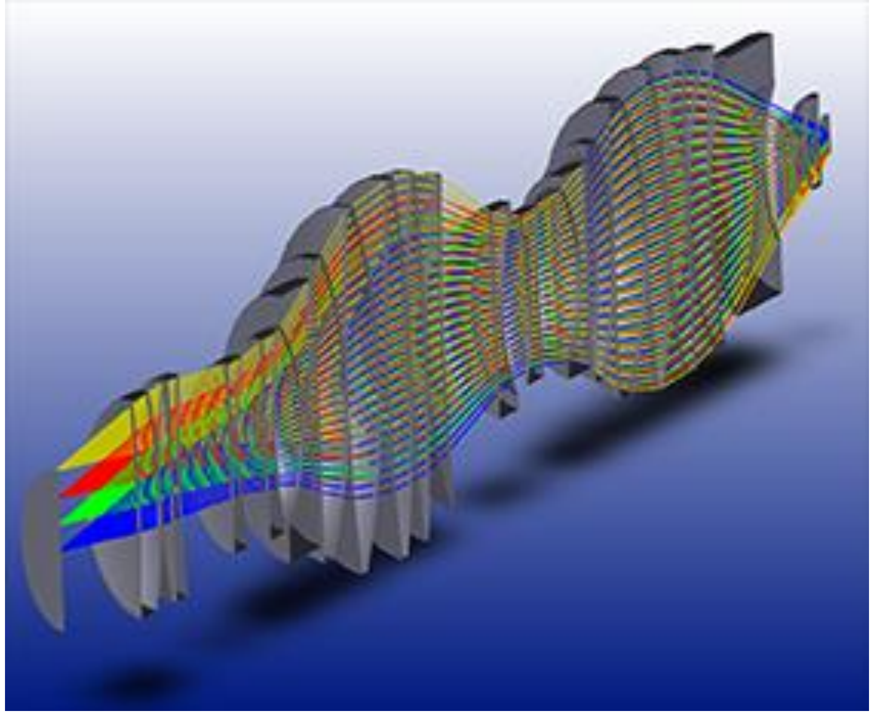
Lectures Notes on Optical Design using Zemax

Fundamental Concepts in Optical Design



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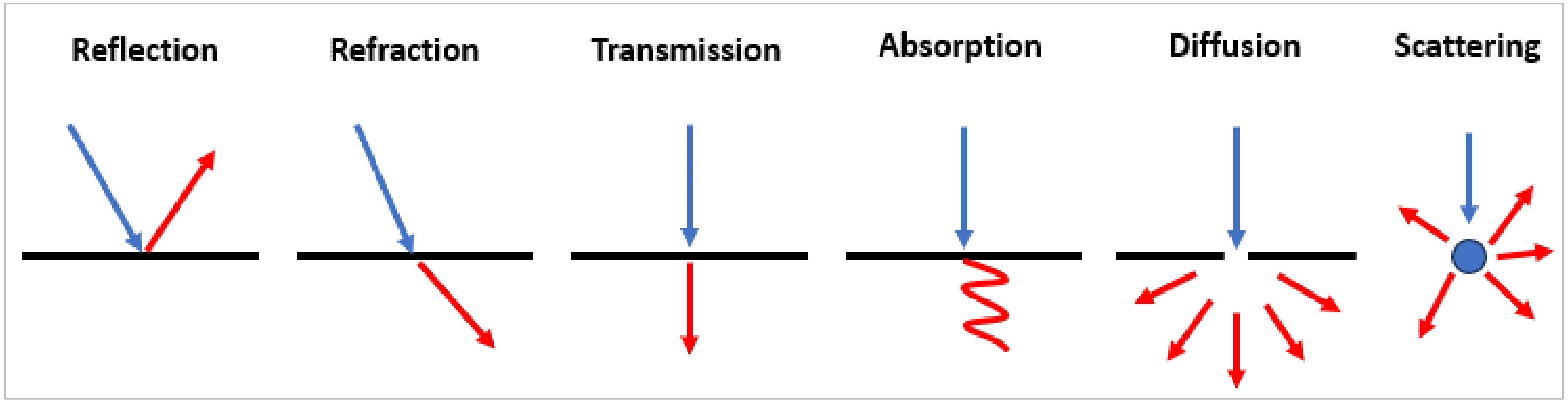
Light Models and Interaction of Optical Photons with Matter

Light is the portion of electromagnetic radiation that is visible to the human eye and is an energy propagating in space as photons.

Table 1: Light Models

Name	Field of Science	Description
Wave Model	Wave Optics	Light is an electromagnetic wave
Ray Model	Geometrical Optics	Light travels in a fixed direction in a straight line called ray
Photon	Physical Optics	Light is a kind of particle carrying momentum.

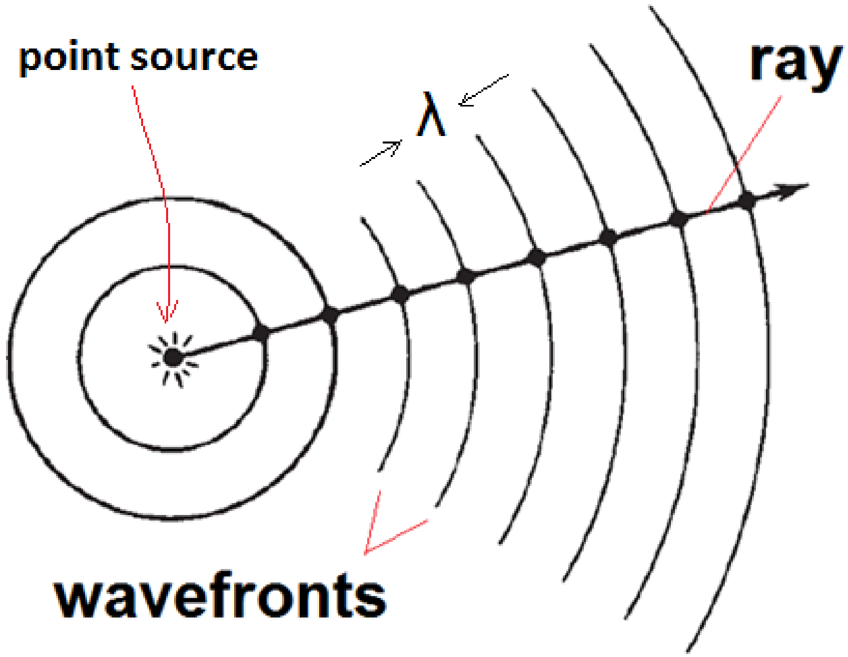


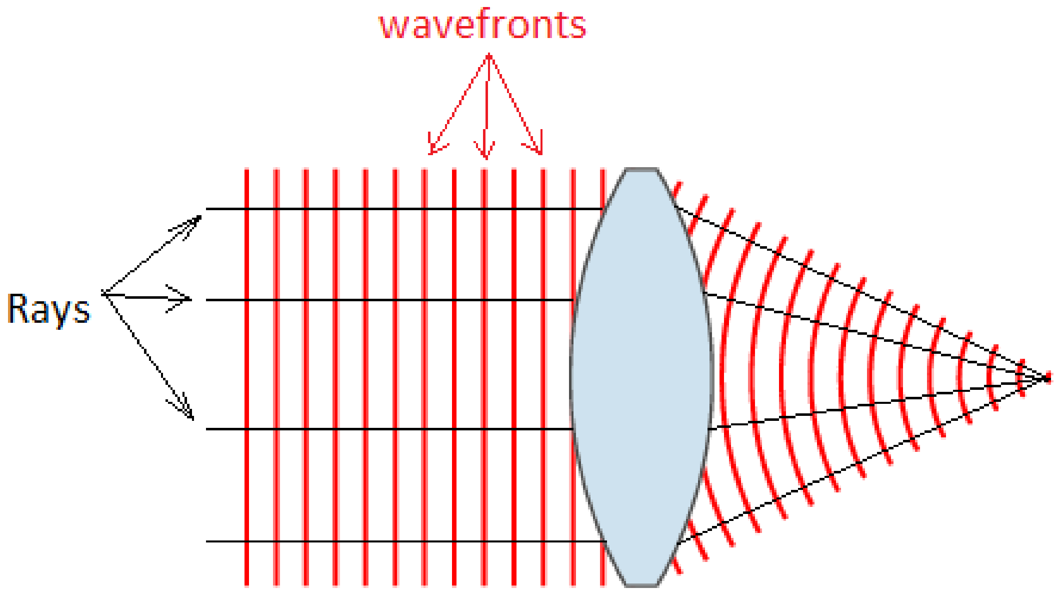
Ray and Wavefront

- The propagation of waves can be described by wavefronts.
- In 1D Plane wave equation is: $\psi(x,t) = A\sin(kx - \omega t) = Ae^{i(kx-\omega t)}$

  In 3D Plane wave equation is: $\psi(\mathbf{r},t) = A\sin(\mathbf{k}\cdot\mathbf{r} - \omega t) = Ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$
- **Wavefront** is the surface across which the phase is constant.
- The path of a point on a wavefront is called a light **ray**.
- Rays are perpendicular to wavefronts.

Amplitude:	*A*
Wave number:	$k = 2\pi/\lambda$
Frequency:	$\omega = 2\pi f$
Phase:	$\varphi = kx - \omega t$





Geometrical Optics

The field of Geometric Optics involves the study of the propagation of light with the **assumption** that:

