Monte Carlo Simulation for the Measurement of $\pi^\pm$ Life Time

Ahmet Bingül
March 2006

University of Gaziantep
Department of Engineering Physics
Introduction

The charged pions decay by the weak interaction (as suggested by $10^{-8}$ s lifetime) into leptons.

The main decay channel ($\text{BR} \approx 100\%$) is:

$$\pi^\pm \rightarrow \mu^\pm + \bar{\nu}_\mu$$
Measuring Lifetime

The most precise measurement of lifetimes of charged pions was done in an experiment reported by Ayres [1].

A counter is moved along the pion beam and measured the number of pions at various distances.
Pion Decay

Radioactive decay law:

\[ N = N_0 \exp(-t/\tau) \]

gives relative number of pions surviving at time \( t \).

Where \( \tau \) \textbf{lab-frame} lifetime.

If beam travels at velocity \( \nu \), the decay law can be written in terms of distance \( x = \nu t \)

\[ N = N_0 \exp(-x/\nu\tau) \]

**In CM-frame**, the lifetime \( \tau_0 \) is not same as \( \tau \)

The relationship is:

\[ \tau = \frac{\tau_0}{\sqrt{1 - \nu^2/c^2}} = \frac{\tau_0}{\sqrt{1 - \beta^2}} = \gamma \tau_0 \]
The population \((N)\) of pions can also be written as:

\[
N = N_0 \exp\left( -\frac{x}{\lambda} \right)
\]

where \(\lambda\) is the mean decay length which is given by:

\[
\lambda = \gamma \beta \tau_0 c
\]

If we know momentum \(p\) in \(\text{MeV/c}\) and mass \(m\) in \(\text{MeV/c}^2\) of the pions:

\[
\gamma = \sqrt{p^2 + m^2} / m
\]

\[
\beta = \sqrt{1 - 1 / \gamma^2}
\]

\[
\tau_0 = 2.602 \times 10^{-8} \text{ s}
\]
Decay length of a pion can be chosen from the probability distribution:

$$\ell = -\lambda \ln(R)$$

where $R$ is a random number selected from a uniform distribution in the range [0,1].

This is a statistical process $e^{-t/\tau}$

More information about random distributions can be found at:

http://www1.gantep.edu.tr/~bingul/seminar/monte-carlo/page11.html
http://www1.gantep.edu.tr/~andrew/ep208/notes?lecture=8
Computer Simulation

- $N_0 = 10,000$ pions are generated for each run
- All pions assumed to have same momentum $p$ and same direction
- The pion counter is moved from 1m to 10m, step 1m
- Decay Rates are calculated by counting pions corresponding to each Distance
- A graph is constructed for Decay Rates vs Distance in a semilog plot of data (linear dependence)
- The slope and intercept is extracted from the plot using Least Square Fitting technique. (see: http://www1.gantep.edu.tr/~andrew/ep208/notes?lecture=3)
- From the slope, mean lifetime is calculated
$N = N_0 \exp\left(-x/\lambda\right)$

We can measure

We can calculate

$$\ln N = \ln N_0 - \left(1/\lambda\right)x$$

Slope $= -1/\lambda$

$$\tau_0 = -\frac{1}{\text{Slope}} \times \frac{1}{\gamma\beta c}$$
Results

*Experiment* is repeated 20 times to get average value and error.

Simulation results:

\[ \tau_0 = (2.600 \pm 0.008) \times 10^{-8} \text{ s} \]
\[ \tau_0 = 26.00 \pm 0.08 \text{ ns} \]

Experimental results[2]:

\[ \tau_0 = 26.02 \pm 0.04 \text{ ns} \]

References:
[2]. Krane, *Introduction to Nuclear Physics*
Computer Programs

You can download the computer implementation of the Simulation at:

**Fortran 90:**
http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.f90

**C:**
http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.c

**ROOT:**
http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.cxx
Sample Output

*** Monte Carlo Simulation for the ********
*** Measurement of Charged Pion Life Time ***

-----------------------------------------------
Pion mom. generated (MeV/c): 5.000e+01
Pion life time (s) : 2.602e-08
Number of Pion generated : 10000
Number of counter position : 10
-----------------------------------------------
Values obtained for each position:
1  6912  8.841e+00
2  4852  8.487e+00
3  3382  8.126e+00
4  2391  7.779e+00
5  1724  7.452e+00
6  1211  7.099e+00
7   832  6.724e+00
8   574  6.353e+00
9   398  5.986e+00
10  270  5.598e+00
-----------------------------------------------
Fitting results:
Slope, A = -3.5804e-01
Intercept, B = 8.8558e+00
-----------------------------------------------
Measured lifetime: 2.5987e-08