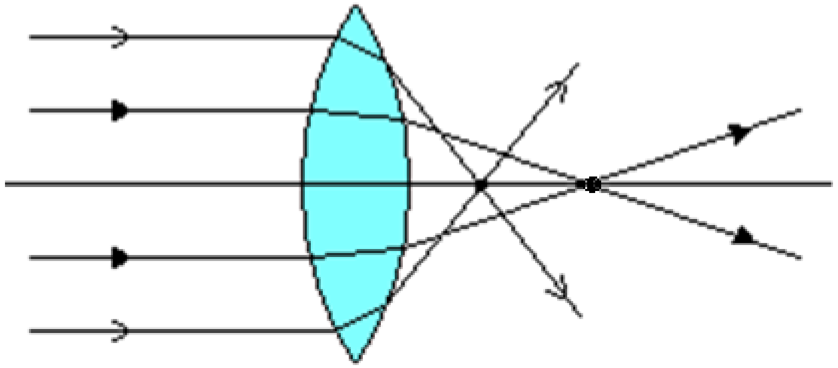
Aberrations



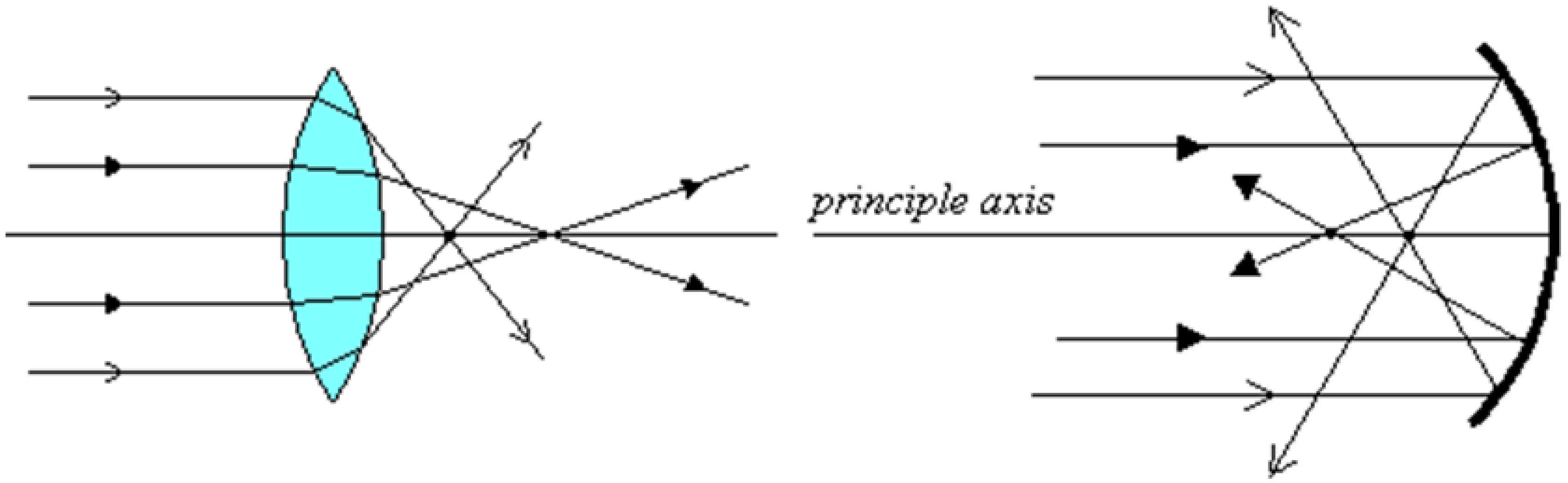
Ahmet Bingül

Gaziantep University
Department of Optical
Engineering

Sep 2024

What is aberration?

- Paraxial approximations result in perfect image!
- Imperfect images caused by geometric factors are called aberrations.
- Aberration leads to blurring of the image produced by an image-forming optical system.



Aberration Types

1. Spherical aberration
2. Coma
3. Astigmatism
4. Field Curvature
5. Distortion
6. Chromatic Aberration

occur with monochromatic light

due to dispersion of optical material

Monochromatic Aberrations

Origin of Aberrations

Snell's law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Taylor series of expansion: $\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$

Taking only first term ➔ We arrive first order optics which is the study of perfect optical systems without aberrations.

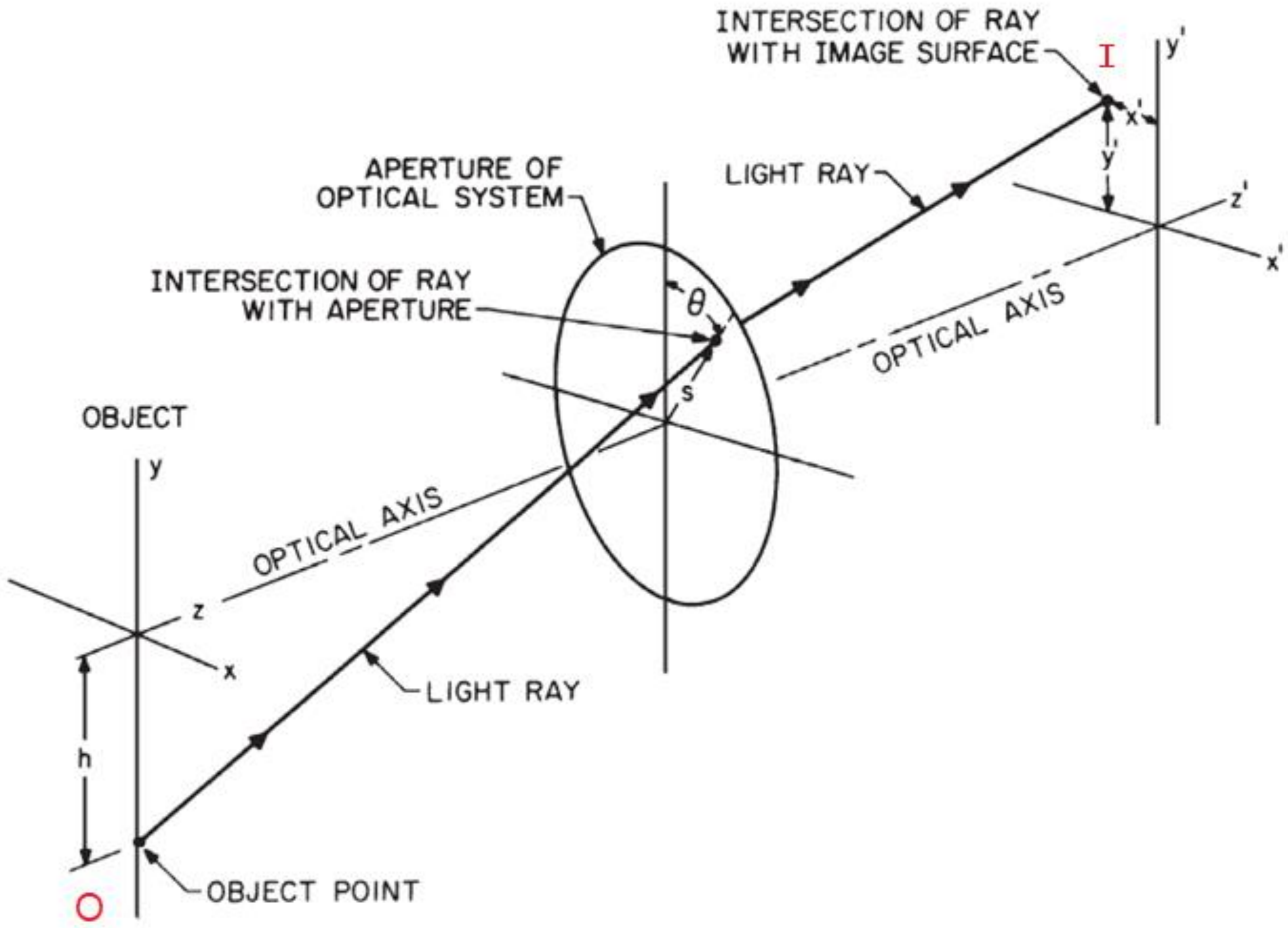
Including third order terms ➔ We arrive third order optics.

- In 3rd order optics, we have set of equations for describing lens aberrations as departures from paraxial theory.
- These equations are called Siedel Aberrations.

Seidel Coefficients

Consider a ray originates from object point at **O**(0, -h, 0). The ray hits image surface at **I**(x’, y’, z).

Question: What are the mathematical relations between these two points?



Seidel Coefficients

The solution is given below. The coefficients of

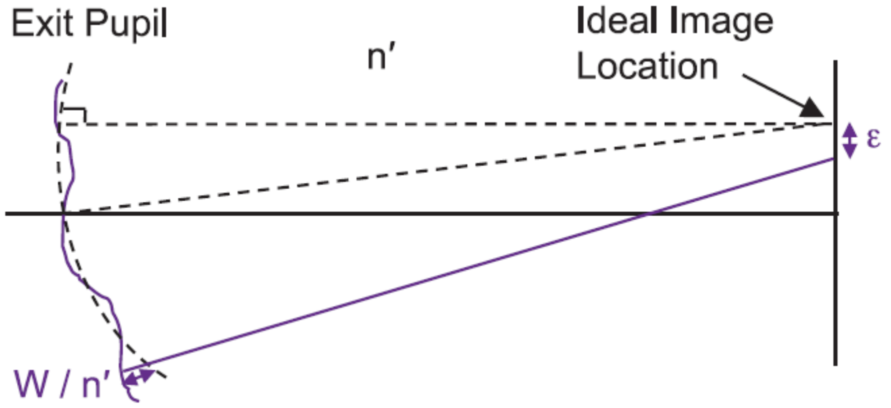
- 1^{st} order terms: A_1, A_2 are related to perfect imaging.
- 3^{rd} order terms: B_1, B_2, B_3, B_4, B_5 are Seidel Coefficients. They are related to 3rd order departure from perfect imaging. (Higher order terms can also be included).

$$
\begin{aligned}
y' &= A_1 s \cos\theta + A_2 h \\
&+ B_1 s^3 \cos\theta + B_2 s^2 h(2+\cos 2\theta) + (3B_3 + B_4) s h^2 \cos\theta + B_5 h^3 \\
&+ C_1 s^5 \cos\theta + (C_2 + C_3 \cos 2\theta) s^4 h + (C_4 + C_6 \cos^2\theta) s^3 h^2 \cos\theta \\
&+ (C_7 + C_8 \cos 2\theta) s^2 h^3 + C_{10} s h^4 \cos\theta + C_{12} h^5 + D_1 s^7 \cos\theta + \cdots
\end{aligned}
$$

$$
\begin{aligned}
x' &= A_1 s \sin\theta \\
&+ B_1 s^3 \sin\theta + B_2 s^2 h \sin 2\theta + (B_3 + B_4) s h^2 \sin\theta \\
&+ C_1 s^5 \sin\theta + C_3 s^4 h \sin 2\theta + (C_5 + C_6 \cos^2\theta) s^3 h^2 \sin\theta \\
&+ C_9 s^2 h^3 \sin 2\theta + C_{11} s h^4 \sin\theta + D_1 s^7 \sin\theta + \cdots
\end{aligned}
$$

Wave Aberration

Wave aberration function **W** is the optical length, measured along a ray, from the aberrated wavefront to the reference sphere.
The distance ε is called the transverse ray error.

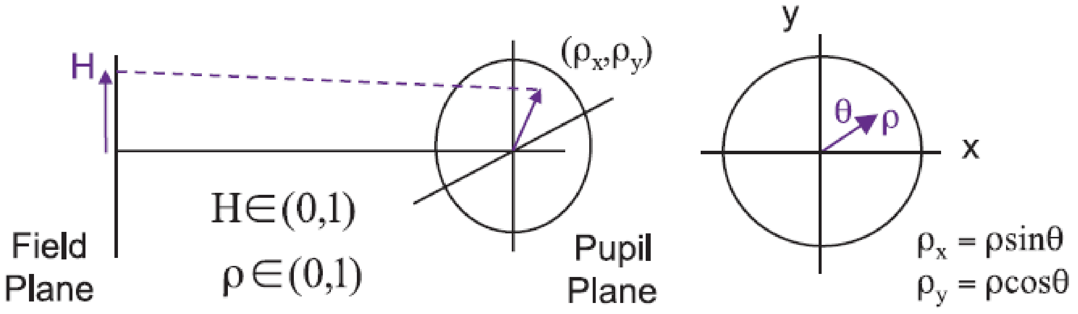


Monochromatic aberrations can also be described by expanding **W** in a power series of aperture and field coordinates, ρ, θ and H:

$$W_{IJK} \Rightarrow H^I \rho^J \cos^K \theta$$

$$W(H,\rho,\theta) = W_{020}\rho^2 + W_{111}H\rho\cos\theta + W_{040}\rho^4 + W_{131}H\rho^3\cos\theta + W_{222}H^2\rho^2\cos^2\theta + W_{220}H^2\rho^2 + W_{311}H^3\rho\cos\theta + O(6)$$

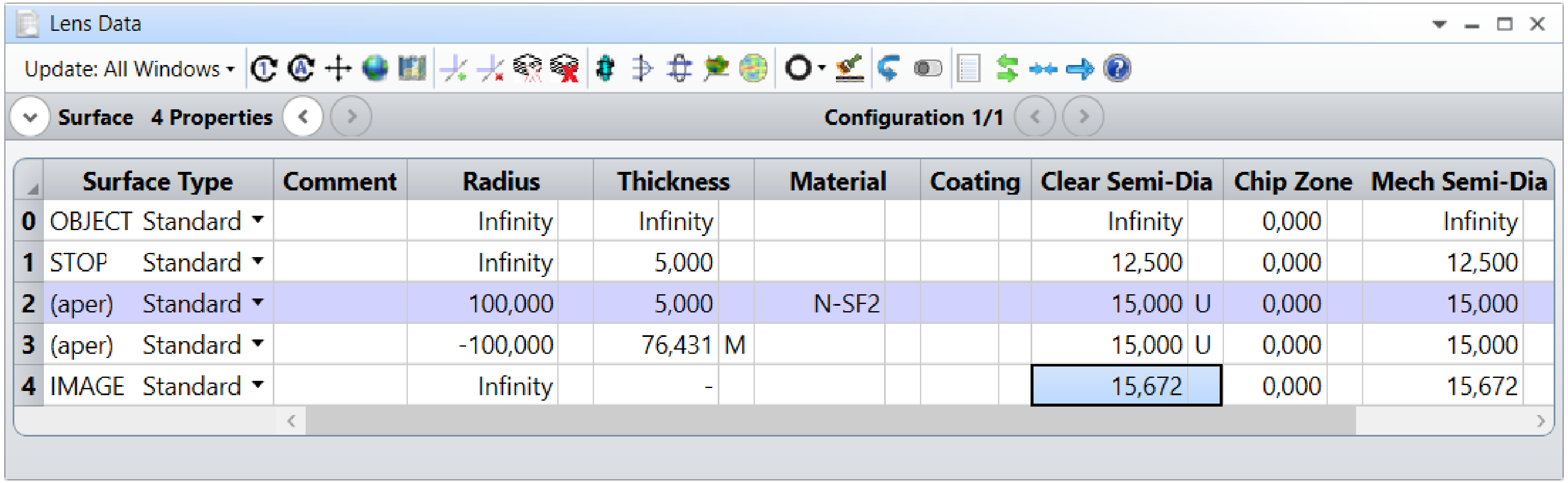
W_{020}: Defocus
W_{111}: Wavefront tilt
W_{040}: Spherical aberration
W_{131}: Coma
W_{222}: Astigmatism
W_{220}: Field curvature
W_{311}: Distortion