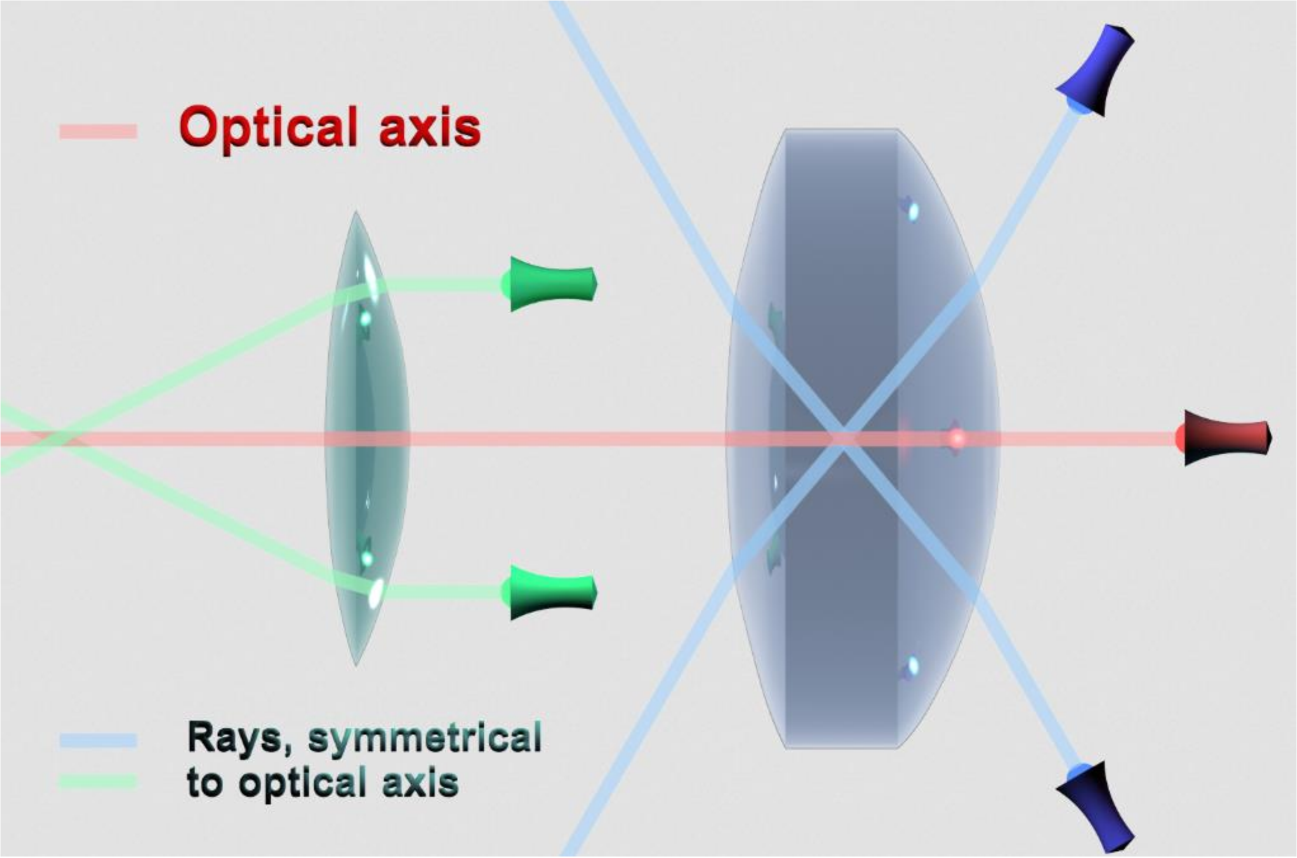
the light travels in a fixed direction in a straight line called ray.

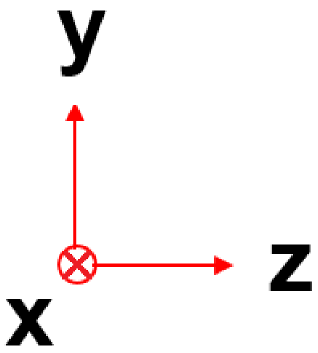
- Rays are defined to propagate in a rectilinear path as they travel in a homogeneous medium.
- Rays bend (and may split in two) at the interface between two dissimilar media.
- Rays may curve in a medium where the refractive index changes.
- Rays may be absorbed and reflected.

Optical Axis

In an optical system, the direction of ray propagation passing through the center of optical components are called the **optical axis**.

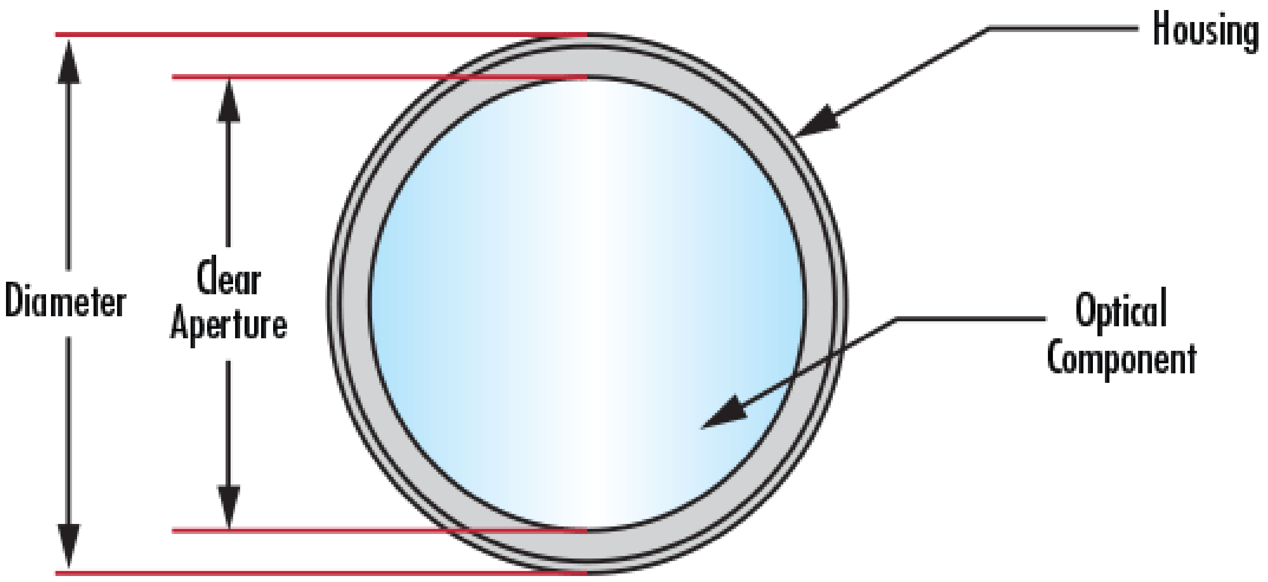
Direction is usually selected in +z axis.





Clear Aperture

- It is the limited opening of a lens for the light collection.
- It is not the mechanical aperture. Clear aperture is a few mm less than mechanical diameter of the lens.



Index of Refraction (Refractive Index)

- In optics, the **refractive index** of an optical medium is a dimensionless number that gives the indication of the light bending ability of that medium. It is defined as:

$$n = \frac{c}{v}$$

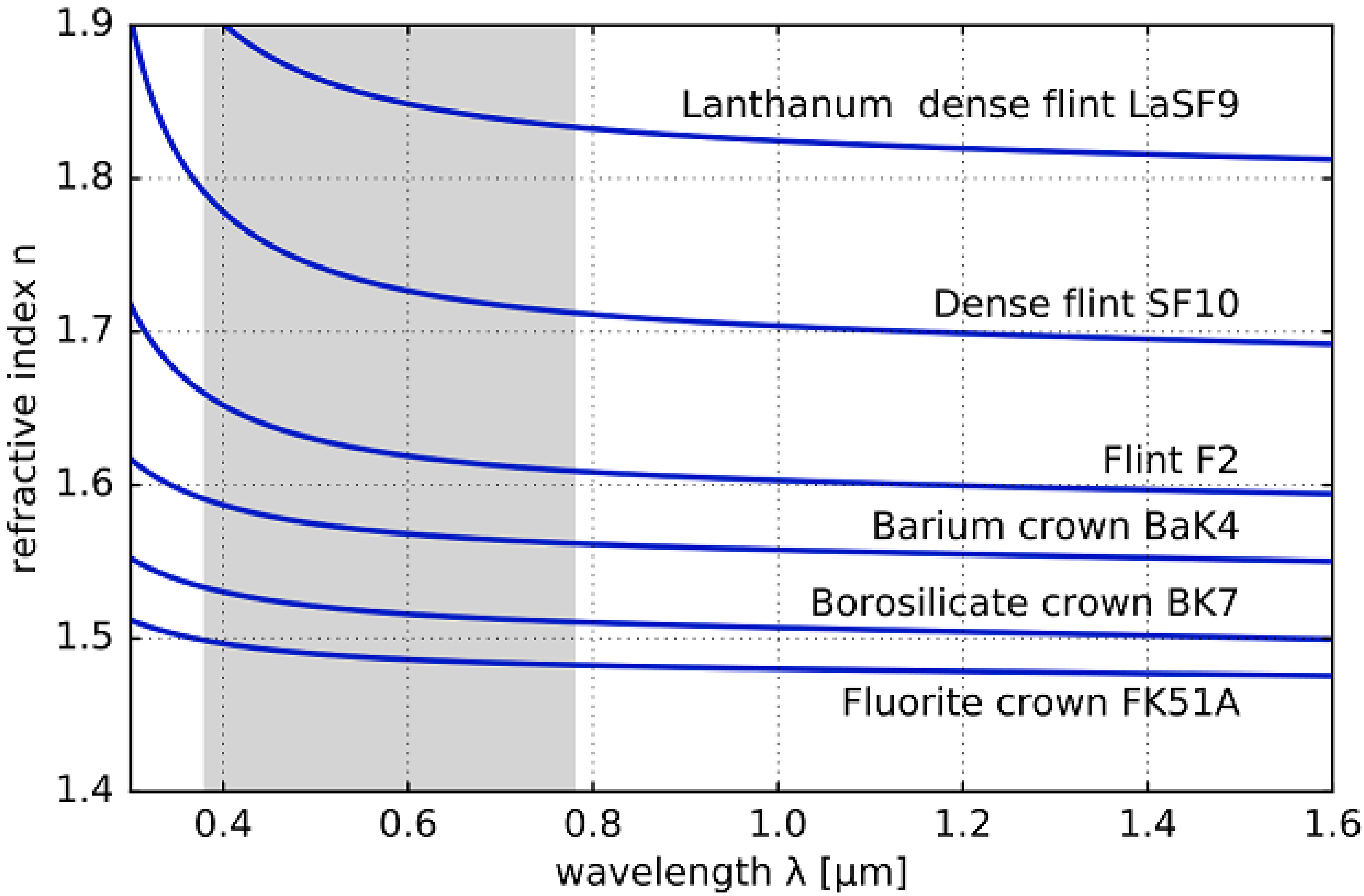
c = speed of light in vacuum
v = speed of light in optical medium
For optical glasses $n > 1$.

- White light contains many wavelengths (colors). Each color will refract in different direction in a lens. That is, index of refraction is function of wavelength,

$$n = n(\lambda)$$

- Variation of index with wavelength is called the **dispersion**. It is evaluated as:

$$\text{dispersion} \equiv \frac{dn}{d\lambda}$$



refractive index n
1.9
1.8
1.7
1.6
1.5
1.4
0.4
0.6
0.8
1.0
1.2
1.4
1.6
wavelength λ [μm]
Lanthanum dense flint LaSF9
Dense flint SF10
Flint F2
Barium crown BaK4
Borosilicate crown BK7
Fluorite crown FK51A

Abbe Number

If a white light falls on prism, each color deviate a different direction due to dispersion, Figure 4.10. As a dispersion measure, three reference colors (called Fraunhoffer F,d,C lines[1]) are defined as follows:

Table 4.2: Fraunhoffer F, d, C lines (wavelengths) and corresponding colors

Name	Wavelength	Color
F	486.1 nm	Blue
D	589.2 nm	Yellow
C	656.3 nm	Red

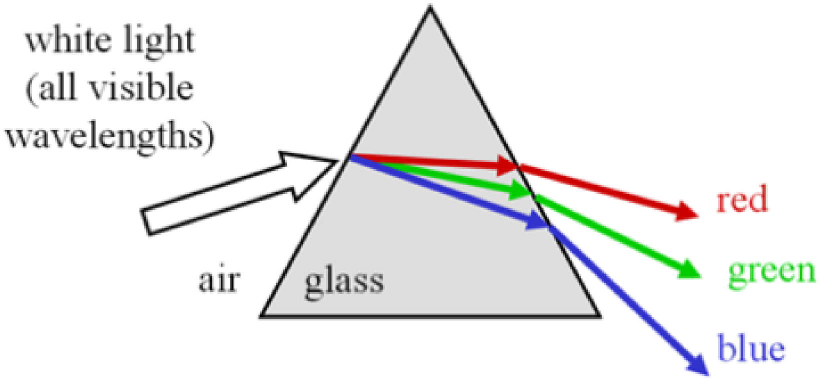


Figure 4.10: Dispersion of a prism.

