Diffraction, OPD
DRI, and MTF

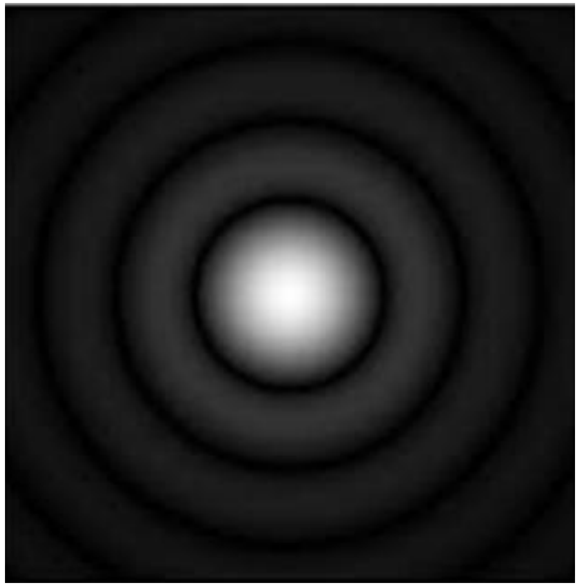

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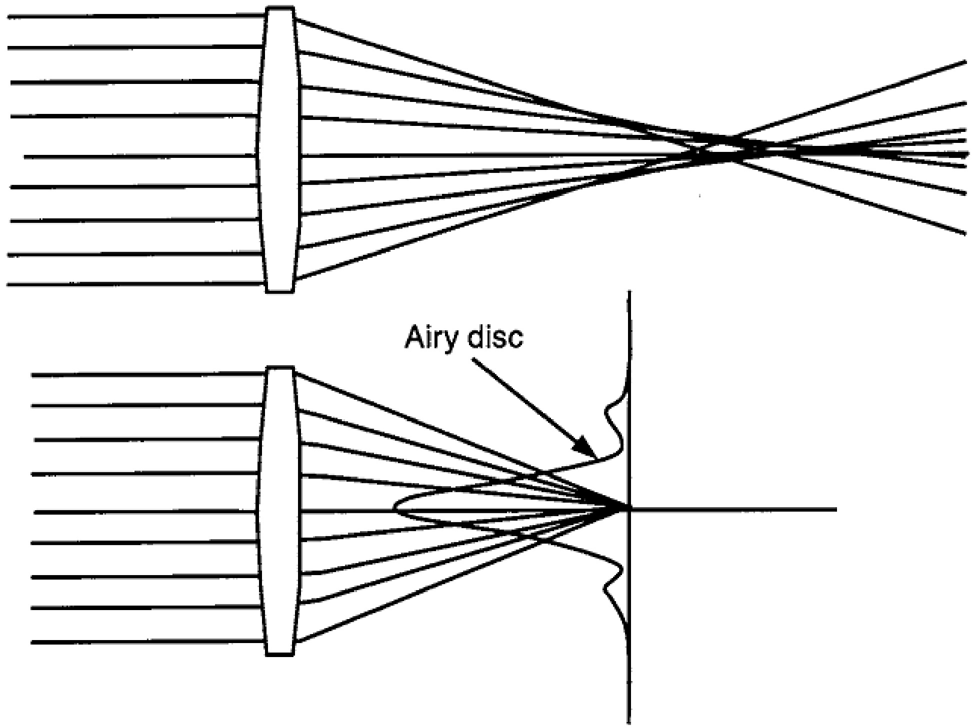
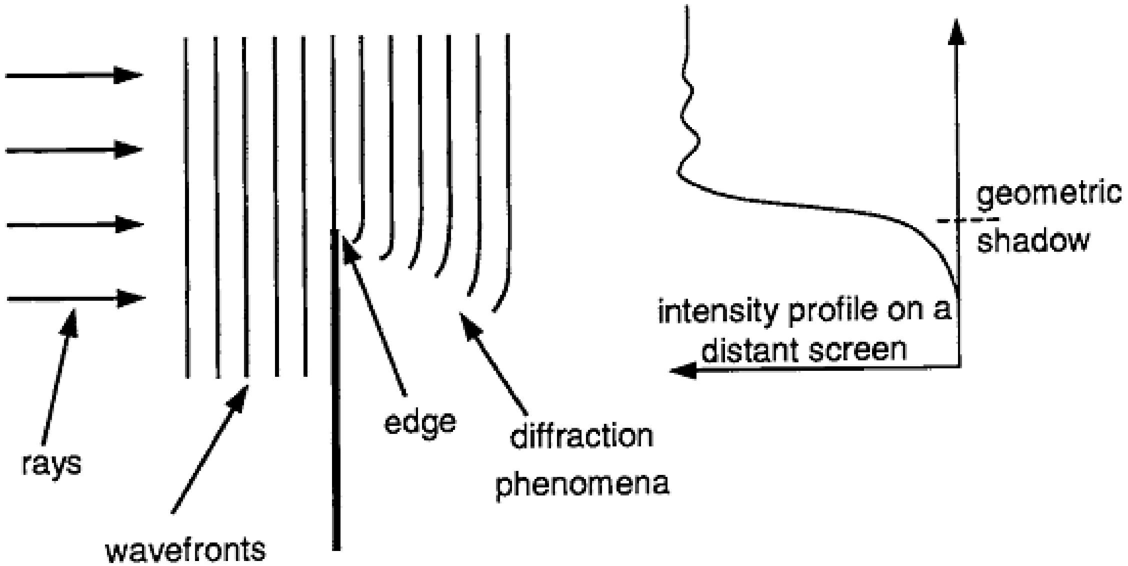
Image Quality

- **Image quality is never perfect**. It would be very nice if the image of a point object could be formed as a perfect point image. This is impossible!
- Image quality is suffered by both
  - geometrical aberrations and
  - diffraction



Diffraction

Diffraction is an effect resulting from the interaction of light wave with the sharp limiting edge or aperture of an optical system.



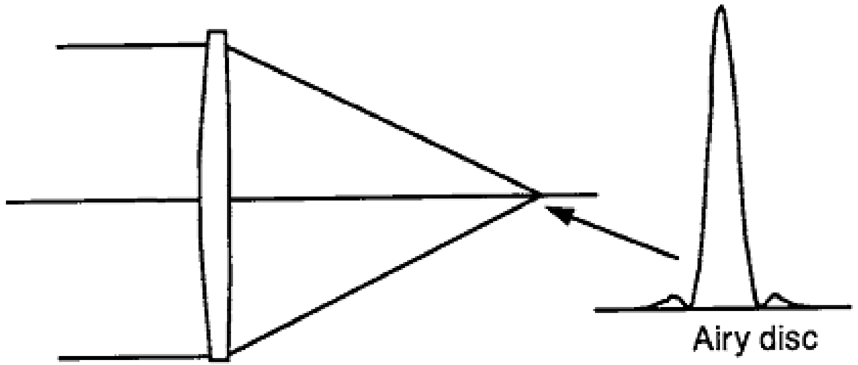
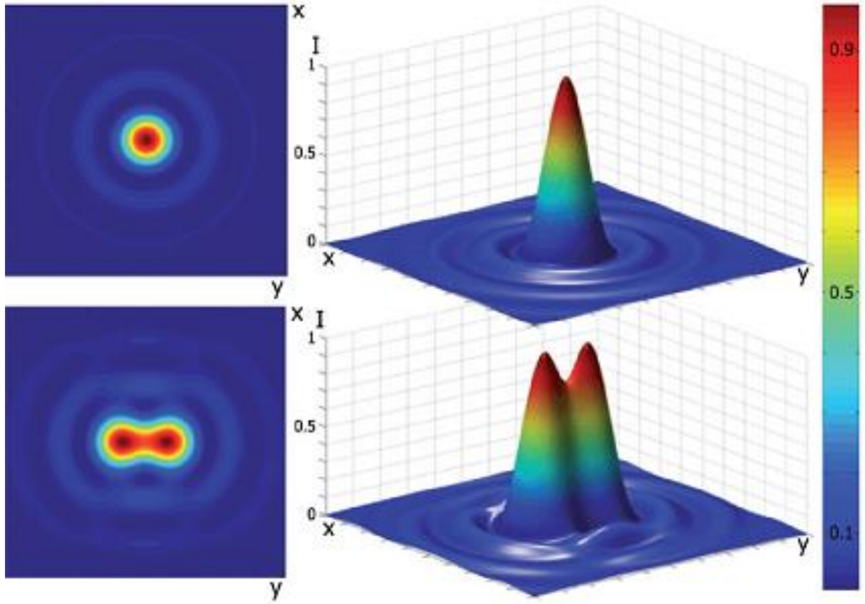


Image Evaluation of a Point Object

- The ideal image of a point object is a point image.
- Observation of the images of point objects produced by aberration free lens systems with circular entrance pupils shows that the images are not pointy, but have a light distribution known as the **Airy pattern, A(r).**
- A(r), is described mathematically by the function:



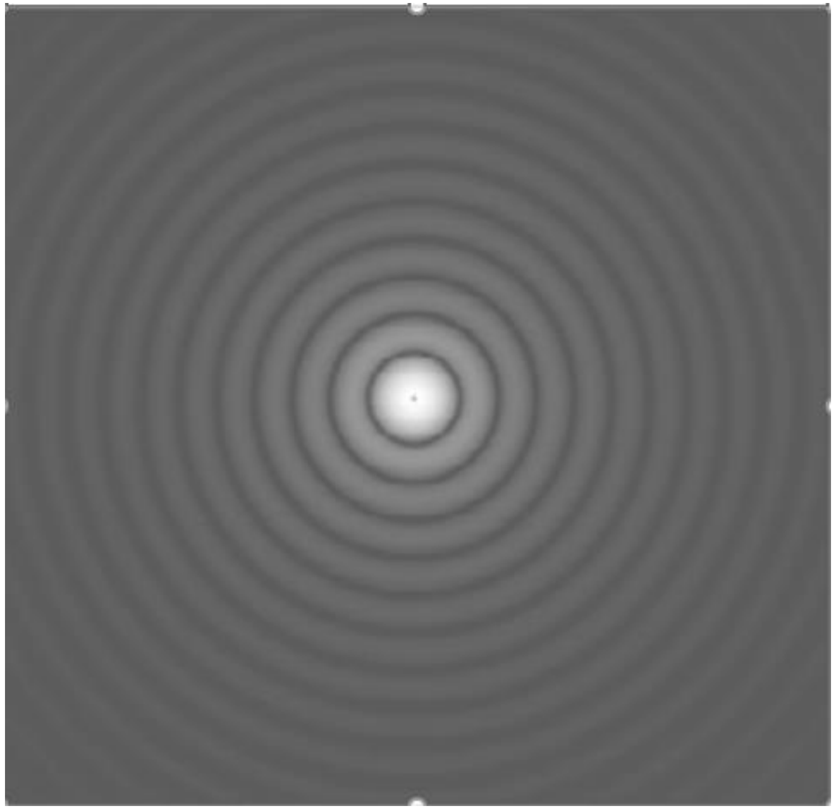
$$A(r) = \left(\frac{2J_1(\pi r)}{\pi r}\right)^2$$

$J_1(r)$ = 1st order Bessel Function of first kind

The radial locations of the first three zeros in the Airy pattern take place at approximately

- r =1.22 => A(r) = 0
- r = 2.23 => A(r) = 0
- r = 3.24 => A(r) = 0

$$A(r) = \left(\frac{2J_1(\pi r)}{\pi r}\right)^2$$



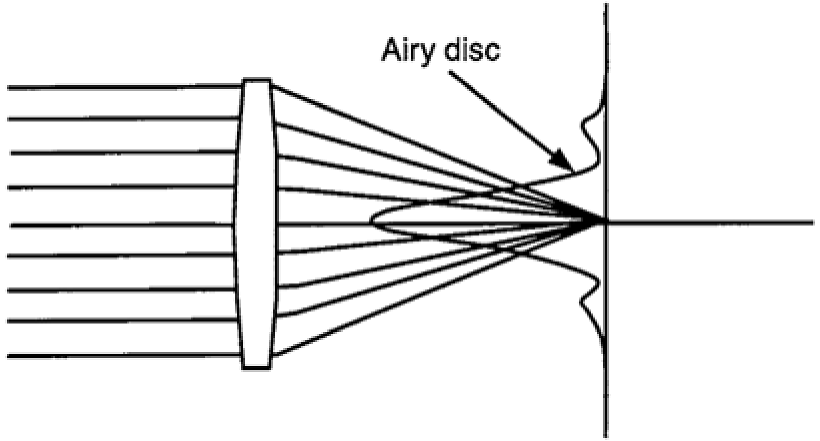
These locations corresponds to dark rings.

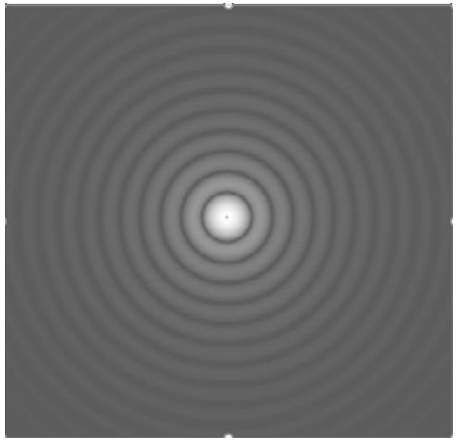
Airy Disk

If the geometrical aberrations are significantly smaller than the diffraction blur, the image is well represented by the Airy disk.