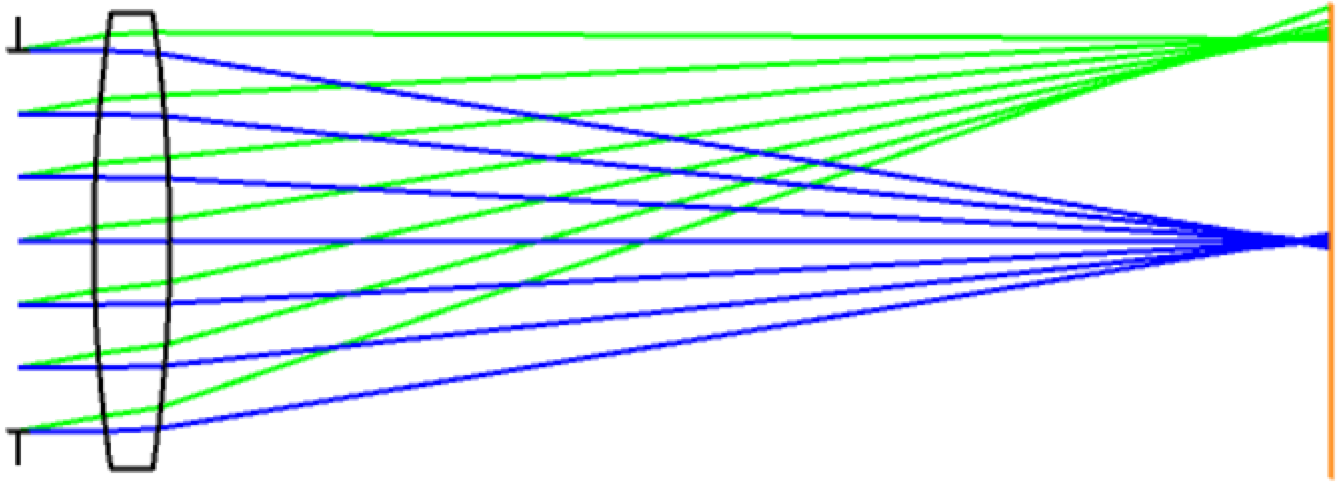
Aberration Plots & Seidel Coefficients

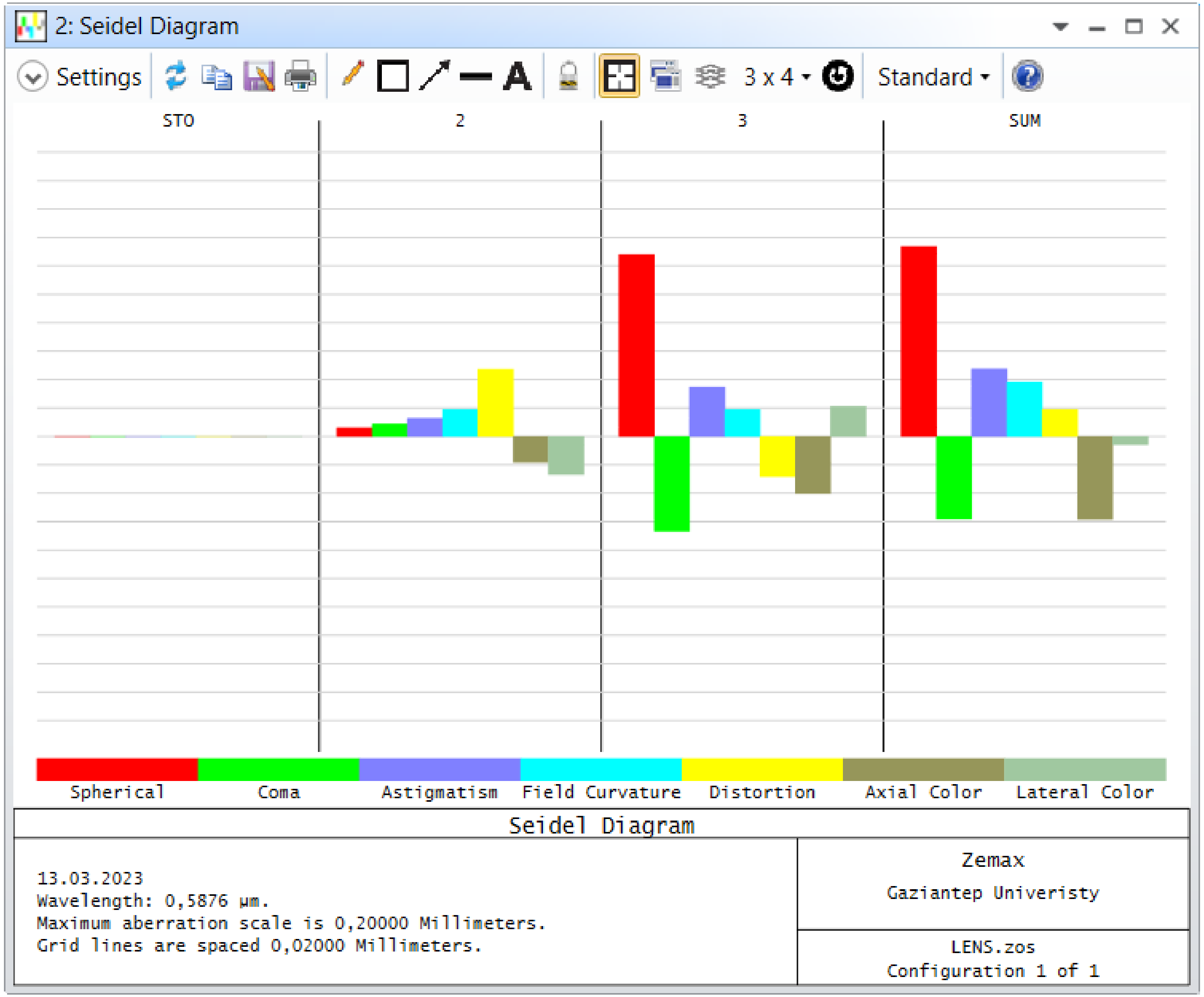
Monochromatic Aberration Demo in Zemax

λ = 550 nm, ENPD = 25 mm, SFOV = 0^{o} and 10^{o}.



	Surface Type		Comment	Radius	Thickness		Material	Coating	Clear Semi-Dia		Chip Zone	Mech Semi-Dia
0	OBJECT	Standard		Infinity	Infinity				Infinity		0,000	Infinity
1	STOP	Standard		Infinity	5,000				12,500		0,000	12,500
2	(aper)	Standard		100,000	5,000		N-SF2		15,000	U	0,000	15,000
3	(aper)	Standard		-100,000	76,431	M			15,000	U	0,000	15,000
4	IMAGE	Standard		Infinity	-				15,672		0,000	15,672




2: Seidel Diagram
Settings
3 x 4
Standard
STO
2
3
SUM
Spherical
Coma
Astigmatism
Field Curvature
Distortion
Axial Color
Lateral Color
Seidel Diagram
13.03.2023
Wavelength: 0,5876 µm.
Maximum aberration scale is 0,20000 Millimeters.
Grid lines are spaced 0,02000 Millimeters.
Zemax
Gaziantep Univeristy
LENS.zos
Configuration 1 of 1

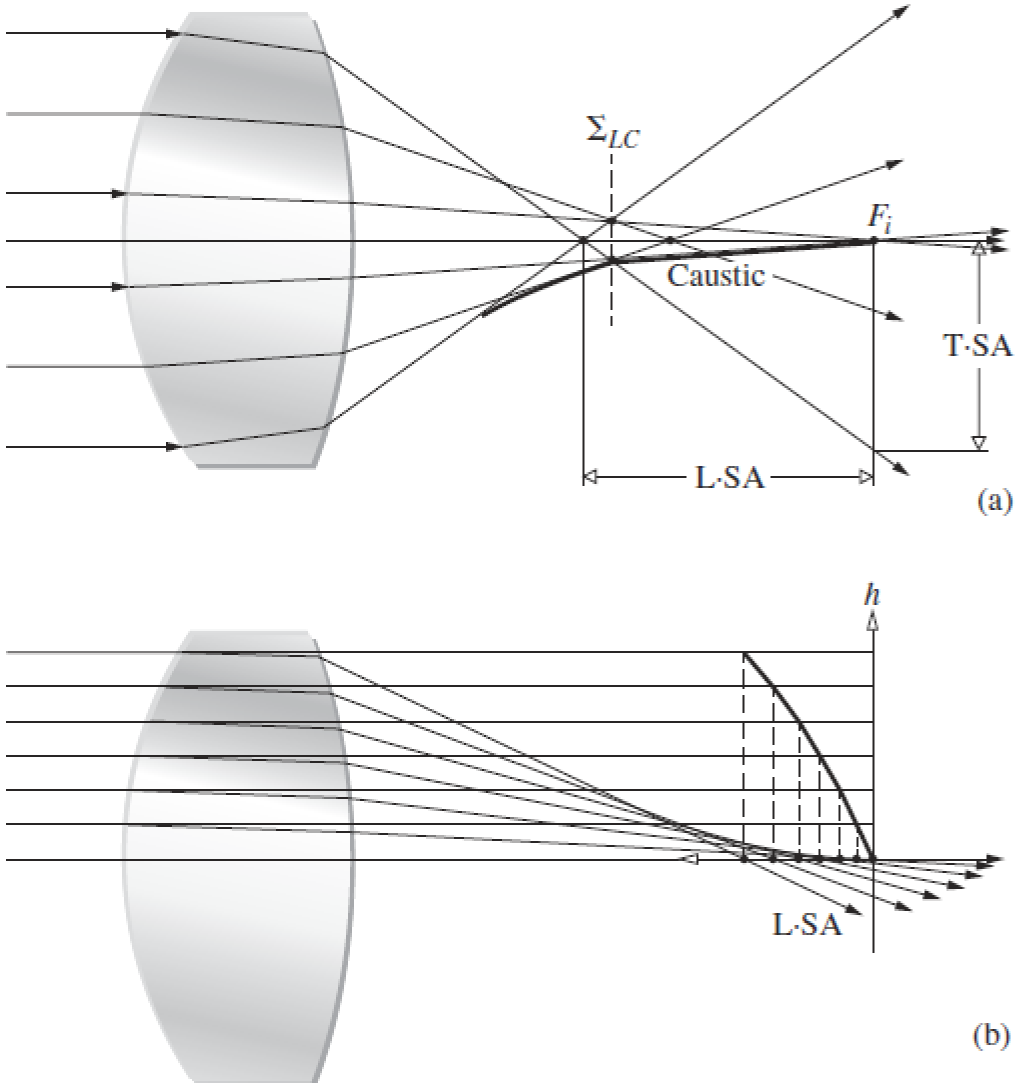
Spherical Aberration

SA is occurs only on-axis.

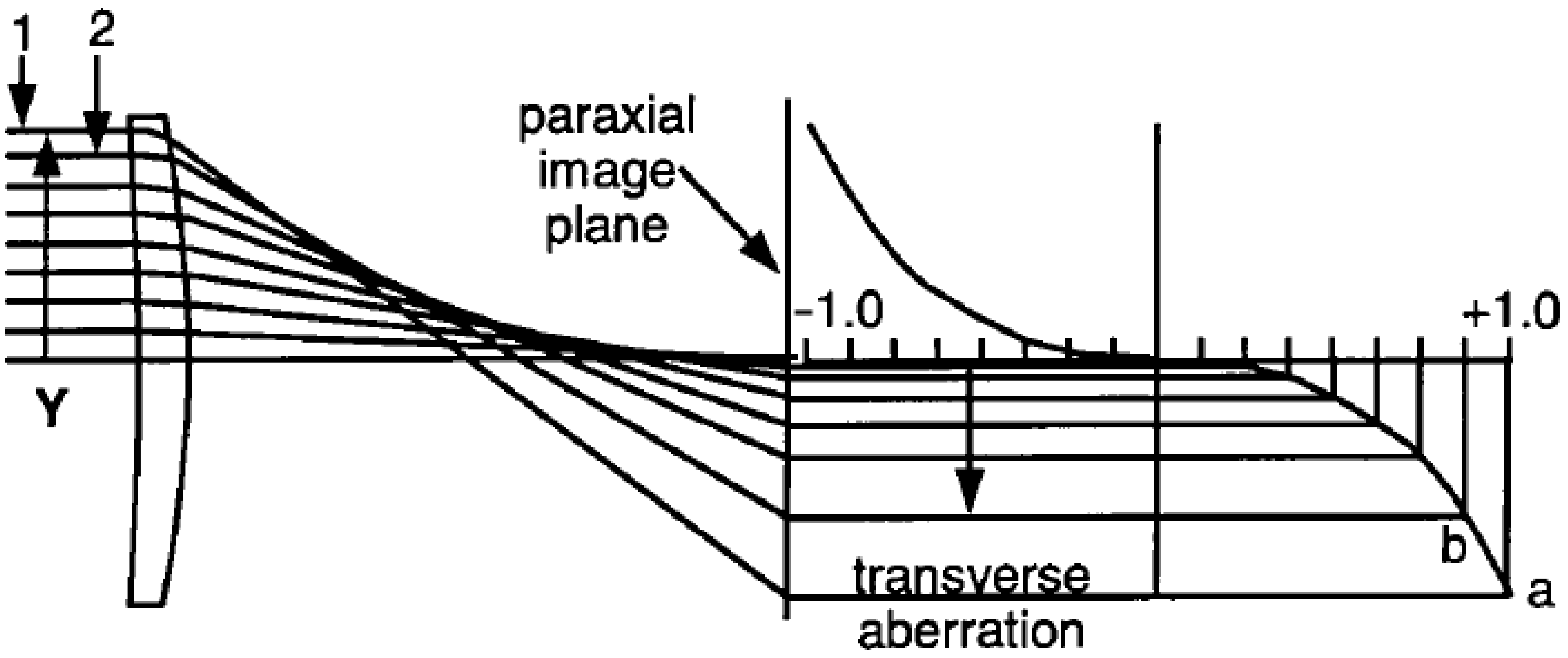
We have two types of spherical aberrations:

Longitudinal Aberration (L.SA)

Transverse Aberration (T.SA)



Ray Fan Plot



Aspherical Surfaces

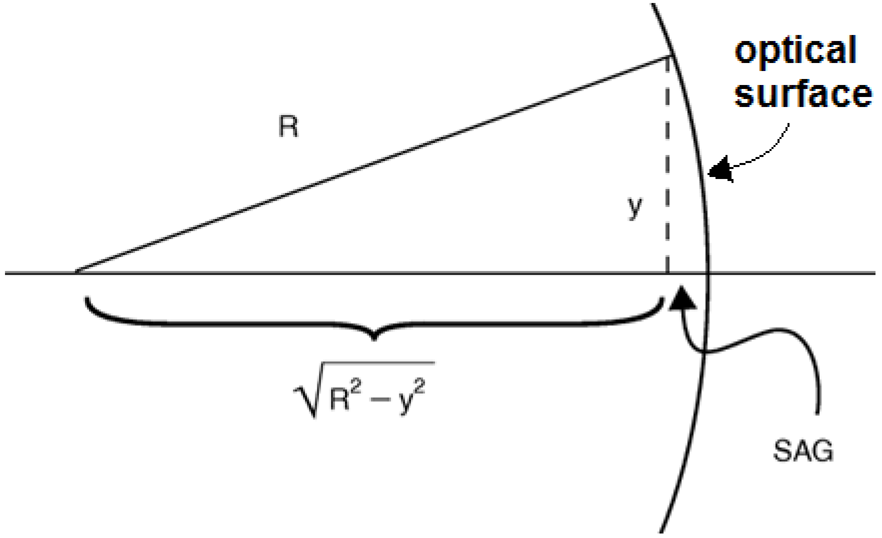
Using Aspherical surfaces one can reduce S.A.

- Aspherical surface is relatively harder to make and measure.
- Aspheric lenses improve image quality and reduce the number of required optical elements.

