ME 534
Intelligent Manufacturing Systems

Chp 4 – Fuzzy Logic
Fuzzy Logic was initiated by Prof. Lotfi A. Zadeh (an Azerbaijani mathematician, electrical engineer, computer scientist, AI researcher and professor emeritus of computer science at the University of California, Berkeley) in 1965.

Experts rely on common sense when they solve problems. How can we represent this knowledge using vague & ambiguous terms in computer?

It is simply a multi-valued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, black/white, etc.

Notions like rather warm or pretty cold can be formulated mathematically and processed by computers. This way, an attempt is made to apply a more human-like way of thinking in the programming of computers.
More Motivation

The concept of fuzzy logic relies on skills of **human reasoning**.

Therefore, it is all about the **relative importance of precision**:
How important is it to be exactly right when a rough answer will do?

Fuzzy logic is a convenient way to map an input space to an output space:

- With your specification of **how hot you want the water**, a system can **adjust the faucet valve to the right setting**.
- With information about **how far away the subject of your photograph is**, a system can **focus the lens for you**.
- With information about **how fast the car is going and how hard the motor is working**, a system can **shift gears for you**.
- Fuzzy logic is a set of mathematical principles for knowledge representation based on degrees of membership.

- Unlike two-valued Boolean logic, fuzzy logic is multi-valued. It deals with degrees of membership and degrees of truth.

- The continuum of logical values between 0 (completely false) and 1 (completely true) is used. Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time.
The fuzzy inference process is **performed in four steps:**

- **Fuzzification:** definition of fuzzy sets and the degree of membership of crisp inputs in these sets.
- **Inference:** Evaluation of fuzzy rules to produce an output for each rule.
- **Composition:** Aggregation or combination of the outputs of all rules.
- **Defuzzification:** Computation of crisp output.
Fuzzy set allows a continuum of possible choices. Degree of membership (also called membership value) of an element in a fuzzy set is represented by a value between 0 & 1.

For instance, in crisp sets, a man with a height of 184 cm is the member of “average men” set only with a degree of membership of 1.0.

In fuzzy sets, he would be the member of “average men” set with a degree of membership of 0.1 and the member of “tall men” set with a degree of 0.4.

Therefore, the universe of discourse (i.e. the domain) of linguistic variable “height” has the range of 150 - 210 cm with fuzzy subsets of short, average and tall.
Different membership functions can be used based on the type of data.
Fuzzification: Operations in Fuzzy Sets

**Complement**

- Not A
- \( \mu(x) \)
- How much do elements **not** belong to the set?
- NOT A

**Containment**

- A ⊆ B
- \( \mu(x) \)
- How much of which sets **belong to** other sets?
- B HAS A

**Intersection**

- A ∩ B
- \( \mu(x) \)
- How much of the element is **in both** sets?
- A AND B

**Union**

- A ∪ B
- \( \mu(x) \)
- How much of the element is **in either** set?
- A OR B
Rule Base: Classical Rules vs Fuzzy-Expert Rules

- The rules are defined as **IF - THEN** type of conditional statements where \( x \) & \( y \) are linguistic variables while \( A \) & \( B \) are linguistic values defined by fuzzy sets with the universe of discourses of \( x \) & \( y \):

\[
\text{IF } x \text{ is } A \quad [\text{antecedent}]
\]
\[
\text{THEN } y \text{ is } B \quad [\text{consequent}]
\]

- **Classical IF - THEN rules** use binary (Boolean) logic:

\[
\begin{align*}
\text{IF} & \quad \text{travelling\_speed} \text{ is } > 100 \text{ mph} \\
\text{THEN} & \quad \text{stopping\_distance} \text{ is } \text{long}
\end{align*}
\]

\[
\begin{align*}
\text{IF} & \quad \text{travelling\_speed} \text{ is } < 40 \text{ mph} \\
\text{THEN} & \quad \text{stopping\_distance} \text{ is } \text{short}
\end{align*}
\]

- **In fuzzy-expert rules**, the linguistic variable (travelling speed) also has a range (universe of discourse) based on fuzzy sets:

\[
\begin{align*}
\text{IF} & \quad \text{travelling\_speed} \text{ is } \text{fast} \\
\text{THEN} & \quad \text{stopping\_distance} \text{ is } \text{long}
\end{align*}
\]

\[
\begin{align*}
\text{IF} & \quad \text{travelling\_speed} \text{ is } \text{slow} \\
\text{THEN} & \quad \text{stopping\_distance} \text{ is } \text{short}
\end{align*}
\]

- There can be multiple antecedents and consequents connected with operations of **AND, OR, NOT**.
Among various reasoning mechanisms in fuzzy inference, two commonly used ones are:

- **Mamdani**: Developed by Prof. Ebrahim Mamdani, this method determines the centroid of a 2D shape by integrating across a continuously varying function.