This form of optics is called diffraction-limited optics.

Radius of Airy disk (Radius of first dark ring) is: $r_A = 1.22\,\lambda\,(f/\#)$

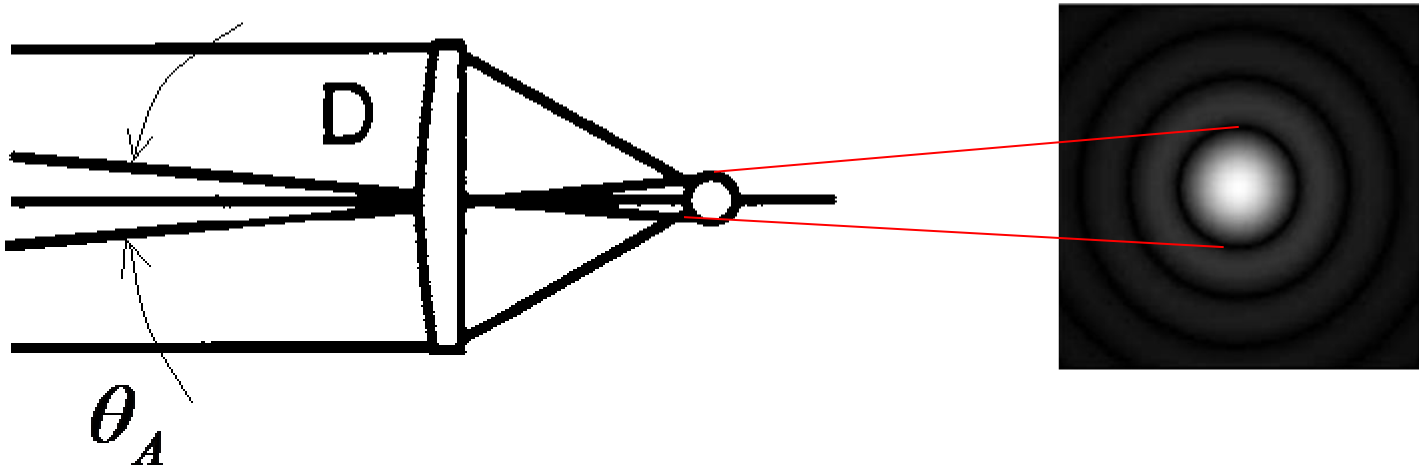




About 83% of energy is encircled within the first dark ring called the Airy Disk

Airy disk is the smallest point to which a beam of light can be focused.

Angular Diameter of Airy Disk

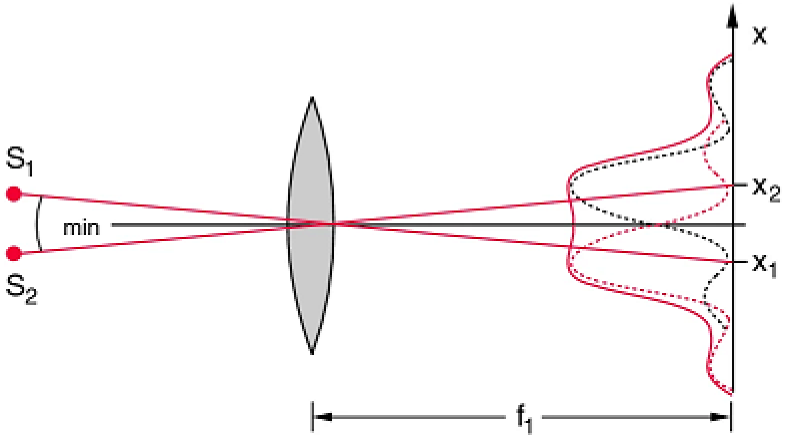


$$\theta_A = \frac{2.44\lambda}{D}$$

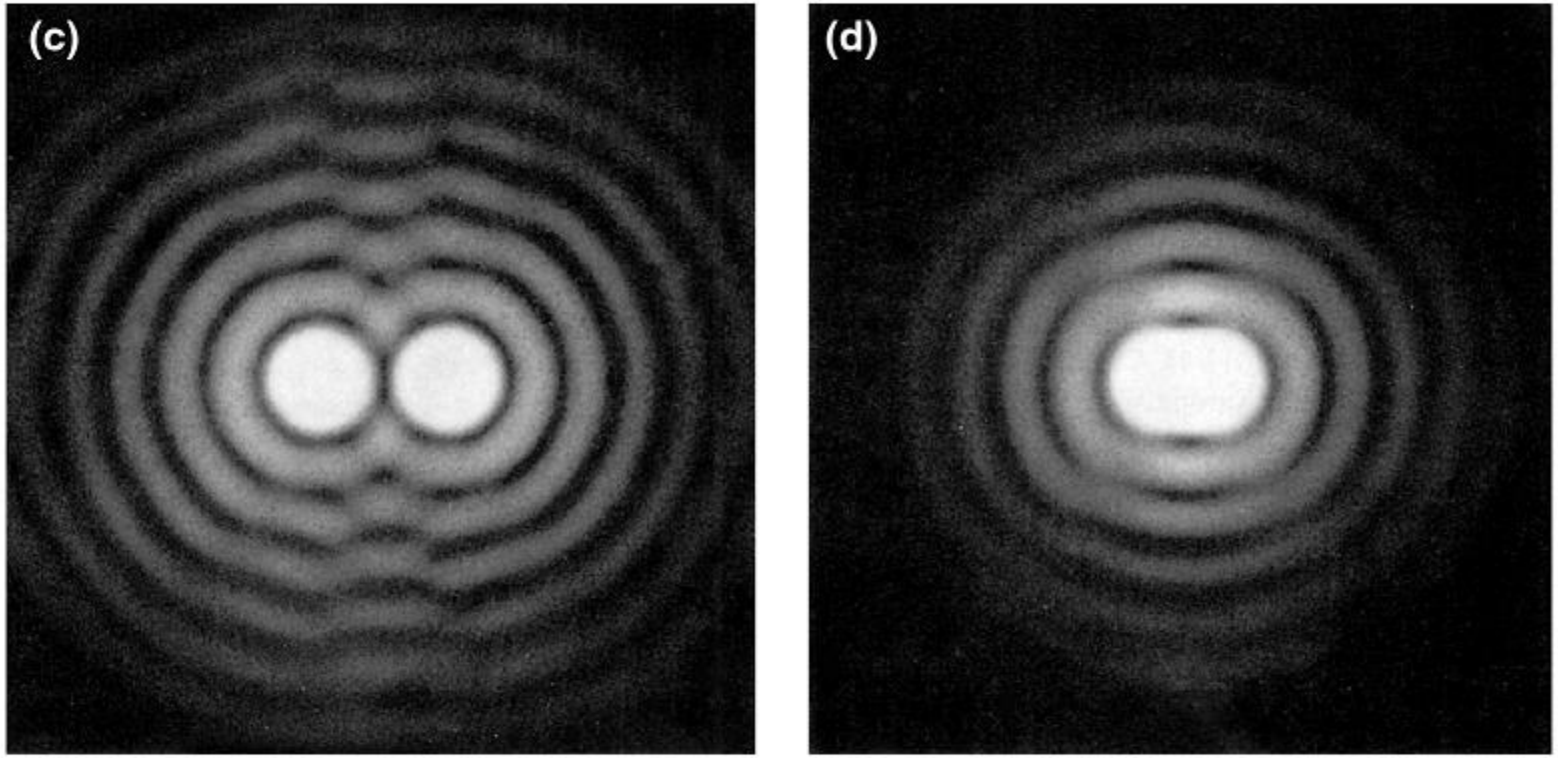
θ_A = Angular diameter of Airy disk

λ = Wavelength used in operation

D = Clear aperture diameter is the diameter of light entering to lens. Usually, it is not the full diameter of the lens.



x
S_1
x_2
min
S_2
x_1
f_1

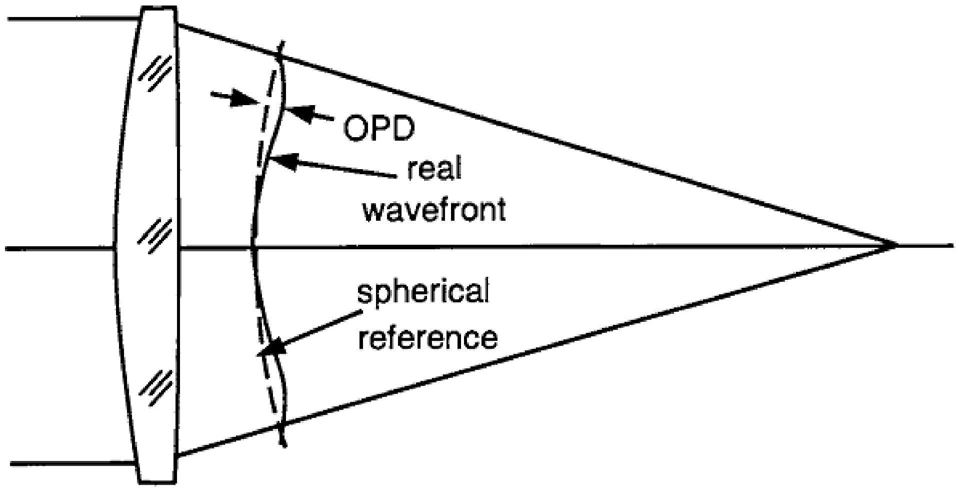


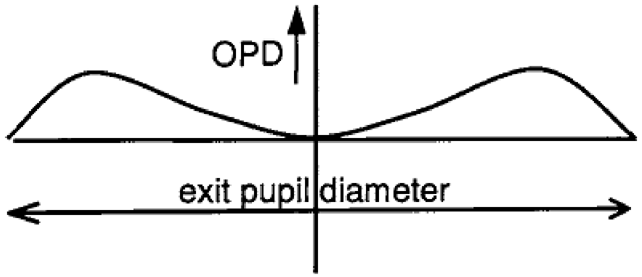
(c)
(d)

Optical Path Difference

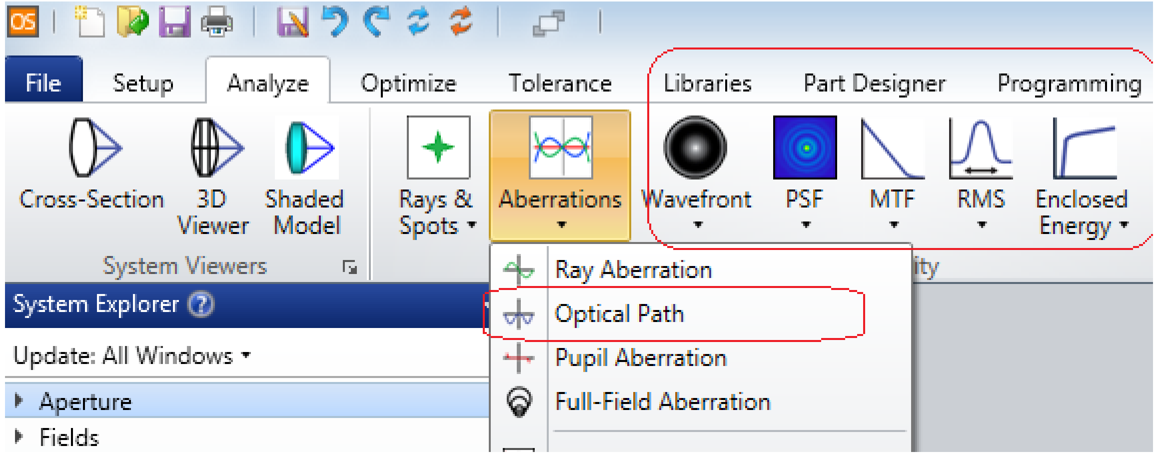
- **OPD** is useful measure of performance of an imaging optical system.

- **Rayleigh Criteria**:
  If the OPD is less than or equal to
  one-quarter of a wave, then
  the performance will be almost
  indistinguishable from perfect.

