Figure shows a right angle triangle.
Find the area of the triangle and its uncertainty for



$$\theta = 37.0^{o} \pm 0.4^{o}$$

$$x = 3.0 \pm 0.2\,\mathrm{m}$$

24. Refractive index (n) of a glass is calculated by using Snell's law:

$$n = n_{\mathrm{AIR}} \frac{\sin\theta_1}{\sin\theta_2}$$

where measured values and their estimated errors are:

$$n_{\mathrm{AIR}} = 1 \qquad \theta_1 = 61 \pm 1^{\mathrm{o}} \qquad \theta_2 = 36 \pm 1^{\mathrm{o}}$$

Calculate the refractive index of the glass and its uncertainty.

25. Refractive index (n) of a glass is calculated by using Snell's law:

$$n = n_{\mathrm{AIR}} \frac{\sin\theta_1}{\sin\theta_2}$$

where measured values and their estimated errors are:

$$n_{\mathrm{AIR}} = 1 \qquad \theta_1 = 61 \pm 1^{\mathrm{o}} \qquad \theta_2 = 36 \pm 1^{\mathrm{o}}$$

Calculate the refractive index of the glass and its uncertainty.

26. Table shows the mass M of several stars and their corresponding luminosity (L = power emitted).

M (in solar mass)	L (in solar luminosity)
1	1 +- 0
3	42 +- 4
5	230 +- 20
12	4700 +- 50
20	26500 +- 300

(a) Plot L against M.

(b) Plot log(L) against log(M). Assume that $L = A.M^n$.
Ignore measurement errors. Using Least square fitting method find the value of n. (You can use linearization)

(c) Plot log(L) against log(M). Assume that $L = A.M^n$.
Do include measurement errors. Using Least square fitting method find the value of n. (You can use linearization)

References

1. "Data Analysis for Physical Science Students", L. Lyons, Cambridge University Press
2. "Physics for the IB Diploma", 6th Ed. K.A. Tsokos, Cambridge University Press
3. http://www.statistics.com
4. http://en.wikipedia.org/wiki/Accuracy_and_precision
5. "Radiation Detection and Measurement", G.F.Knoll, Wiley
6. http://en.wikipedia.org/wiki/Propagation_of_uncertainty
7. http://www1.gantep.edu.tr/~andrew/eee484
8. http://www.shef.ac.uk/polopoly_fs/1.14221!/file/IntroToExperimentalErrors_y2.pdf