Abbe value of a glass is defined as:

$$V = \frac{n_D - 1}{n_F - n_C} \qquad (4.22)$$

Using the dispersion curve in Figure 4.9, the Abbe value of BK7 can be found as $V_{BK7} \approx 64.2$. See also `https://refractiveindex.info`.

[1]These colors are observed first in the absorption band of the Sun spectra.

Optical Path Length

Optical Path Length for a light beam is defined as follows:

$$
\begin{aligned}
OPL &= \text{index of refraction} \times \text{path travelled by light} \\
&= n\ s
\end{aligned}
$$

If there are a number of mediums then

$$OPL = n_1 s_1 + n_2 s_2 + \cdots + n_k s_k = \sum_{i=1}^{k} n_i s_i \tag{4.13}$$

Finally, if the medium consists of continues materials then:

$$OPL = \int n(s) ds \tag{4.14}$$

Distance traveled in time t by light in optical medium, whose index of refraction is n, is

$$s = vt$$

or time traveled by light in the same medium

$$t = \frac{s}{v} = \frac{s}{c/n} = \frac{ns}{c} = \frac{OPL}{c} \tag{4.15}$$

Fermat’s Principle

Fermat's Principle of Least Time states that:

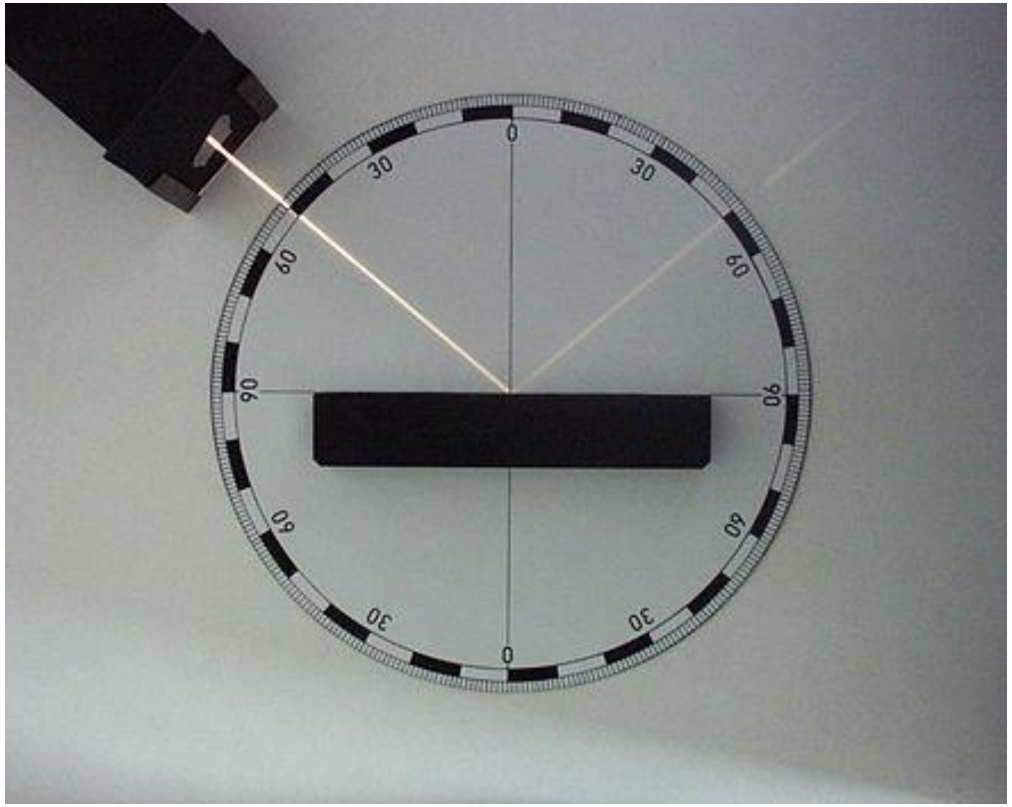
light takes the path which requires the shortest time.

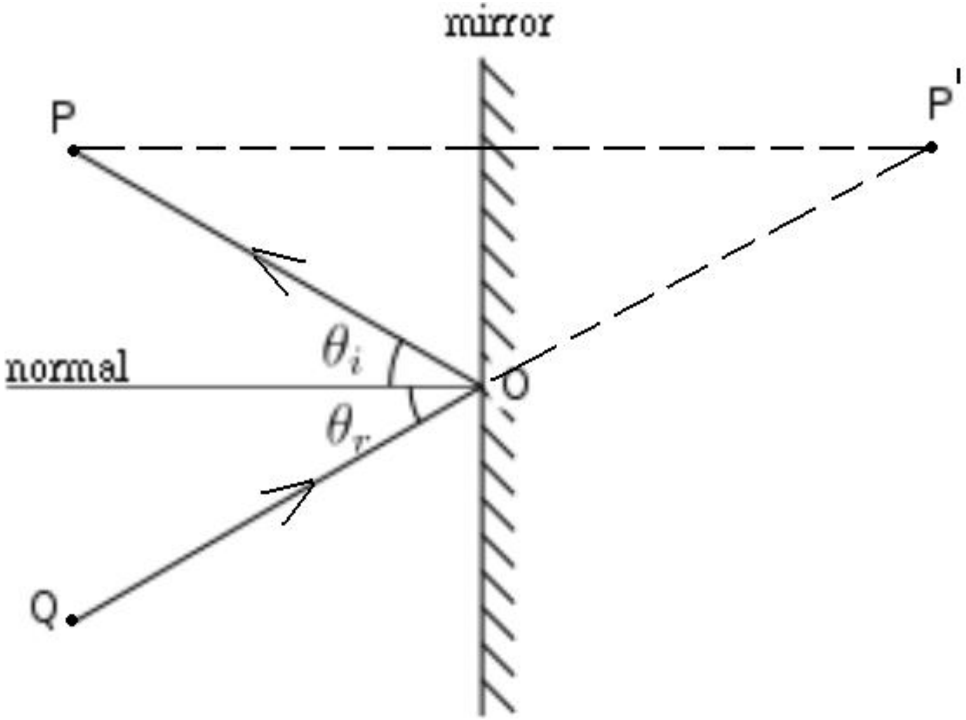
Fermat's principle is related to optimum time. That is, Fermat's principle is equivalently related to optimum OPL since $t = OPL/c$. Therefore, last form of the Fermat's principle can be written as:

light travels in medium such that its total optical path length is optimum.

Reflection

The law of reflection: $\theta_i = \theta_r$

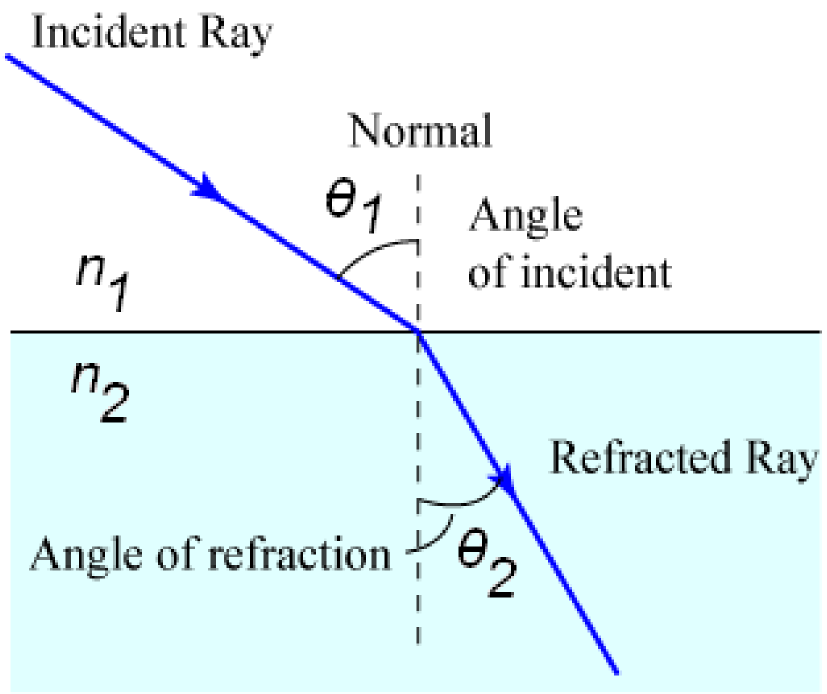




Refraction

- The refractive index determines how much the path of light is bent, or refracted, when entering an optical material.
- The rule for a refraction for a ray is described by **Snell's law**:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



- Snell’s law can be derived from Fermat’s Principle.