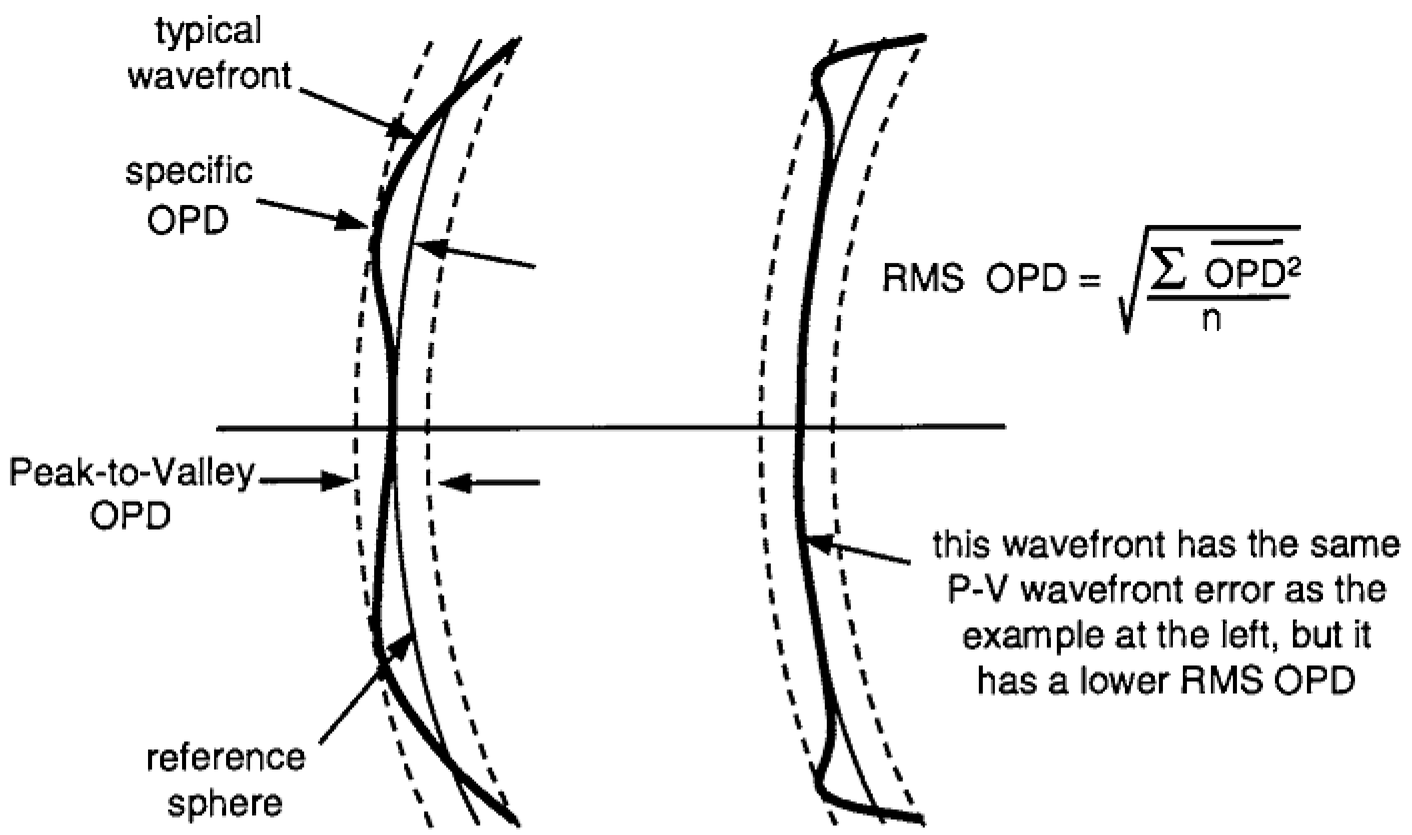




- Zemax menu



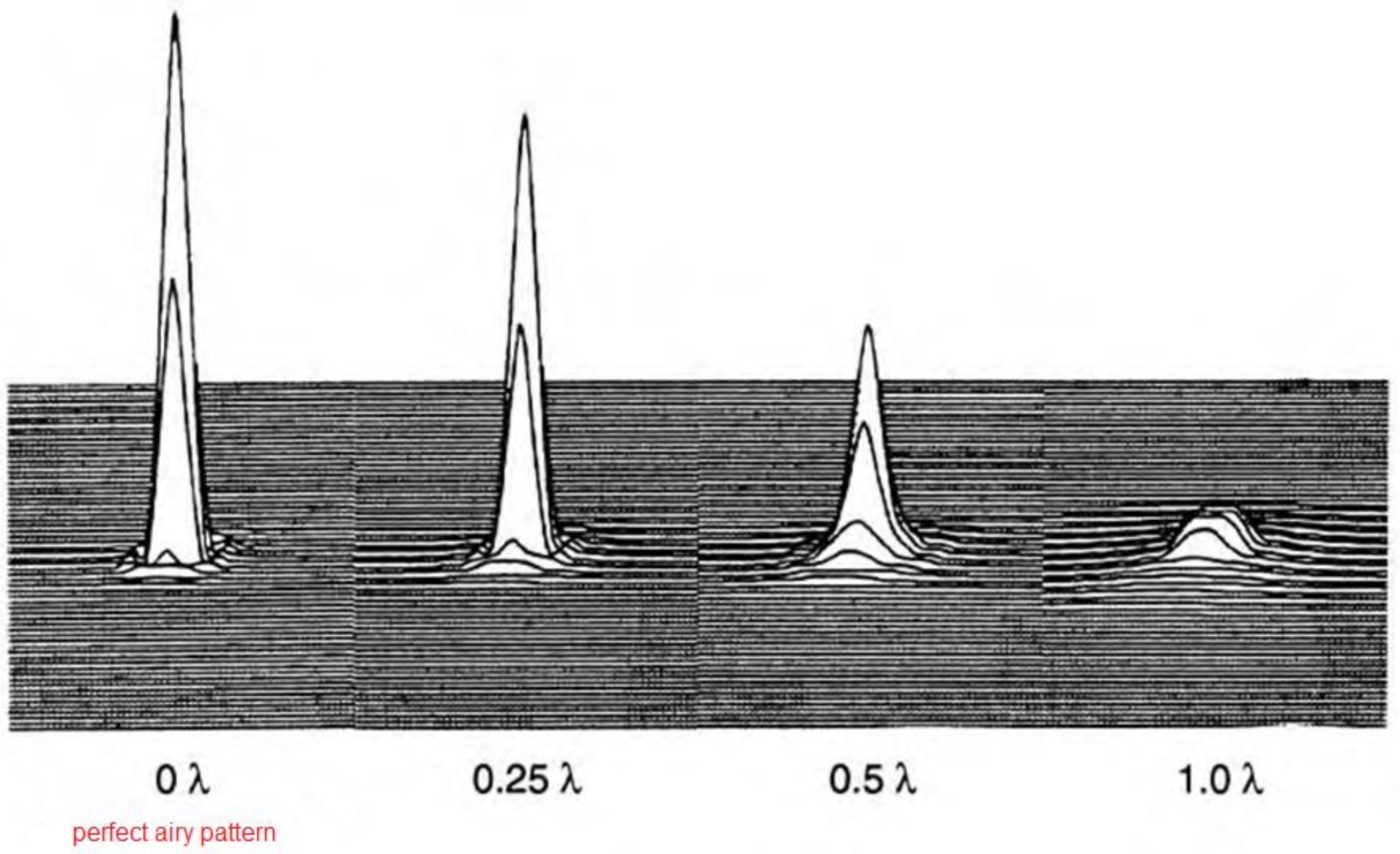
Peak-to-Valey OPD



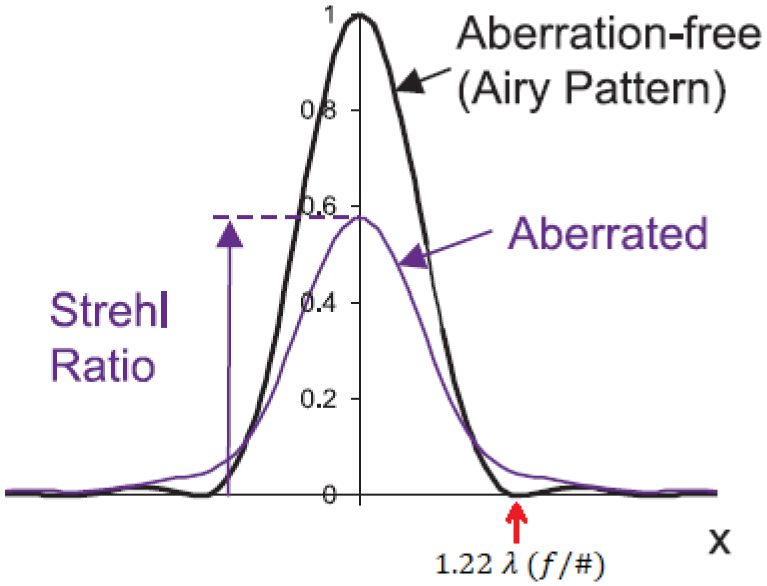
RMS wavefront error = square root of the sum of the squares of the OPDs as measured from a best-fit reference spherical wavefront over the total wavefront area

PSF (Point Spread Function)

Figure left shows the appearance of the image of a point source, which is known as a PSF, for optical path differences of 0 wave, 0.25 wave, 0.5 wave, and 1.0 wave.



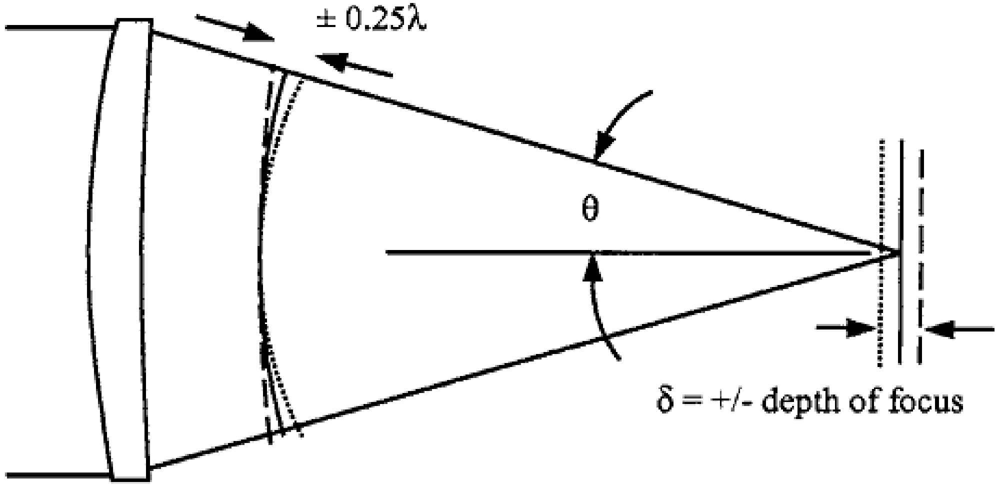
The **Strehl ratio** is defined as the ratio of the peak intensity of an aberrated lens to the peak intensity of an aberration-free lens.

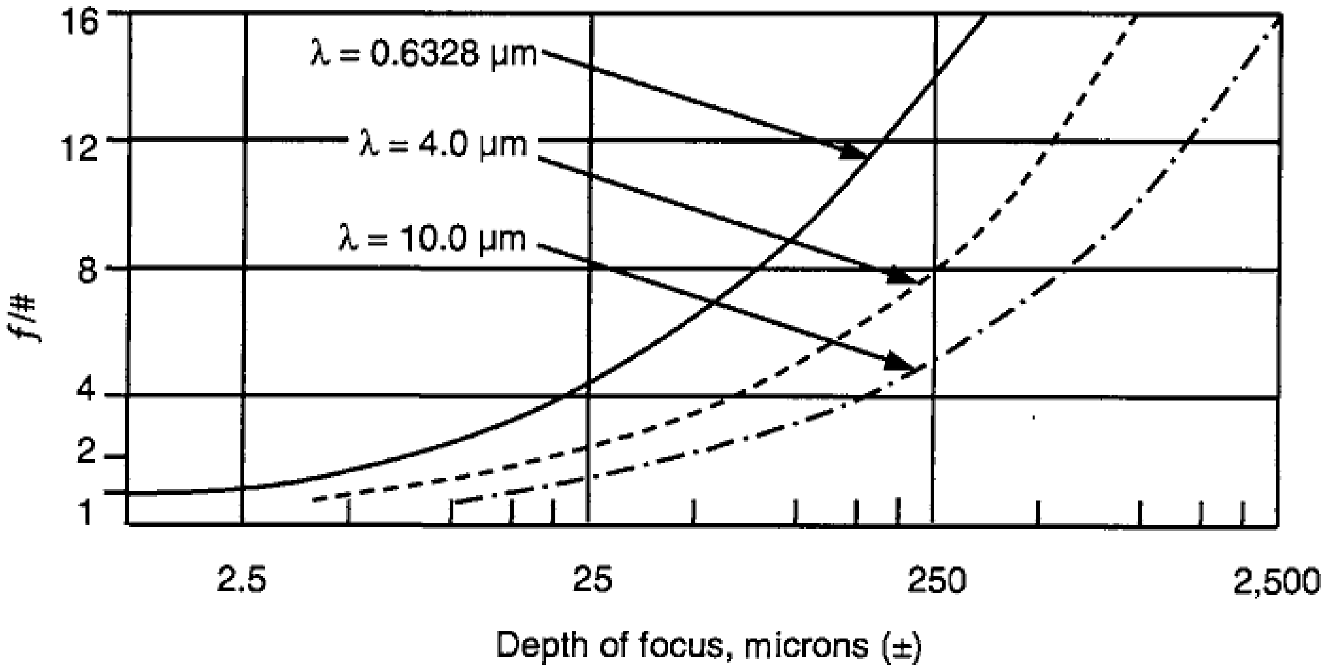


Depth of Focus

- Rayleigh Criteria can be used to determine just how much defocus is tolerable to maintain diffraction limited performance.
- Depth of focus corresponding to OPD = λ / 4 is

$$\delta = \pm 2\,\lambda (f/\#)^2$$





Difraction Limited Optics

Diffraction-limited lens is limited solely by diffraction and refers to a zero-wave RMS OPD; however, these criteria are rarely the intent of a “diffraction-limited” specification.

