

Monte Carlo Simulation for the Measurement of π^\pm Life Time

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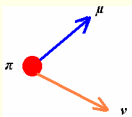
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March 2006
Page 2/11

Introduction

The charged pions decay by the weak interaction (as suggested by 10^{-8} s lifetime) into leptons.

The main decay channel (BR \approx 100%) is:

$$\pi^\pm \rightarrow \mu^\pm + \bar{\nu}_\mu$$


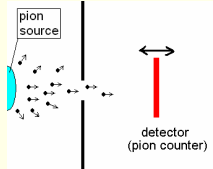
The diagram shows a red dot labeled π at the origin. A blue arrow labeled μ points upwards and to the right. An orange arrow labeled $\bar{\nu}$ points downwards and to the right.

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March 2006
Page 3/11

Measuring Lifetime

The most precise measurement of lifetimes of charged pions was done in an experiment reported by Ayres [1].



The diagram shows a pion source on the left emitting a beam of pions (represented by small circles) towards a detector (pion counter) on the right. A red vertical bar represents the detector, and a double-headed arrow above it indicates its movement along the beam path.

A counter is moved along the pion beam and measured the number of pions at various distances.

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Department of Engineering Physics
March 2008
Page 4/11

Pion Decay

Radioactive decay law:

$$N = N_0 \exp(-t/\tau)$$

gives relative number of pions surviving at time t .
Where τ **lab-frame** lifetime.

If beam travels at velocity v , the decay law can be written in terms of distance $x = vt$

$$N = N_0 \exp(-x/v\tau)$$

In CM-frame, the lifetime τ_0 is not same as τ
The relationship is:

$$\tau = \frac{\tau_0}{\sqrt{1-v^2/c^2}} = \frac{\tau_0}{\sqrt{1-\beta^2}} = \gamma\tau_0$$

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Department of Engineering Physics
March 2008
Page 5/11

The population (N) of pions can also be written as:

$$N = N_0 \exp(-x/\lambda)$$

where λ is the mean decay length which is given by:

$$\lambda = \gamma\beta\tau_0 c$$

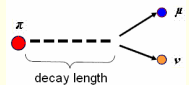
If we know momentum p in MeV/c and mass m in MeV/c^2 of the pions:

$$\gamma = \sqrt{p^2 + m^2} / m$$

$$\beta = \sqrt{1 - 1/\gamma^2}$$

$$\tau_0 = 2.602 \times 10^{-8} \text{ s}$$

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Department of Engineering Physics
March 2008
Page 6/11



Decay length of a pion can be chosen from the probability distribution:

$$\ell = -\lambda \ln(R)$$

where R is a random number selected from a uniform distribution in the range $[0,1]$.

This is a statistical process $e^{-t/\tau}$

More information about random distributions can be found at:

<http://www1.gantep.edu.tr/~binqul/seminar/monte-carlo/page11.html>
<http://www1.gantep.edu.tr/~andrew/ep208/notes?lecture=8>

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Department of Engineering Physics
March 2008
Page 7/11

Computer Simulation

- $N_0 = 10,000$ pions are generated for each run
- All pions assumed to have **same** momentum p and **same** direction
- The pion counter is moved from 1m to 10m, step 1m
- Decay Rates are calculated by counting pions corresponding to each Distance
- A graph is constructed for Decay Rates vs Distance in a *semilog* plot of data (linear dependence)
- The slope and intercept is extracted from the plot using *Least Square Fitting* technique. (see: <http://www1.gantep.edu.tr/~andrew/ep208/notes?lecture=3>)
- From the slope, mean lifetime is calculated

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March 2008
Page 8/11

Decay Rate vs Counter Distance

Decay rate vs Distance

$N = N_0 \exp(-x / \lambda) \longrightarrow \ln N = \ln N_0 - (1 / \lambda)x$
We can measure \longrightarrow Slope $= -1 / \lambda$
We can calculate $\longrightarrow \tau_0 = \frac{-1}{\text{Slope}} \times \frac{1}{\beta c}$

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March 2008
Page 9/11


Results

Experiment is repeated 20 times to get average value and error.

Simulation results: $\tau_0 = (2.600 \pm 0.008) \times 10^{-8} \text{ s}$
 $\tau_0 = 26.00 \pm 0.08 \text{ ns}$

Experimental results[2]: $\tau_0 = 26.02 \pm 0.04 \text{ ns}$

References:
 [1]. Ayres et al., *Phys. Rev. D* 3, 1051 (1971)
 [2]. Krane, *Introduction to Nuclear Physics*


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 March 2008
 Page 10/11


Computer Programs

You can download the computer implementation of the Simulation at:

Fortran 90:
<http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.f90>

C:
<http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.c>

ROOT:
<http://www1.gantep.edu.tr/~bingul/seminar/pion-lifetime/plt.C>


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 Department of Engineering Physics

 March 2008
 Page 11/11

Sample Output

```

*** Monte Carlo Simulation for the *****
*** Measurement of Charged Pion Life Time ***
-----
Pion mom. generated (MeV/c): 5.000e+01
Pion life time (ns) : 2.402e-08
Number of Pion generated : 10000
Number of counter position : 10
-----
Values obtained for each position:
1  4912  8.841e+00
2  4852  8.487e+00
3  3382  8.126e+00
4  2391  7.779e+00
5  1724  7.452e+00
6  1211  7.099e+00
7   832  6.724e+00
8   574  6.353e+00
9   398  5.986e+00
10  270  5.598e+00
-----
Fitting results:
Slope, A = -3.5804e-01
Intercept, B = 8.8558e+00
-----
Measured lifetime: 2.5987e-08
        
```