Snell's Law in 3D

$$\mathbf{t} = \sqrt{1-\mu^2\left[1-(\mathbf{n}\cdot\mathbf{i})^2\right]}\,\mathbf{n} + \mu\left[\mathbf{i}-(\mathbf{n}\cdot\mathbf{i})\,\mathbf{n}\right]$$

$$(\mathbf{i}+\mathbf{r})\cdot\mathbf{n} = 0$$

$\mu = n_1 / n_2$

i = incident ray (vector)

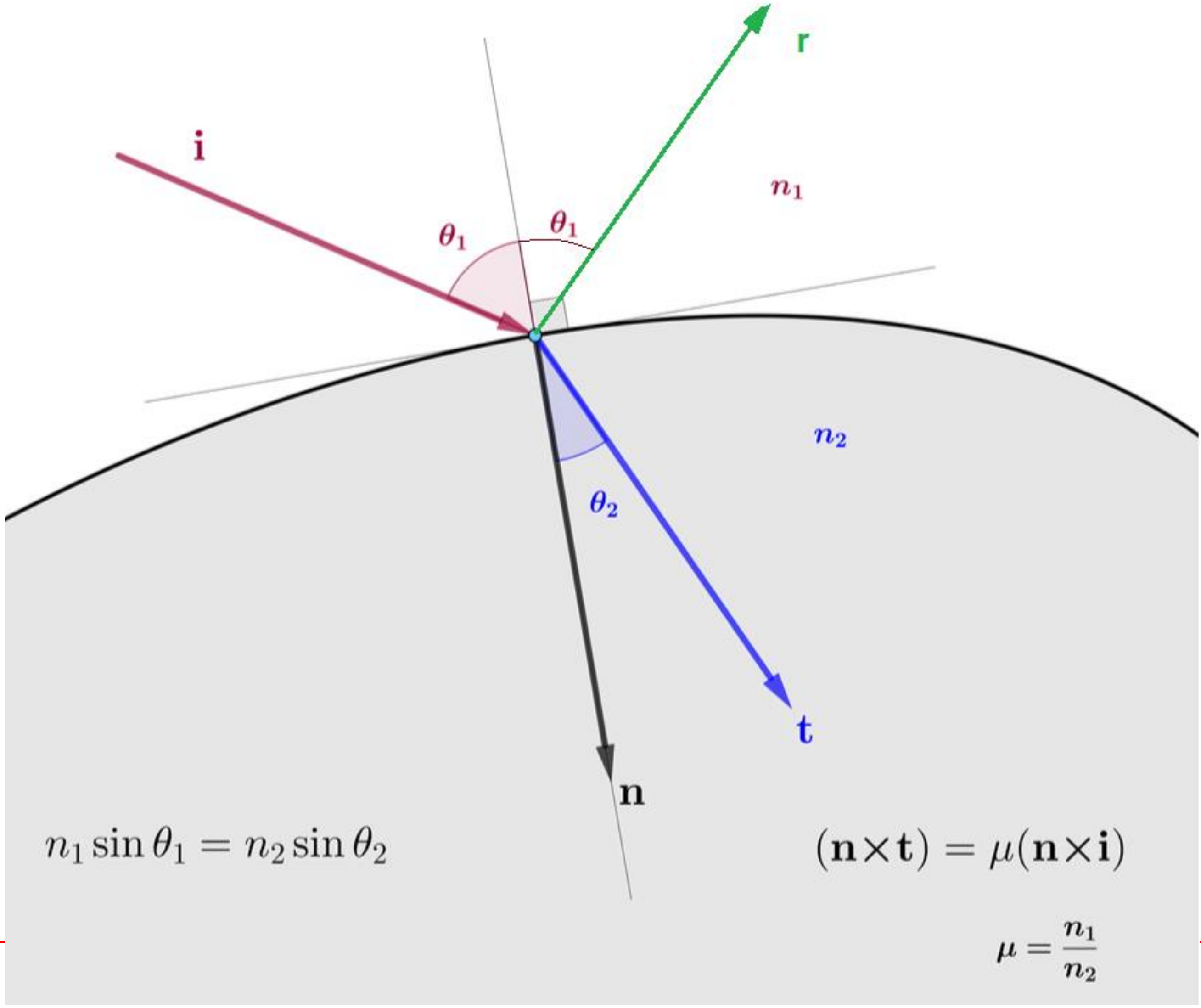
t = transmitted ray (vector)

r = reflected ray (voctor)

n = $\frac{\nabla f}{|\nabla f|}$ = surface normal vector

$$f = f(x, y, z) = c$$

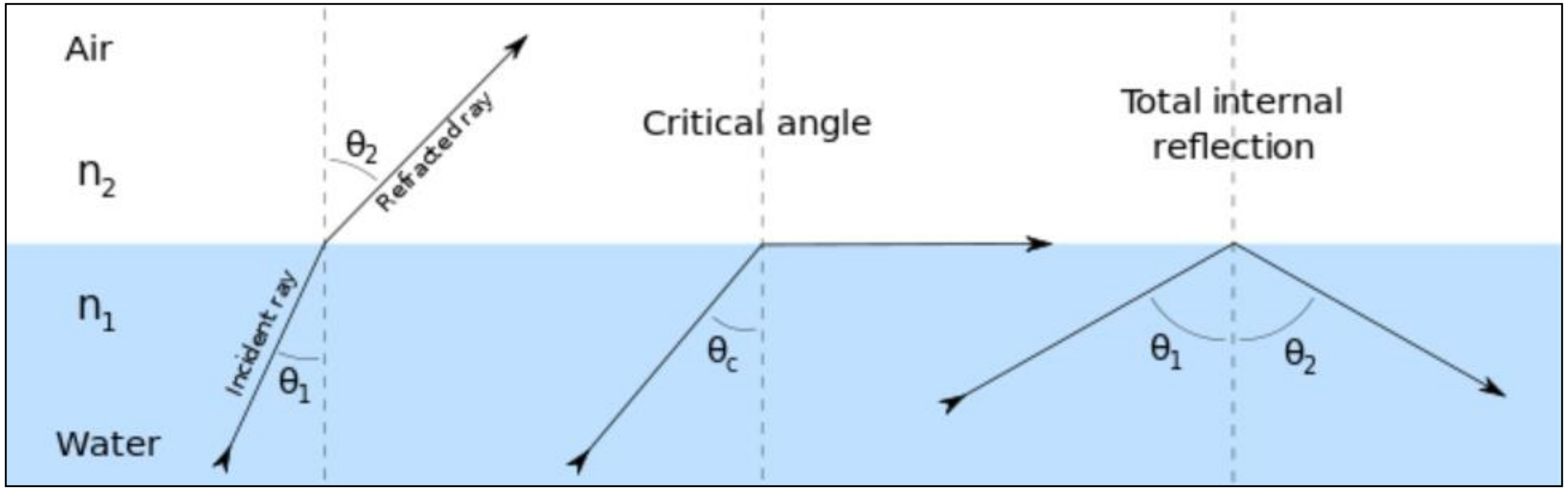
i.e. $f = x^2 + y^2 + (z-5)^2 = 144$



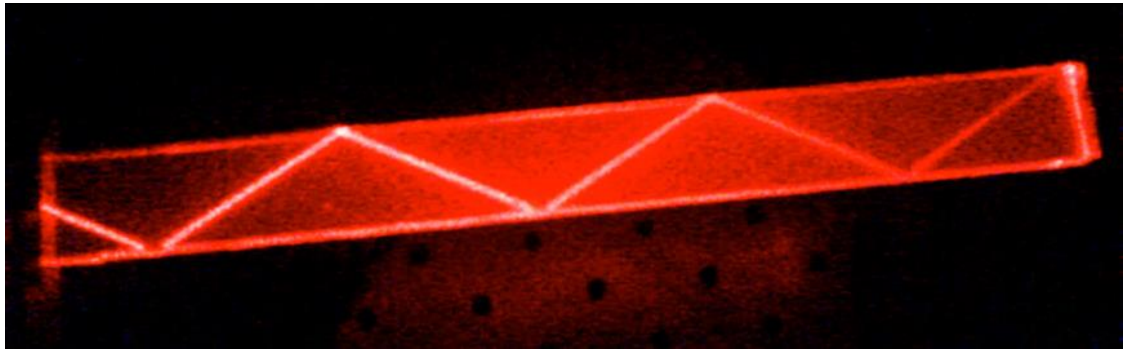
Total Internal Reflection (TIR)

TIR happens when a ray strikes a medium boundary at an angle larger than a particular critical angle given by:

$$\sin\theta_c = n_2/n_1$$



TIRs in a block of acrylic ==>

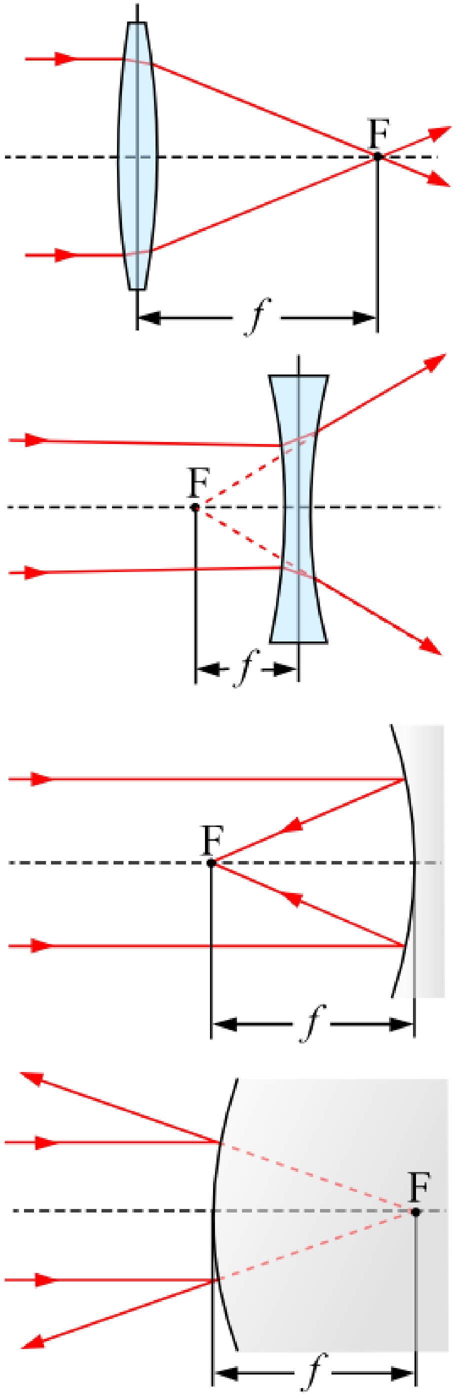


Focal Length

- The **focal length** (f) of an optical system is a measure of how strongly the system converges or diverges light.
- Inverse of focal length is called the **optical power** (*P*).

