Monte Carlo Simulation for the Measurement of $\pi^{\pm}$ Life Time

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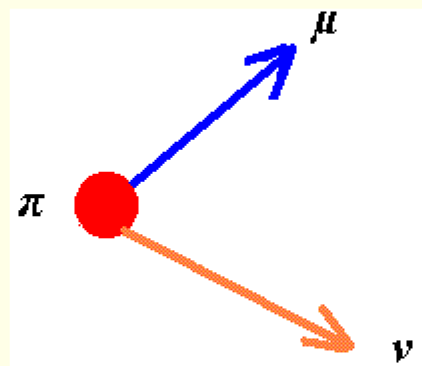
Introduction

The charged pions decay by the weak interaction (as suggested by 10^{-8} s lifetime) into leptons.

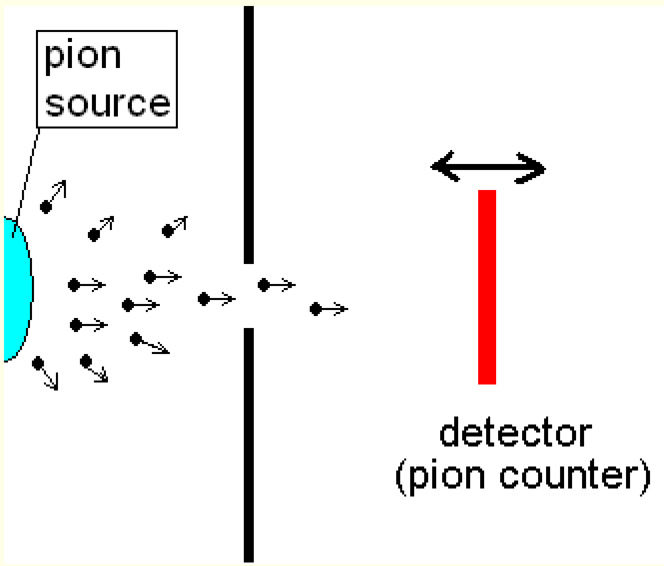
The main decay channel (BR ≈ 100%) is:

$$\pi^{\pm} \rightarrow \mu^{\pm} + \bar{\nu}_{\mu}$$



Measuring Lifetime

The most precise measurement of lifetimes of charged pions was done in an experiment reported by Ayres [1].



A counter is moved along the pion beam and measured the number of pions at various distances.

Pion Decay

Radioactive decay law:

$$N = N_0 \exp(-t/\tau)$$

gives realtive number of pions surviving at time t.
Where τ **lab-frame** lifetime.

If beam traves at velocity υ, the decay law can be written in terms of distance $x = \upsilon t$

$$N = N_0 \exp(-x / \upsilon\tau)$$

In CM-frame, the lifetime τ_0 is not same as τ
The relationship is:

$$\tau = \frac{\tau_0}{\sqrt{1 - v^2 / c^2}} = \frac{\tau_0}{\sqrt{1 - \beta^2}} = \gamma\tau_0$$

The population (N) of pions can also be written as:

$$N = N_0 \exp(-x/\lambda)$$

where λ is the mean decay length which is given by:

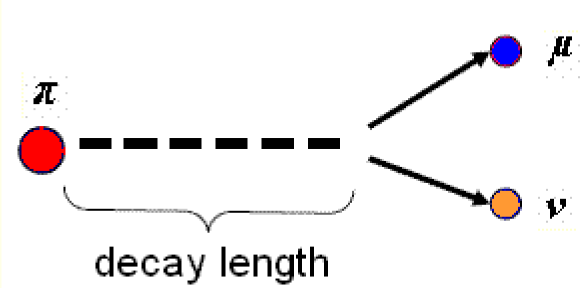
$$\lambda = \gamma\beta\tau_0 c$$

If we know momentum p in *MeV/c* and mass m in *MeV/c*^{2} of the pions:

$$\gamma = \sqrt{p^2 + m^2} / m$$

$$\beta = \sqrt{1 - 1/\gamma^2}$$

$$\tau_0 = 2.602 \times 10^{-8} \text{ s}$$



Decay length of a pion can be choosen from the probability distribution:

$$\ell = -\lambda \ln(R)$$

where R is a random number selected from a uniform distribution in the range [0,1].