Common approximations for diffraction-limited performance include:

- **peak-to-valley OPD < λ / 4** (Rayleigh criterion), or
- **RMS OPD < 0.07 λ** (Maréchal criterion), or
- **Strehl ratio > 0.8**

Example 1: Simple LWIR Objective Design

Consider an objective used in the long-wave infrared (LWIR) range.

* Spectral band: 8 μm to 12 μm
* We need to resolve: 0.25 mrad in object space
* Detector used: CCD with 50 μm pixel size (pitch)

Determine f/#, clear aperture diameter, focal length and depth of focus.

$$\textbf{Diameter of Airy disk} = \textbf{CCD pixel size}$$

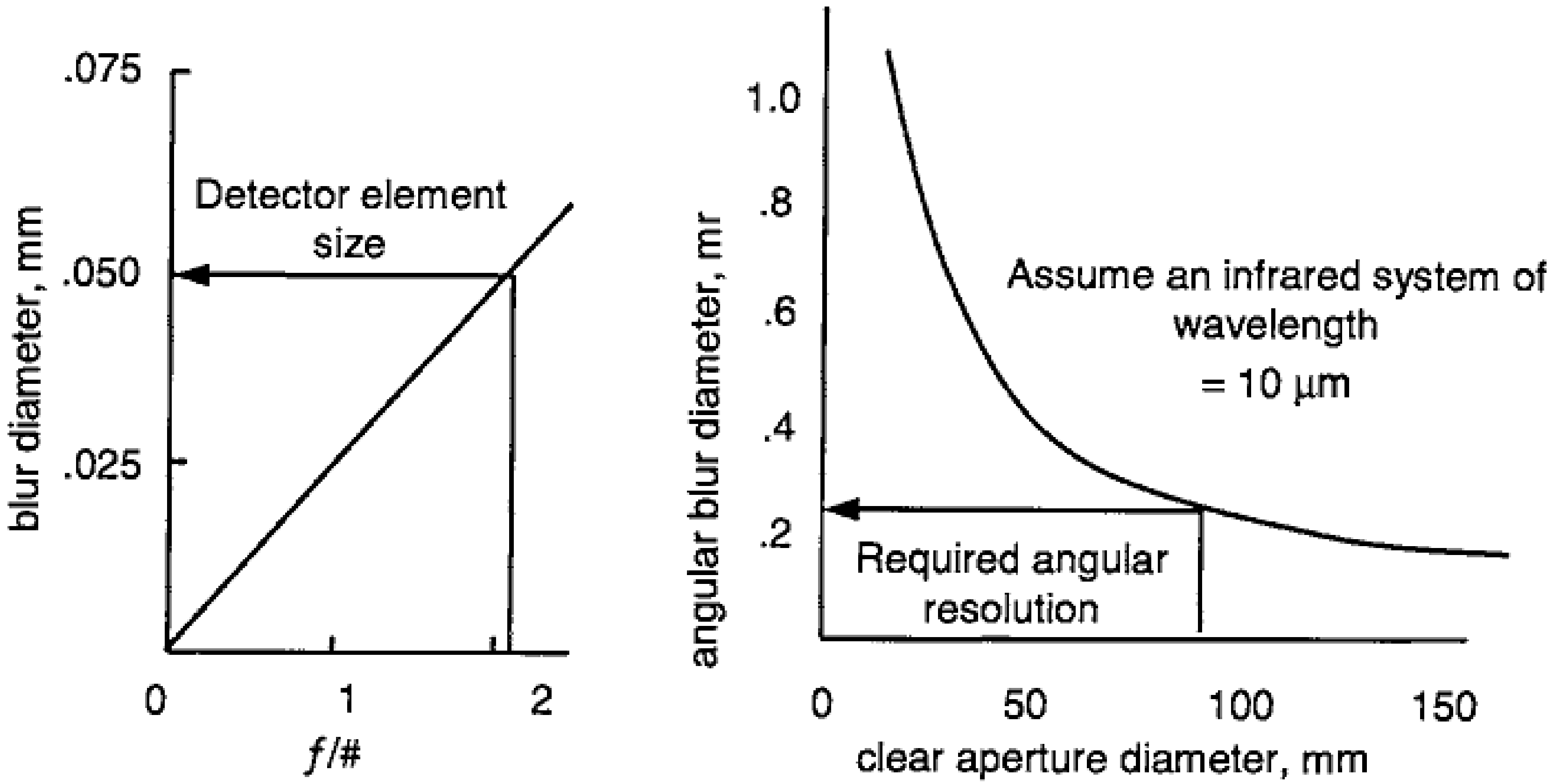
$$2.44\,\lambda\,(f/\#) = \textbf{CCD pixel size}$$

$$2.44\,(10\ \mu\text{m})\,(f/\#) = 50\ \mu\text{m} \rightarrow \boxed{f/2.2}$$

$$\boxed{f = 220\ \text{mm}}$$

$$D = \frac{2.44\lambda}{\theta_A} = \frac{2.44(10\ \mu\text{m})}{0.25\times 10^{-3}} = 100\ \text{mm} \rightarrow \boxed{D = 100\ \text{mm}}$$

$$\delta = \pm 2\,(10\ \mu\text{m})\,(2.2)^2 \approx \pm 100\ \mu\text{m}$$



.075
.050
.025
blur diameter, mm
Detector element size
0
1
2
f/#
1.0
.8
.6
.4
.2
angular blur diameter, mr
Assume an infrared system of wavelength = 10 µm
Required angular resolution
0
50
100
150
clear aperture diameter, mm

Example 2: Zemax Implementation of Example 1

Before Optimization

Lens Data — Update: All Windows — Surface 6 Properties — Configuration 1/1

	Surface Type		Comment	Radius		Thickness		Material	Clear Semi-		Chip Zone	Mech Semi-Dia
0	OBJECT	Standard		Infinity		Infinity			Infinity		0.000	Infinity
1	STOP	Standard		Infinity		0.000			50.000		0.000	50.000
2	(aper)	Standard	L1	Infinity	V	10.000	V	GERMANIUM	55.000	U	0.000	55.000
3	(aper)	Standard		Infinity	V	15.000	V		55.000	U	0.000	55.000
4	(aper)	Standard	L2	Infinity	V	10.000	V	GERMANIUM	55.000	U	0.000	55.000
5	(aper)	Standard		Infinity	V	200.000	V		55.000	U	0.000	55.000
6	IMAGE	Standard		Infinity		-			55.761		0.000	55.761

Merit Function Editor — Wizards and Operands — Merit Function: 0.159707119812836

Optimization Wizard / Current Operand (10)

Optimization Function: Image Quality: Spot; Spatial Frequency: 30; X Weight: 1; Y Weight: 1; Type: RMS; Reference: Centroid; Max Distortion (%): 1; Ignore Lateral Color

Pupil Integration: Gaussian Quadrature (selected); Rectangular Array; Rings: 5; Arms: 6; Obscuration: 0

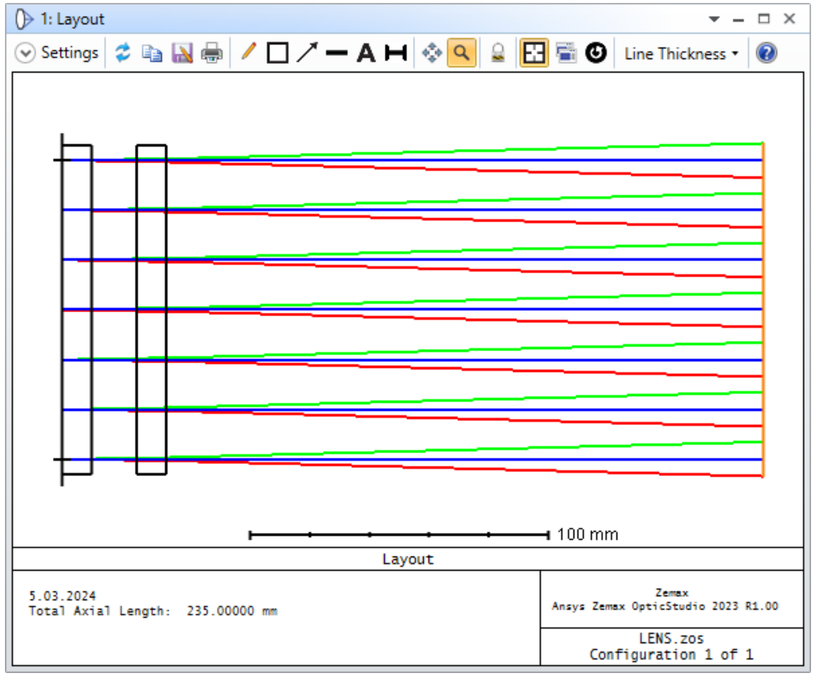
Boundary Values: Glass (checked) Min: 8, Max: 15, Edge Thickness: 2; Air (checked) Min: 1, Max: 1e+03, Edge Thickness: 1

Optimization Goal: Best Nominal Performance (selected); Improve Manufacturing Yield; Weight: 1

Start At: 5; Overall Weight: 1; Configuration: All; Field: All; Assume Axial Symmetry: (checked); Add Favorite Operands: (unchecked)

OK | Apply | Close | Save Settings | Load Settings | Reset Settings

	Type	Surf1	Surf2	Target	Weight	Value	% Contrib
1	EFFL		2	220.000	1.000	220.000	1.627E-11
2	CTGT	3		2.000	1.000	2.000	0.000
3	CTLT	3		20.000	1.000	20.000	1.843E-10
4	DMFS						
5	BLNK	Sequential merit function: RMS spot x+y centroid X Wgt = 1.0000 Y Wgt = 1.0000 GQ 5 rings 6 arms					