An important property of an optical surface is <u>sag</u> defined by:

$$z = \frac{Cy^2}{\sqrt{1+(1+k)C^2y^2}} + A_2y^2 + A_4y^4 + A_6y^6 + \cdots$$

z = sag of surface parallel to the optical axis
y = radial distance from the optical axis
C = curvature, inverse of radius ($C = 1/R$)
k = conic constant
A_i = i^{th} order aspheric coefficient



Geometric meaning of conic constant:

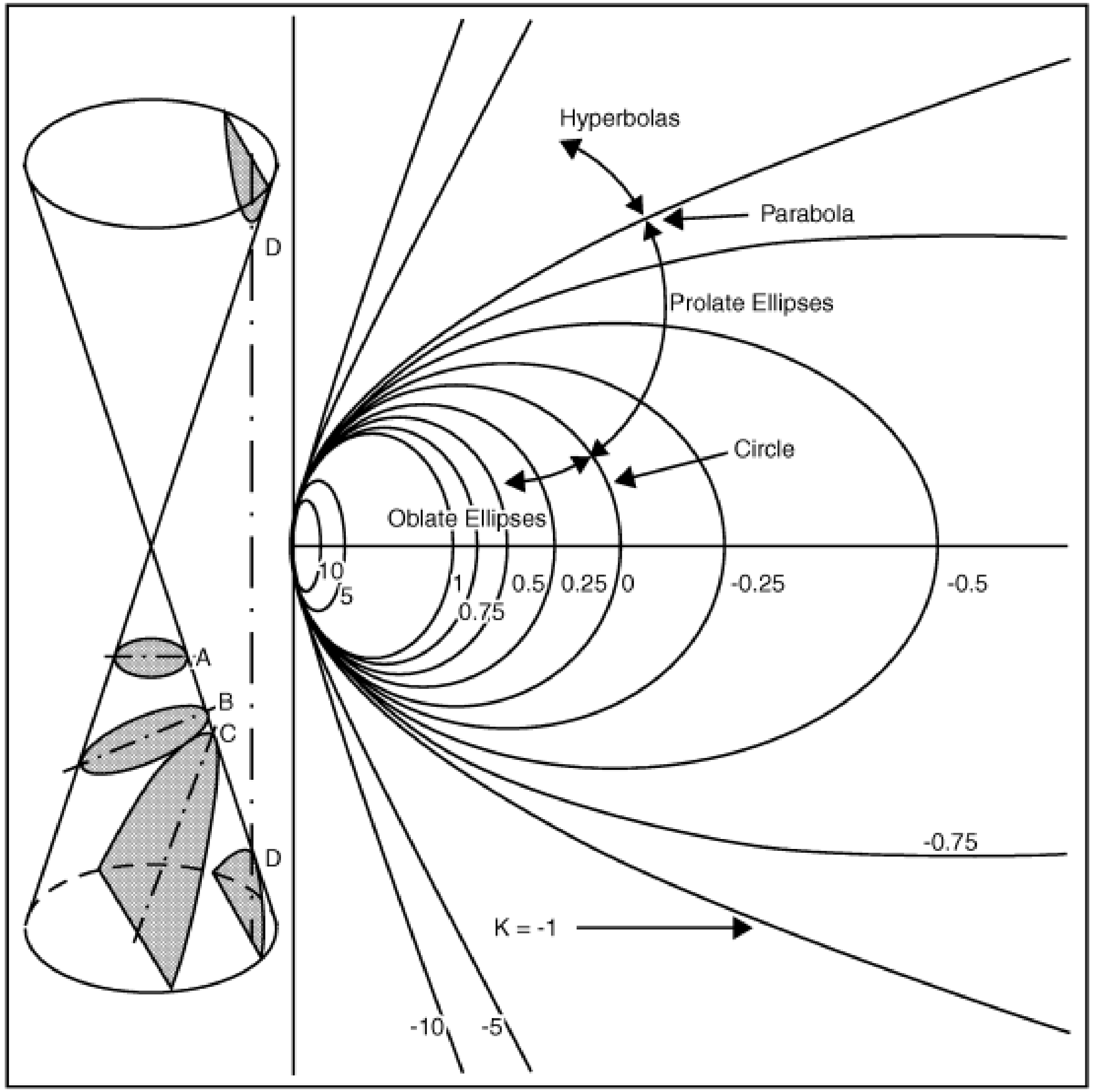
k = 0 => *circle*

k = -1 => *parabola*

k < -1 => *hyperbola*

k > 0 => *ellipse*

-1 < *k* < 0 => *ellipse*



How to Get Rid Off Spherical Aberration

To reduce spherical aberration:

- Reduce size (diameter) of the lens
- Change bending (radii of curvatures) of the lens
- Use more than one spherical lens
- Use aspherical surface(s)

Example 1: Three Lenses to reduce SA

In this example, we will use three N-BK7 glasses separated by 5 mm and 7 mm.

ENPD = 25 mm, F/# = 4 and λ = 550 nm.

Diameter of each lens is D = 30 mm and $ct_1 = ct_2 = ct_3$ = 6 mm

(*Note that for optomechanical reasons center thickness must satisfy ct > D/10*).

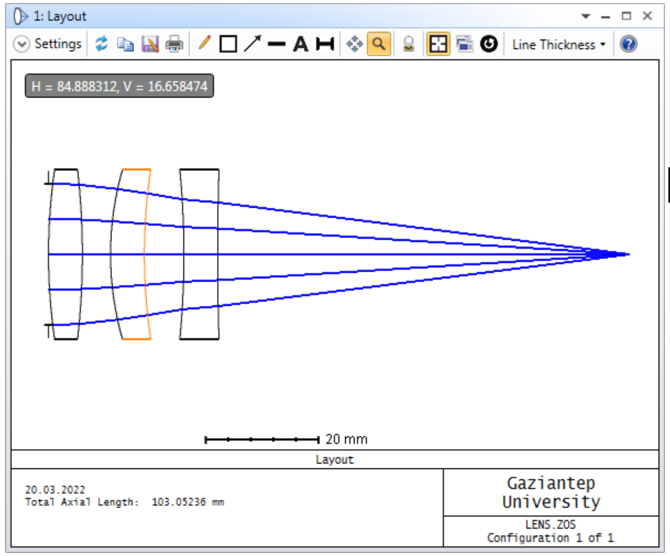
An example recipie is as follows:

Step 1: We have only one lens. Do not insert other lenses.
R_{11} = 90 mm and R_{12} is variable.
Optimize (min spot) such that focal length of the lens is f_1 = 120 mm.

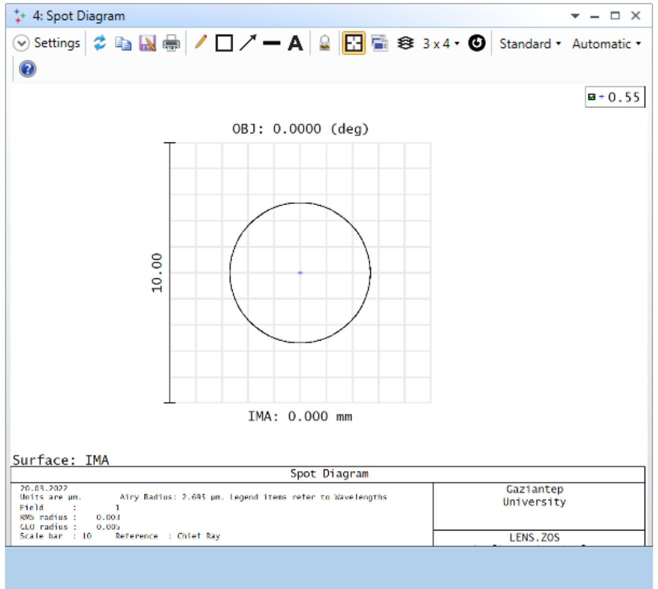
Step 2: Insert new lens 5 mm away from first lens. Now, we have two lenses.
R_{11}, R_{12} are fixed. R_{21} and R_{22} are variable.
Optimize (min spot) such that focal length of the two lenses is f_{12} = 80 mm.

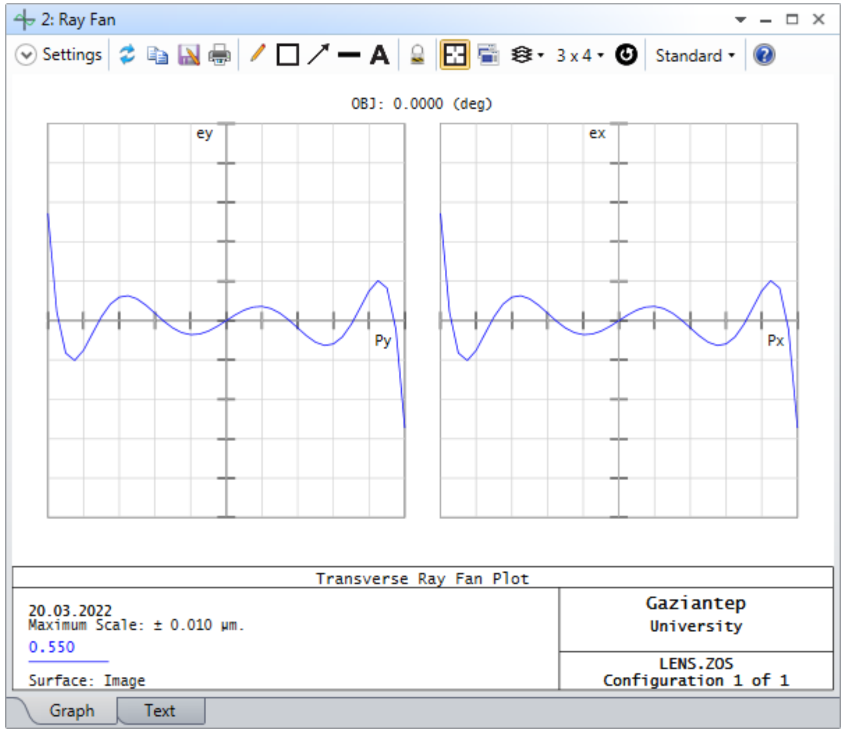
Step 3: Insert new lens 7 mm away from second lens. Now, we have three lenses.
R_{11}, R_{12}, R_{21}, R_{22} are fixed. R_{31} and R_{32} are variable.
Optimize (min spot) such that focal length of lenses is f_{123} = 100 mm.

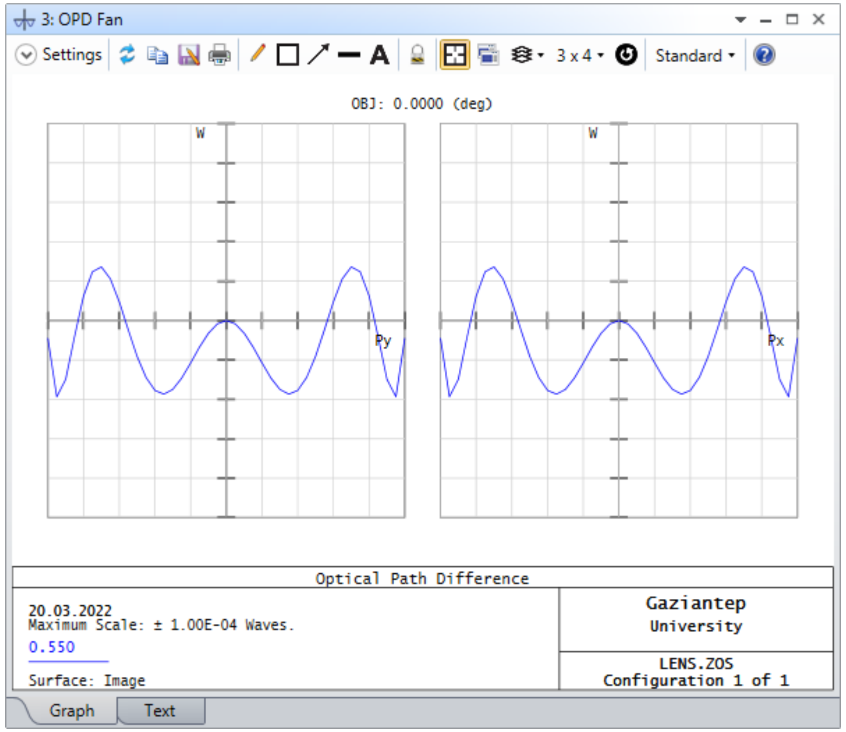
Step 4: Set all 6 radii variables.
Optimize (min spot) such that focal length of lenses is f_{123} = 100 mm.



ptimization:







Example 2: Even Aspheric Surface

In some cases, use of the conic constant may not be enough to remove S.A. An alternative way is to use even aspheric surface which is a standard surface plus polynomial asphere terms (See Page 13). In Zemax OpticStudio, this surface is defined as **Even Asphere**. In this example, we'll consider a plano convex aspherical lens whose focal length is 100 mm and ENPD = 25 mm.

Lens Data

Update: All Windows

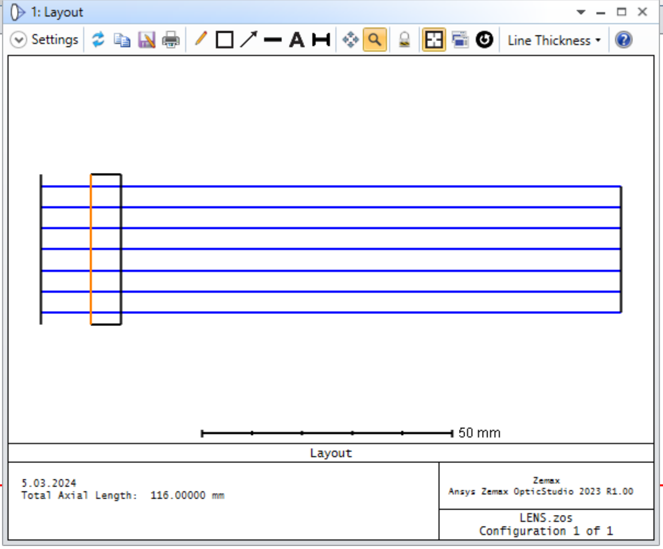
Surface 2 Properties — Configuration 1/1

	Surface Type		Co	Radius	Thickness	Material	Clear Semi-	Chip Zone	Mech Semi-Dia	Conic	Coati	TCE x 1E-6	2nd Order Term	4th Order Term	6t
0	OBJECT	Standard		Infinity	Infinity		0.000	0.000	0.000	0.000		0.000			
1	STOP	Standard		Infinity	10.000		12.500	0.000	12.500	0.000		0.000			
2	(aper)	Even Asphere		Infinity V	6.000	N-BK7	15.000 U	0.000	15.000	0.000		-	0.000 V	0.000 V	
3	(aper)	Standard		Infinity	100.000 V		15.000 U	0.000	15.000	0.000		0.000			
4	IMAGE	Standard		Infinity	-		12.500	0.000	12.500	0.000		0.000			

Merit Function Editor

Wizards and Operands — Merit Function: 3705435992.59739

	Type		Wave	Hx	Hy	Px	Py		Target	Weight	Value	% Contrib
1	EFFL		1						100.000	1.000	1.000E+10	100.000
2	DMFS											
3	BLNK	Sequential merit function: RMS spot x+y centroid X Wgt = 1.0000 Y Wgt = 1.0000 GQ 3 rings 6 arms										
4	BLNK	No air or glass constraints.										
5	BLNK	Operands for field 1.										
6	TRCX		1	0.0...	0.0...	0.3...	0.000		0.000	0.873	4.196	1.537E-17
7	TRCY		1	0.0...	0.0...	0.3...	0.000		0.000	0.873	0.000	0.000
8	TRCX		1	0.0...	0.0...	0.7...	0.000		0.000	1.396	8.839	1.091E-16
9	TRCY		1	0.0...	0.0...	0.7...	0.000		0.000	1.396	0.000	0.000
10	TRCX		1	0.0...	0.0...	0.9...	0.000		0.000	0.873	11.775	1.210E-16
11	TRCY		1	0.0...	0.0...	0.9...	0.000		0.000	0.873	0.000	0.000



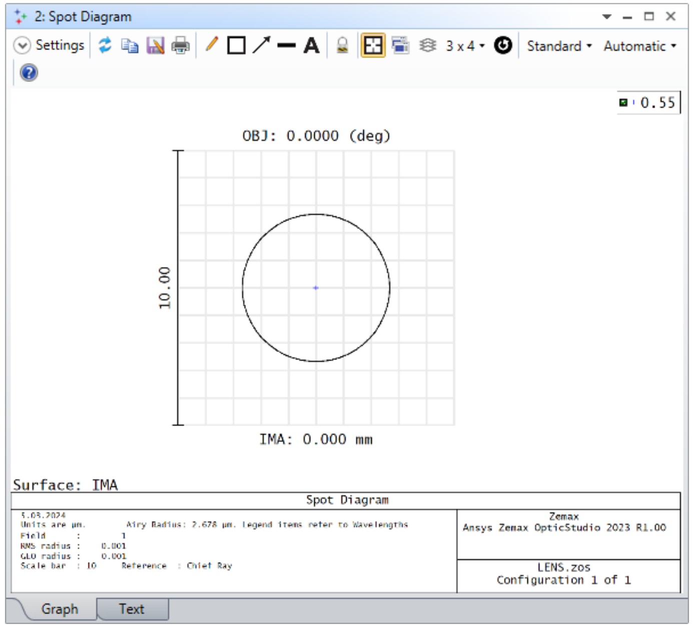
After optimization, we have a perfect form. Compare the solution with Example 1.