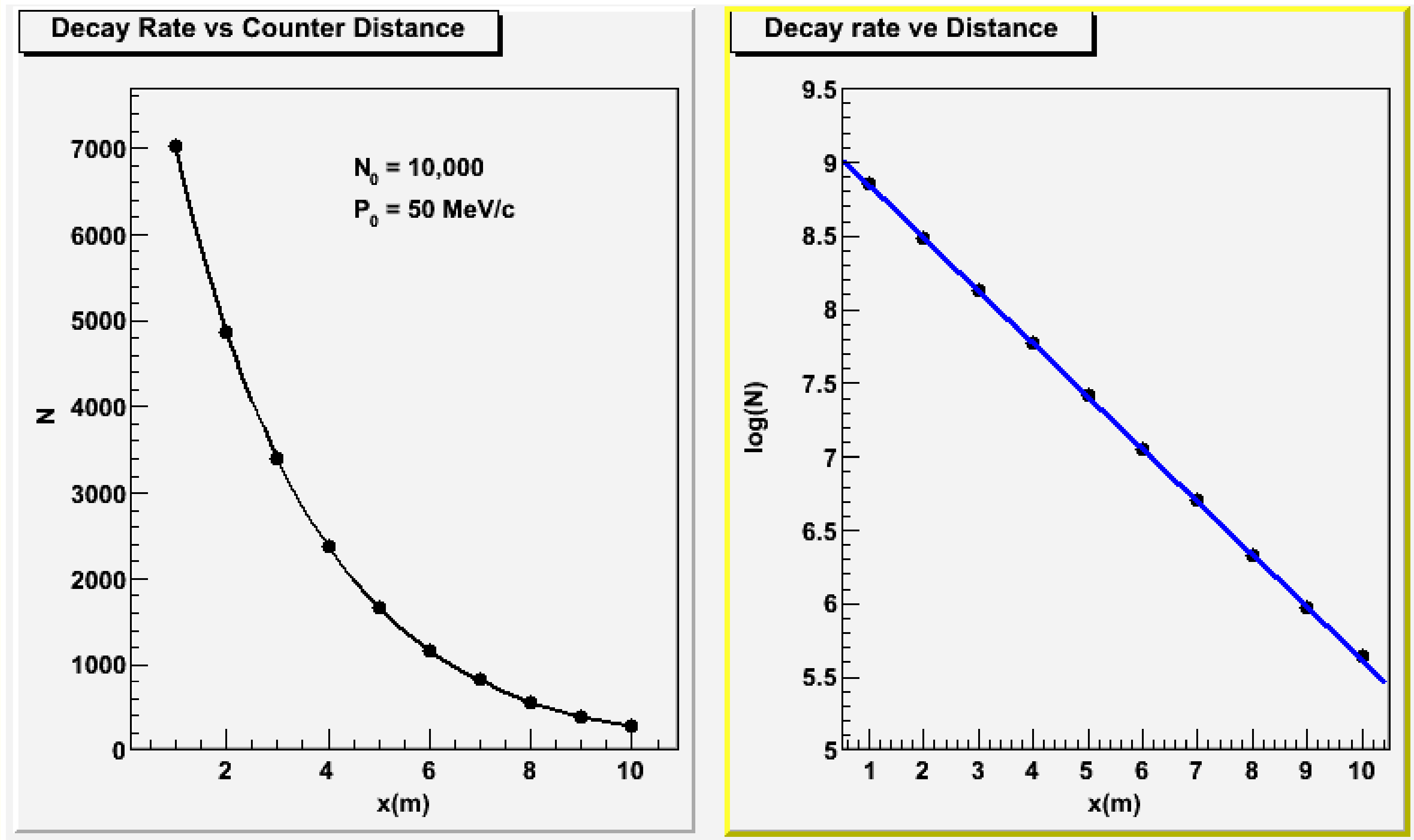
This is a statistical process $e^{-t/\tau}$

More information about random distributions can be found at:

http://www1.gantep.edu.tr/~bingul/seminar/monte-carlo/page11.html
http://www1.gantep.edu.tr/~andrew/ep208/notes?lecture=8

Computer Simulation

- N_0 = 10,000 pions are generated for each run
- All pions assumed to have **same** momentum *p* and **same** direction
- The pion counter is moved from 1m to 10m, step 1m
- Decay Rates are calculated by counting pions corresponding to each Distance
- A graph is constructed for Decay Rates vs Distance in a *semilog* plot of data (linear dependence)
- The slope and intercept is extracted from the plot using *Least Square Fitting* technique.
  (see: http://www1.gantep.edu.tr/~andrew/ep208/notes?lecture=3)
- From the slope, mean lifetime is calculated



$$N = N_0 \exp(-x/\lambda) \longrightarrow \ln N = \ln N_0 - (1/\lambda)x$$

We can measure $\longrightarrow$ $\text{Slope} = -1/\lambda$

We can calculate $\longrightarrow$ $\tau_0 = \dfrac{-1}{\text{Slope}} \times \dfrac{1}{\gamma\beta c}$

Results

Experiment is repeated 20 times to get average value and error.

Simulation results:

$$\tau_0 = (2.600 \pm 0.008) \times 10^{-8}\ \text{s}$$

$$\tau_0 = 26.00 \pm 0.08\ \text{ns}$$

Experimental results[2]:

$$\tau_0 = 26.02 \pm 0.04\ \text{ns}$$

References:

[1]. Ayres et al., *Phys. Rev.* D 3, 1051 (1971)
[2]. Krane, *Introduction to Nuclear Physics*

Computer Programs

You can download the computer implementation of the Simulation at:

Fortran 90:

http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.f90

C:

http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.c

ROOT:

http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.cxx

Sample Output

```
*** Monte Carlo Simulation for the **********
*** Measurement of Charged Pion Life Time ***

---------------------------------------------
Pion mom. generated (MeV/c):  5.000e+01
Pion life time      (s)    :  2.602e-08
Number of Pion generated   :    10000
Number of counter positon  :       10
---------------------------------------------
Values obtained for each position:
 1      6912       8.841e+00
 2      4852       8.487e+00
 3      3382       8.126e+00
 4      2391       7.779e+00
 5      1724       7.452e+00
 6      1211       7.099e+00
 7       832       6.724e+00
 8       574       6.353e+00
 9       398       5.986e+00
10       270       5.598e+00
---------------------------------------------
Fitting results:
Slope,     A =     -3.5804e-01
Intercept, B =      8.8558e+00
---------------------------------------------


Measured lifetime:      2.5987e-08
```