ENPD = 100 mm

EFFL = 220 mm

λ = 8, **10**, 12 µm

FOV = 0, 3 deg

ct = [8, 15] mm

Example 2: Zemax Implementation

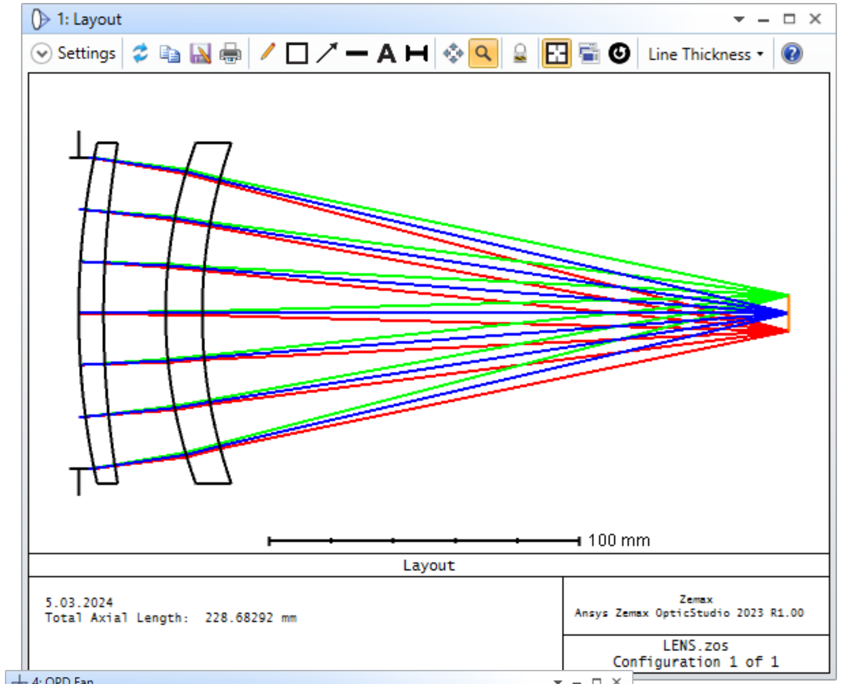
After Local Optimization

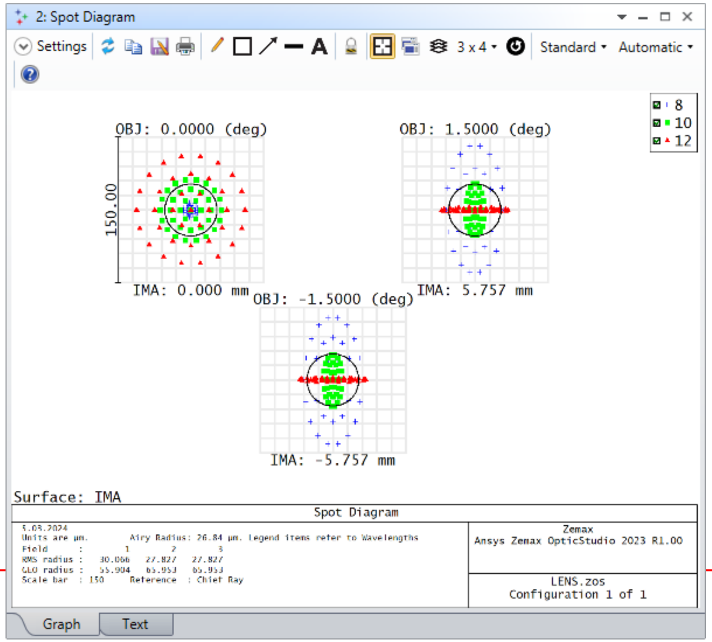
Lens Data

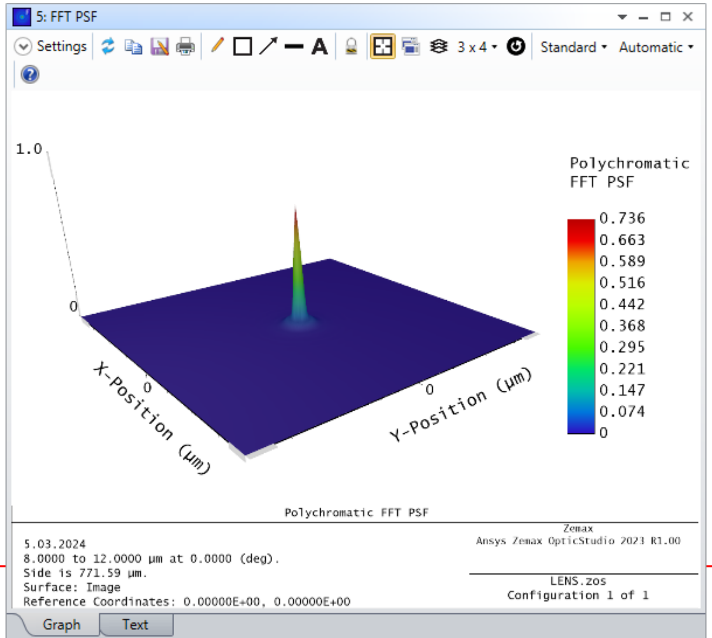
Update: All Windows

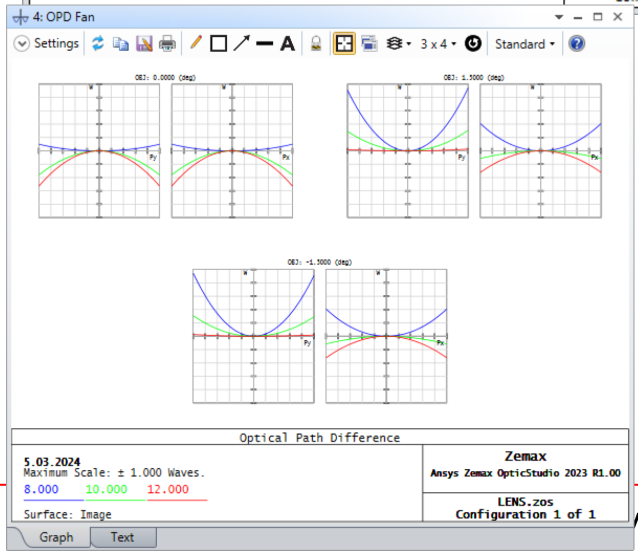
Surface 6 Properties — Configuration 1/1

#	Surface Type		Comment	Radius		Thickness		Material	Clear Semi-		Chip Zone	Mech Semi-Dia
0	OBJECT	Standard		Infinity		Infinity			Infinity		0.000	Infinity
1	STOP	Standard		Infinity		0.000			50.000		0.000	50.000
2	(aper)	Standard	L1	256.008	V	8.000	V	GERMANIUM	55.000	U	0.000	55.000
3	(aper)	Standard		332.218	V	20.000	V		55.000	U	0.000	55.000
4	(aper)	Standard	L2	160.337	V	11.782	V	GERMANIUM	55.000	U	0.000	55.000
5	(aper)	Standard		165.820	V	188.901	V		55.000	U	0.000	55.000
6	IMAGE	Standard		Infinity		-			5.824		0.000	5.824



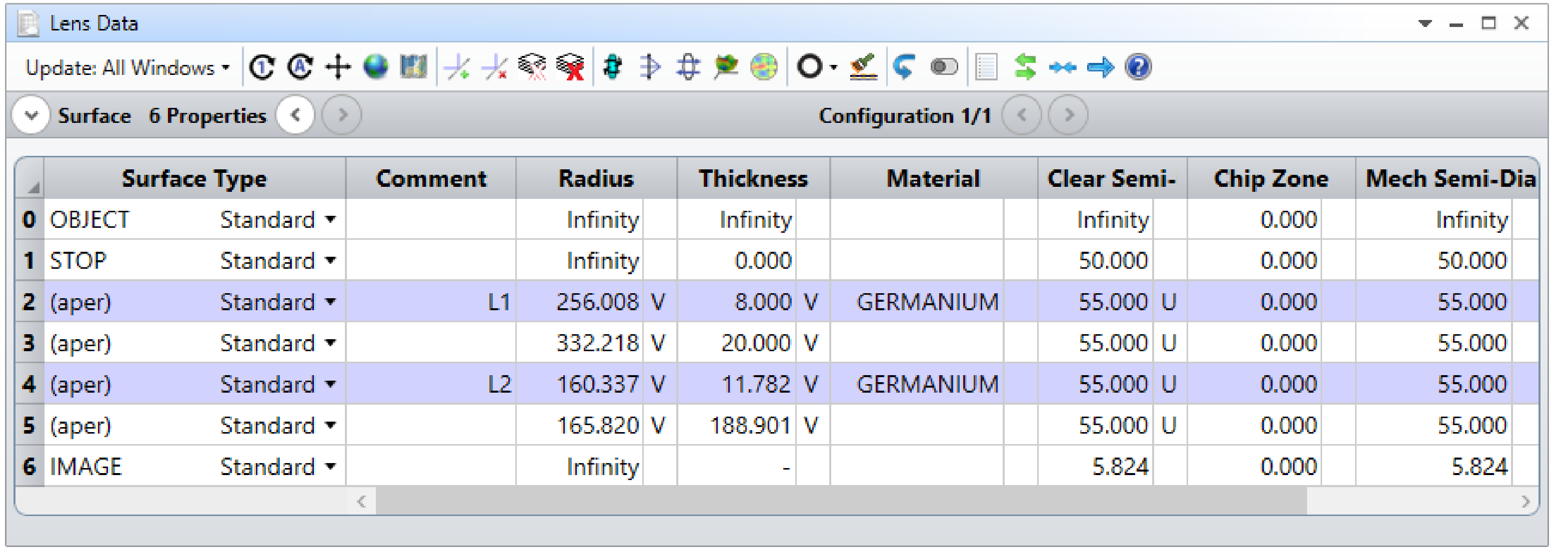






Example 2: Zemax Implementation

After Local Optimization

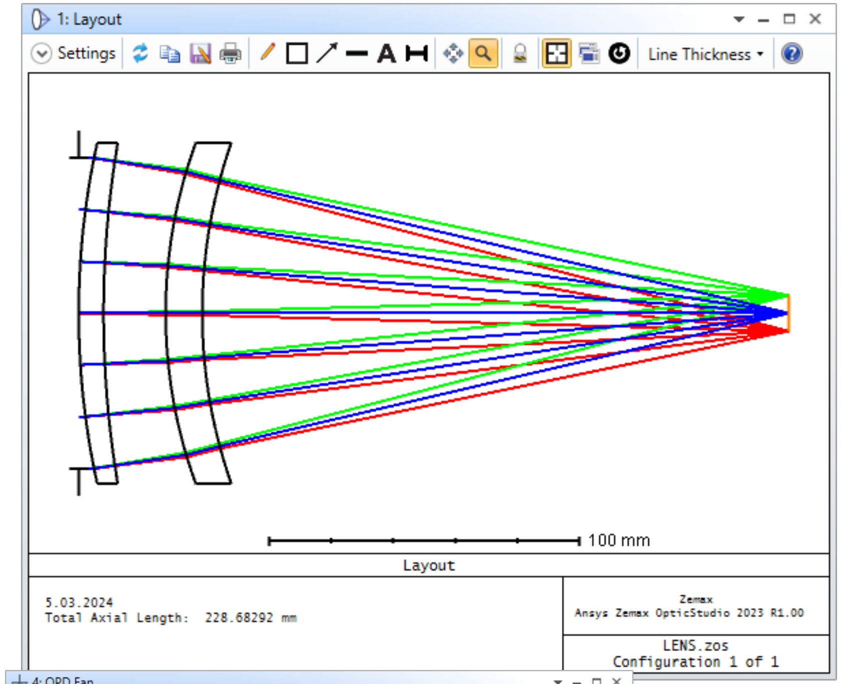


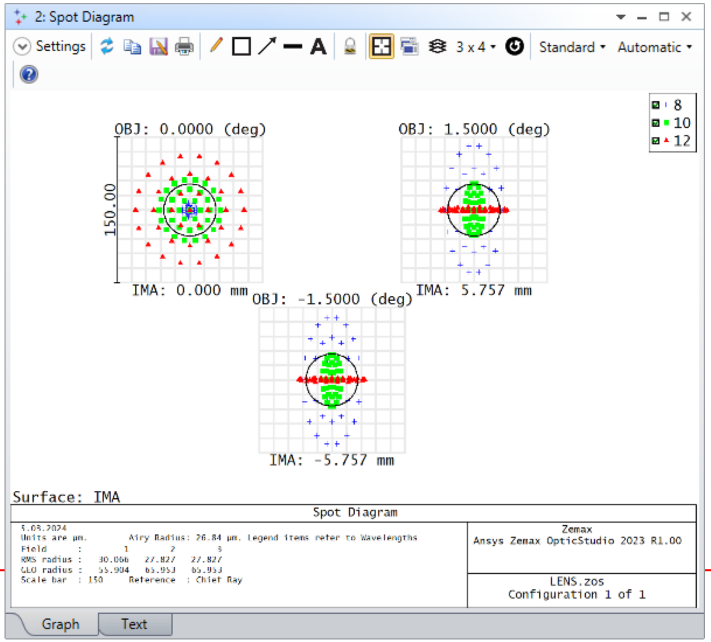
Lens Data

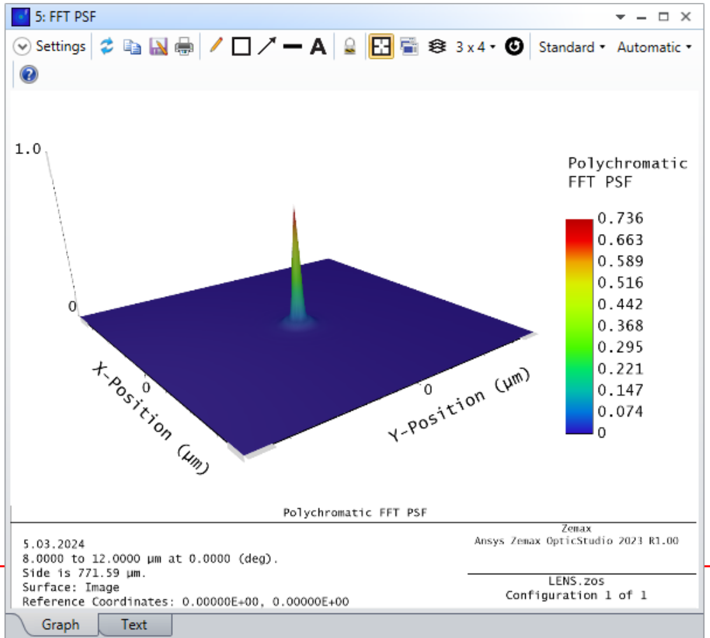
Update: All Windows

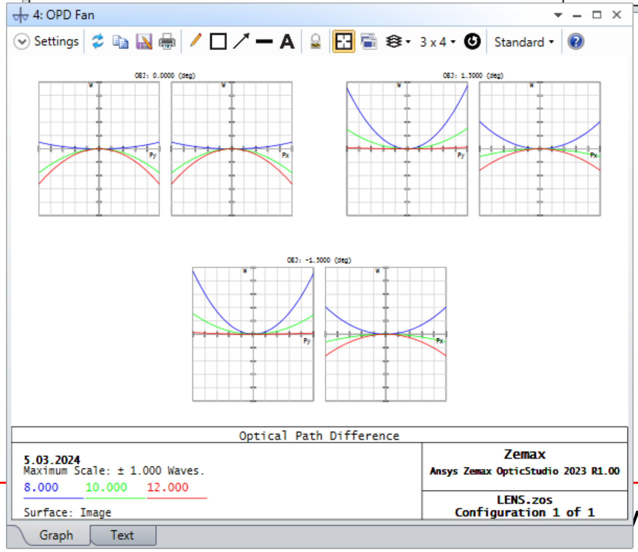
Surface 6 Properties — Configuration 1/1

	Surface Type		Comment	Radius		Thickness		Material	Clear Semi-		Chip Zone	Mech Semi-Dia
0	OBJECT	Standard		Infinity		Infinity			Infinity		0.000	Infinity
1	STOP	Standard		Infinity		0.000			50.000		0.000	50.000
2	(aper)	Standard	L1	256.008	V	8.000	V	GERMANIUM	55.000	U	0.000	55.000
3	(aper)	Standard		332.218	V	20.000	V		55.000	U	0.000	55.000
4	(aper)	Standard	L2	160.337	V	11.782	V	GERMANIUM	55.000	U	0.000	55.000
5	(aper)	Standard		165.820	V	188.901	V		55.000	U	0.000	55.000
6	IMAGE	Standard		Infinity		-			5.824		0.000	5.824









DRI

How far can a camera see an object?
What does it mean to see an object clearly in an electro-optical system (night vision, thermal etc.)?