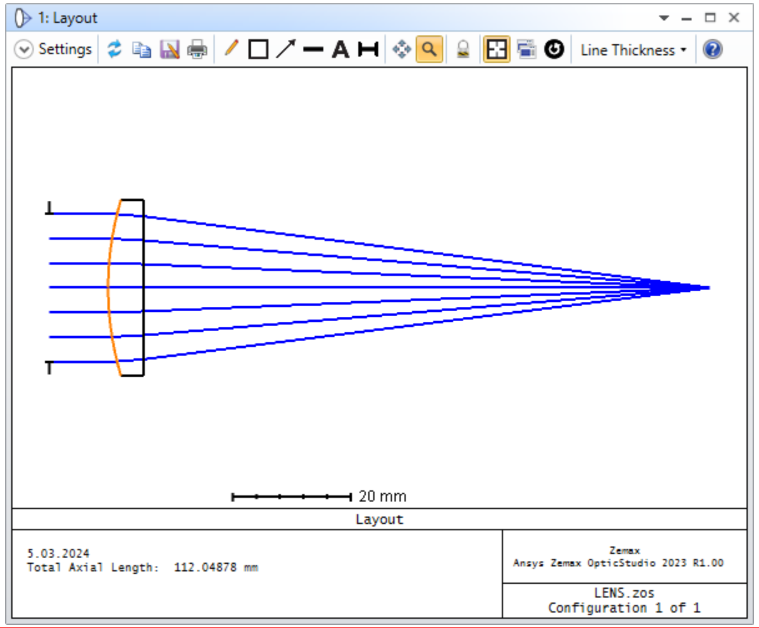
Lens Data

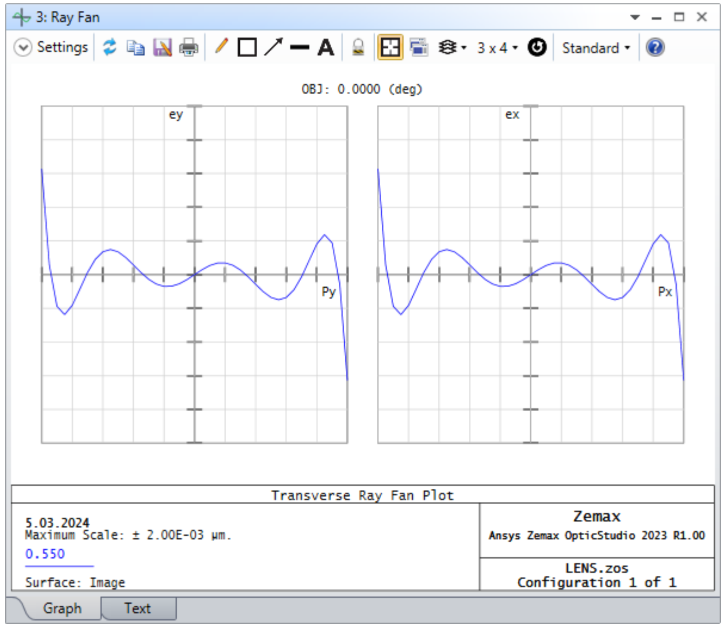
Update: All Windows

Surface 2 Properties | Configuration 1/1

#	Surface Type		Co	Radius	Thickness	Material	Clear Semi-	Chip Zone	Mech Semi-Dia	Conic	Coati	TCE x 1E-6	2nd Order Term	4th Order Term	6t
0	OBJECT	Standard		Infinity	Infinity		0.000	0.000	0.000	0.000		0.000			
1	STOP	Standard		Infinity	10.000		12.500	0.000	12.500	0.000		0.000			
2	(aper)	Even Asphere		78.802 V	6.000	N-BK7	15.000 U	0.000	15.000	0.000		-	3.298E-03 V	1.212E-07 V	
3	(aper)	Standard		Infinity	96.049 V		15.000 U	0.000	15.000	0.000		0.000			
4	IMAGE	Standard		Infinity	-		1.255E-06	0.000	1.255E-06	0.000		0.000			



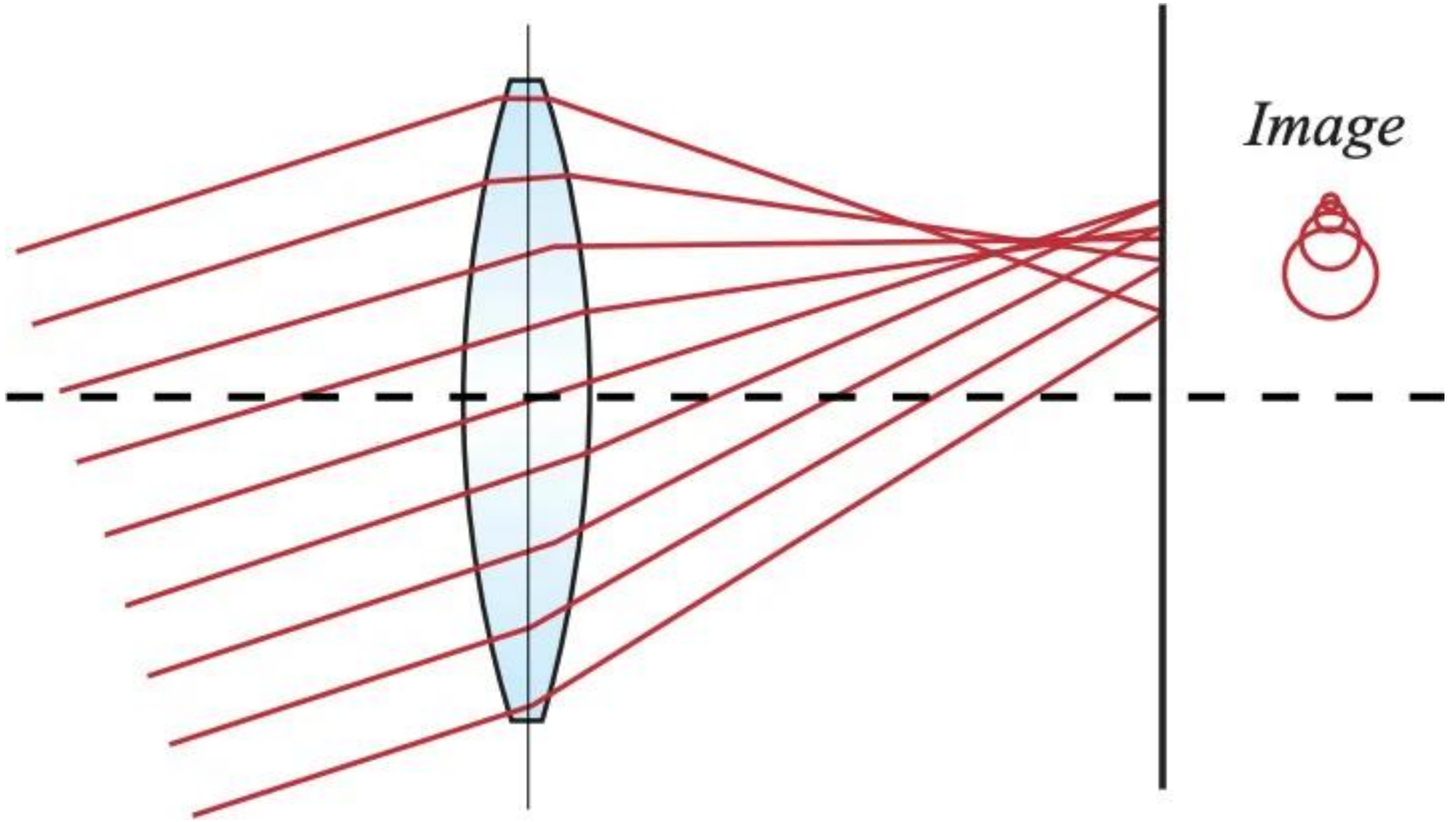




Coma

Coma is similar to SA but in addition the rays come from off-axis points.

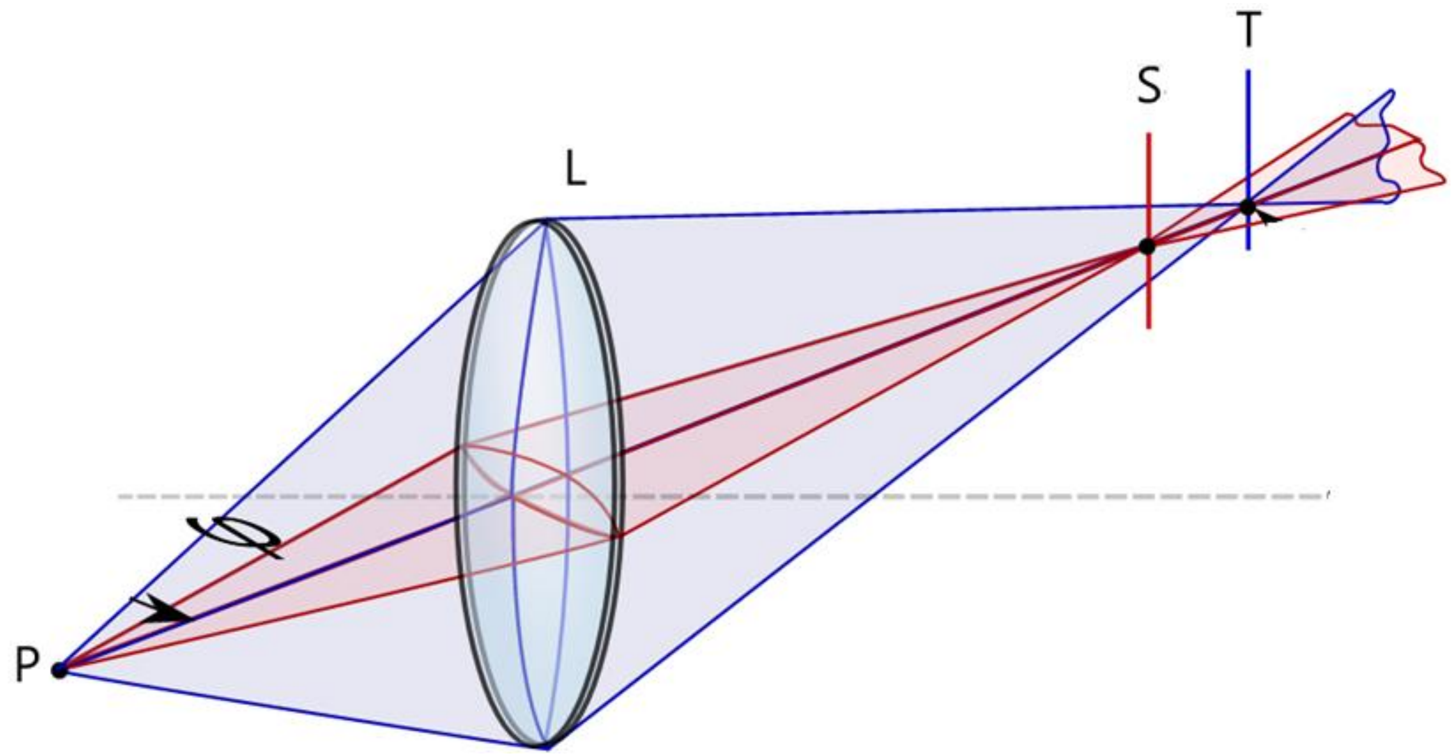
Coma increases rapidly as the third power of the lens aperture.



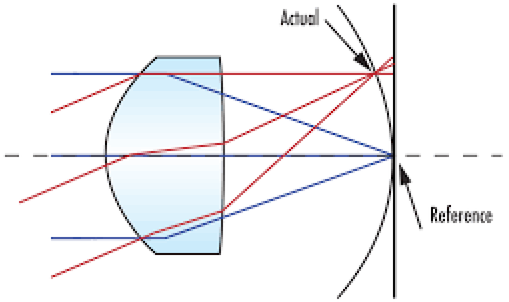
The term coma comes from comet due to the shape of the spot diagram.

Astigmatizm

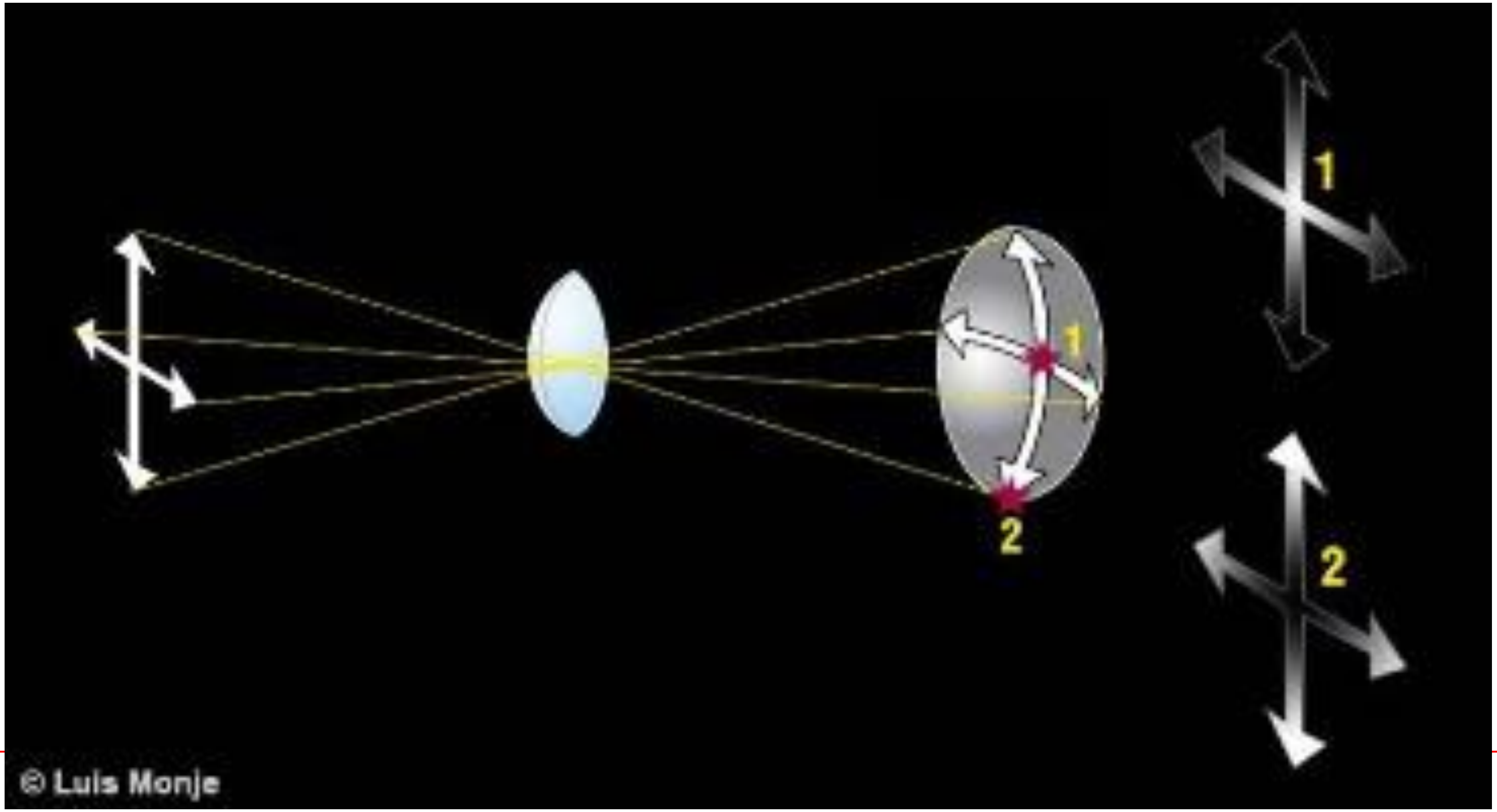
- Astigmatism is another off-axis aberration.
- Tangential and Sagital rays for oblique rays are focused at dierent points.



Field Curvature



- For a light ray comes from off-axis, a positive lens appears to be thicker than really it is.
- As a results, for oblique rays we have different focal lengths.
- The points on a plane object, then form a curved image, a deciency is called **field curvature.**
- This aberration is important in camera systems and projectors: **image is expected to be flat**.



Distortion

- Distortion occurs as variation in the lateral magnication.
- If the magnication decreases with distance from the axis, the images appears as barrel distortion.
- If the magnication increases with the from axis, the image appears as pincushion distortion.
- Distortion increases with the cube of the field of view.