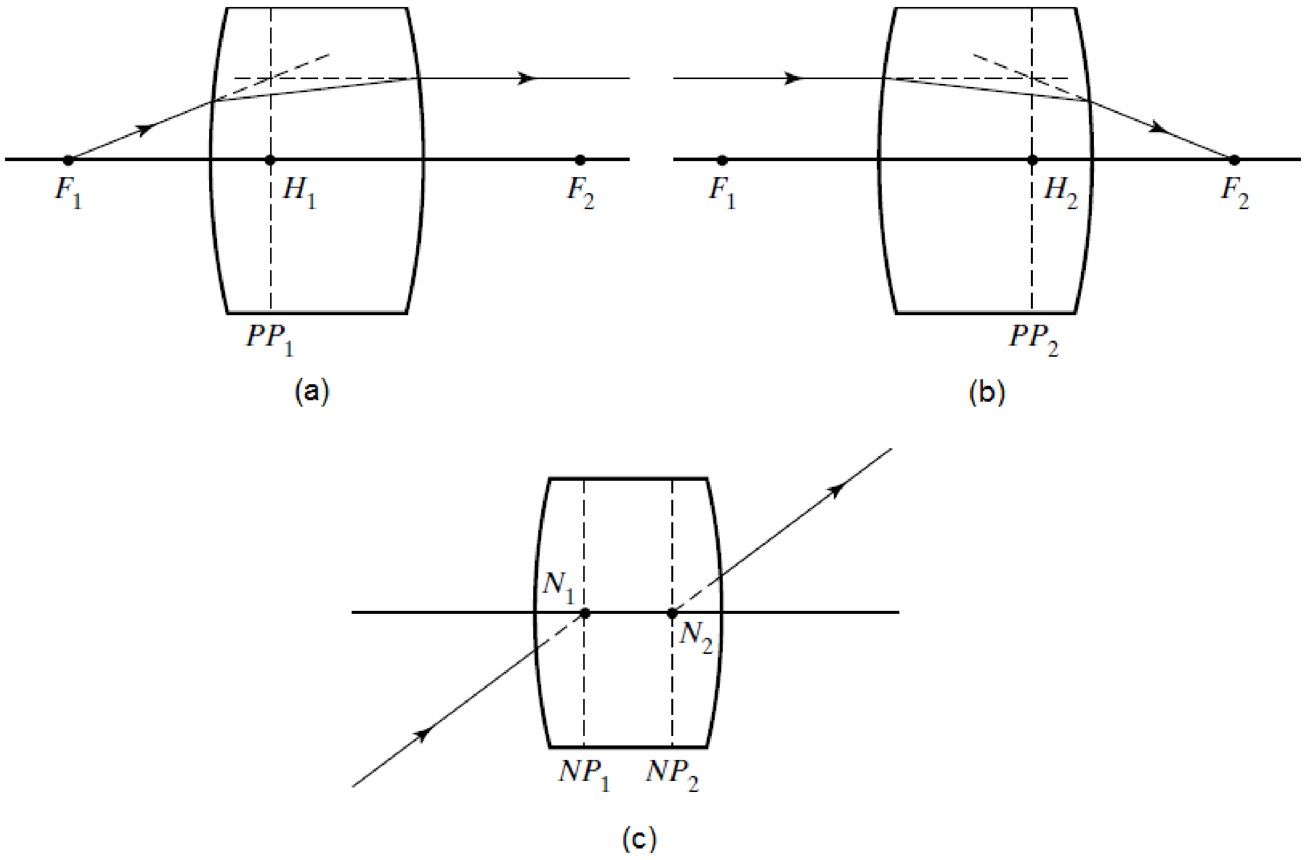
$$P = 1/f$$

- SI unit of focal length is meter, m.
  SI unit of power is diopter, D = 1 / m



Focal Length of Thick Lens

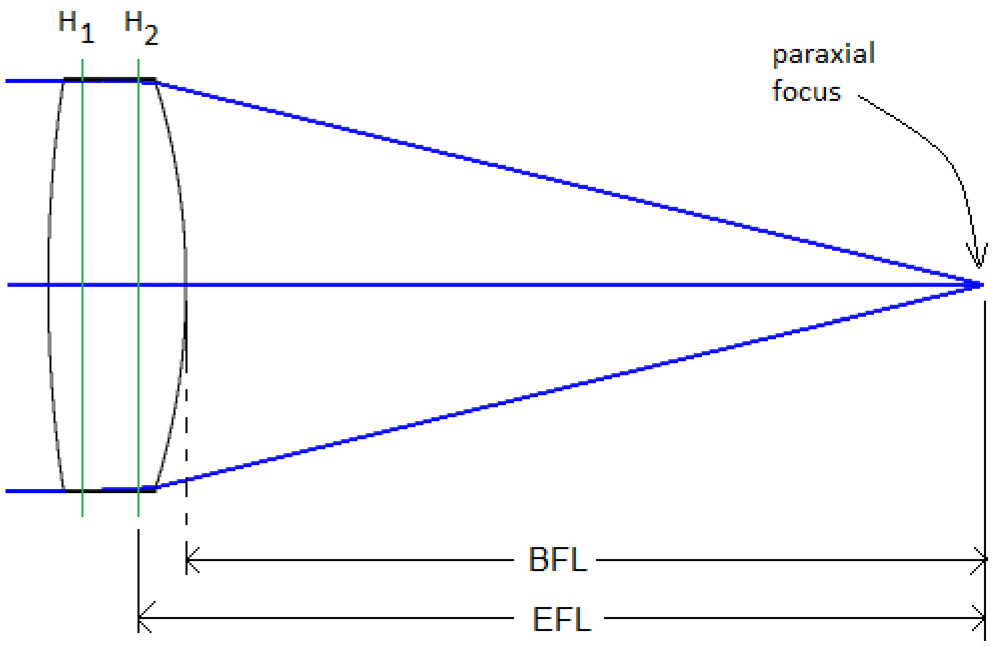
- There are six **cardinal points which** are widely used to approximate the behavior of real optical systems.
  - the first and second system focal points; F_1 and F_2.
  - the first and second principal points; H_1 and H_2.
  - the first and second nodal points; N_1 and N_2.



Effective Focal Length (EFL)
of a lens in air is calculated from:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)t}{nR_1R_2}\right]$$

R_1, R_2 are radius of curvatures
t is the center thickness
n is the refractive index of the lens.

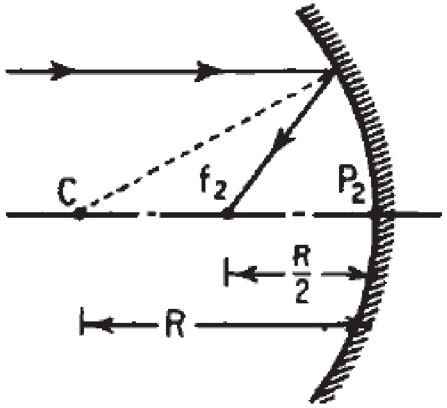


EFL is measured from second principle plane.
Back Focal Length (BFL) is measured from the second vertex.

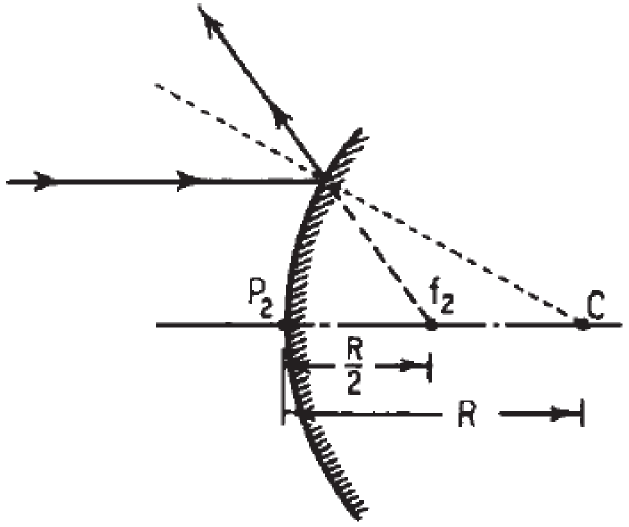
Focal Length of Spherical Mirror

Effective Focal Length (EFL) of a mirror is calculated from:

$$f = \frac{R}{2}$$



CONCAVE MIRROR
(CONVERGING)



CONVEX MIRROR
(DIVERGING)

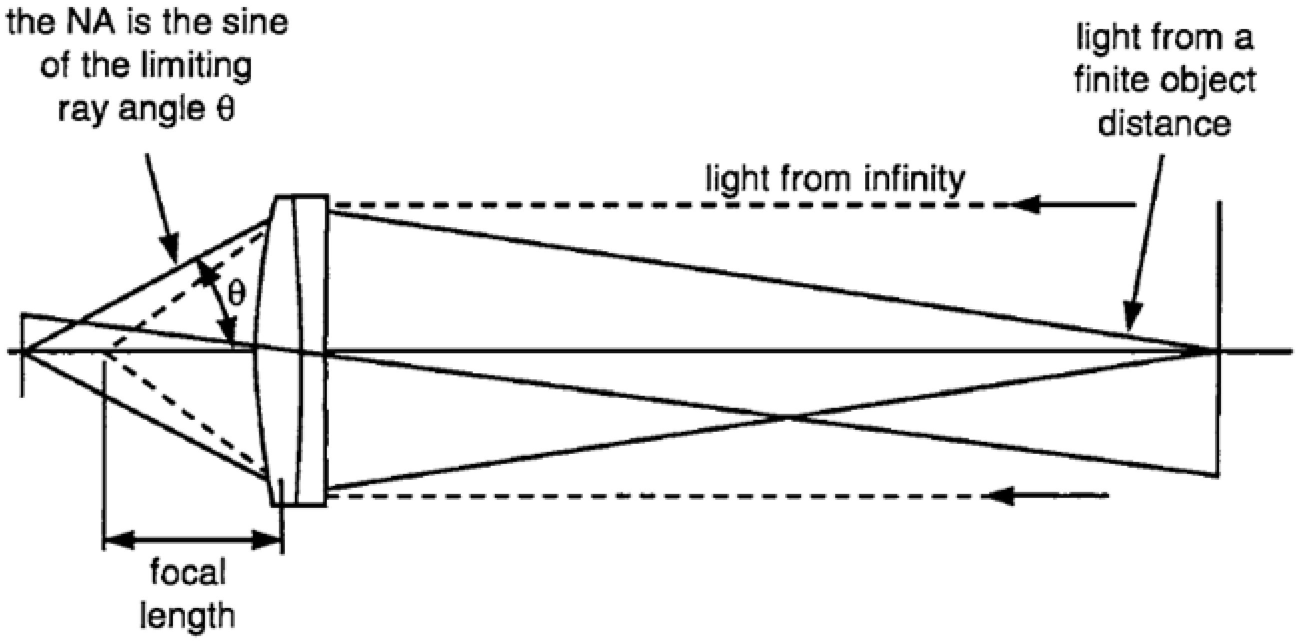
Numerical Aperture

The numerical aperture is simply the sine of the half cone angle coming from the axial object point. *This term is one of important the system parameter in microscope designs*.