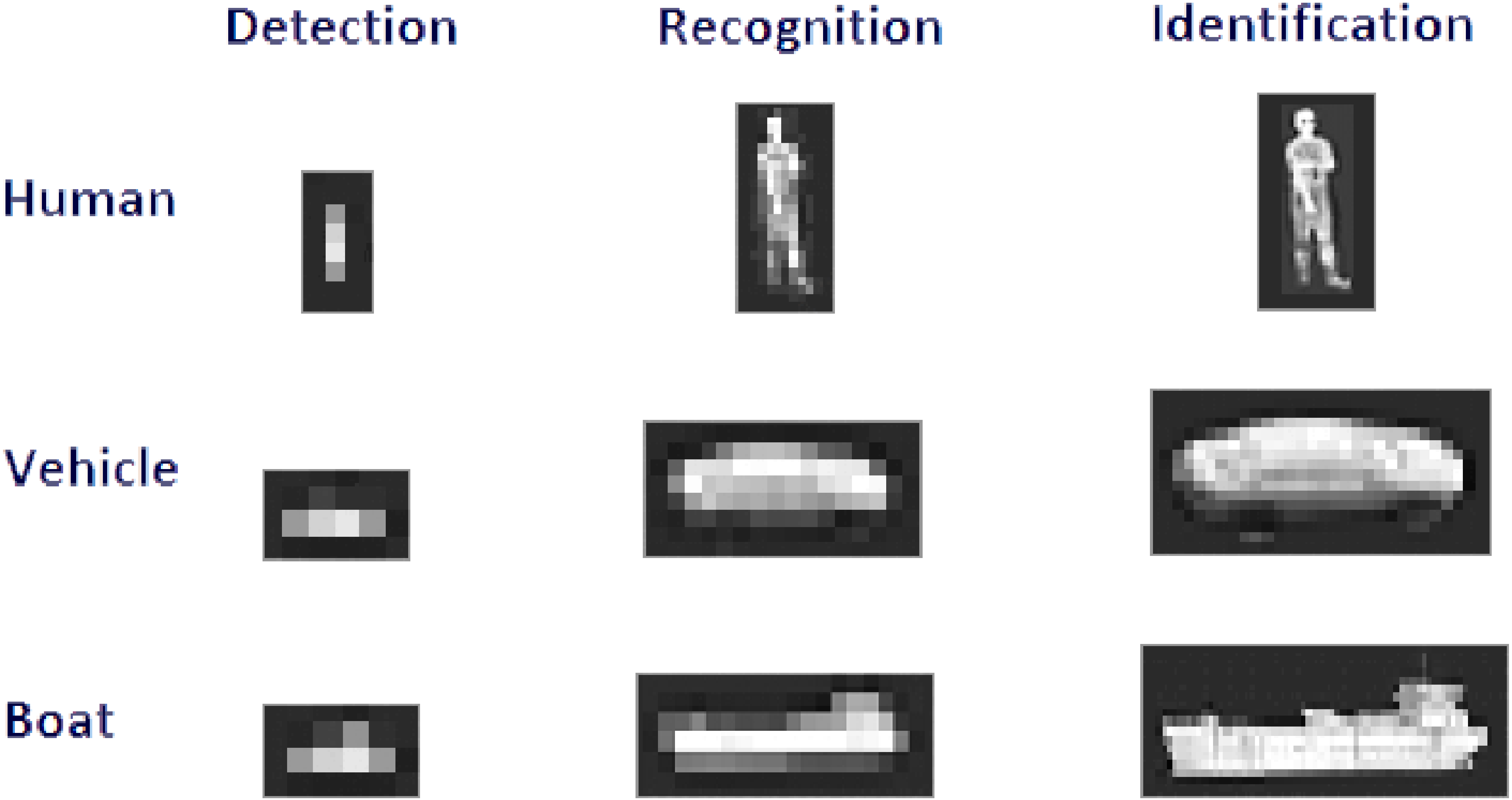
In optical and imaging systems,
DRI stands for **Detection**, **Recognition**, and **Identification**.
These terms represent different levels of information acquisition in surveillance, target acquisition, and other imaging applications.

DRI distance is the universal standard for describing both spatial domain and frequency domain approaches to analyze the ability of observers to perform visual tasks using image intensifier technology.

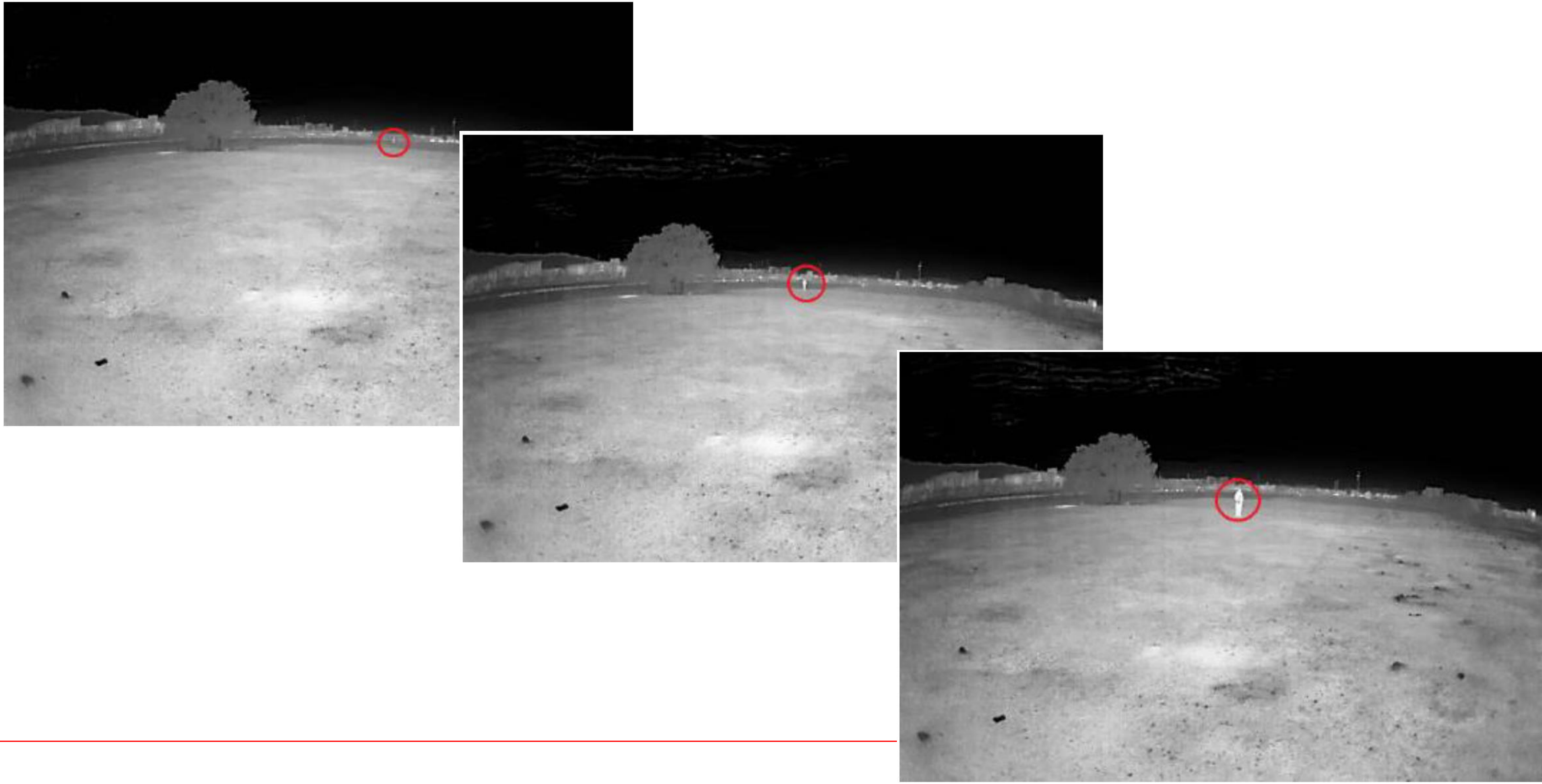
Johnson’s Criteria

The minimum required resolution according to **Johnson's criteria** are expressed in terms of line pairs of image resolution across a target in terms of several tasks:

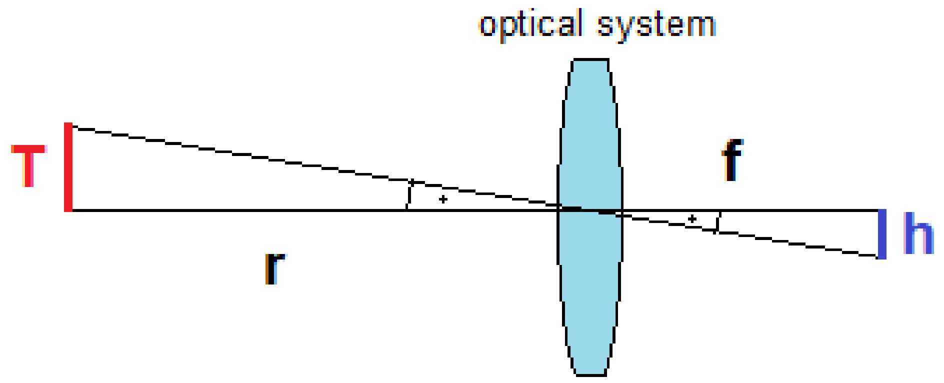
Criteria Name	Description	Number of lines pairs (lp)	# of pixel on on target
Detection	an object is present	**1.0 ± 0.25**	**2**
Recognition	the type object can be discerned, *a person versus a car*	**4.0 ± 0.8**	**8**
Identification	a specific object can be discerned, *a woman versus a man, a specific car*	**6.4 ± 1.5**	**12.8**


Detection
Recognition
Identification
Human
Vehicle
Boat

Detection, Recognition and Identification



DRI Range



Consider an imaging optical system.

Using similar triangles, range of target can be calculated as:

T = *target size*
h = *image size*
r = *target range*
f = *focal length*

$$r = \frac{Tf}{h}$$

In image plane, if we have a CCD sensor with pixel pitch *p*, then image height is $h = N \times p$ where *N* is number pixel on target (see table). Hence, DRI range can be computed as:

$$r = \frac{Tf}{Np}$$

Here DRI range (*r*) is the minimum distance required to see target clearly (with certain DRI value).

Example 3: DRI Calculation

Compute the DRI ranges for the optical system given in Example 2 for the target about 2.5 m wide.

Detection: $$r = \frac{(2500\ \text{mm})(220\ \text{mm})}{(2)(0.050\ \text{mm})} = 5500\ \text{m}$$

Recognition: $$r = \frac{(2500\ \text{mm})(220\ \text{mm})}{(8)(0.050\ \text{mm})} = 1375\ \text{m}$$

Identification: $$r = \frac{(2500\ \text{mm})(220\ \text{mm})}{(12.8)(0.050\ \text{mm})} = 860\ \text{m}$$

Example

Consider human target with dimensions 1.8 m x 0.5 m.

Compute the DRI ranges for each dimensions. Assume f = 100 mm and p = 30 μm.

```
%------------------------------------------------
% dri.m
% all values are in mm
%------------------------------------------------
clear; clc;
T = 1800;  % target size
f = 100;   % focal length
p = 0.03;  % pixel pitch
N = [2, 4, 12.8]; % DRI values, Johnson’s criteria
r = T*f./(N*p)/1000; % in meters

fprintf('Detection     : r = %5d m\n',round(r(1)))
fprintf('Recognition   : r = %5d m\n',round(r(2)))
fprintf('Identification: r = %5d m\n',round(r(3)))
```

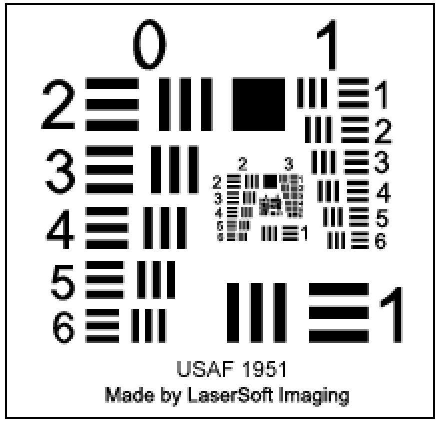
```
For 1.8 m
Detection     : r =  3000 m
Recognition   : r =  1500 m
Identification: r =   469 m

For 0.5 m
Detection     : r =   833 m
Recognition   : r =   417 m
Identification: r =   130 m
```

MTF

The most used metric for characterizing the optical system's performance is the **Modulation Transfer Function** (MTF)*. MTF is a measure of how well a lens relays contrast from object to image.

The spatial frequency (line-pair per millimeter (lp/mm)) is the standard unit of measurement for resolution. A line pair consists of one black line and one white line.