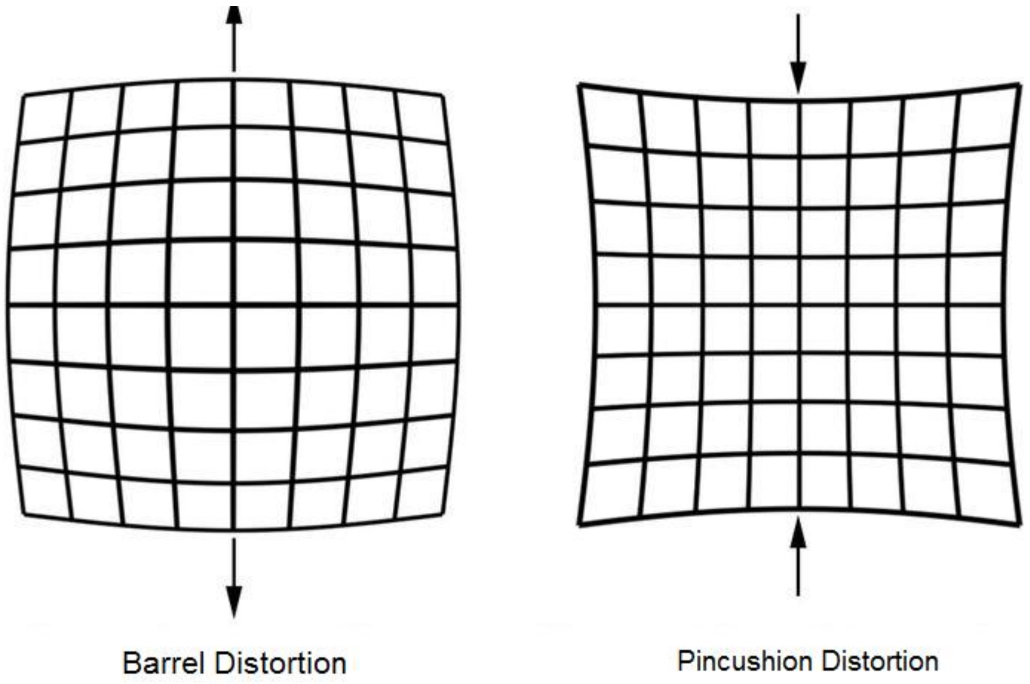
- Distortion is defined as

$$D = \frac{y - y_p}{y_p}$$

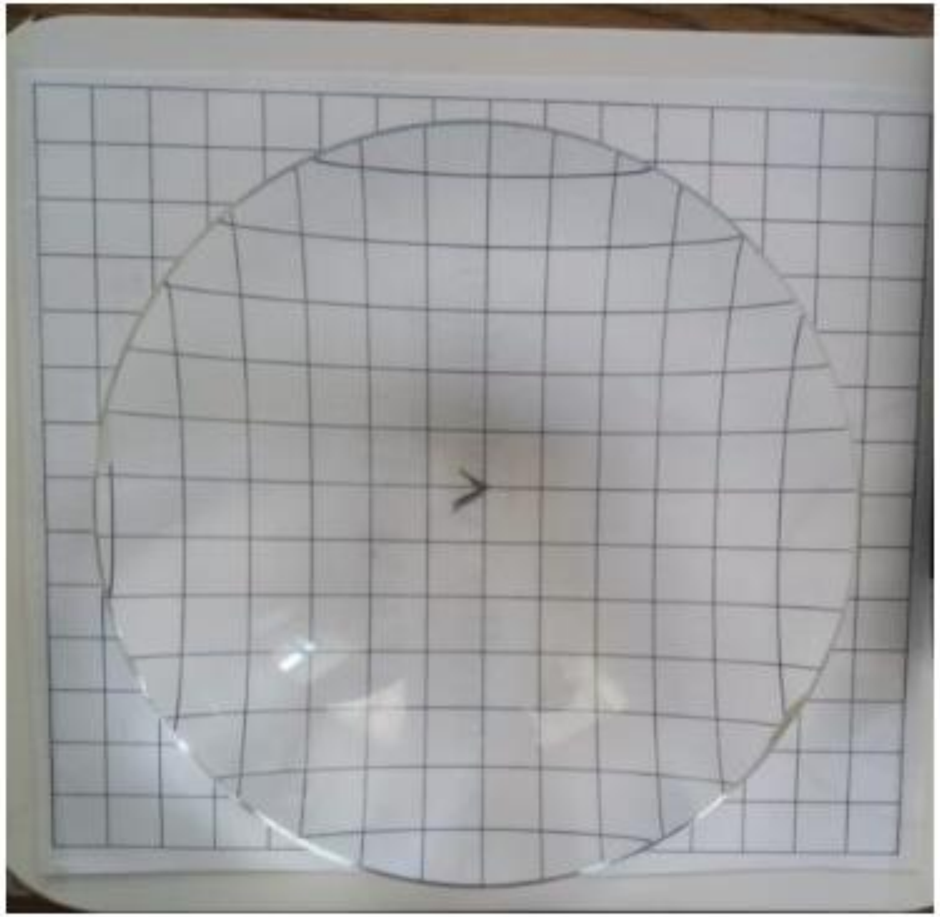
y is the height in the image plane

y_p is the paraxial height

Generally, distortion in the order of 2-3% is acceptable visually.



Distortion

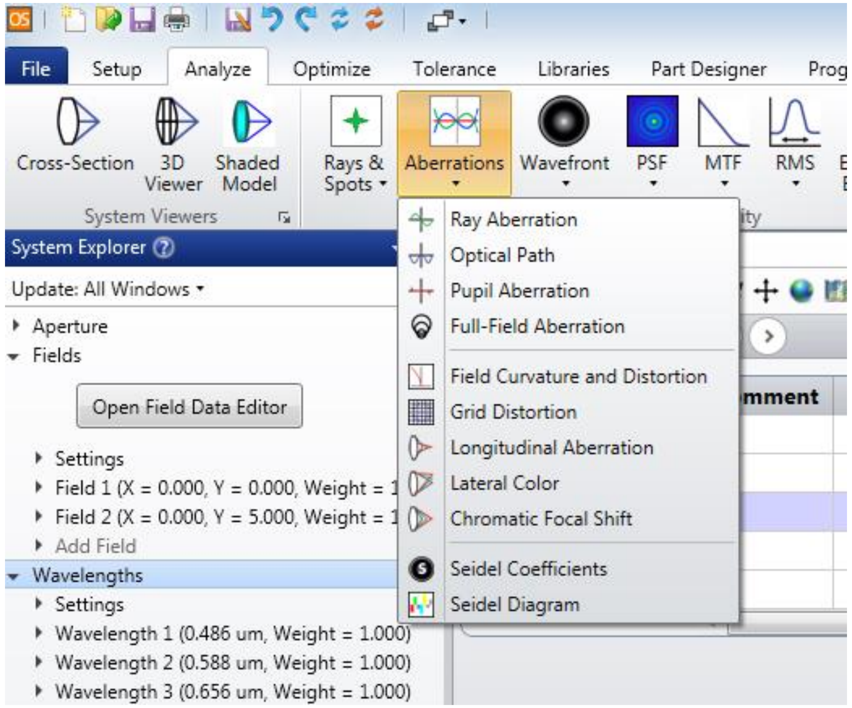


Summary of Monochomatic Aberrations

Summary of Third-Order Monochromatic Aberration Dependence on Aperture and Field Angle

Aberration	Aperture Dependence	Field Dependence
Spherical	Cubic	—
Coma	Quadratic	Linear
Astigmatism	Linear	Quadratic
Field curvature	Linear	Quadratic
Distortion	—	Cubic

Aberration Performance Plots in Zemax OpticStudio

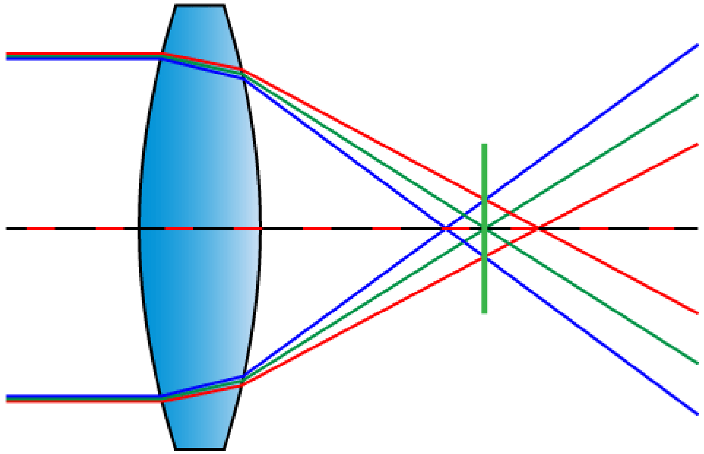


Chromatic Aberration

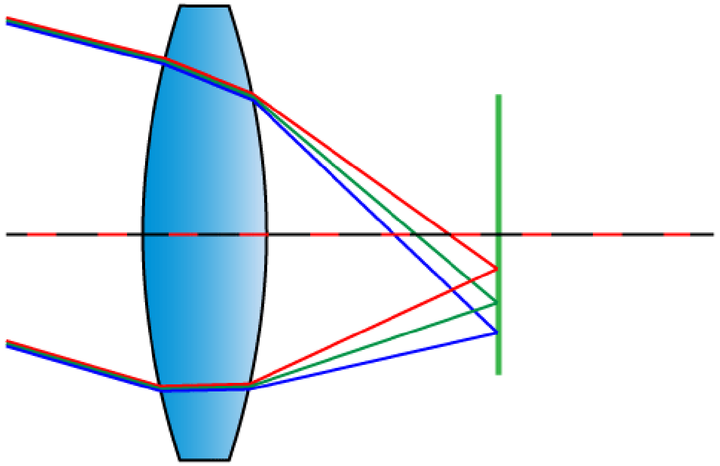
What is Chromatic Aberration?

- A lens will not focus different colors (wavelengths) at the same place on the optical axis since focal length depends on refractive index of the material.
- This color dependent deficiency is called the <u>chromatic aberration</u>.

Longitudinal / Axial
Chromatic Aberration



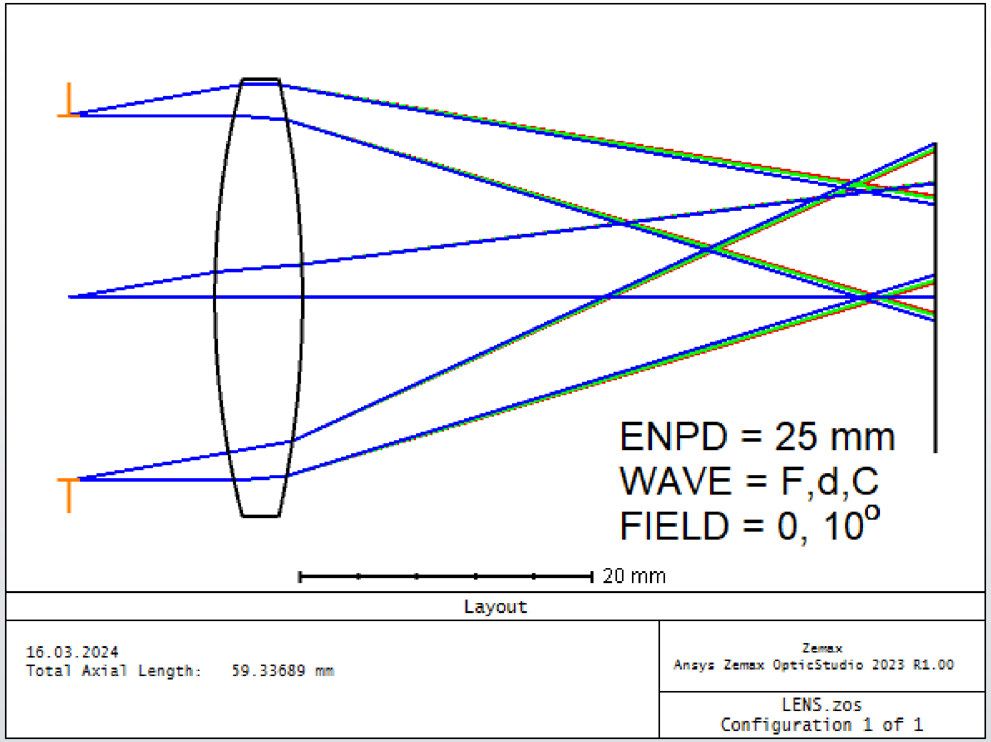
Lateral / Transverse
Chromatic Aberration

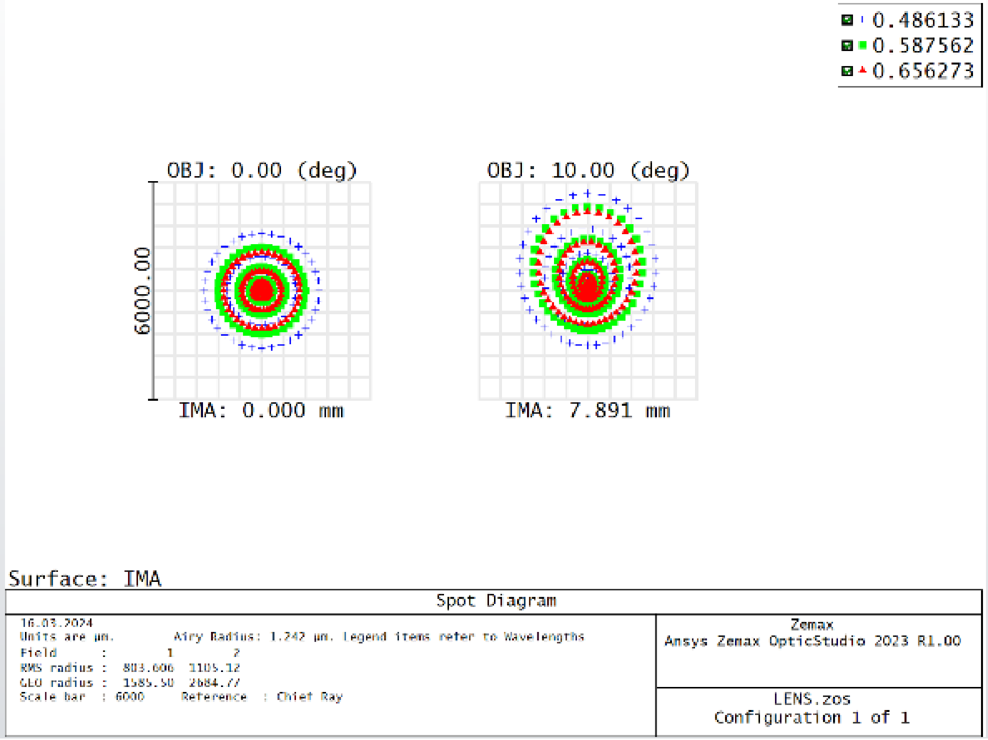



Uncorrected
Corrected

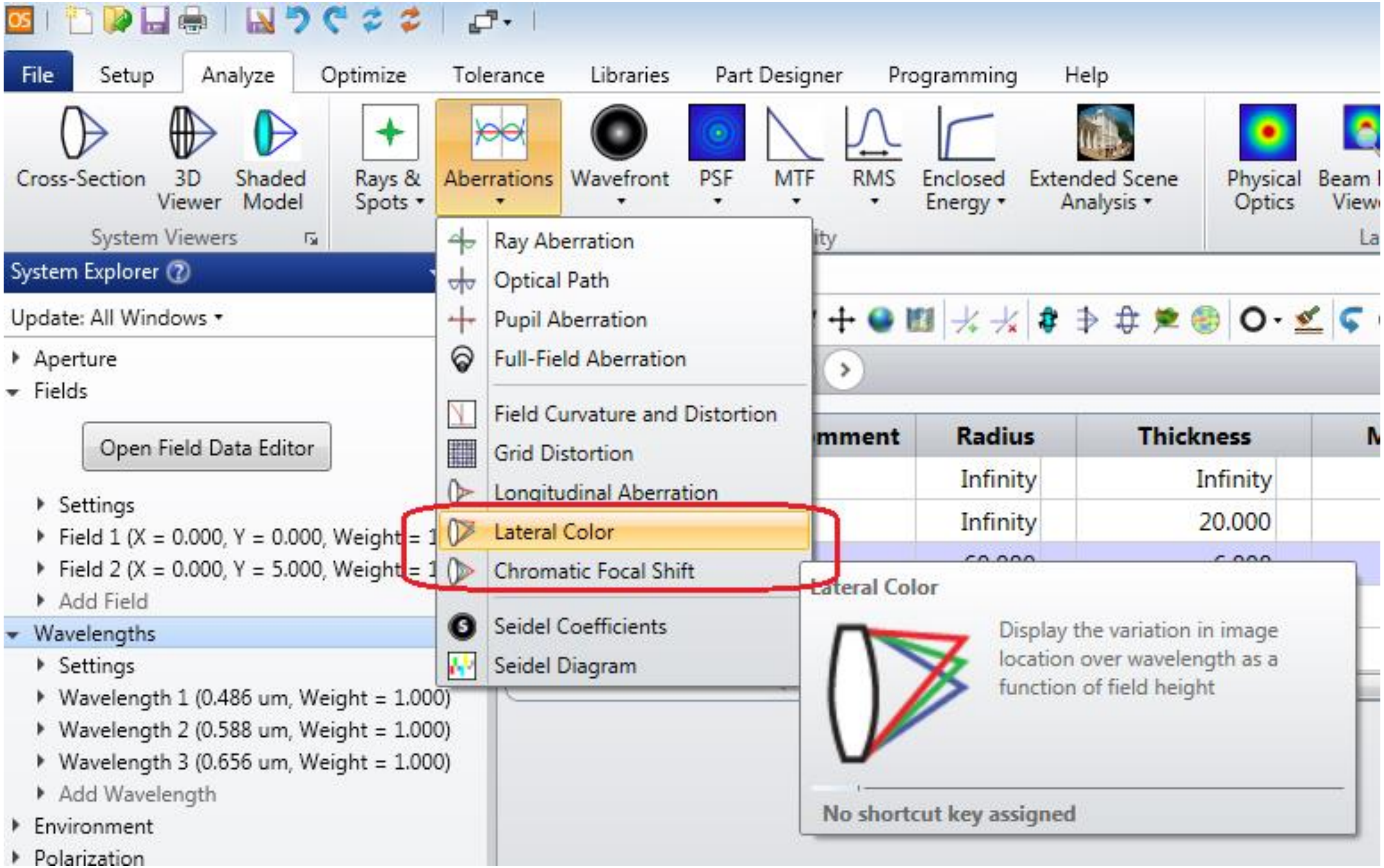
Example 1: Demo for Chromatic Aberration

	Surface Type		Comment	Radius	Thickness	Material	Clear Semi-	Chip Zone	Mech Semi-Dia	Conic	Coati	TCE x 1E-6
0	OBJECT	Standard		Infinity	Infinity		Infinity	0.000	Infinity	0.000		0.000
1	STOP	Standard		Infinity	10.000		12.500	0.000	12.500	0.000		0.000
2	(aper)	Standard		60.000	6.000	SF10	15.000 U	0.000	15.000	0.000		-
3	(aper)	Standard		-70.000	43.337 M		15.000 U	0.000	15.000	0.000		0.000
4	IMAGE	Standard		Infinity	-		10.576	0.000	10.576	0.000		0.000