Numerical aperture is defined as:

$$NA = n_0 \sin(\theta)$$

n_0 **is index of medium**
(**usually** $n_0 = 1$)



Focal length implies light from infinity

F-number (f/#)

Paraxial f/#: $$f/\# = \frac{f}{D} = \frac{\text{Effective Focal Length}}{\text{Entrance Pupil Diameter}}$$

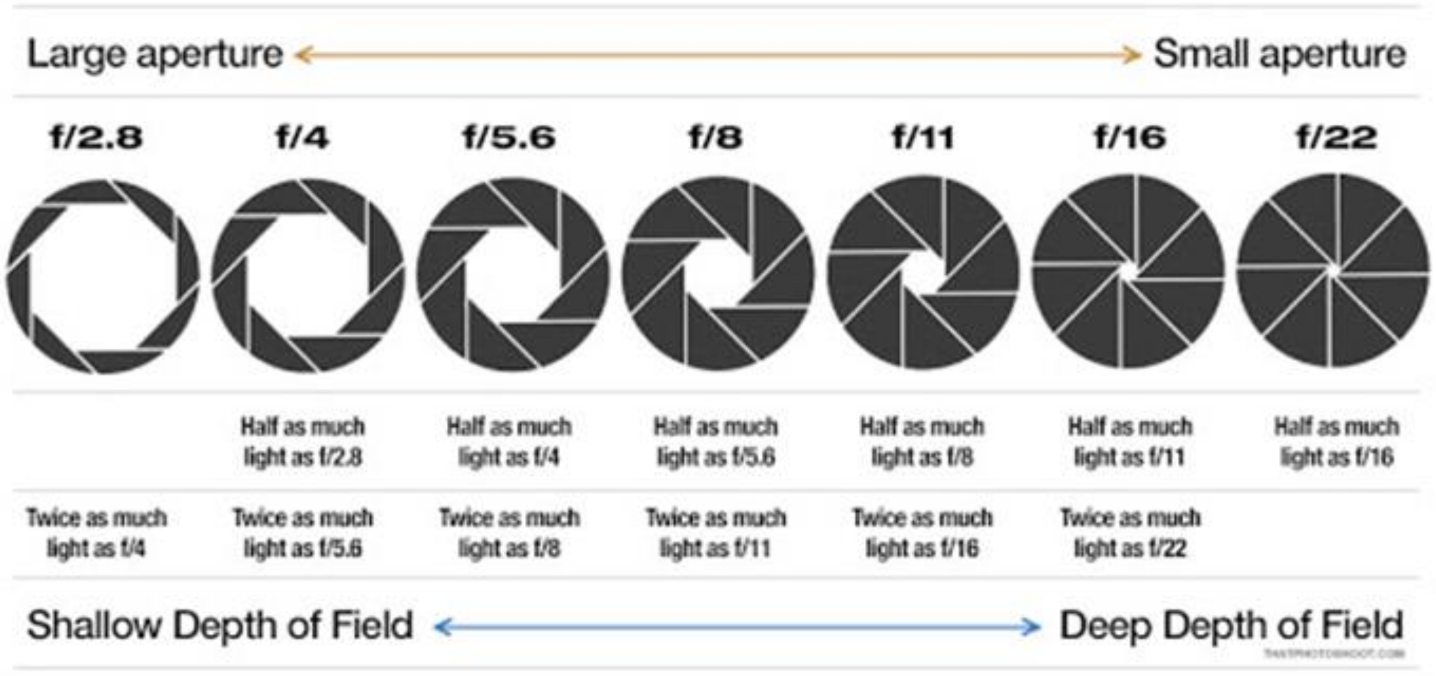
Working f/#: $$W = \frac{1}{2NA} = \frac{1}{2n_0 \sin(\theta)}$$

W	NA
1	0.5
1.5	0.3333
2	0.25
4	0.125
8	0.0625

NOTES:

- If the angle θ is small, definitions are equivalent.
- In application, we usually use paraxial f/#.

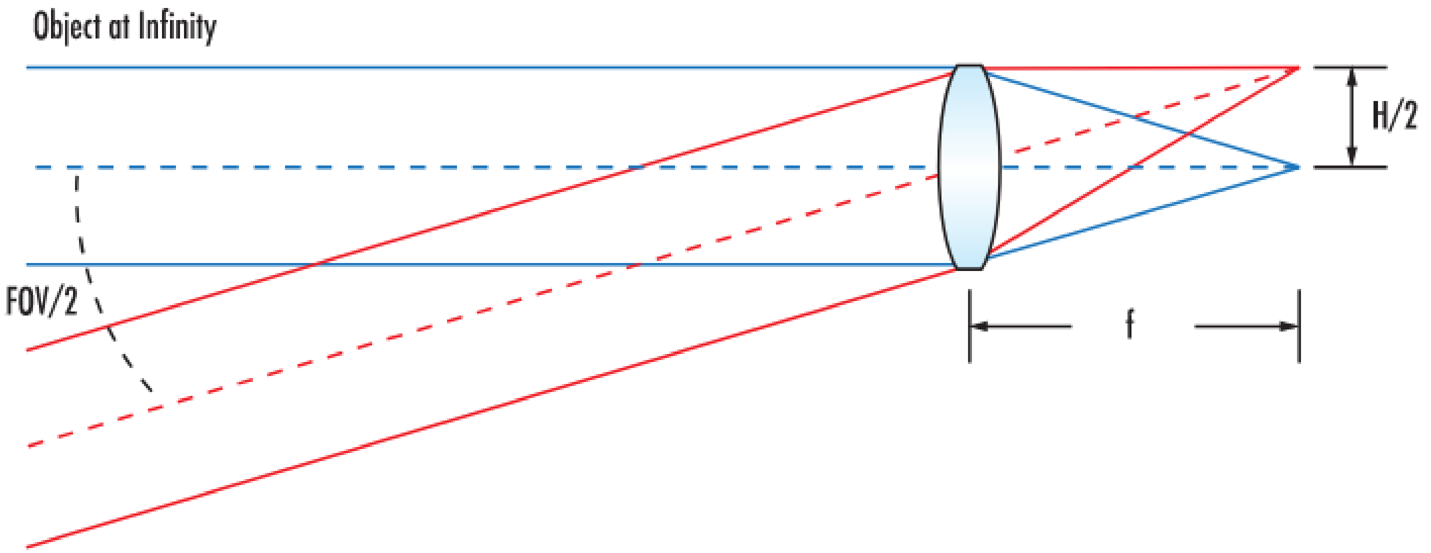
F-Number vs Depth of Field



Field of View (FOV)

It defines angle 'seen' by an optical system. The sensor size and focal length of the lens determines FOV:

$$FOV = 2\tan^{-1}\left(\frac{H/2}{f}\right)$$



We will use the following notation:

FOV = Full Field of View
FOV/2 = SFOV = Semi Field of View

e.g: FOV = 20^o (FOV ≡ ±10^o)
SFOV = 10^o.

Example 1:

if sensor size is *H* = 5 mm and focal length is *f* = 50 mm, then

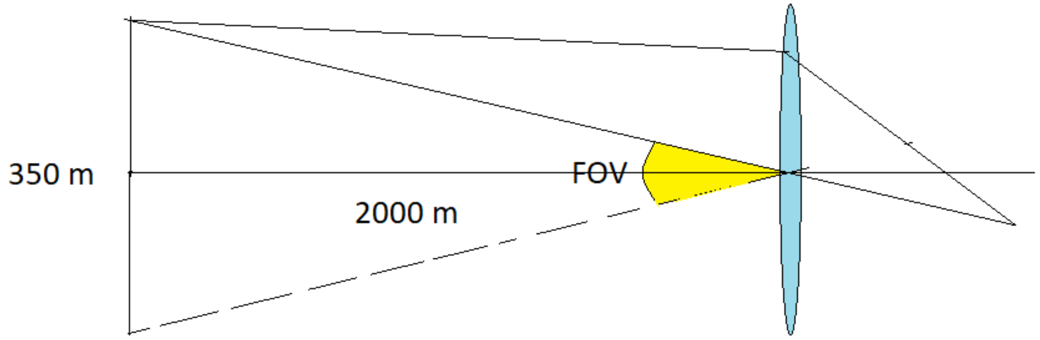
$$FOV = 2\tan^{-1}\left(\frac{5/2}{50}\right) = 5.7^{o}$$

Example 2:

In same applications, FOV is defined in terms of distance units. For instance, '*the field of view can be expressed as 350 m at a distance of 2 km'.*

What is angular FOV?

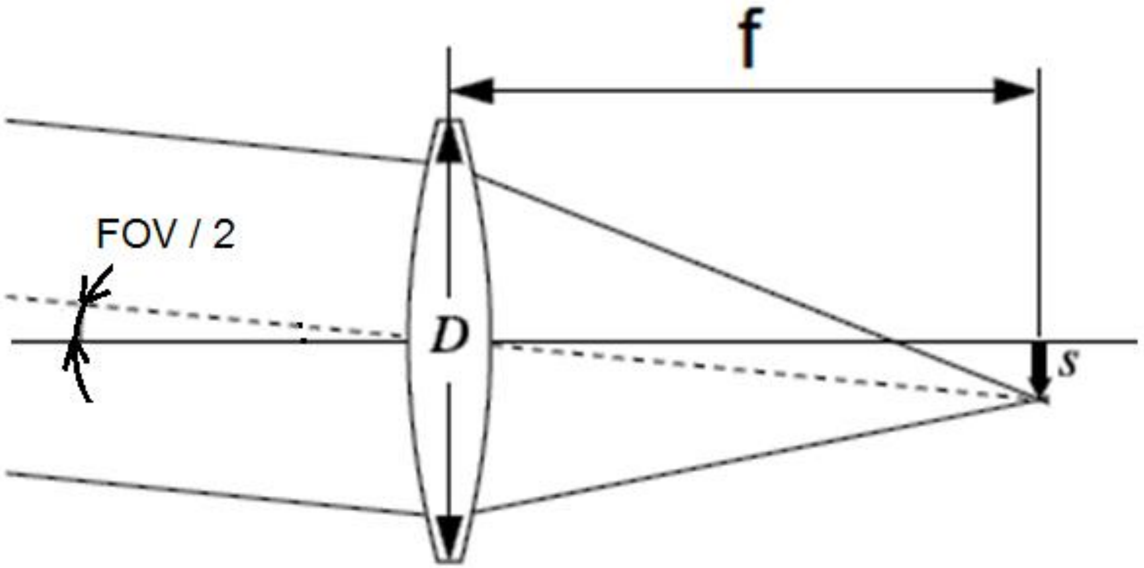
$$FOV = 2\tan^{-1}\left(\frac{350/2}{2000}\right) = 10^{o}$$



FOV and Size of Image

The **scale** of the image formed in the focal plane of an optical system can be geometrically determined. When the object is seen at the angle ***FOV***, it forms an image of height ***s***:

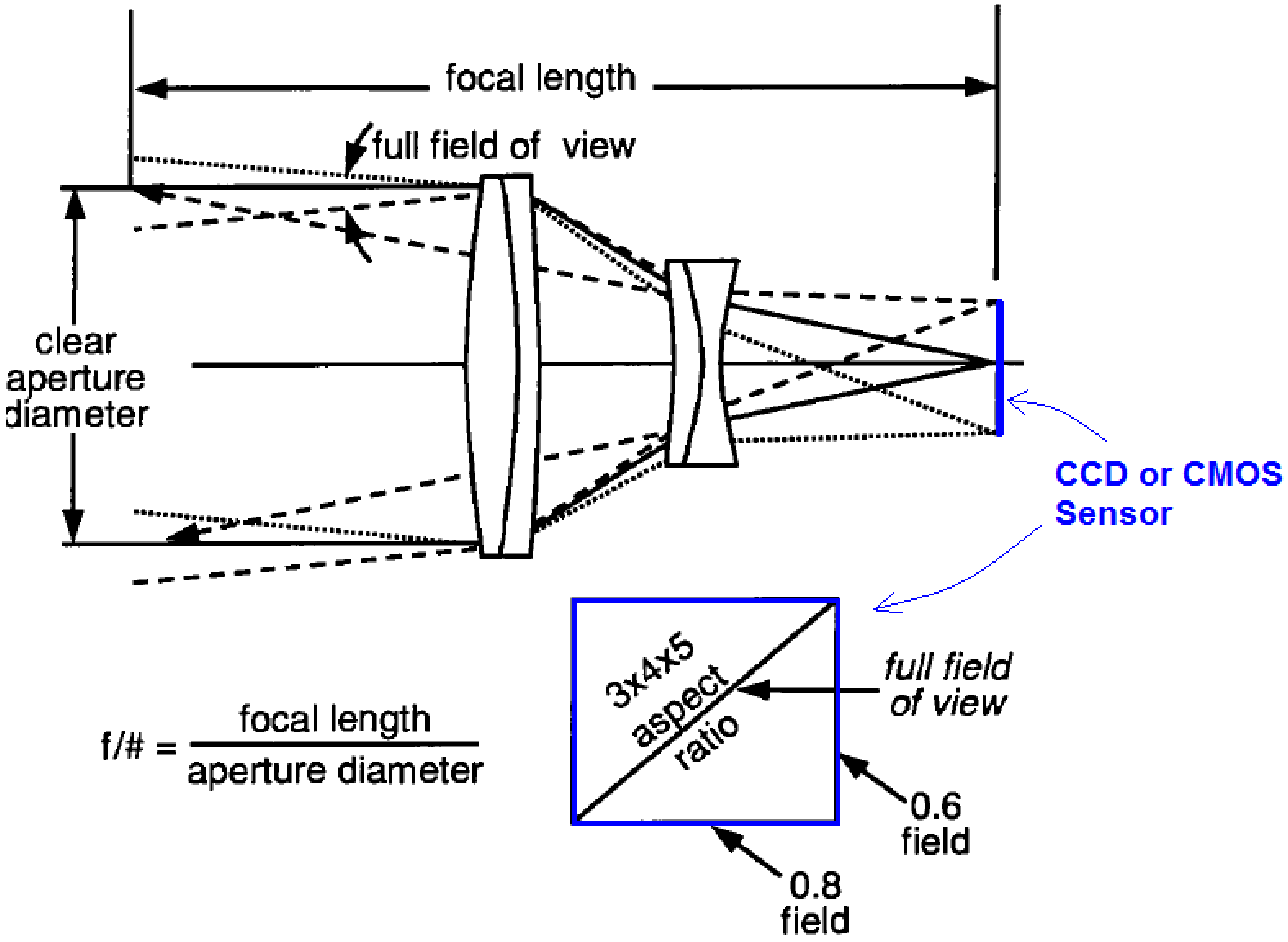
$$s = f \tan(FOV/2)$$



Numerical example:

f = 17 mm and $FOV = 0.5^{o}$ => s = 0.074 mm.

Basic Parameters in an Optical System



Stops and Pupils

- **Aperture Stop (AS):** controls number of rays from object to image plane.

- **Field Stop (FS):** do or do not obstruct rays entirely.

- **Enterance pupil (EnP):** The image of the aperture stop as seen from object space is called the Entrance Pupil (EnP) of the system.
  For telescopes or binoculars EnP = diameter of objective

- **Exit pupil (ExP):** The image of the aperture stop as seen from image space is known as the Exit Pupil (ExP) of the system.
  Exit Pupil of telescopes and microscopes are usually selected as the size of human pupil (~ 2-8 mm).

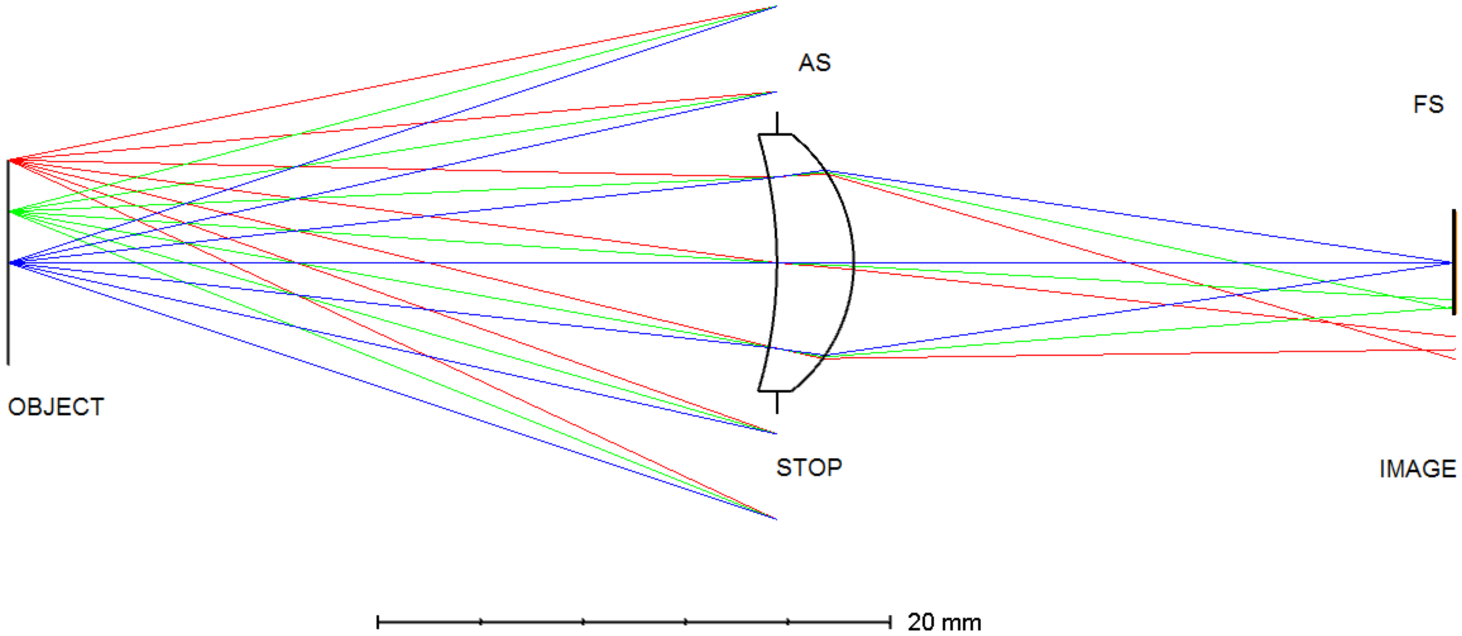
Fields 0, 2 and 4 degreess

Lens Data

Update: All Windows

Surface 4 Properties | Configuration 1/1

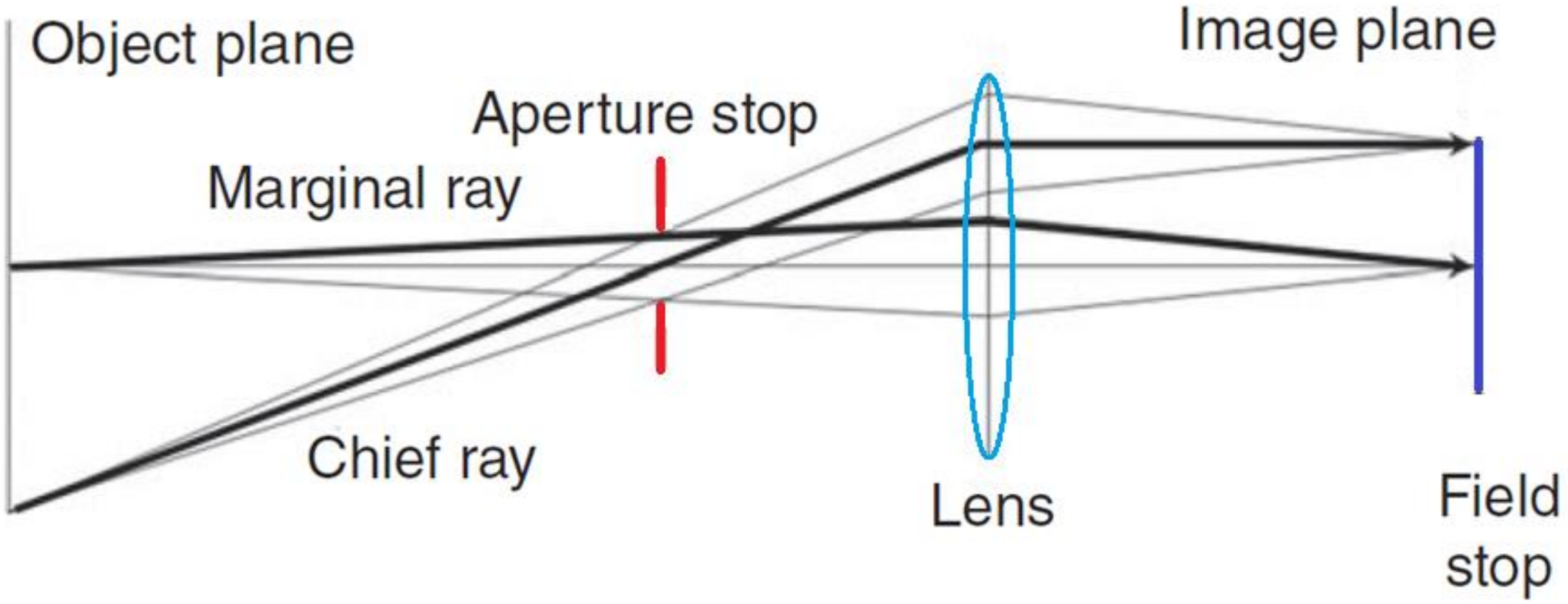
	Surface Type		Comment	Radius	Thickness	Material	Coating	Clear Sem	Chi	Mech Semi	Conic	TCE x 1E-6
0	OBJECT	Standard		Infinity	30.000			4.000	0.0..	4.000	0.0...	0.000
1	STOP	Standard		Infinity	0.000			5.000 U	0.0..	5.000	0.0...	0.000
2	(aper)	Standard		-17.500	3.000	N-BK7		5.000 U	0.0..	5.000	0.0...	-
3	(aper)	Standard		-6.364	23.500			5.000 U	0.0..	5.000	0.0...	0.000
4	IMAGE	Standard		Infinity	-			2.000 U	0.0..	2.000	0.0...	0.000