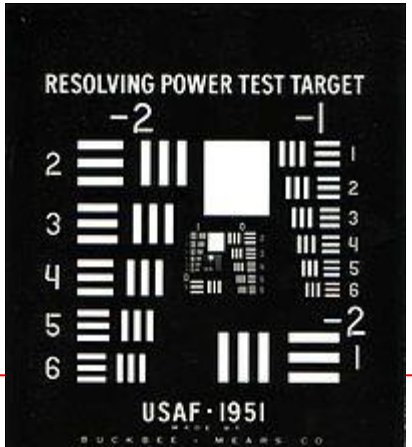
Contrast is difference between a black line and a white line.

The constrast modulation is defined as:

$$M = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

I_{max} = maximum intensity level (of bright value)

I_{min} = minimum intensity level (of dark value)

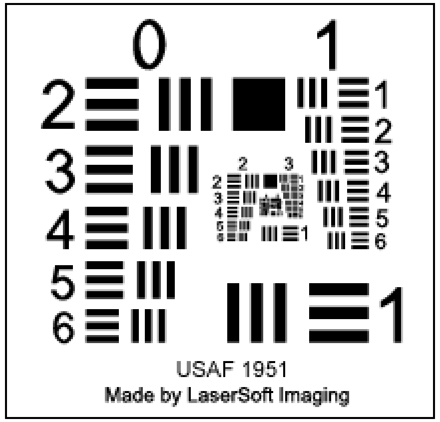


** MTF is the magnitude of the complex optical transfer function (OTF)*

Contrast Transfer

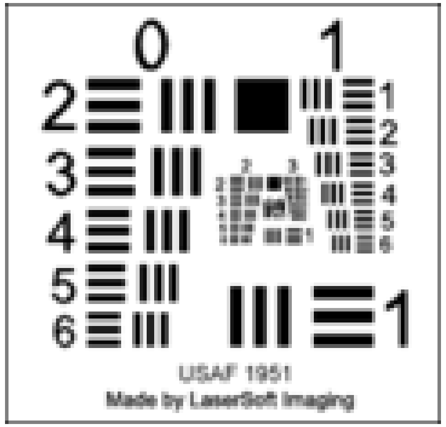
The contrast in object plane can be transferred to image plane by a transfer function called Modulation Transfer Function (MTF).

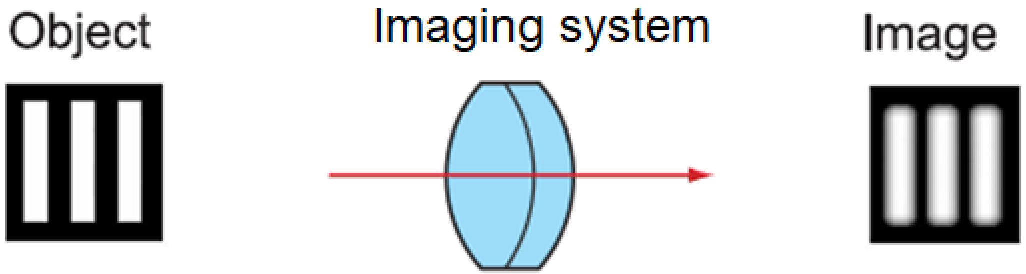
Object (M_{obj})



$$M_{img} = MTF \times M_{obj}$$

Image (M_{img})





Perfect line edges before (left) and after (right) passing through a low resolution imaging lens

Diffraction-limited MTF of a Circular Lens

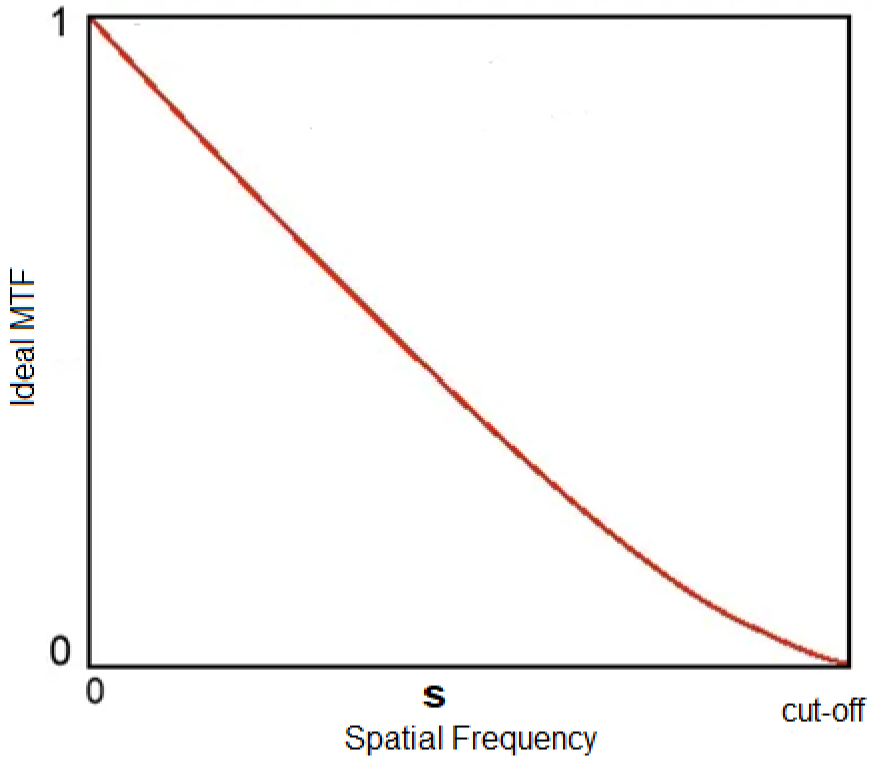
MTF is a function of spatial frequency (*s*):

s = number of lines within a given length.

Usually we used *lines / mm* or *line pairs / mm* unit.

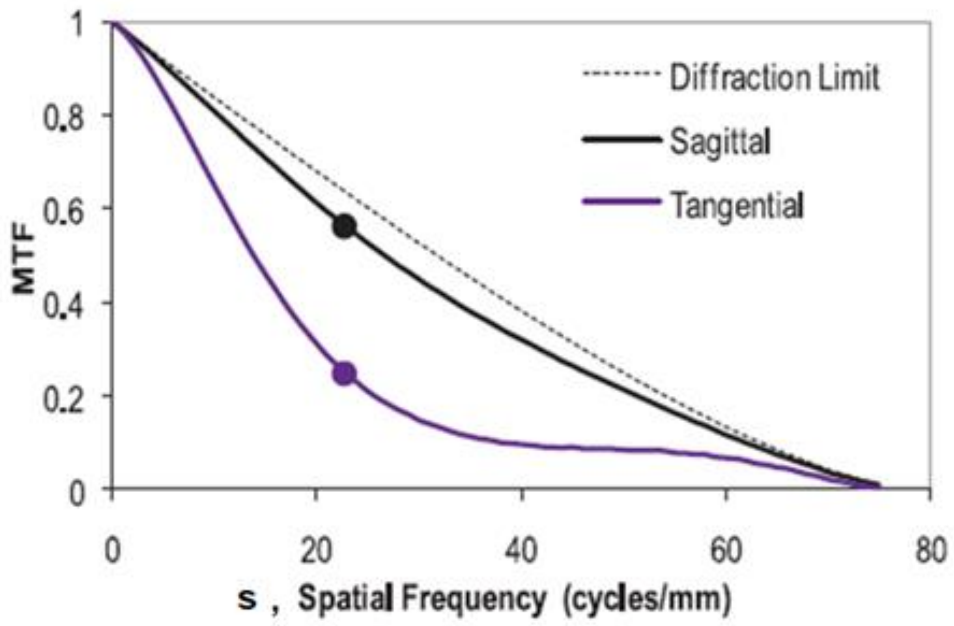
The diffraction-limited incoherent MTF for a lens having a circular pupil is given by:

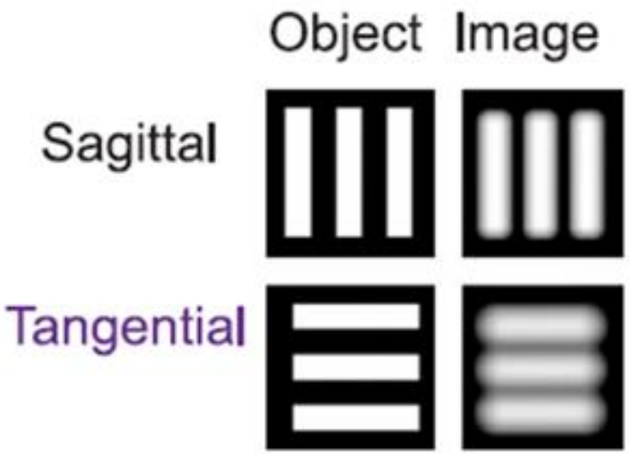
$$MTF(s) = \frac{2}{\pi}\left\{\operatorname{acos}\left(\frac{s}{s_c}\right) - \frac{s}{s_c}\sqrt{1-\left(\frac{s}{s_c}\right)^2}\right\}$$

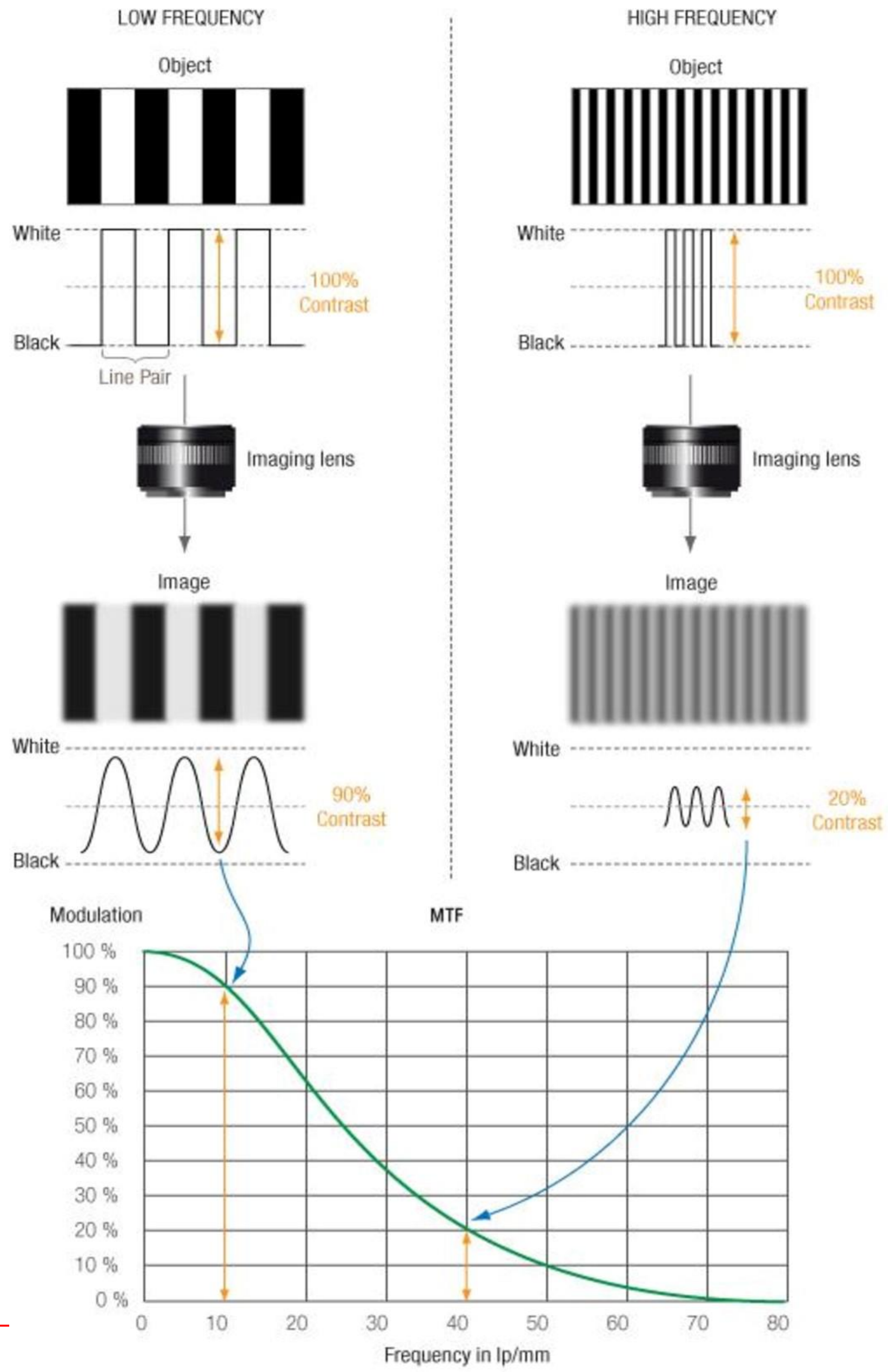


The cut-off resolution can be found by:

$$s_c = \frac{1}{\lambda \times (f/\#)}$$


Diffraction Limit
Sagittal
Tangential
MTF
s , Spatial Frequency (cycles/mm)


Object
Image
Sagittal
Tangential


LOW FREQUENCY
HIGH FREQUENCY
Object
White
Black
100% Contrast
Line Pair
Imaging lens
Image
90% Contrast
20% Contrast
Modulation
MTF
Frequency in lp/mm