Aberration Plots

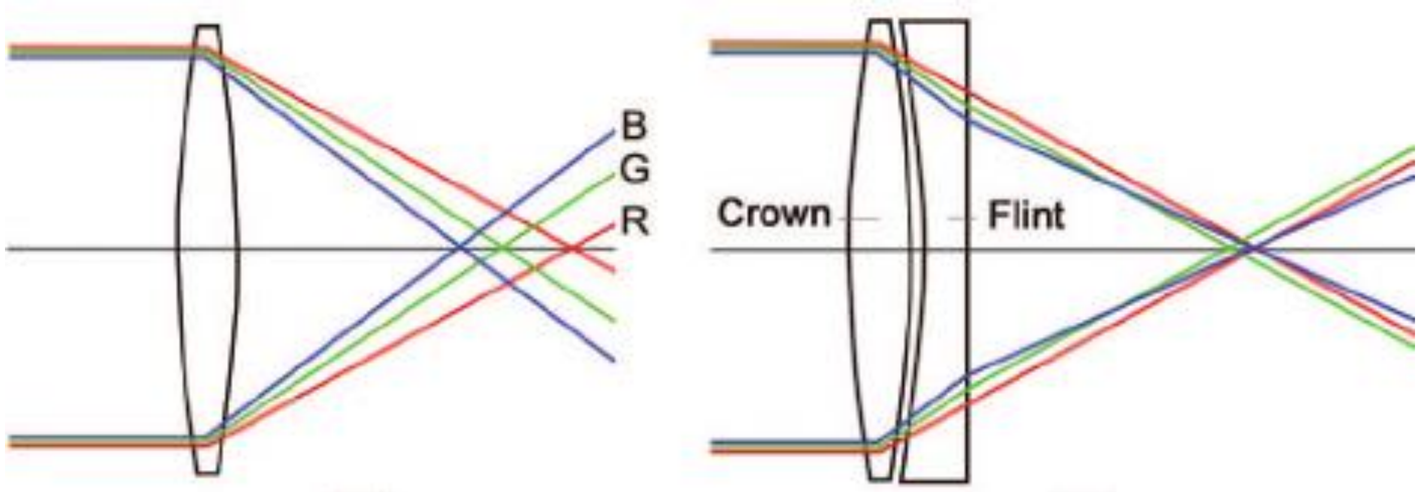


How to Correct Chromatic Aberration

- One way to minimize this aberration is to use glasses of different dispersion in a doublet or triplet. We will mostly investigate **Achromatic Doublet.**

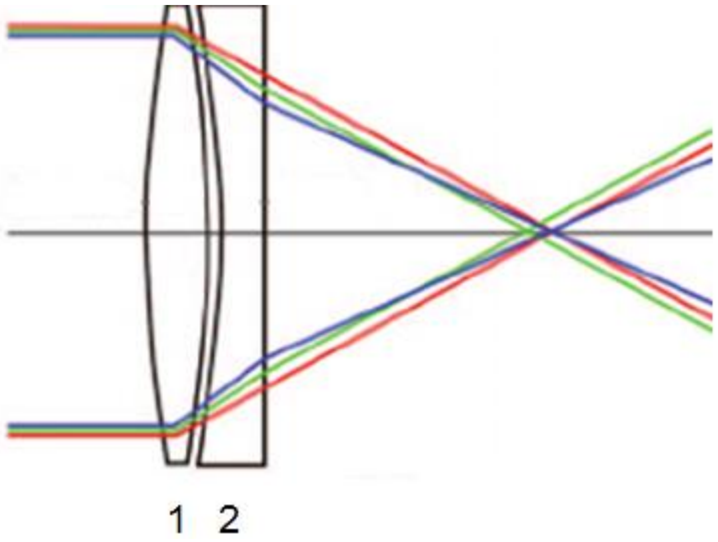
- The use of a strong positive lens made from a low dispersion glass like crown glass (like BK7) coupled with a weaker high dispersion glass like flint glass (like SF2) can correct the chromatic aberration for two colors; e.g., red and blue.

- Such doublets are often cemented together and called <u>achromatic lens</u>.

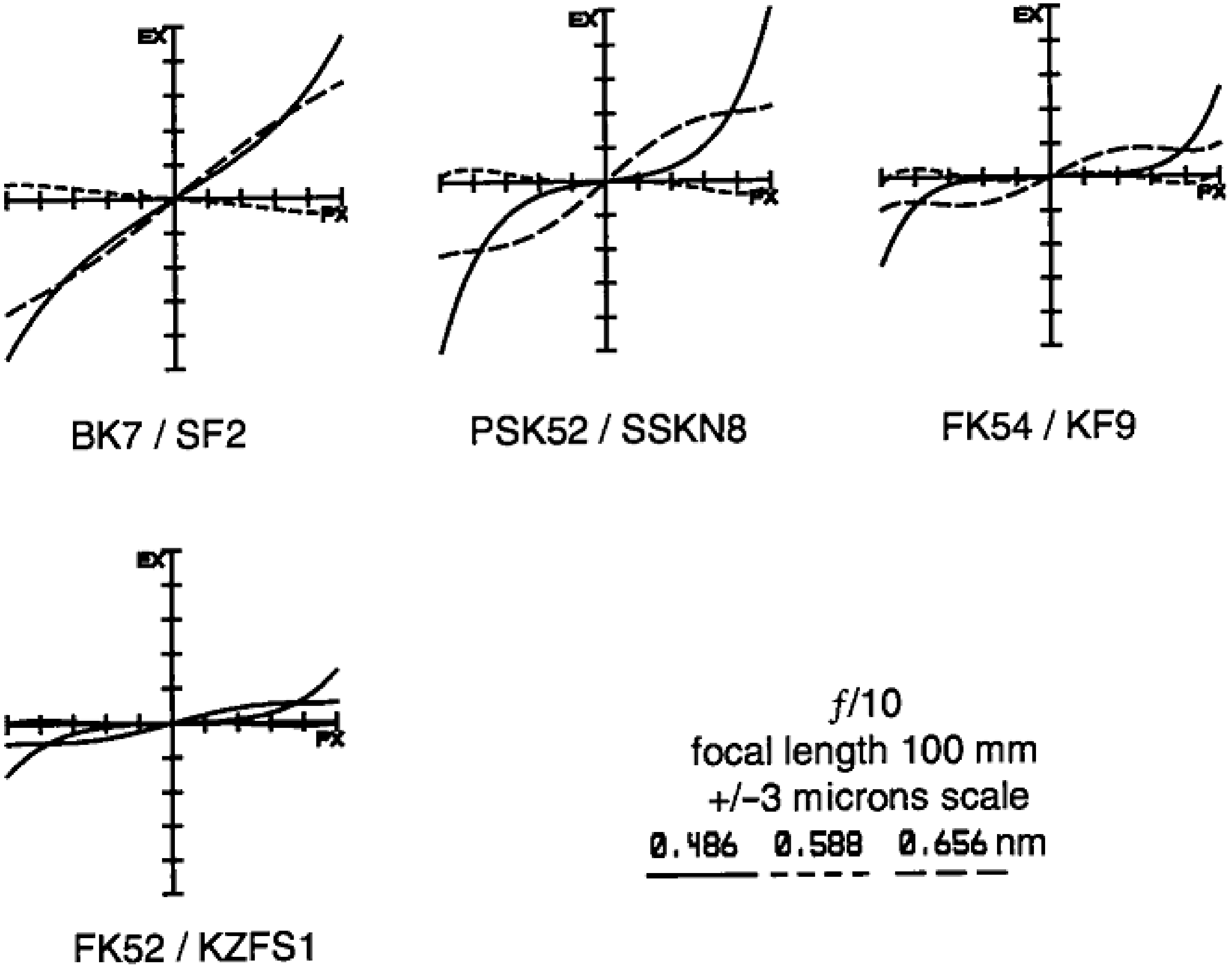


Suggested Glass Pairs for Achoromatic Lens

Glass1	Glass2
BK7	SF2
PSK52	SSKN8
FK54	KF9
FK52	KZFS1

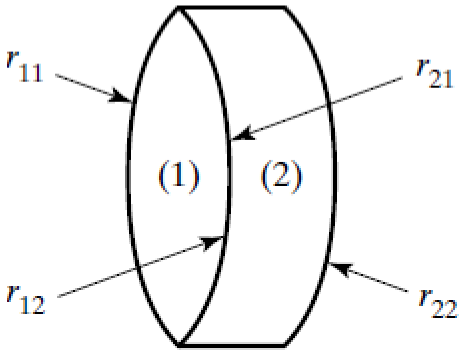


Ray Fan Plots fot Glass Pairs



Achromatic Doublet Design

- Consider two thin lenses cemented as shown.
- For d-line (λ = 587.6 nm)
  Let P_1, P_2, V_1 and V_2 be powers and Abbe values of glasses, repectively.



Best correction occurs for the condition:

$$\boxed{P_1 V_2 + P_2 V_1 = 0} \text{ here } P_i = 1/f_i$$

$$P_1 = P\frac{-V_1}{V_2 - V_1} \qquad P_2 = P\frac{V_2}{V_2 - V_1}$$

$$P = P_1 + P_2$$

$$K_1 = \frac{P_1}{n_1 - 1} \qquad K_2 = \frac{P_2}{n_2 - 1}$$

Suggested of radius of curvatures:

r_{11} = system focal length / 2

$$r_{12} = -r_{11}$$

$$r_{21} = -r_{11}$$

$$r_{22} = \frac{r_{12}}{1 - K_2 r_{12}}$$

Download `achromate.m` in course web page for implementation of the solution.

Example 2: 300mm-Doublet Design

Design an achromatic doublet to satisfy the following specifications:

EFFL = 300 mm

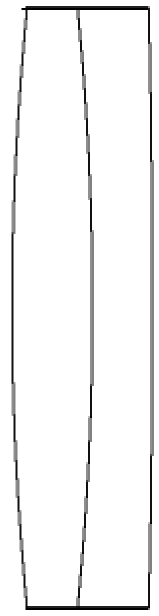
ENPD = 30 mm

Wavelengths = F, d, C (visible)

Lens1: N-BK7, ct = 4 mm

Lens2: N-SF2, ct = 3 mm

Optimize doublet to get minimum spot size and minimum axial color error in the image plane. [Hint: start with R_{11} = EFFL / 2 = 150 mm]



Using thin lens equations, we can obtain radii of curvatures as follows:

R_{11} = +150.000

R_{12} = -150.000

R_{21} = -150.000

R_{22} = -602.307

Before optimization

Lens Data

Update: All Windows

Surface 5 Properties — Configuration 1/1

	Surface Type	Comment	Radius	Thickness	Material	Coating	Clear Semi-Dia	Chi
0	OBJE(Standard		Infinity	Infinity			0.000	
1	STOP Standard		Infinity	0.000			15.000	
2	(aper) Standard	R11	150.000	4.000	N-BK7		15.000 U	
3	(aper) Standard	R12 & R21	-150.000	3.000	N-SF2		15.000 U	
4	(aper) Standard	R22	-602.307 V	90.000 V			15.000 U	
5	IMAG Standard		Infinity	-			9.896	

Merit Function Editor

Wizards and Operands — Merit Function: 9.81291470515264

	Type	Wave1	Wave2	Zone						Target	Weight	Value	% Contrib
1	DMFS												
2	EFFL		2							300.000	1.000	274.636	80.660
3	AXCL	1	3	0.000						0.000	1.000	0.880	0.097
4	BLNK	Operands for field 1.											
5	TRCX		1	0.000	0.000	0.336	0.000			0.000	0.291	3.313	0.400

After optimization

Lens Data

Update: All Windows

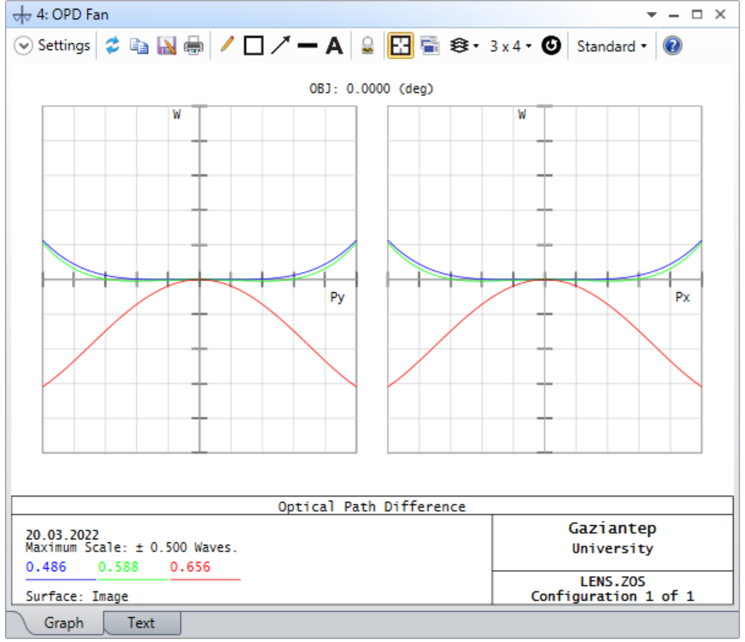
Surface 5 Properties — Configuration 1/1

	Surface Type	Comment	Radius		Thickness		Material		Coating	Clear Semi-Dia		Chi
0	OBJE(Standard		Infinity		Infinity					0.000		
1	STOP Standard		Infinity		0.000					15.000		
2	(aper) Standard	R11	150.000		4.000		N-BK7			15.000	U	
3	(aper) Standard	R12 & R21	-150.000		3.000		N-SF2			15.000	U	
4	(aper) Standard	R22	-848.745	V	295.845	V				15.000	U	
5	IMAG Standard		Infinity		-					6.105E-03		

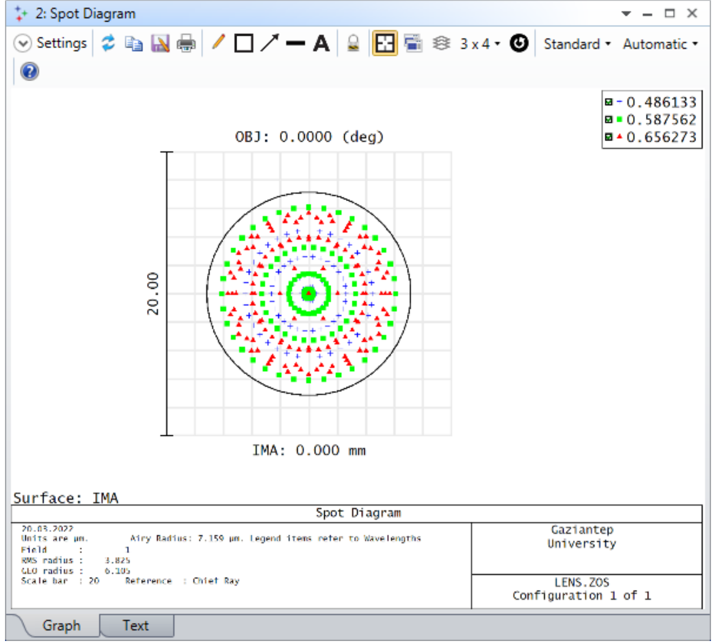
Merit Function Editor

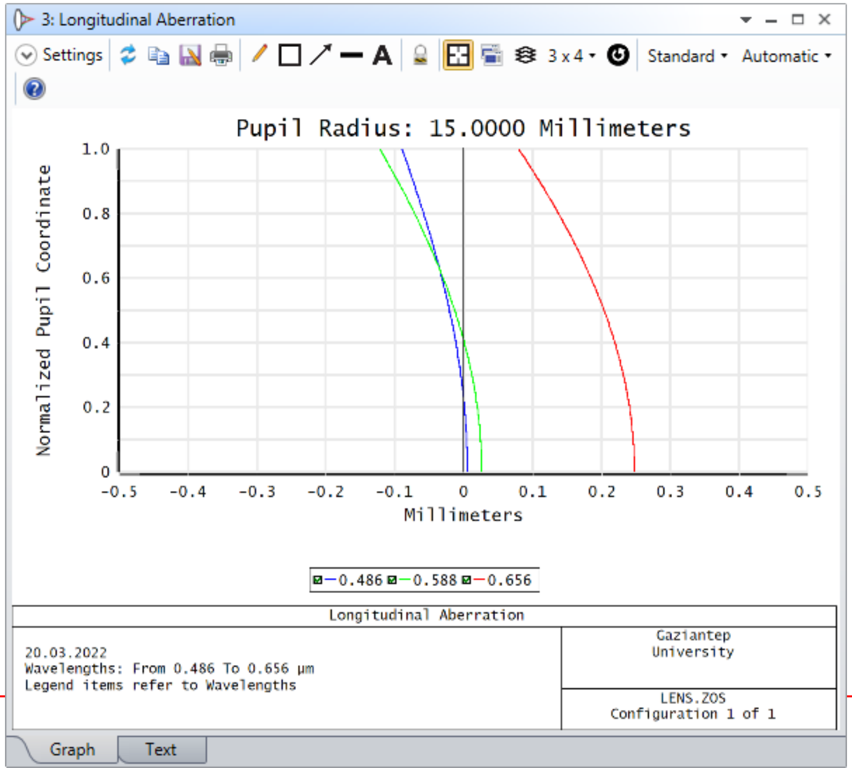
Wizards and Operands — Merit Function: 0.0843456571358175