- **Sugeno**: Suggested by Michio Sugeno, this method uses a single spike (a singleton) as MF of rule consequent. A singleton is a fuzzy set with a membership degree of unity at a single particular point on the universe of discourse and zero everywhere else.

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Which one shall we use: Mamdani or Sugeno?

- **Mamdani method** is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, this method is not computationally efficient.

- **Sugeno method** is computationally effective and works well with optimization & adaptive techniques, which makes it very attractive in applications of system control particularly for dynamic nonlinear systems.
An Example: Air Conditioner

Air-conditioning involves the delivery of air which can be warmed or cooled and have its humidity raised or lowered. An air-conditioner typically has a fan that blows/cools/circulates the fresh air via a cooler under thermostatic control. Generally speaking, the amount of air being compressed is proportional to the ambient temperature.

Our air conditioner has five temperature levels: COLD, COOL, PLEASANT, WARM, HOT.

There are also five corresponding levels for speed of motor controlling the fan: MINIMAL, SLOW, MEDIUM, FAST, BLAST.

Therefore, the fuzzy-expert rules are constructed as follows:

<table>
<thead>
<tr>
<th>Rule</th>
<th>IF</th>
<th>THEN</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1</td>
<td>Temperature is COLD</td>
<td>Speed is MINIMAL</td>
<td></td>
</tr>
<tr>
<td>Rule 2</td>
<td>Temperature is COOL</td>
<td>Speed is SLOW</td>
<td></td>
</tr>
<tr>
<td>Rule 3</td>
<td>Temperature is PLEASANT</td>
<td>Speed is MEDIUM</td>
<td></td>
</tr>
<tr>
<td>Rule 4</td>
<td>Temperature is WARM</td>
<td>Speed is FAST</td>
<td></td>
</tr>
<tr>
<td>Rule 5</td>
<td>Temperature is HOT</td>
<td>Speed is BLAST</td>
<td></td>
</tr>
</tbody>
</table>
The temperature graduations are related to our perception of ambient temperatures:

- **Y**: Value belongs to the set \((0 < \mu < 1)\)
- **Y***: Value is ideal member of the set \((\mu = 1)\)
- **N**: Value is not a member of the set \((\mu = 0)\)

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>COLD</th>
<th>COOL</th>
<th>PLEASANT</th>
<th>WARM</th>
<th>HOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Y*</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>5</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>12.5</td>
<td>N</td>
<td>Y*</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>15</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>17.5</td>
<td>N</td>
<td>N</td>
<td>Y*</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>20</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>22.5</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y*</td>
<td>N</td>
</tr>
<tr>
<td>25</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>27.5</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>30</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y*</td>
</tr>
</tbody>
</table>
Similarly, perceptions of **speed** of motor are:

<table>
<thead>
<tr>
<th>Y : Value belongs to the set ( 0 &lt; μ &lt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y* : Value is ideal member of the set ( μ = 1 )</td>
</tr>
<tr>
<td>N : Value is not a member of the set ( μ = 0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>MINIMAL</th>
<th>SLOW</th>
<th>MEDIUM</th>
<th>FAST</th>
<th>BLAST</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Y*</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>20</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>30</td>
<td>N</td>
<td>Y*</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>40</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>50</td>
<td>N</td>
<td>N</td>
<td>Y*</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>60</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>70</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y*</td>
<td>N</td>
</tr>
<tr>
<td>80</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>90</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>100</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y*</td>
</tr>
</tbody>
</table>
Air Conditioner: Creating Membership Functions

- **Triangular MFs (trimf)** for the fuzzy subsets of temperature are analytically calculated based on a linear trendline (i.e. \( y = ax + b \)). For instance, such calculations for COLD and COOL are as follows:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Membership Function</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COLD</strong></td>
<td>( \mu_{\text{COLD}}(T) = -\frac{T}{10} + 1 ) (for ( 0 \leq T \leq 10 ))</td>
</tr>
<tr>
<td></td>
<td>( \mu_{\text{COOL}}(T) = -\frac{T}{12.5} ) (for ( 0 \leq T \leq 12.5 ))</td>
</tr>
<tr>
<td></td>
<td>( \mu_{\text{COOL}}(T) = -\frac{T}{5} + 3.5 ) (for ( 12.5 \leq T \leq 17.5 ))</td>
</tr>
</tbody>
</table>

- Therefore, MFs for all fuzzy subsets of temperature can be plotted as seen below:
Similarly, **triangular MFs** for the fuzzy subsets of **speed** are also calculated based on linear trendline. For instance, such calculations for MINIMAL and SLOW are as follows:

<table>
<thead>
<tr>
<th>MINIMAL</th>
<th>SLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{MINIMAL}}(V) = -V/30 + 1$ (for $0 \leq V \leq 30$)</td>
<td>$\mu_{\text{SLOW}}(V) = V/20 - 0.5$ (for $10 \leq T \leq 30$)</td>
</tr>
<tr>
<td></td>
<td>$\mu_{\text{SLOW}}(V) = -V/20 + 2.5$ (for $30 \leq T \leq 50$)</td>
</tr>
</tbody>
</table>

Then, MFs for all fuzzy subsets of speed are plotted as seen below:
Let's compute the optimal fan speed for temperature of 16 °C.