Nyquist Frequency

The Nyquist frequency (f_N) is referred to as the **sampling frequency**.
In a digital system f_N is calculated as:

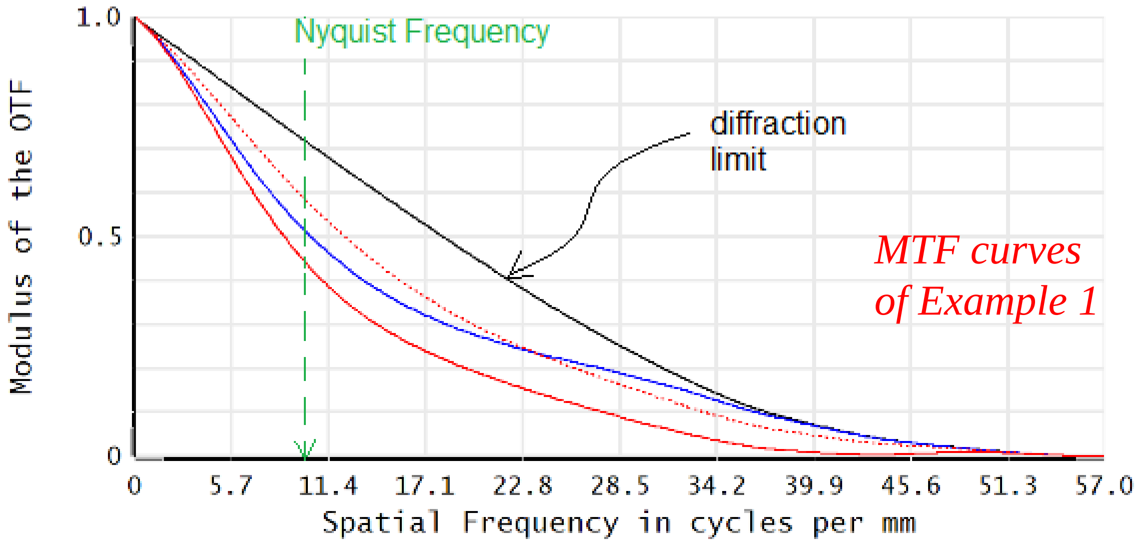
$$f_N = \frac{1}{2p}$$

p = the pixel size in mm of the sensor.

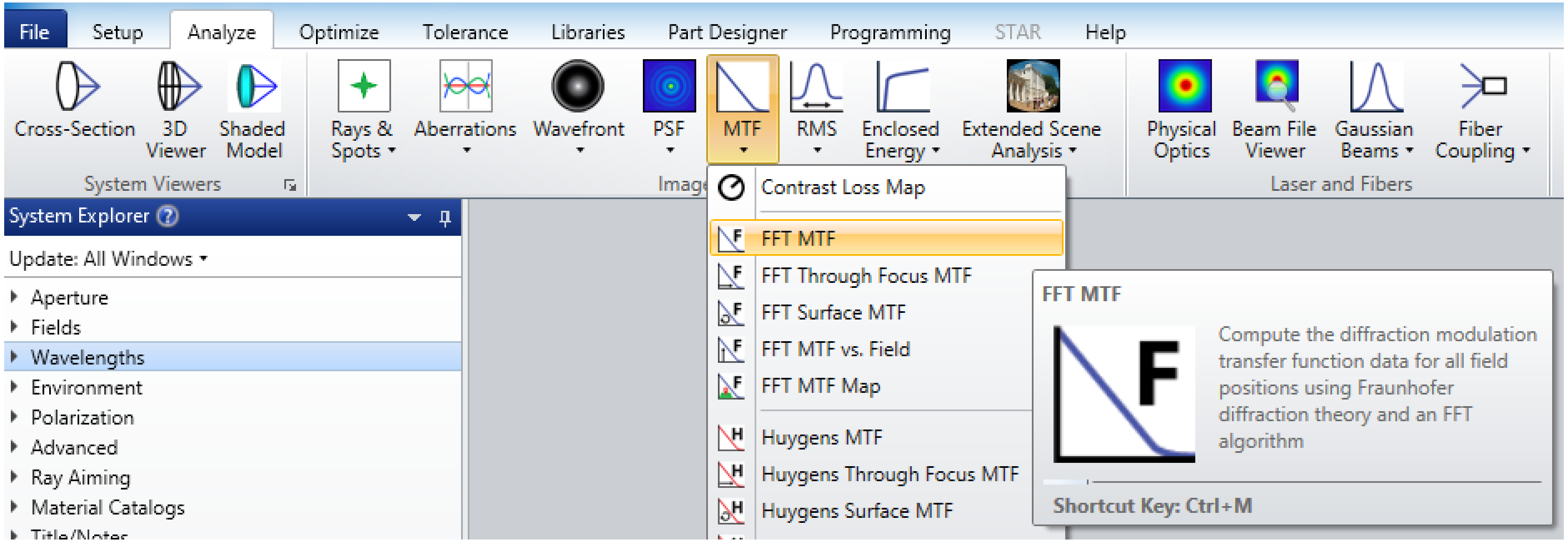
Performance of the MTF is subsequently evaluated using **this value** as a reference parameter.

For the sensor in Example 1,
$p = 50\ \mu\text{m} \Rightarrow f_N = 10\ \text{lp/mm}.$
and

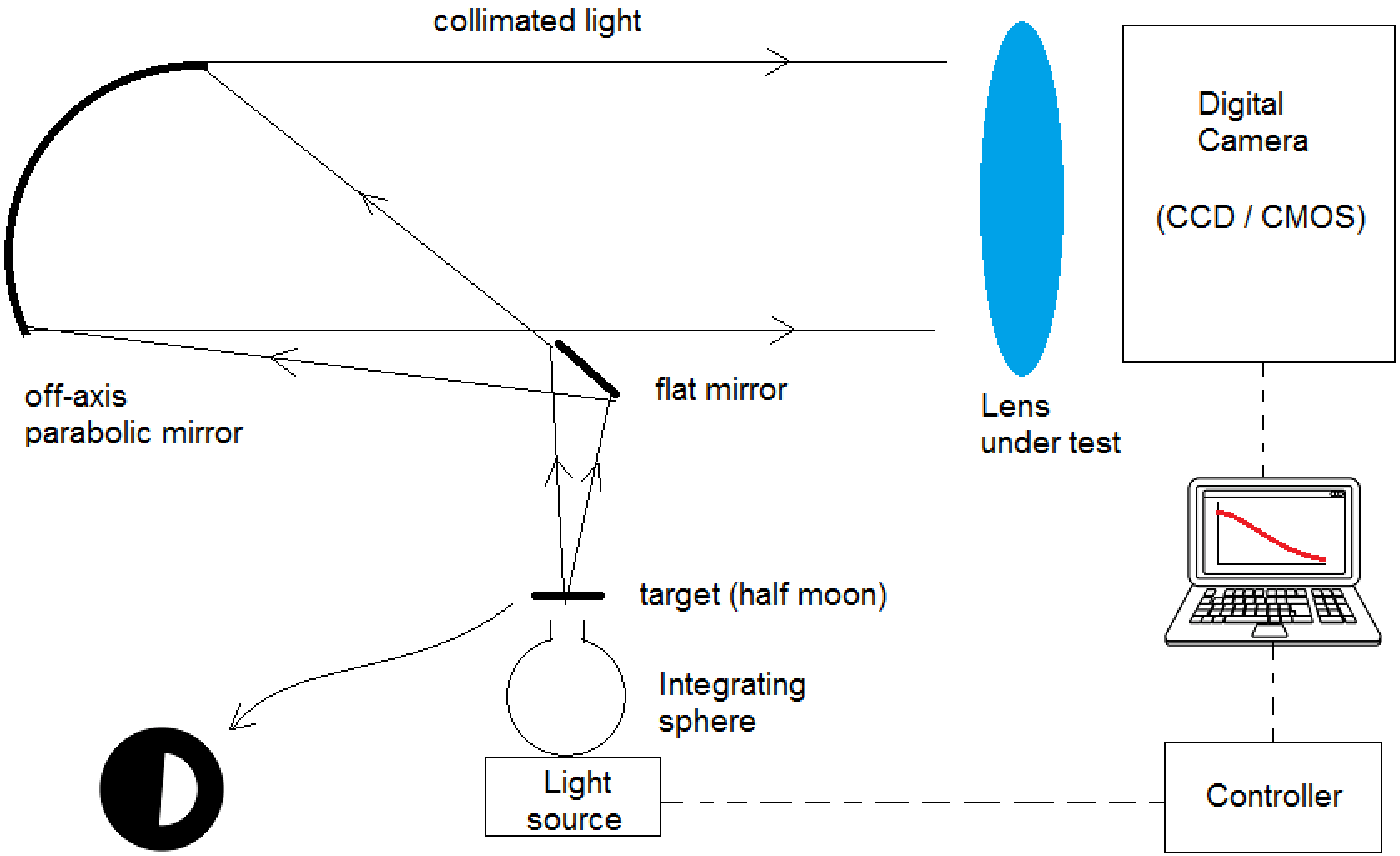
$$\text{cutoff resolution} = \frac{1}{\lambda(f/\#)} = 57\ \text{lp/mm}$$



MTF Analysis in Zemax



Measuring MTF via Slanted Edge

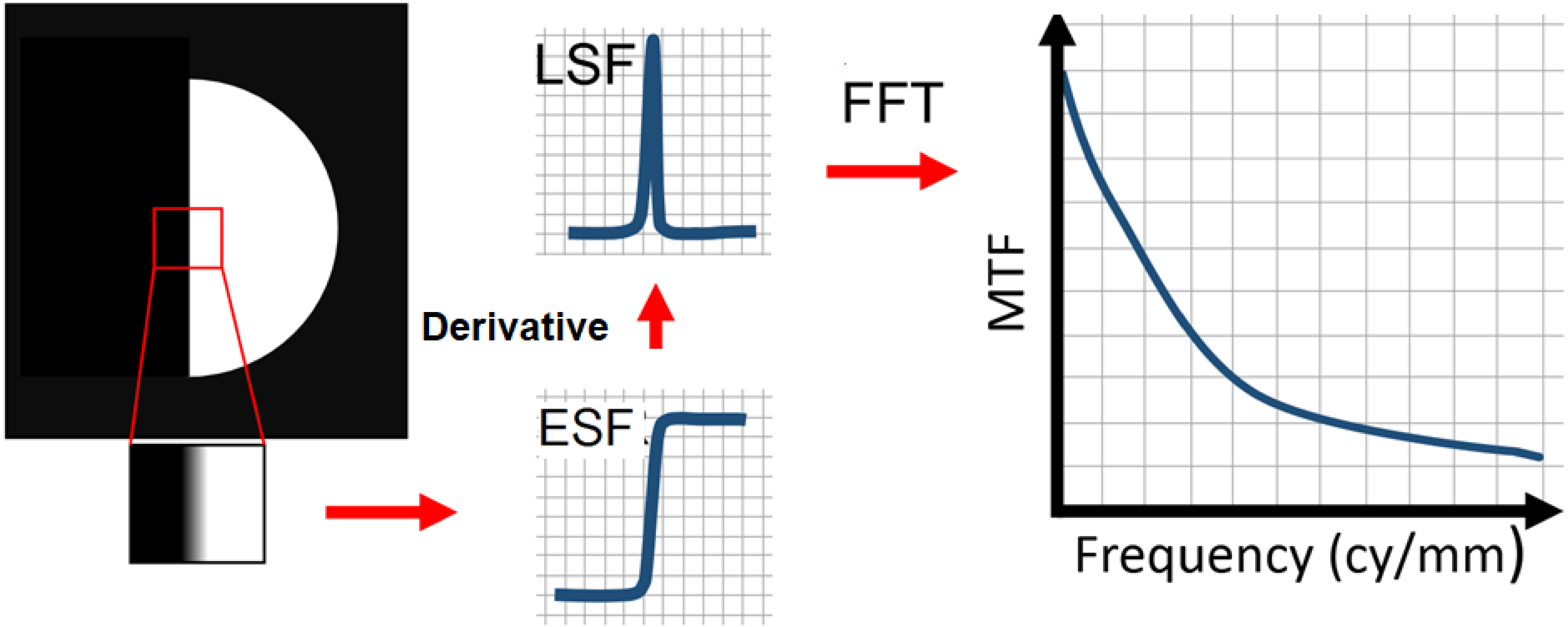


See related scientific paper from web page.

Measuring MTF via Slanted Edge

MTF is calculated from ESF and LSF using FFT.

- **Edge Spread Function** (ESF) is measured using a half-Moon target which is tilted a few degrees. This target gives differences between blackbody and background in a single image. SFR measurement readings is a step-function whose derivative is **Line Spread Function** (LSF).
- LSF is a function of the angle of view which describes the sharpness of the camera.
- MTF comes as a result of the **Fast Fourier Transform** of the LSF.



System MTF

Each factor influencing performance should be reviewed to determine the MTF budget of the optical system.

$$MTF_{system} = MTF_{atmosphere} \times MTF_{optics} \times MTF_{detector} \times \cdots$$