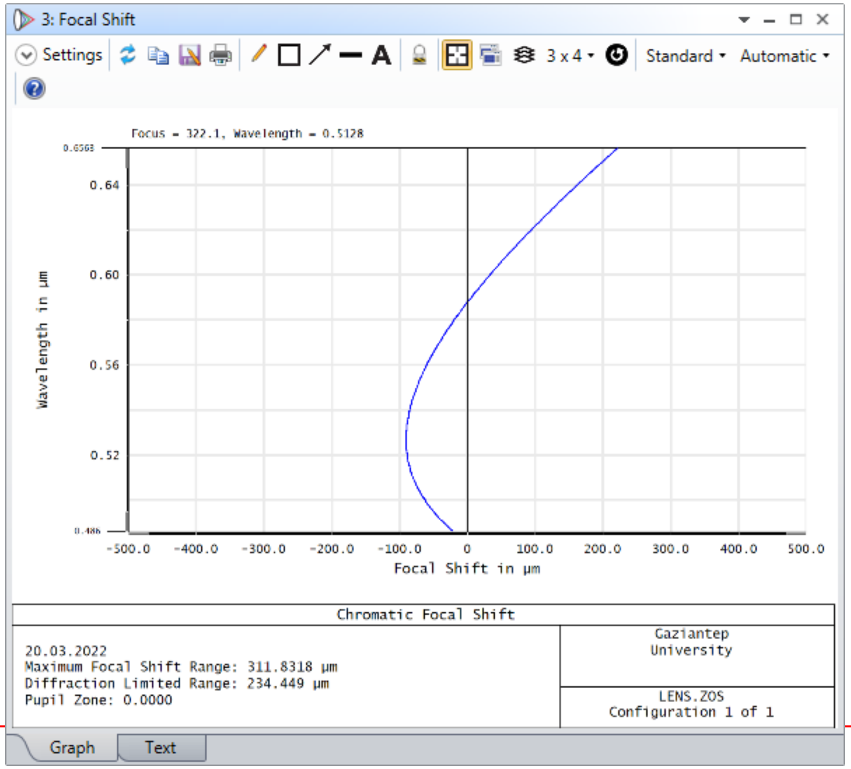
	Type		Wave	Hx	Hy	Px	Py			Target	Weight	Value	% Contrib
1	DMFS												
2	EFFL		2							300.000	1.000	300.007	0.076
3	AXCL	1	3	0.000						0.000	1.000	0.243	99.857
4	BLNK	Operands for field 1.											
5	TRCX		1	0.000	0.000	0.336	0.000			0.000	0.291	-1.021E-04	5.143E-06



final
optimization







Apochromatic Lenses (Triplet)

If we use thin lenses, Achromatic Doublet must satisfy:

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

$$f_1 V_1 + f_2 V_2 = 0$$

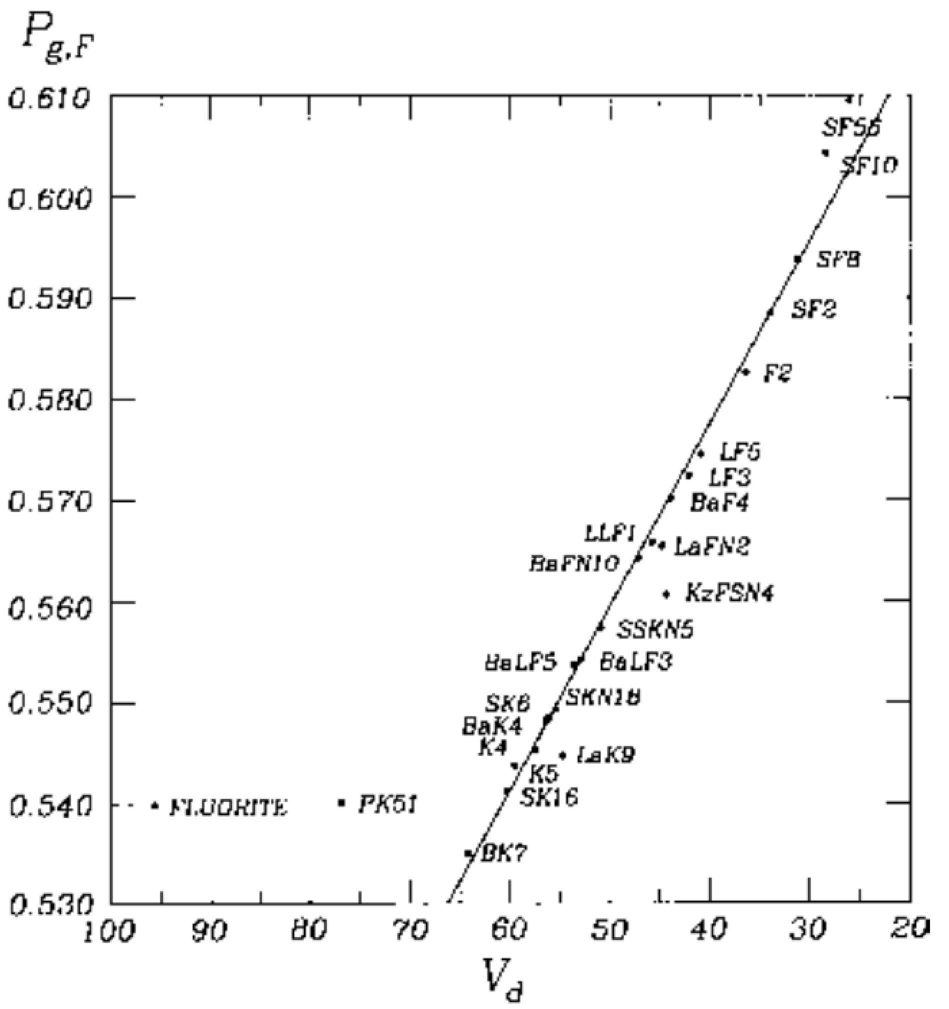
In order to achieve Apochromatic Correction,
a lens system with <u>three elements</u> and
overall focal length of f must satisfy the following
conditions chromatic lens must satisfy:

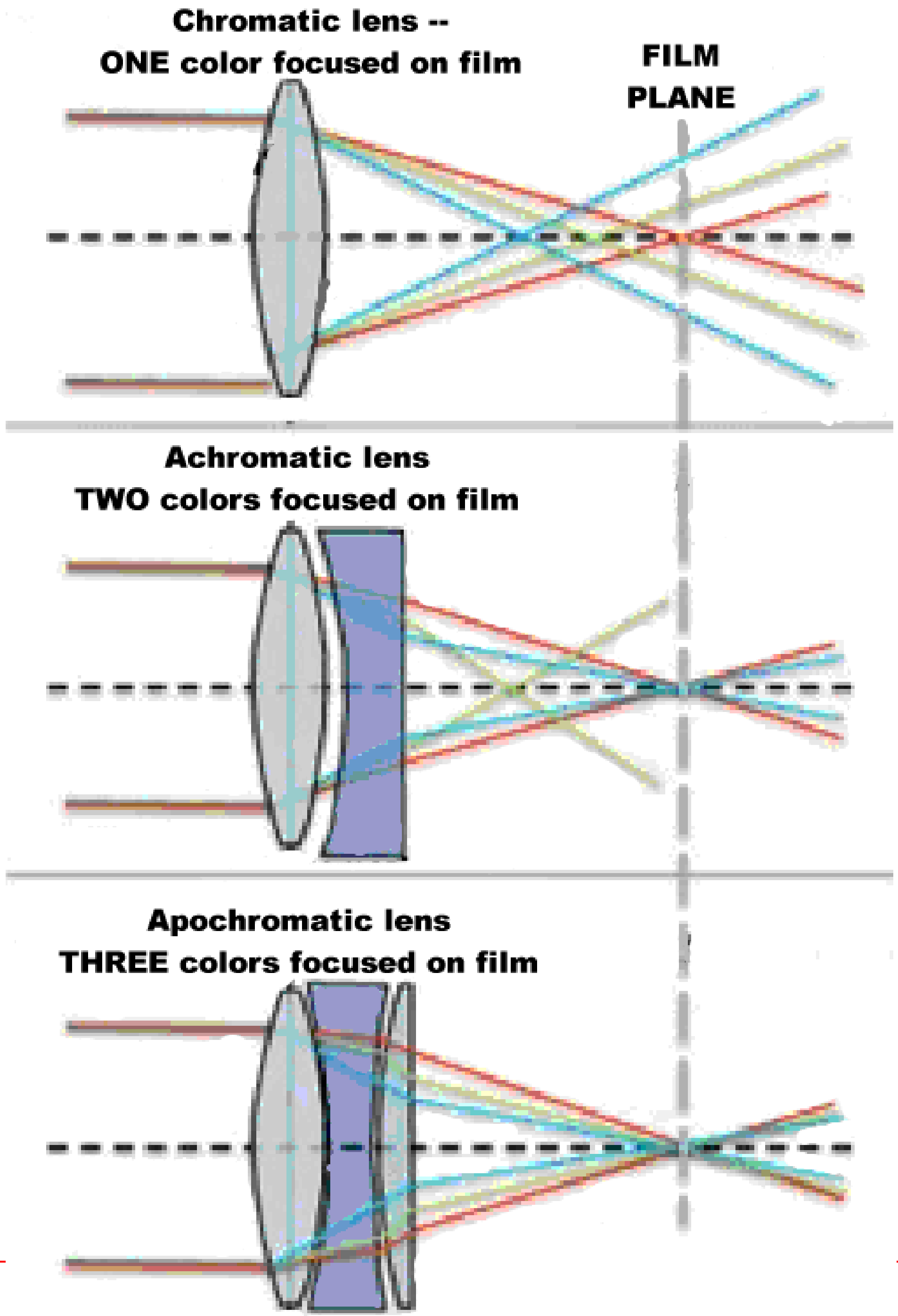
$$\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{f}$$

$$\frac{1}{f_1 V_1} + \frac{1}{f_2 V_2} + \frac{1}{f_3 V_3} = 0$$

$$\frac{P_1}{f_1 V_1} + \frac{1}{f_2 V_2} + \frac{1}{f_3 V_3} = 0$$

- *P is partial dispersion and it is a linear function of Abbe Value:* $P = aV + b$
- Suggested sturcture: PNP
- Suggested glasses:
  (PK51, KZFS4, SF15)
  (PK51, LAF21, SF15)

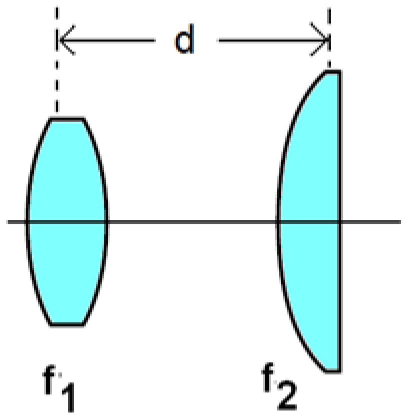



Chromatic lens --
ONE color focused on film
FILM
PLANE
Achromatic lens
TWO colors focused on film
Apochromatic lens
THREE colors focused on film

Spaced Doublet

Another method of making a system achromatic is to use two positive lenses made of same type of glass. Doublet must be separated by a distance equal to one-half the sum of their focal lengths.

$$d = \frac{f_1 + f_2}{2}$$



Effective focal length (f) of the lens system can be found by:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Subsituting first equation into second one yields:

$$\frac{2}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The spaced doublets are mostly used in eyepieces.

Eyepieces

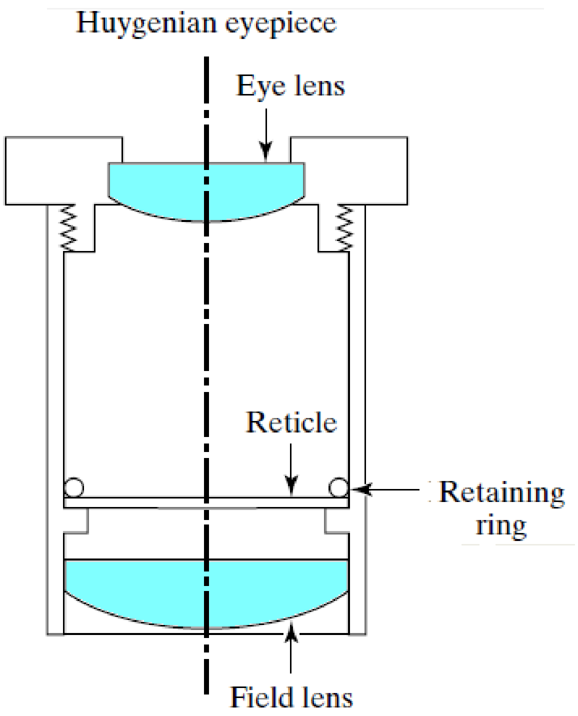
Eyepieces are used in microscpoes, telescopes, and binoculars.
There are simple designs known as Huygenian and Ramsden.
Both designs use two plano-convex lenses.
In Ramsden design, the following relation is suggested:

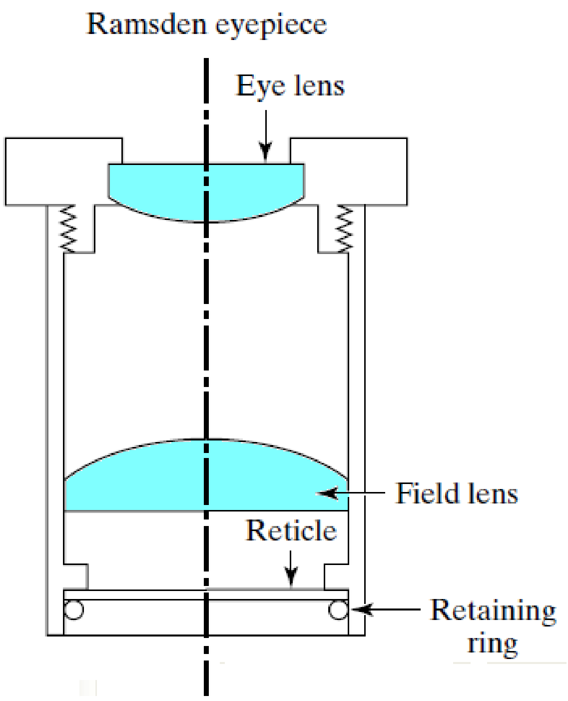
$$f_1 = \frac{3f_2}{2}$$

Final equations for each focal length become:

$$f_1 = \frac{5f}{4} \qquad f_2 = \frac{5f}{6}$$

where *f* is the eyepiece focal length.





Eyepieces

- **Reticle:**
  is a pattern of fine markings
  built into the eyepiece.

- **Eye relief (Göz konumu):**
  is exit pupil position where you observe full FOV.

Example 3: Ramsden Eyepiece Design

We want to design f = 28 mm Ramsden Eyepiece using N-BK7 glasses.
ENPD = 3.5 mm, λ = F,d,C, FOV = 10^o, ER = 12 mm, TOTR < 60 mm.