Affected fuzzy sets are COOL and PLEASANT.

During fuzzification process, membership values (μ) are calculated accordingly:

\[ \mu_{COOL}(T) = -\frac{T}{5} + 3.5 = -\frac{16}{5} + 3.5 = 0.3 \]

\[ \mu_{PLEASANT}(T) = -\frac{T}{2.5} - 6 = -\frac{16}{2.5} - 6 = 0.4 \]

At inference stage, Rule 2 and Rule 3 are fired. Hence, the affected fuzzy sets for the output variable (speed) are SLOW and MEDIUM:

Rule 2: IF Temperature is COOL THEN Speed is SLOW
Rule 3: IF Temperature is PLEASANT THEN Speed is MEDIUM
Air Conditioner: Mamdani Inference

- Mamdani inference defines the areas in consequent membership functions:

**Rule 2:** IF Temperature is COOL (0.3) THEN Speed is SLOW (0.3)

**Rule 3:** IF Temperature is PLEASANT (0.4) THEN Speed is MEDIUM (0.4)
The next stage is aggregation of rule outputs (i.e. composition). This is the process of unification of the outputs of all rules. Thus, MFs of all rule consequents are combined into a single fuzzy set.

The last step is the defuzzification, which gives a crisp number as the final output. The most popular defuzzification technique is the centroid method, which finds centre of gravity (COG).

Thus, the resulting speed value based on Mamdani inference:

\[
S = \frac{0.125 \ (12.5) + 0.25 \ (15) + 0.3 \ (17.5 + 20 + \ldots + 40 + 42.5) + 0.4 \ (45 + 47.5 + \ldots + 52.5 + 55) + 0.25 \ (57.5)}{0.125 + 0.25 + 0.3 \ (11) + 0.4 \ (5) + 0.25} = 45.54 \text{ rpm}
\]
Air Conditioner: Sugeno Inference & Defuzzification

- Sugeno inference calculates **the exact values** in consequent MFs:

  \[ \mu_{\text{SLOW}}(S) = -0.05S + 2.5 \text{ where } \mu_{\text{SLOW}}(S) = 0.3, \text{ hence } S_{\text{SLOW}} = 44 \text{ rpm} \]

  \[ \mu_{\text{MEDIUM}}(S) = 0.1S - 4 \text{ where } \mu_{\text{MEDIUM}}(S) = 0.4, \text{ hence } S_{\text{MEDIUM}} = 44 \text{ rpm} \]

- Thus, composition and defuzzification using Sugano inference gives a **crisp value of speed**:

  \[ S = \frac{0.3(44) + 0.4(44)}{0.3 + 0.4} = 44 \text{ rpm} \]
Why shall we use Fuzzy Logic?

- **It is conceptually easy to understand:** The mathematical concepts behind fuzzy reasoning are very simple. Fuzzy logic is a more intuitive approach without the far-reaching complexity.

- **It is flexible:** With any given system, it is easy to add more functions without starting from scratch.

- **It is tolerant of imprecise data:** Everything is imprecise if you look closely enough. More than that, most things are imprecise even on careful inspection. Fuzzy reasoning builds this understanding into the process rather than tacking it onto the end.

- **It can be built on top of the experience of experts:** In direct contrast to neural networks which take training data and generate opaque, impenetrable models; fuzzy logic lets you rely on the experience of people who already understand your system.

- **It can be blended with conventional control techniques:** Fuzzy systems do not necessarily replace conventional control methods. In many cases, fuzzy systems simplify their implementation.

- **It is based on natural language:** The basis for fuzzy logic is the basis for human communication. This observation underpins many of the other statements about fuzzy logic. Since fuzzy logic is built on the structures of qualitative description used in everyday language, it is easy to use.
Fuzzy logic is **not a cure-all**. It is a convenient way to map an input space to an output space. If you find it is not convenient, try something else. If a simpler solution already exists, use it!

Fuzzy logic is **the codification of common sense**. Use this common sense, and you will probably make the right decision. Many controllers, for instance, do a fine job without using fuzzy logic.

If you take the time to become familiar with fuzzy logic, you will see that it can be a very powerful tool to deal with imprecision and nonlinearity in a quick and efficient way. Otherwise, for straight-forward linear problems, try using simple analytical solutions.