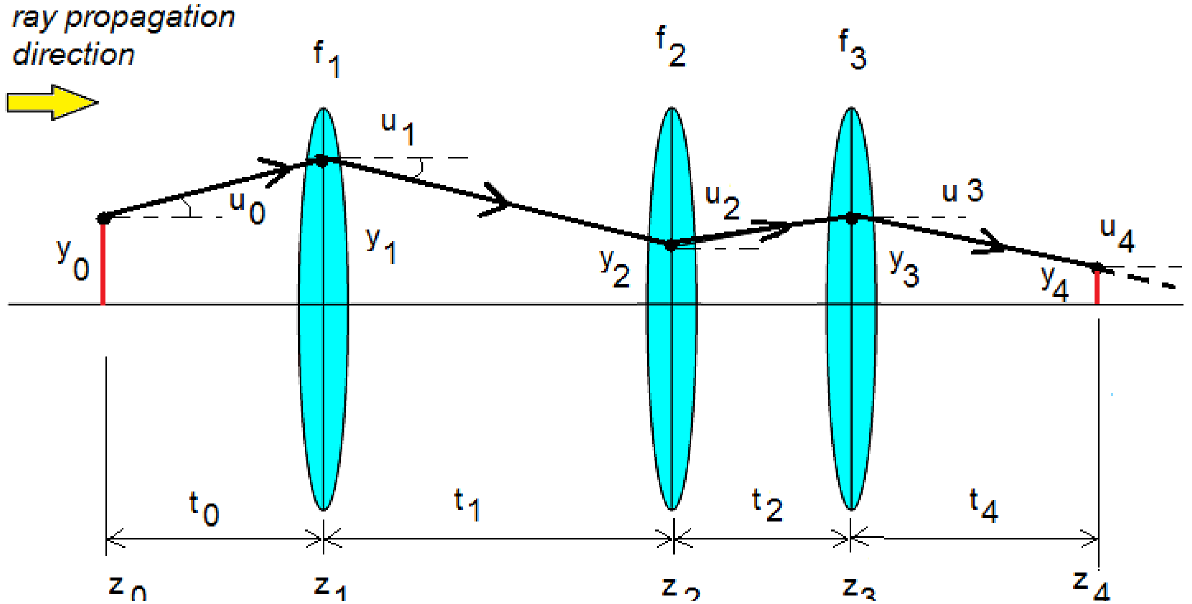
Marginal and Chief Rays

Marginal Ray: are rays from object and passing through the edge of AS.

Chief Ray: are rays from object passing throgh the center of AS.



Ideal Thin Lens Ray Tracing (y-u method)



Using paraxial rays ($\tan u \approx u$) in Figure the ray transfer equation from one lens to another is given by:

$$y_{k+1} = y_k + u_k t_k \tag{5.1}$$

slope angle equation can be obtained from

$$u_{k+1} = u_k - y_{k+1} p_{k+1} \tag{5.2}$$

and ray position along z-axis:

$$z_{k+1} = z_k + t_k \tag{5.3}$$

Starting with an initial ray position (z_0, y_0, u_0), this iterative procedure is used to obtain final ray position (z_n, y_n, u_n).

Here

- $k = 0, 1, 2, \cdots, n$
- y_k = ray height at k^{th} lens (surface)
- u_k = ray slope (angle in radian) at k^{th} lens (surface)
- t_k = distance between k^{th} and $(k+1)^{\text{th}}$ lens (surface)
- z_k = z-position of the k^{th} lens (surface). Usually we start with $z_0 = 0$.
- $p_k = 1/f_k$ is the power of the k^{th} lens whose focal length is f_k. Note that for object (OBJ) and image (IMG) planes $p_k = 0$.

Focal Length and Magnification

By using a ray which is parallel to optical axis, namely $(z_0, y_0, u_0) = (0, y_0, 0)$, the effective focal length and back focal length of the optical system can be obtained from:

$$\text{efl} = f = -\frac{\text{initial ray height}}{\text{final ray angle}} = -\frac{y_0}{u_n} \tag{7.3}$$

and

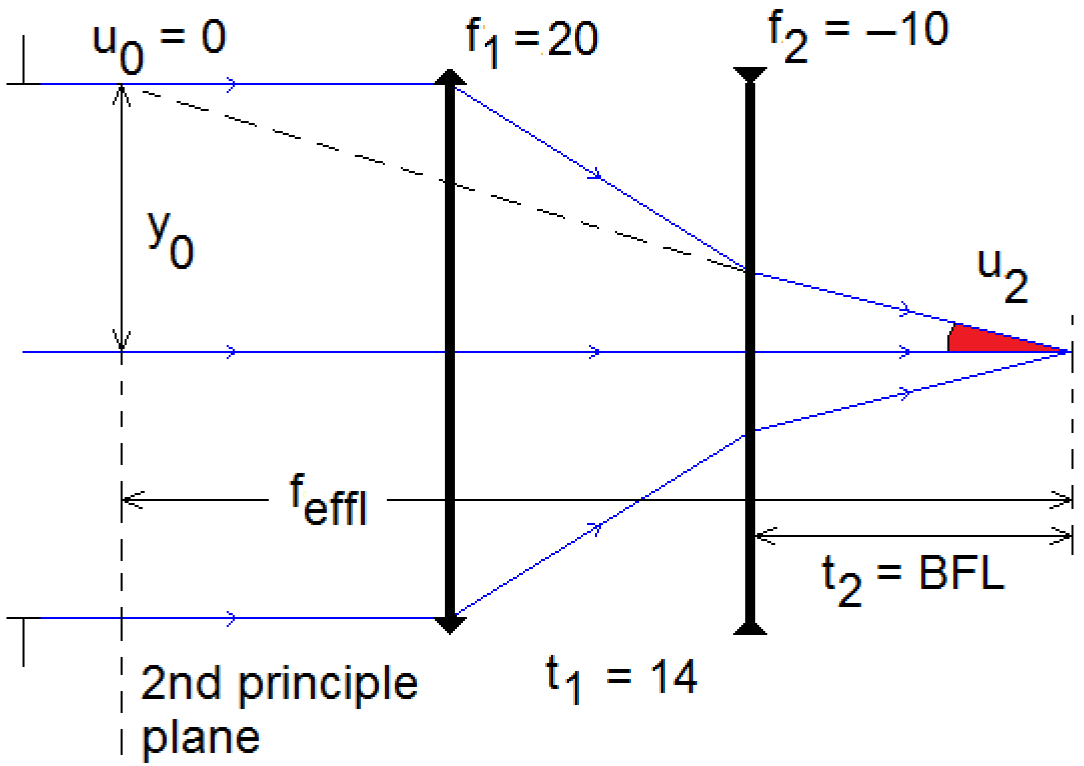
$$\text{bfl} = -\frac{\text{final lens surface ray height}}{\text{final ray angle}} = -\frac{y_{n\text{-}1}}{u_n} \tag{7.4}$$

Also, for parallel incident rays, the magnification may be defined as:

$$m = \frac{y_n}{y_0} \tag{7.5}$$

Example

Figure shows a simple telephoto lens. Calculate effective focal length and back focal length of the system using y-u trace.



Solution

$$y_{k+1} = y_k + u_k t_k$$

$$u_{k+1} = u_k - y_{k+1} p_{k+1}$$

Given: $f_1 = 20$, $f_2 = -10$, $t_1 = 14$
$p_0 = 0$, $p_1 = 1/20$, $p_2 = -1/10$, $p_3 = 0$
Let's start with $y_0 = 1$, $u_0 = 0$, $t_0 = 1$

$y_1 = y_0 + u_0 t_0 = 1 + (0)(1) = 1.0$
$u_1 = u_0 - y_1/f_1 = 0 - 1/20 = -0.05$

$y_2 = y_1 + u_1 t_1 = 1 + (-0.05)(14) = 0.3$
$u_2 = u_1 - y_2/f_2 = -0.05 - 0.3/(-10) = -0.02$

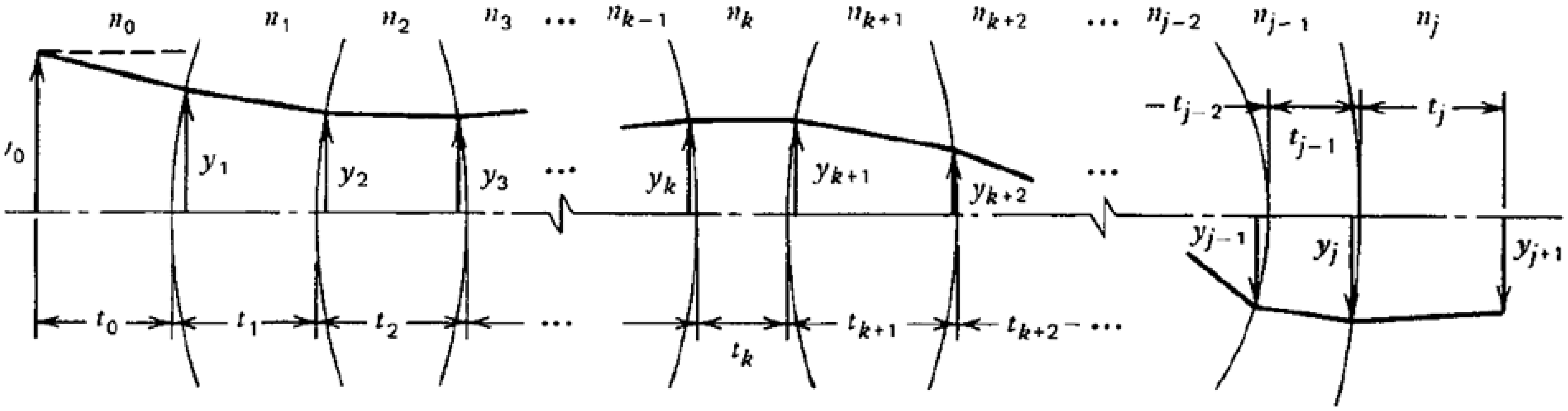
$y_3 = y_2 + u_2 t_2 = 0.3 + (-0.02)(t_2) = 0.0$ => $t_2 = 15$
$u_3 = u_2 - y_3/f_3 = -0.02 - 0.10/(-\text{inf}) = -0.02$

BFL = $-y_2/u_3 = -0.3/-0.02 = 15$ mm
EFFL = $-y_0/u_3 = -1.0/-0.02 = 50$ mm
Telephoto ratio = EFFL/(t_1+t_2) = 50/(14+15) = 1.7

Summary of y-u trace:

z	y	u
0.0000	1.0000	0.0000
1.0000	1.0000	-0.0500
15.0000	0.3000	-0.0200
29.0000	0.0000	-0.0200

Thick Lens Ray Tracing (y-nu method)



As in the y-u trace, the ray transfer equation is as follows:

$$y_{k+1} = y_k + u_k t_k \tag{7.1}$$

Then, the slope angle (or refraction) equation is given by:

$$n_{k+1}u_{k+1} = n_k u_k - (n_{k+1} - n_k)\frac{y_{k+1}}{R_{k+1}} \tag{7.2}$$

n = index of refraction

R = radius of curvature

Diffraction

Diffraction is an effect resulting from the interaction of light wave with the sharp limiting edge or aperture of an optical system.