Starting point is to use thin lens equations:

$$f_1 = 5f/4 = 35.0 \text{ mm}$$

$$f_2 = 5f/6 = 23.3 \text{ mm}$$

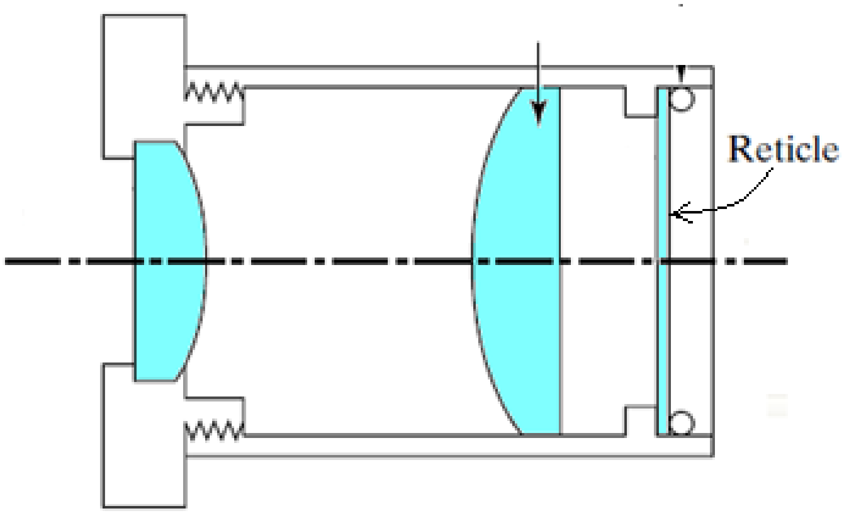
If the lenses are plano-convex, then radius of curvatures for n = 1.52 are as follows:

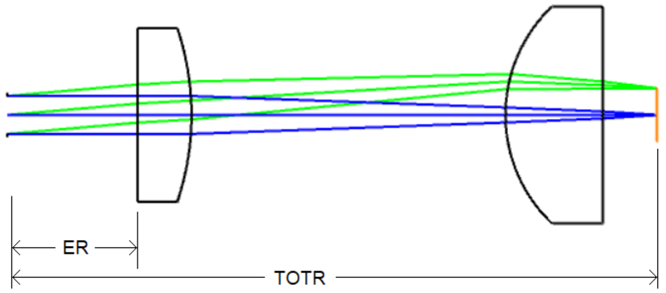
$$|R_1| = (n-1)f_1 = 18.2 \text{ mm}$$

$$|R_2| = (n-1)f_2 = 12.1 \text{ mm}$$

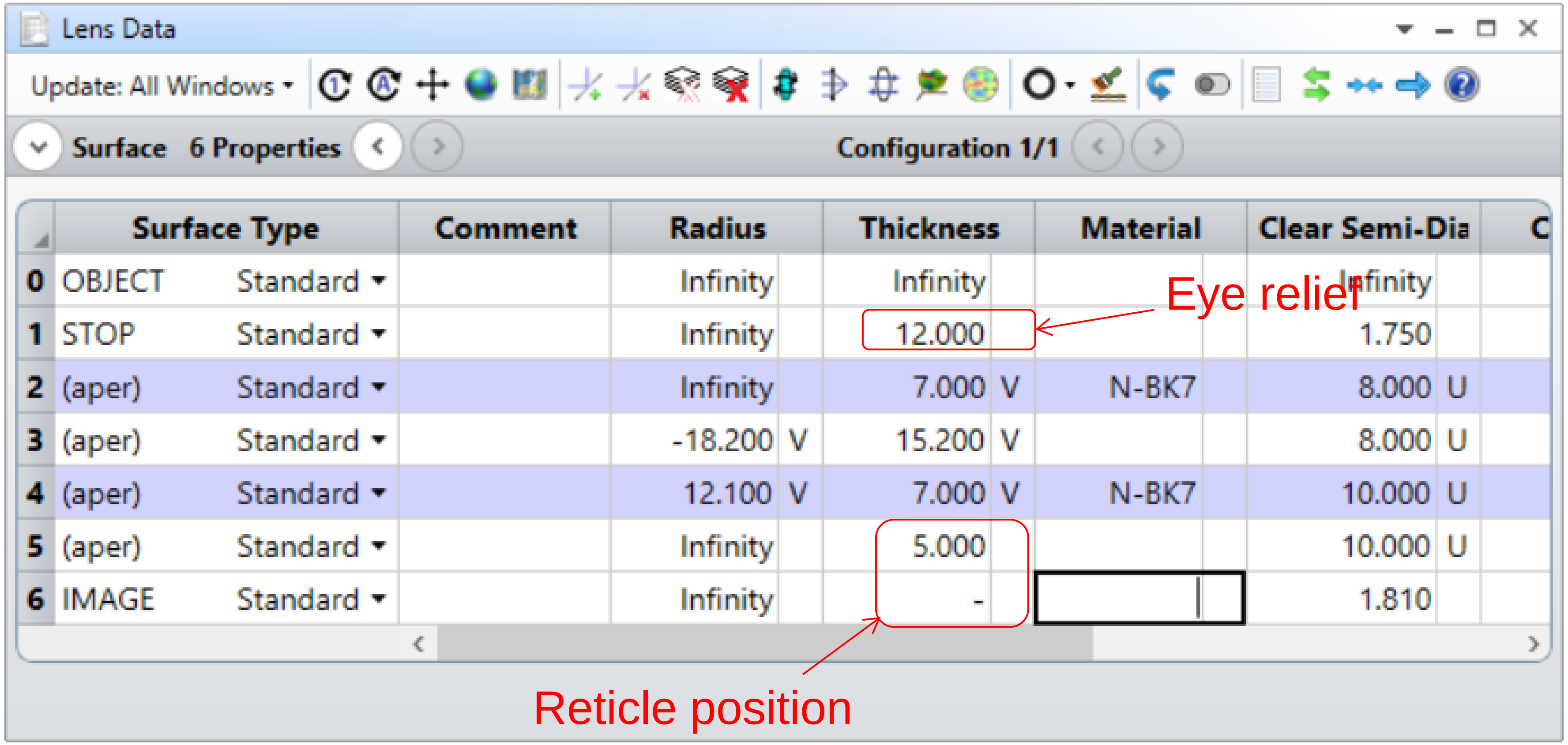
Distance between lenses:

$$d = (f_1 + f_2)/2 = 15.2 \text{ mm}$$



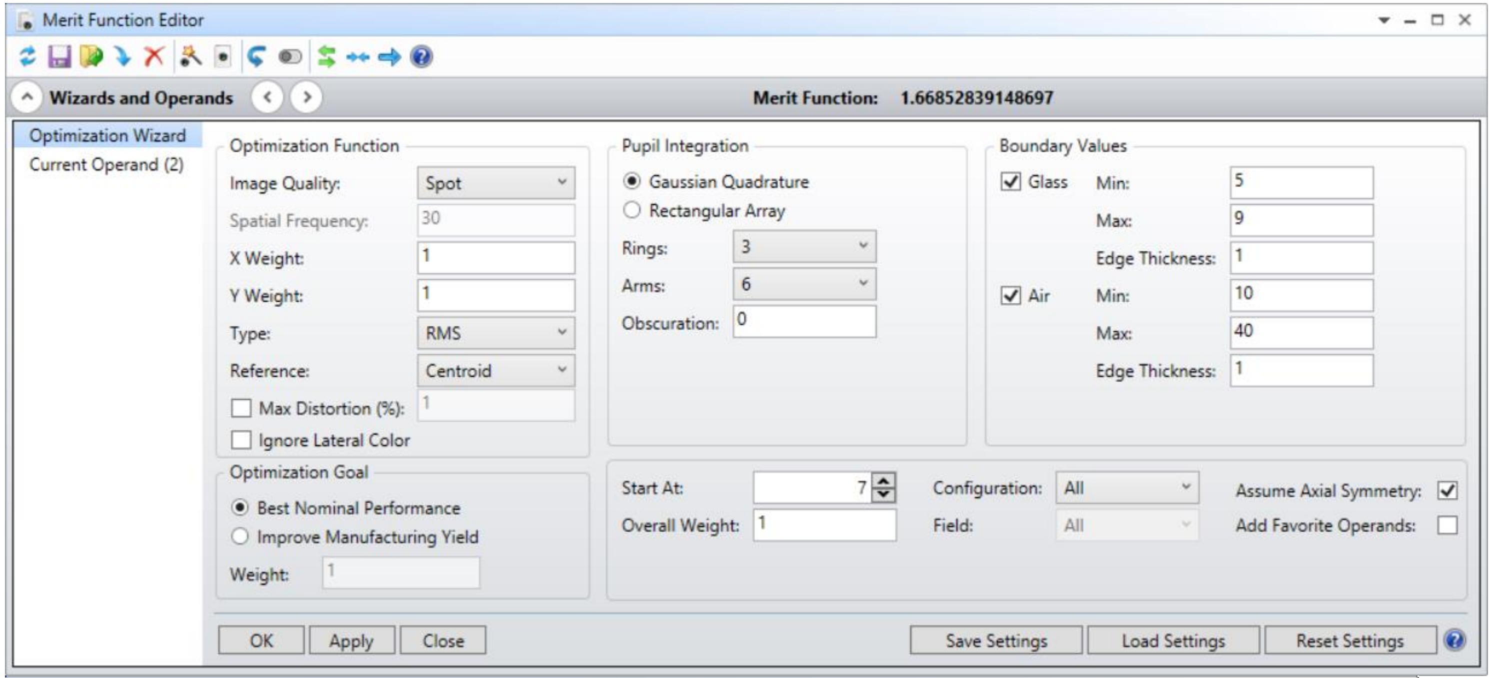


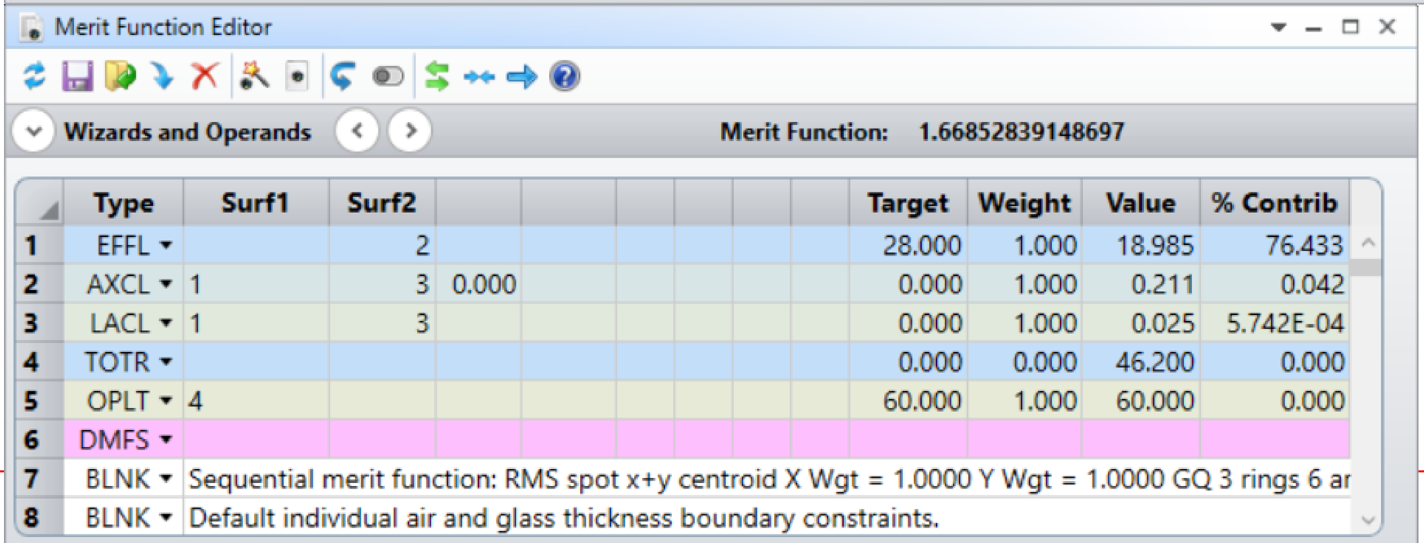
Before Optimization:



	Surface Type		Comment	Radius		Thickness		Material	Clear Semi-Dia		C
0	OBJECT	Standard		Infinity		Infinity			Infinity		
1	STOP	Standard		Infinity		12.000			1.750		
2	(aper)	Standard		Infinity		7.000	V	N-BK7	8.000	U	
3	(aper)	Standard		-18.200	V	15.200	V		8.000	U	
4	(aper)	Standard		12.100	V	7.000	V	N-BK7	10.000	U	
5	(aper)	Standard		Infinity		5.000			10.000	U	
6	IMAGE	Standard		Infinity		-			1.810		

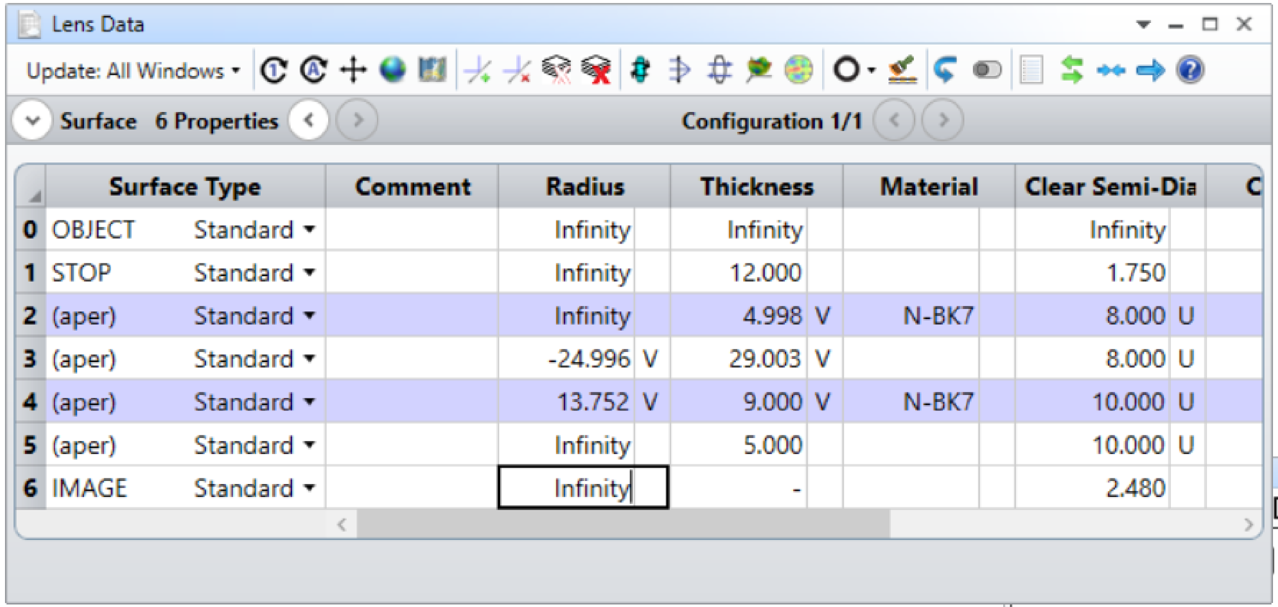
Optimization:



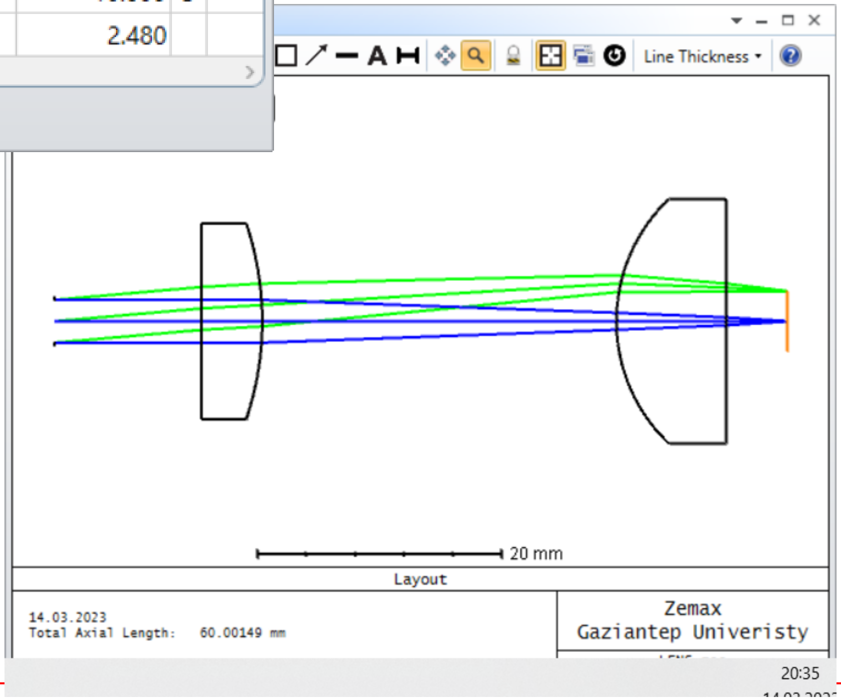


	Type	Surf1	Surf2							Target	Weight	Value	% Contrib
1	EFFL		2							28.000	1.000	18.985	76.433
2	AXCL	1	3	0.000						0.000	1.000	0.211	0.042
3	LACL	1	3							0.000	1.000	0.025	5.742E-04
4	TOTR									0.000	0.000	46.200	0.000
5	OPLT	4								60.000	1.000	60.000	0.000
6	DMFS												
7	BLNK	Sequential merit function: RMS spot x+y centroid X Wgt = 1.0000 Y Wgt = 1.0000 GQ 3 rings 6 ar											
8	BLNK	Default individual air and glass thickness boundary constraints.											

After Optimization:



	Surface Type		Comment	Radius		Thickness		Material	Clear Semi-Dia	
0	OBJECT	Standard		Infinity		Infinity			Infinity	
1	STOP	Standard		Infinity		12.000			1.750	
2	(aper)	Standard		Infinity		4.998	V	N-BK7	8.000	U
3	(aper)	Standard		-24.996	V	29.003	V		8.000	U
4	(aper)	Standard		13.752	V	9.000	V	N-BK7	10.000	U
5	(aper)	Standard		Infinity		5.000			10.000	U
6	IMAGE	Standard		Infinity		-			2.480	



After Optimization: