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Dynamic optimization of multipass milling operations via geometric programming

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Abstract

This paper outlines the development of an optimization strategy to determine the optimum cutting parameters for multipass milling operations like plain milling and face milling. The developed strategy is based on the "maximum production rate" criterion and incorporates eight technological constraints. The optimum number of passes is determined via dynamic programming, and the optimal values of the cutting conditions are found based on the objective function developed for the typified criterion by using a non-linear programming technique called "geometric programming". This paper also underlies the importance of using optimization strategies rather than handbook recommendations as well as pointing out the superiority of the multipass over the single-pass optimization approach. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Multipass operations; Milling; Dynamic programming; Geometric programming; Constrained optimization

Nomenclature

a	Depth of cut for its pass (mm)
a_{max}	Maximum depth cut for machine tool workpiece system
a_{min}	Minimum depth cut for machine tool workpiece system
a_T	Total depth of cut (mm)
b_v, b_z	Exponents determined empirically
B	Milling width (mm)

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B_m	Correction coefficient of tool life equation
B_t	Correction coefficient of tool life equation
B_h	Correction coefficient of tool life equation
B_p	Correction coefficient of tool life equation
C_v	A constant taking into account the influence of all factors that are appearing separately in the tool life formula
C_{zp}	Constant of the cutting force equation
d_{a}	Arbor diameter (mm)
D	Outer diameter of the cutter (mm)
e	Permissible values of arbor deflection (mm)
e_v, e_z	Exponents determined empirically
E	Modulus of elasticity of arbor material (kg/mm^2)
E_{s}	Modulus of elasticity of stub arbor material (MPa)
f	Feed rate (mm/min)
f_z	Feed per tooth (mm/tooth)
F_{c}	Mean peripheral cutting force (kg)
F_{d}	Permissible force with regard to arbor deflection (kg)
F_{s}	Permissible force with regard to arbor strength (kg)
I_{s}	Moment of inertia of stub arbor (mm^4)
k_{b}	Permissible bending stress of the arbor material (kg/mm^2)
k_{t}	Permissible torsional stress of the arbor material (kg/mm^2)
L	Length of cut (mm)
L_{a}	Arbor length between supports (mm)
L_{s}	Length of stub arbor (mm)
m	Exponent determined empirically
n_v, n_z	Exponents determined empirically
N_{b}	Total number of components in the batch
N	Spindle speed (rpm)
$\mathrm{N_p}$	Number of passes
N	Number of sections
q, q_v	Exponents determined empirically
P	Exponent determined empirically
P_{c}	Cutting power (kW)
P_{m}	Nominal motor power (kW)
r_v, r_z	Exponents determined empirically
T	Tool life (min)
T_{p}	Machine preparation time per component (min)
T_{s}	Set up time of the machine for a new batch (min)
T_{L}	Loading and unloading time (min)
T_{a}	Process adjusting time and quick return time
T_{c}	Tool changing time per component (min)
T_{d}	Time for changing a dull cutting edge or tool (min)
T_{m}	Machining time (min)
T_{pr}	Total production time per component (min)

u_v	Exponent determined empirically
u_z	Exponent determined empirically
V	Cutting speed (m/min)
z	Number of teeth on the cutter
η	Overall efficiency
δ	Permissible deflection of stub arbor at the end (mm)
λ_s	Cutting inclination angle

1. Introduction

Multipass operations are generally used to machine stocks that cannot be removed in a single pass. Some turning operations like external step turning and boring, and some of the milling operations, such as face milling and deep shoulder milling in which a significant amount of stock material is removed, are good examples of the operations which are commonly required to be machined using multipass operations. Determination of the optimal cutting parameters (cutting conditions) like the number of passes, depth of cut for each pass, speed, and feed is considered as a crucial stage of multipass machining as in the case of all chip removal processes and especially in process planning. The effective optimization of these parameters affects dramatically the cost and production time of machined components as well as the quality of the final products.

Although in the early 1900s, Taylor [1] recognised that an optimum value for the speed can be achieved by maximizing the material removal rate in a single pass operation, the progress in developing optimization strategies has been very slow. Indeed, there have not been many studies on the optimization of machining conditions in the literature [2,3]. This is mainly due to the complex nature of optimization of machining operations that require the following,

- Knowledge of machining (i.e., turning or milling);
- Empirical equations relating the tool life, forces, power, surface finish and arbor deflection, etc., to develop realistic constraints;
- Specification of machine tool capabilities, (i.e., maximum power or maximum feed available from a machine tool);
- Development of an effective optimization criterion, (e.g., maximum production rate, minimum production cost, maximum profit or a combination of these);
- Knowledge of mathematical and numerical optimization techniques, like the Simplex method, Search method, Geometric programming and dynamic programming, etc.

The studies on the subject were first initiated in the early 1970s, and were generally trying to answer the question of whether the multipass scheme is superior to the single pass or not? After the published studies showed that the multipass optimization strategies can yield superior machining parameters than single-pass optimization techniques, in the 1980s this time, the multipass strategies had to prove their significant and higher efficiency over the handbook recommendations, which include rough machining performance data and machine tool specifications. Recent optimization techniques have also been shown to be more and more optimal than the handbook recommendations which can only be considered as rough and initial values.

The literature in the area of the optimization of multipass machining operations has therefore been quite limited. Crookall and Venkataramani [4], Kals et al., [5] and Lambert and Walvekar [6] have analysed the optimization of multipass turning operations. However, they have all selected the number of passes arbitrarily, thus their results could not be considered as the optimal values. Hitomi [7–9] has optimized the optimum cutting speeds and the cycle time without considering the other cutting parameters in his first three attempts. In his later publication Hitomi [10], he applied an approximation in order to find the optimum values of the cutting speed and feed. Rao and Hati [11] as well as Iwata et al. [12], have also developed an optimization model for the turning based on either minimum production time or minimum production cost. Devor et al. [13], and Ermer and Kromodihardjo [14], have investigated the optimization of a two-pass operation using the “minimum production time” criterion. However, their methods attempted to optimize the cutting conditions through a trial and error method, and some non-flexible assumptions like using equal passes for the machining, although optimal values can be achieved by using unequal cutting conditions. Agapiou [3] has developed a multipass optimization system for turning processes in which the number of passes have been found by dynamic programming technique, and then the cutting conditions are optimized through the Nelder–Mead Simplex Search method by using a combined objective function.

It has also been recognized that the progress in developing constrained optimization systems for milling operations has been even slower than for turning operations, since the milling has a more complex cutting mechanism than that of turning. Today, there are only a few works on the optimization of multipass milling operations cited in the literature. There are currently two approaches to solve the problem:

- Using computer aided mathematical programming techniques, and
- Using numerical search techniques.

Recently, Wang [15] has developed an optimization software for multipass peripheral and end milling operations which use a combination of the above two considerations based on the objective function of “maximum production rate”. He has also verified the superiority of multipass over the singlepass by carrying out some simulation tests. In this paper, the development of a constrained optimization system for multipass face milling operations is outlined. The optimum number of passes is first determined via dynamic programming, and then the optimal values of the cutting parameters are found based on the objective function “maximum production rate” and using a non-linear programming technique “geometric programming”.

The algorithm used in this study is adopted from the study of Agapiou [3] which is proposed for the multipass turning operations. However, the methodology used in this work is rather different because the geometric programming is preferred in lieu of the Nelder–Mead Simplex Search method in the optimization of each stage of the dynamic programming. Also, it is well worth pointing out that the cutting mechanism and constraints to the face milling problem are quite different from those of turning processes. An example is presented to illustrate the procedure. The factors that effect the efficiency of the method are also discussed.

2. Formulation of objective function

Production time for a component is the total time required to produce a component and is composed of following items:

(i) Machine preparation time, T_p (min)

$$T_p = \frac{T_s}{N_b} \tag{1}$$

where T_p = machine preparation time per component (min), T_s = set up time of the machine for a new batch (min), and N_b = total number of components in the batch;

(ii) Loading and unloading time, T_L, (min);

(iii) Process adjusting and quick return time, T_a (min);

(iv) Machining time, T_m (min); and

(v) Tool changing time per component, T_c (min):

$$T_c = \frac{T_d \cdot T_m}{T} \tag{2}$$

where T_c = tool changing time per component (min), T_d = time for changing a dull cutting edge or tool (min), and T = tool life (min).

Total production time per component for a single pass operation T_{pr}, is the sum of the above time elements and can be written as:

$$T_{pr} = T_p + T_L + T_a + T_m + T_c \tag{3}$$

or

$$T_{pr} = \frac{T_s}{N_b} + T_L + T_a + T_m + T_d\left(\frac{T_m}{T}\right). \tag{4}$$

For a multi-pass operation in which N_p passes are required to remove the total depth of cut, T_{pr} becomes as follows;

$$T_{pr} = \frac{T_s}{N_b} + T_L + \sum_{i=1}^{N_p}\left(T_{a_i} + T_{m_i} + T_d \frac{T_{m_i}}{T_i}\right) \tag{5}$$

where, N_p is the total number of passes and subscript i denotes the ith pass. Set up time T_s, loading and unloading time T_L, process adjusting and quick return time T_a, and tool (or cutting edge) changing time T_d, can be obtained from the standard time tables prepared for standard processes, or determined directly measuring the required time for the related process. However, machining time T_m and tool life T depend on the cutting conditions and should be calculated. Machining time and tool life can be expressed in terms of cutting parameters. For a particular milling operation, the machining time is given as:

$$T_m = \frac{L}{f} \tag{6}$$

where L = length of cut (mm) and f = feed rate (mm/min).

The feed is specified as below:

$$f = f_z \cdot z \cdot N \tag{7}$$

where f_z = feed per tooth (mm/tooth), z = number of teeth on the cutter, N = spindle speed (rpm), and N is given as

$$N = \frac{1000 \cdot V}{\pi D} \tag{8}$$

where V = cutting speed (m/min) and D = outer diameter of cutter (mm).

For a particular milling operation, the tool life formula given by Kaczmarek [16] can be safely used:

$$T = \frac{C_v^{1/m} D^{\frac{b_v}{m}}}{V^{1/m} a^{e_v/m} f_z^{u_v/m} B^{r_v/m} z^{n_v/m} \lambda_s^{q_v/m}} (B_m B_h B_p B_t)^{\frac{1}{m}} \tag{9}$$

where T = tool life (min), C_v = a constant taking into account the influence of all factors, a = depth of cut (min), V = cutting speed (m/min), D = outer diameter of the cutter (mm), f_z = feed per tooth (mm/tooth), B = milling width (min), z = number of teeth on the cutter, λ_s = cutting inclination angle (degrees), $b_v,m,e_v,u_v,r_v,n_v,q_v$ = exponents, and B_m,B_h,B_p,B_t = correction coefficients.

The influence of cutting speed on tool life largely depends on the value of exponent m. It follows from experiments that for high speed steel teeth m = 0.15–0.33 and for sintered carbide teeth m = 0.2–0.6. These values indicate that cutting speed has a stronger influence on the tooth life of high speed steel than on that of sintered carbide teeth. Average values of the constant C_v, and of exponents b_v,m,e_v,u_v,r_v, and n_v, for plain and face milling can be found in Zümrüt [17] for milling steel and cast iron with high speed steel and sintered carbide teeth. Influence of the helix angle of cutters with helical teeth or of the cutting edge inclination angle within practically applied limits is quite small. However, it can be observed that tool life shortens with increasing angle λ_s, because then the number of teeth cutting simultaneously increase. At the same time, although the cutting edges lengthen, the general amount of cutting teeth increase and tooth temperature rises. You can consider that in cutters with helical teeth the angle λ_s = 20–45°, the influence of the cutting edge inclination angle may be neglected. Values of the correction factor B_m taking into account the type of workpiece material are also given in [17]. For the same type of material, increased hardness or tensile strength causes a decrease of tool life for the same cutting conditions. This may be corrected by using a correction factor B_h. For ductile materials it is equal to:

$$B_h = \left(\frac{\sigma_1}{\sigma_2}\right)^p \tag{10}$$

where σ_1 = tensile strength of reference material (MPa), σ_2 = tensile strength of the material being machined (MPa), and p = exponent for various materials.The same relationship for cast iron and other brittle metals can also be expressed by:

$$B_h = \left(\frac{H_{B_1}}{H_{B_2}}\right)^q \tag{11}$$

where H_{B_1} = Brinell hardness number of the reference material, H_{B_2} = Brinell hardness number of the material being machined, and q = exponent different for the various materials.

Tool life is also affected by the production engineering methods used in the preparation of semi-finished products (casting, forging, rolling etc.), the condition of the material and of its surface layer. The influence of these factors is expressed by the correction factor B_p which can be found in Arshinov and Aleksiev [18] and Zümrüt [17]. The kind of machining is taken into account by the correction factor B_t (for rough milling $B_t = 1.0$, for accurate milling $B_t = 0.8$).

By substituting Eq. (6),Eq. (7),Eq. (8),Eq. (9) into Eq. (5), the objective function in multipass milling operations can be expressed by the following:

$$T_{\mathrm{pr}} = \frac{T_{\mathrm{s}}}{N_{\mathrm{b}}} + T_{\mathrm{L}} + \mathrm{N}_p T_{\mathrm{a}} + \sum_{i=1}^{\mathrm{N}_p} \left(\frac{\pi D L}{f_{z_i} z 1000 V_i} + \frac{T_{\mathrm{d}} \pi L V_i^{1/m-1} a_i^{e_v/m} f_{z_i}^{u_v/m-1} B^{r_v/m} z^{n_v/m-1} \lambda_s^{q_v/m}}{1000 C_v^{1/m} D^{b_v/m-1} (B_m B_h B_p B_t)^{1/m}} \right). \tag{12}$$

3. Evaluation of constraints

Optimal cutting conditions should satisfy some technological constraints. Machine tool, cutting tool and workpiece specifications are the sources of these restrictions. These constraints can be determined experimentally for a given workpiece as a function of tool material and geometry, etc. Otherwise, the constraints related to speed, feed and depth of cut for the particular tool workpiece combination should be used in order to proceed with optimization of cutting parameters. Below are constraints considered in this work.

3.1. Available feed and speeds

Optimum feed and cutting speed (or spindle speed) must be in the range determined by the minimum and maximum feed rates and spindle speeds of the machine:

$$f_{z_{\min}} \le f_z \le f_{z_{\max}} = \frac{f_{\max}}{z N_{\max}}, \tag{13}$$

$$V_{\min} \le V \le V_{\max} = \frac{\pi D N_{\max}}{1000} \tag{14}$$

where $f_{z_{\min}}$ and $f_{z_{\max}}$ are minimum and maximum feed per tooth of the machine tool respectively, whereas $V_{\min}$ and $V_{\max}$ are minimum and maximum cutting speed.

3.2. Power

Power required for the cutting operation should not exceed the effective power transmitted to cutting point by the machine tool:

$$P_{\mathrm{m}} \ge \frac{P_{\mathrm{c}}}{\eta} \tag{15}$$

where P_m = nominal motor power (kW), P_c = cutting power (kW), and η = overall efficiency of machine tool.

In milling, mean value of power is given as:

$$P_c = \frac{F_c V}{6120} \tag{16}$$

where F_c is the mean peripheral cutting force (kg).

For the plain milling case, cutting force F_c is given by:

$$F_c = C_{zp} \cdot B \cdot z \cdot D^{b_z} a^{e_z} f_z^{u_z} \tag{17}$$

For the face milling case, cutting force F_c is given by:

$$F_c = C_{zp} \cdot B^{r_z} \cdot z^{n_z} \cdot D^{b_z} a^{e_z} f_z^{u_z} \tag{18}$$

Coefficients and exponents appearing in cutting force equations (Eq. (17) and Eq. (18)) are found in [18] and [17]. Eq. (15) concerns the milling of steel with ultimate tensile strength σ_{UTS} = 750 MPa. Since the strength of the material strongly influences the cutting resistance, a correction coefficient, K_H should be introduced into the calculation of the peripheral cutting force and cutting power, if the ultimate tensile strength of the steel is other than 750 MPa. This correction factor for the different tensile strengths of steel is given in Table 1. Within the range of the cutting edge angle x = 15–90° studied so far, an optimum value of this angle appears, amounting to x = 60° for face mills with sintered carbide teeth. For greater and smaller cutting edge angles, correction coefficient K_Y is used according to Table 1.

3.3. Arbor rigidity (arbor strength)

The arbor is also subject to torsion from the action of resistance to cutting. Therefore selected feed rate should be checked with regard to arbor strength:

$$F_c \leq F_s \tag{19}$$

where F_c = mean peripheral cutting force (kg), F_s = permissible force with regard to arbor strength (kg):

Table 1
Correction factors for peripheral cutting forces

Correction coefficient, K_H, for the peripheral cutting force taking into account the tensile strength of the steel							
σ_{UTS} (MPa)	550	650	750	850	950	1050	1150
K_H	0.85	0.925	1.00	1.075	1.15	1.125	1.30
Correction coefficient K_Y for the peripheral cutting force taking into account the cutting edge angle, x							
x°	15	30	45	60	75	90	
K_Y	1.23	1.15	1.06	1.0	1.06	1.14	

$$F_s = \frac{0.1k_b d_a^3}{0.08L_a + 0.65\sqrt{(0.25L_a)^2 + (0.5\alpha D)^2}} \tag{20}$$

where k_b = permissible bending stress of the arbor material (kg/mm^2), d_a = arbor diameter (mm), L_a = arbor length between supports (mm), $\alpha = k_b/(1.3k_t)$ coefficient in which k_t is the permissible torsional stress (kg/mm^2), and D = cutter diameter (mm).

3.4. Arbor deflection

Selected feed rate should be checked for arbor deflection as follows:

$$F_c \leq F_d \tag{21}$$

where F_d = permissible force with regard to arbor deflection (kg):

$$F_d = \frac{4Eed_a^4}{L_a^3} \tag{22}$$

where E = modulus of elasticity of arbor material (kg/mm^2), and e = permissible value of arbor deflection (in roughing operations, e = 0.2 mm, in finishing operations, e = 0.05 mm).

Similar to the plain milling case, in face milling, arbor deflection should also be kept below a certain value. Assuming the arbor as a cantilever beam, deflection at the end can be calculated by:

$$\delta = \frac{F_c L_s^3}{3E_s I_s} \tag{23}$$

where δ = permissible deflection of stub arbor at the end (mm), F_c = mean peripheral cutting force (N), L_s = length of stub arbor (mm), E_s = modulus of elasticity of stub arbor material (MPa), and I_s = moment of inertia of stub arbor (mm^4).

Therefore, for mean peripheral cutting force, the inequality can be rewritten as:

$$F_c \leq \frac{3\delta E_s I_s}{L_s^2}\,. \tag{24}$$

Permissible value of δ can be taken as 0.2 mm in roughing and 0.05 mm in finishing operations.

4. Optimization methodology

The optimum value of the number of passes and the corresponding speed, feed and depth of cut for each pass are all obtained in a dynamic multipass process. The problem, here, involves four variables. The number of passes and depth of cut for each pass are determined through the dynamic programming procedure, while the optimum cutting speed and feed for each pass are determined by using the geometric programming method. The dynamic programming can be considered as a multistage decision process in which each single-stage optimization problem can be stated, i.e., as a geometric problem. On one hand, geometric programming is a useful method that can be used for solving non-linear programming problems subject to non-linear constraints.

It is actually a formulation of the problem (the objective function) expressed in terms of a class of function called positive polynomial or posynomial. One can find the detailed information on the dynamic programming and geometric programming in Duffin [19] and Nemhause [20], respectively.

The decision variable in the dynamic programming (in the optimization problem) is the depth of cut (a_i) to be taken in the ith pass, which is represented as $a(i,j)$. Total depth of cut (a_T) is divided into N equal sections which are actually the N discrete decision states. The minimum increment of depth of cut is equal to:

$$\frac{a_T}{N} = a \tag{25}$$

It is defined arbitrarily and should always smaller than the maximum depth of cut allowed for a machine tool workpiece system a_{max}. The optimum number of passes N_p is determined by the dynamic programming approach and a pass consists of a certain number of sections a.

The dynamic programming procedure applied to multipass milling operations can be summarized as follows:

1. The dynamic programming procedure is started from the Nth section. First, variables when the stock is machined from the outer end of the Nth section to the inner end of the Nth section are optimized. The depth of cut is the thickness of each section and calculated by $a = a_T/N$. At this stage, the problem is an optimization problem of the single pass operation. Optimum cutting speed V(N,N) and feed per tooth f_z(N,N) which minimize the objective function T_{pr}(N,N) are through the geometric programming module.
2. At the second stage, variables when the stock is machined from the outer end of the (N-1)th section to the inner end of the (N-1)th section are optimized. But, in this case there are two alternatives. The first one is to machine from the outer end of the (N-1) section to the inner end of the same section and then from the outer end of the Nth section to the inner of the same section in two passes. Secondly, you can also machine from the outer end of the (N-1)th section to the inner end of the Nth section in a single pass. In this situation, for the (N-1,N-1) and (N-1,N) processes optimum strategies and objective function values are determined. Then, the minimum of T_{pr}(N-1,N-1) + T_{pr}(N,N) and T_{pr}(N-1,N) determines the optimum strategy at the second stage. Note that, at the end of each stage all alternatives should be stored for future use.
3. The next stage is the (N-2)th section. Variables when the stock is machined from the outer end of the (N-2)th section to the inner end of the (N-2)th section are optimized. As you rapidly catch the situation, here there are three alternatives.
4. The same procedure is continued successively until reaching the first section for other dynamic problem stages. Optimum values for the first section become the optimum values of the entire cutting process, i.e. gives the optimal speed and feed values as well as the optimum number of passes and the thickness of each pass. As seen from the procedure, when the dynamic programming technique is active, the optimization of single pass machining for each section is also accomplished by using the geometric programming module interconnected to the dynamic programming program. The dynamic programming strategy used in this study is illustrated in Fig. 1. A simplified flowchart for this strategy is also presented in Fig. 2.

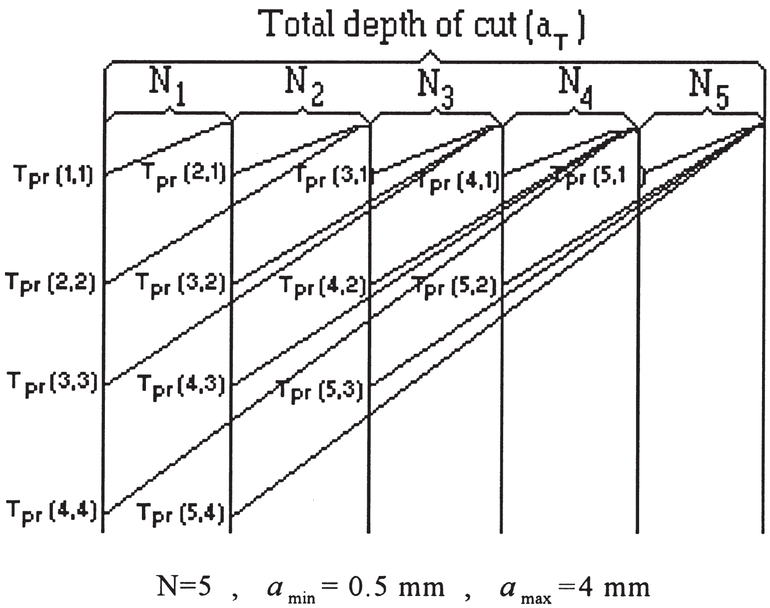

Fig. 1. Schematic representation of dynamic programming strategy.

According to Fig. 1, to minimize the production time and to reach to the first section from the 5th section, we have several alternatives. These alternatives are:

1. T_{pr} (5,4) + T_{pr} (1,1),
2. T_{pr} (5,3) + T_{pr} (2,2),
3. T_{pr} (5,3) + T_{pr} (2,1) + T_{pr} (1,1),
4. T_{pr} (5,2) + T_{pr} (3,3),
5. T_{pr} (5,2) + T_{pr} (3,2) + T_{pr} (1,1),
6. T_{pr} (5,2) + T_{pr} (3,1) + T_{pr} (2,2),
7. T_{pr} (5,2) + T_{pr} (3,1) + T_{pr} (2,1) + T_{pr} (1,1),
8. T_{pr} (5,1) + T_{pr} (4,4),
9. T_{pr} (5,1) + T_{pr} (4,3) + T_{pr} (1,1),
10. T_{pr} (5,1) + T_{pr} (4,2) + T_{pr} (2,2),
11. T_{pr} (5,1) + T_{pr} (4,2) + T_{pr} (2,1) + T_{pr} (1,1),
12. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,3),
13. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,2) + T_{pr} (1,1),
14. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,1) + T_{pr} (2,2),
15. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,1) + T_{pr} (2,1) + T_{pr} (1,1).

In a milling problem, for example, T_{pr} (1,1) and T_{pr} (2,1) are equal because in the milling operations the workpiece is fixed while the cutter turns as opposed to the turning. As a result of this we have:

T_{pr} (5,4) = T_{pr} (4,4),
T_{pr} (5,3) = T_{pr} (4,3) = T_{pr} (3,3),

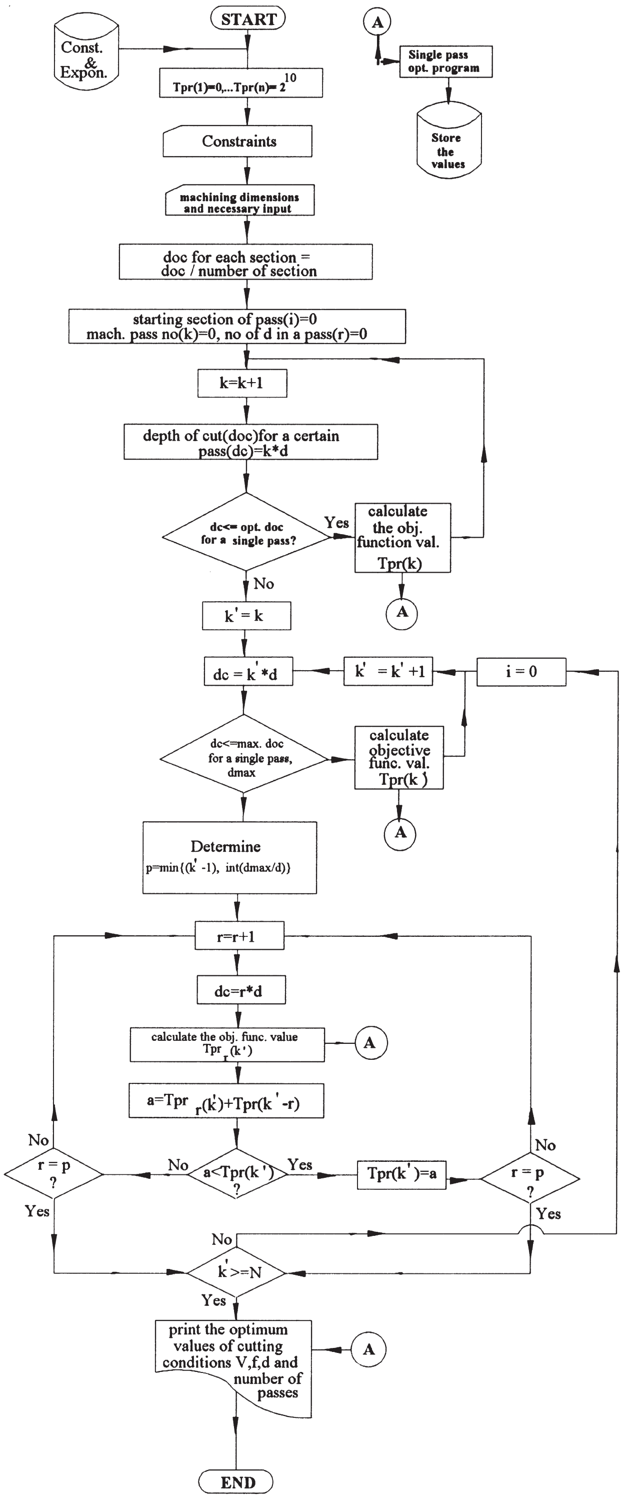

Fig. 2. Flowchart of the developed optimization strategy.

T_{pr} (5,2) = T_{pr} (4,2) = T_{pr} (3,2) = T_{pr} (2,2),
T_{pr} (5,1) = T_{pr} (4,1) = T_{pr} (3,1) = T_{pr} (2,1) = T_{pr} (1,1).

When the optimum cutting conditions are found for $a = 1$, $a = 2$, $a = 3$, $a = 4$, and so on, the optimum solution is reached, such as optimum depths of cut for minimum production time criteria. The stages of the dynamic iteration can also be given in a lower triangular matrix form [N × N] for ease of calculating. The (i,j) elements of the matrix represent the starting diameter section and the number of sections to be machined for a particular pass. A schematic representation of the dynamic programming procedure is given by the diagram depicted in Fig. 1, where the total depth of cut a_T is divided into five sections (N = 5). Assuming that the maximum and minimum depths of cut for a machine tool workpiece system are known, the lower triangular matrix of the example depicted in Fig. 1 for the objective function values, is given below:

$$T_{pr}(i,j) = \begin{vmatrix} X & 0 & 0 & 0 & 0 \\ X & X & 0 & 0 & 0 \\ X & X & X & 0 & 0 \\ X & X & X & X & 0 \\ X & X & X & X & 0 \end{vmatrix}$$

where X represents an entry and the diagonal elements represent the possible single passes. The cutting speed and feed values can also be stored in separate lower triangular matrices in a similar manner [3].

5. Optimization of each pass by geometric programming

5.1. Development of primal program

The general form of the primal objective function of the geometric programming is given in Eq. (12) and can be written in terms of the geometric programming formulation as:

$$g_0(t) = C_{01}{*}t_1^{a_{011}}{*}t_2^{a_{012}} + C_{02}{*}t_1^{a_{021}}{*}t_2^{a_{022}} \tag{26}$$

where t_1 = cutting speed, V (m/min), t_2 = feed per tooth, f_z (mm).

$$C_{01} - \frac{\pi DL}{1000z} \tag{27}$$

$$a_{011} = -1 \tag{28}$$

$$a_{012} = -1 \tag{29}$$

$$C_{02} = \frac{T_{\mathrm{d}}\pi La^{e_v/m}\, B^{r_v/m} z^{n_v/m-1} \lambda_s^{q_v/m}}{1000{*}C_v^{1/m}\, D^{b_v/m-1} (B_m B_h B_p B_t)^{1/m}} \tag{30}$$

$$a_{201} = \frac{1}{m} - 1 \tag{31}$$

$$a_{022} = \frac{u_v}{m} - 1 \tag{32}$$

The constraint functions formulated in the previous section should also be rewritten in such a way that they should be in terms of forced constraint functions. For this purpose, the first constraint is formulated as:

$$C_{11} t_1^{a_{111}} t_2^{a_{112}} \leq 1 \tag{33}$$

where

$$C_{11} = \frac{1}{f_{z_{\max}}} = \frac{zN_{\max}}{f_{\max}} \tag{34}$$

$$a_{111} = 0 \tag{35}$$

$$a_{112} = 1. \tag{36}$$

The second constraint is formulated as:

$$C_{21} t_1^{a_{211}} t_2^{a_{212}} \leq 1 \tag{37}$$

where

$$C_{21} = \frac{1}{V_{\max}} = \frac{1000}{\pi DN_{\max}} \tag{38}$$

$$a_{211} = 1 \tag{39}$$

$$a_{212} = 0. \tag{40}$$

The third constraint is formulated as:

$$C_{31} t_1^{a_{311}} t_2^{a_{312}} \leq 1 \tag{41}$$

where

$$C_{31} = \frac{C_{zp} BzD^{b_z} a^{e_z}}{K_1} \tag{42}$$

$$K_1 = P_{\mathrm{m}} \eta 6120 \tag{43}$$

$$a_{311} = 1 \tag{44}$$

$$a_{312} = u_z. \tag{45}$$

The fourth constraint is formulated as:

$$C_{41} t_1^{a_{411}} t_2^{a_{412}} \leq 1 \tag{46}$$

where

$$C_{41} = \frac{C_{zp} BzD^{b_z} a^{e_z}}{K_2} \tag{47}$$

$$K_2 = F_{\mathrm{s}} = \frac{0.1 k_{\mathrm{b}} d_{\mathrm{a}}^3}{0.08 L_{\mathrm{a}} + 0.65\sqrt{(0.25 L_{\mathrm{a}})^2 + (0.5 \alpha D)^2}} \tag{48}$$

$$a_{411} = 0 \tag{49}$$

$$a_{412} = u_z \tag{50}$$

The fifth constraint is formulated as:

$$C_{51}t_1^{a_{511}}t_2^{a_{512}} \leq 1 \tag{51}$$

where

$$C_{51} = \frac{C_{zp}BzD^{b_z}a^{e_z}}{K_3} \tag{52}$$

$$K_3 = F_\mathrm{d} = \frac{4Eed_\mathrm{a}^4}{L_\mathrm{a}^3} \tag{53}$$

$$a_{511} = 0 \tag{54}$$

$$a_{512} = u_z. \tag{55}$$

The sixth constraint is formulated as:

$$C_{61}t_1^{a_{611}}t_2^{a_{612}} \leq 1 \tag{56}$$

where

$$C_{61} = \frac{C_{zp}B^{r_z}z^{n_z}D^{b_z}a^{e_z}}{K_1} \tag{57}$$

$$K_1 = P_\mathrm{m}\eta 6120 \tag{58}$$

$$a_{611} = 1 \tag{59}$$

$$a_{612} = u_z. \tag{60}$$

The seventh constraint is formulated as:

$$C_{71}t_1^{a_{711}}t_2^{a_{712}} \leq 1 \tag{61}$$

where

$$C_{71} = \frac{C_{zp}B^{r_z}z^{n_z}D^{b_z}a^{e_z}}{K_2} \tag{62}$$

$$K_2 = F_s = \frac{0.1k_b d_a^3}{0.08L_a + 0.65\sqrt{(0.25L_a)^2 + (0.5\alpha D)^2}} \tag{63}$$

$$a_{711} = 0 \tag{64}$$

$$a_{712} = u_z. \tag{65}$$

The eighth constraint is formulated as:

$$C_{81}t_1^{a_{811}}t_2^{a_{812}} \leq 1 \tag{66}$$

where

$$C_{81} = \frac{C_{zp}B^{r_z}z^{n_z}D^{b_z}a^{e_z}}{K_4} \tag{67}$$

$$K_4 = \frac{3\delta E_s I_s}{L_s^3} \tag{68}$$

$$a_{811} = 0 \tag{69}$$

$$a_{812} = u_z. \tag{70}$$

6. Dual program

It is seen from the primal objective function that the objective function has two terms and there are eight constraint functions. All of the constraint functions have single terms. So the dual objective function turns out to be:

$$U(\delta) = \left(\frac{C_{01}}{\delta_{01}}\right)^{\delta_{01}}\left(\frac{C_{02}}{\delta_{02}}\right)^{\delta_{02}} C_{11}^{\delta_{11}}C_{21}^{\delta_{21}}C_{31}^{\delta_{31}}C_{41}^{\delta_{41}}C_{51}^{\delta_{51}}C_{61}^{\delta_{61}}C_{71}^{\delta_{71}}C_{81}^{\delta_{81}} \tag{71}$$

where δ_{01} and δ_{02} are the dual variables of the objective function. The dual variables are subjected to the linear constraints. According to the normality condition of the geometric programming, the first dual constraint function is given by:

$$\delta_{01} + \delta_{02} = 1. \tag{72}$$

The other constraints functions are according to the orthogonality condition:

$$a_{011}\delta_{01} + a_{021}\delta_{02} + a_{111}\delta_{11} + a_{211}\delta_{21} + a_{311}\delta_{31} + a_{411}\delta_{41} + a_{511}\delta_{51} + a_{611}\delta_{61} + a_{711}\delta_{71} + a_{811}\delta_{81} = 0 \tag{73}$$

$$a_{012}\delta_{01} + a_{022}\delta_{02} + a_{112}\delta_{11} + a_{212}\delta_{21} + a_{312}\delta_{31} + a_{412}\delta_{41} + a_{512}\delta_{51} + a_{612}\delta_{61} + a_{712}\delta_{71} + a_{812}\delta_{81} = 0 \tag{74}$$

and the non-negative constraints are:

$$\delta_{01} \geq 0,\ \delta_{02} \geq 0,\ \delta_{11} \geq 0,\ \delta_{21} \geq 0,\ \delta_{31} \geq 0,\ \delta_{41} \geq 0,\ \delta_{51} \geq 0,\ \delta_{61} \geq 0,\ \delta_{71} \geq 0,\ \delta_{81} \geq 0. \tag{75}$$

The dual objective function $U(\delta)$ has to be maximized by using the dual constraint functions which are formulated in Eqs. (72)–(75). The maximum point obtained from the dual objective function is the minimum value of the original objective function. However, the degree of difficulty of the problem is seven, therefore an additional method is necessary to solve the problem. For this purpose, the natural algorithm of the dual objective function is taken first. The resulting non-linear optimization problem can be solved by using the "Generalized Lagrange Multipliers" method. The detailed information on the "Generalized Lagrange Multipliers" method can be found in [21]. After the required manipulations, although there are thirteen equations and thirteen unknowns, the problem cannot be solved easily due to the existence of non-linear terms in the constraint equations. However, some equations are only functions of certain variables that can be found by solving those equations simultaneously, there remaining two equations and eight unknowns. Finally, to solve them, a "direct elimination" should be carried out according to the type of milling operation. If the type of milling is plain milling, constraints 6, 7, 8 can be eliminated directly ($\delta_{61} = \delta_{71} = \delta_{81} = 0$), whereas, if the type of milling is face milling, constraints 3, 4, 5 can also be eliminated ($\delta_{31} = \delta_{41} = \delta_{51} = 0$). After the direct elimination process, in any plain milling optimization problem, if two of the constraints are active and the others are redundant then possible combinations of the dual vectors are as follows:

Com	For C.1	For C.2	For C.3	For C.4	For C.5
1	$\delta_{11}\neq 0$	$\delta_{21}\neq 0$	$\delta_{31} = 0$	$\delta_{41} = 0$	$\delta_{51} = 0$
2	$\delta_{11}\neq 0$	$\delta_{21} = 0$	$\delta_{31}\neq 0$	$\delta_{41} = 0$	$\delta_{51} = 0$
3	$\delta_{11}\neq 0$	$\delta_{21} = 0$	$\delta_{31} = 0$	$\delta_{41}\neq 0$	$\delta_{51} = 0$
4	$\delta_{11}\neq 0$	$\delta_{21} = 0$	$\delta_{31} = 0$	$\delta_{41} = 0$	$\delta_{51}\neq 0$
5	$\delta_{11} = 0$	$\delta_{21}\neq 0$	$\delta_{31}\neq 0$	$\delta_{41} = 0$	$\delta_{51} = 0$
6	$\delta_{11} = 0$	$\delta_{21}\neq 0$	$\delta_{31} = 0$	$\delta_{41}\neq 0$	$\delta_{51} = 0$
7	$\delta_{11} = 0$	$\delta_{21}\neq 0$	$\delta_{31} = 0$	$\delta_{41} = 0$	$\delta_{51}\neq 0$
8	$\delta_{11} = 0$	$\delta_{21} = 0$	$\delta_{31}\neq 0$	$\delta_{41}\neq 0$	$\delta_{51} = 0$
9	$\delta_{11} = 0$	$\delta_{21} = 0$	$\delta_{31}\neq 0$	$\delta_{41} = 0$	$\delta_{51}\neq 0$
10	$\delta_{11} = 0$	$\delta_{21} = 0$	$\delta_{31} = 0$	$\delta_{41}\neq 0$	$\delta_{51}\neq 0$

From these ten possibilities, one which gives the maximum value of $U(\delta)$ is the optimum solution of the original function $g(t)$ and the dual vectors for this solution are the terms. The original objective function is obtained as follows:

$$g(V,f_z) = C_{01}V^{a_{011}}f_z^{a_{012}} + C_{02}V^{a_{021}}f_z^{a_{022}} \tag{76}$$

$$g(V,f_z) = g(\delta_{01}) + g(\delta_{02}) \tag{77}$$

$$g(\delta_{01}) = C_{01}V^{a_{011}}f_z^{a_{012}} \tag{78}$$

$$d(\delta_{02}) = C_{02}V^{a_{021}}f_z^{a_{022}}. \tag{79}$$

There are two equations and two unknowns, which can be solved simultaneously to find V and f_z. Using Eqs. (28), (29), (78) and (79), the optimal cutting speed can be found by the following formula:

$$V = \left[\left(\frac{C_{02}}{g\delta_{02}}\right)\left(\frac{C_{01}}{g\delta_{01}}\right)^{a_{022}}\right]^{\left(\frac{1}{a_{022}-a_{021}}\right)}. \tag{80}$$

And similarly, the feed per tooth can be calculated by the formula:

$$f_z = \left[\left(\frac{C_{02}}{g\delta_{02}}\right)\left(\frac{C_{01}}{g\delta_{01}}\right)^{a_{021}}\right]^{\left(\frac{1}{a_{021}-a_{022}}\right)}. \tag{81}$$

The procedure used in both dynamic and geometric programming stages of the optimization problem of the multipass milling operations are given below.

7. Dynamic programming procedure

1. Determine depth of cut each section ($a = a_T/\mathrm{N}$)
2. $k = k + 1$ and $a_s = k \cdot a$
3. If $a_s \leq a_{min}$, calculate T_{pr} go to 2
4. If $a_s \geq a_{min}$, take $k' = k$
5. $k' = k + 1$ and $a_s = k' \cdot a$
6. If $a_s \leq a_{max}$, calculate T_{pr} go to 5
7. If $a_s \geq a_{max}$, take $a_s = a_{max}$, calculate T_{pr}
8. Determine minimum T_{pr} according to dynamic programming alternatives. Take V_{opt} and $f_{z,opt}$ from this T_{pr}.

8. Geometric programming procedure

1. Calculate coefficients (C_{01}, C_{02}, C_{11}, C_{21}, . . ., and a_{011}, a_{012}, a_{012}, a_{022}, a_{111}, a_{112}, . . .)
2. State differential equations
3. Obtain possible combinations to decide which constraints are active or redundant. For constraint elimination make a like iteration method.
4. Calculate (δ_{01}, δ_{02}, δ_{11}, δ_{21}, . . .)
5. Calculate $U(\delta)$ from
6. Calculate V_{opt}, $f_{z,\mathrm{opt}}$ and $T_{\mathrm{pr,opt}}$.

9. An application example

Given:

Type of machining: plain milling (U_{p})
Motor power (P_{m}) = 5.5 kW, efficiency = 0.7
Arbor diameter = 27 mm, arbor length between supports = 210 mm
Permissible bending stress of arbor: 140 MPa = 14.27 kg/mm^2
Permissible torsional stress of arbor: 120 MPa = 12.23 kg/mm^2
Modulus of elasticity of arbor material = 200 GPa = 20,387 kg/mm^2
Spindle speed range: (31.5–2000) rpm, feed rate range: (14–900) mm/min
Tool material: HSS, tool diameter = 63 mm, number of teeth = 8
Material: structural carbon steel (C $\leq$ 0.6%)
Scale on the hot rolled workpiece.
Tensile strength: 750 MPa, Brinell hardness number = 150
Length of cut = 160 mm, width of cut = 50 mm, depth of cut = 5 mm
Loading and unloading time of one workpiece = 1.5 min
Set-up time of fixtures and machine tool = 10 min
Tool change time = 5 min
Process adjusting and quick return time = 0.1 (min/part)
Lot size (number of parts in the batch) = 100
Cutting inclination = 30°

Find the optimal cutting parameters and the number of passes with depth of cuts. For this, first you have to find the necessary constants from related tables like the following:

$C_v = 35.4$
$m = 0.33$
$b_v = 0.45$
$e_v = 0.3,\ B_m = 1.0,\ B_h = \left(\dfrac{\sigma_1}{\sigma_2}\right)^p = \left(\dfrac{750}{750}\right)^1 = 1,\ B_p = 0.8,\ B_t = 0.8,\ e = 0.2$ mm
$u_v = 0.4$
$r_v = 0.1$

$n_v = 0.1$

$q_v = 0,\ C_{zp} = 68.2,$

$$b_z = -0.86$$
$$e_z = 0.86$$
$$u_z = 0.72$$

Let's take the number of sections for dynamic iteration as 5, and suppose that:

$$a_{\min} = 0.5\ mm$$

$$a_{\max} = 4\ mm$$

Through the given procedures, dynamic iteration trials for production time, speed and feed values are calculated and stored in the lower triangular matrices separately:

$$V = \begin{vmatrix} 32.25 & 0 & 0 & 0 & 0 \\ 32.25 & 25.16 & 0 & 0 & 0 \\ 32.25 & 25.16 & 26.4 & 0 & 0 \\ 32.25 & 25.16 & 26.4 & 30.95 & 0 \\ 32.25 & 26.15 & 26.4 & 30.95 & 0 \end{vmatrix}$$

$$f_z = \begin{vmatrix} 0.7044 & 0 & 0 & 0 & 0 \\ 0.7044 & 0.57 & 0 & 0 & 0 \\ 07044 & 0.57 & 0.338 & 0 & 0 \\ 0.7044 & 0.57 & 0.338 & 0.149 & 0 \\ 0.7044 & 0.57 & 0.338 & 0.149 & 0 \end{vmatrix}$$

$$T_{pr} = \begin{vmatrix} 1.9067 & 0 & 0 & 0 & 0 \\ 1.9067 & 2.01 & 0 & 0 & 0 \\ 1.9067 & 2.01 & 2.195 & 0 & 0 \\ 1.9067 & 2.01 & 2.195 & 2.636 & 0 \\ 1.9067 & 2.01 & 2.195 & 2.636 & 0 \end{vmatrix}$$

T_{pr} (5,4) = T_{pr} (4,4) = 2.636 min
T_{pr} (5,3) = T_{pr} (4,3) = T_{pr} (3,3) = 2.195 min
T_{pr} (5,2) = T_{pr} (4,2) = T_{pr} (3,2) = T_{pr} (2,2) = 2.01 min
T_{pr} (5,1) = T_{pr} (4,1) = T_{pr} (3,1) = T_{pr} (2,1) = T_{pr} (1,1) = 1.843 min
For total depth of cut a = 5 mm total production time alternatives:

1. T_{pr} (5,4) + T_{pr} (1,1) = 4.479 min
2. T_{pr} (5,3) + T_{pr} (2,2) = 4.205 min
3. T_{pr} (5,3) + T_{pr} (2,1) + T_{pr} (1,1) = 5.881 min
4. T_{pr} (5,2) + T_{pr} (3,3) = 4.205 min
5. T_{pr} (5,2) + T_{pr} (3,2) + T_{pr} (1,1) = 5.863 min
6. T_{pr} (5,2) + T_{pr} (3,1) + T_{pr} (2,2) = 5.863 min
7. T_{pr} (5,2) + T_{pr} (3,1) + T_{pr} (2,1) + T_{pr} (1,1) = 7.539 min
8. T_{pr} (5,1) + T_{pr} (4.4) = 4.479 min
9. T_{pr} (5,1) + T_{pr} (4,3) + T_{pr} (1,1) = 5.881 min
10. T_{pr} (5,1) + T_{pr} (4,2) + T_{pr} (2,2) = 5.863 min
11. T_{pr} (5,1) + T_{pr} (4,2) + T_{pr} (2,1) + T_{pr} (1,1) = 7.539 min
12. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,3) = 5.881 min
13. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,2) + T_{pr} (1,1) = 7.539 min
14. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,1) + T_{pr} (2,2) = 7.539 min
15. T_{pr} (5,1) + T_{pr} (4,1) + T_{pr} (3,1) + T_{pr} (2,1) + T_{pr} (1,1) = 9.215 min

From the above alternatives, the 2nd and 4th are found to be the optimum solutions based on our objective function. Where:
Optimum unit production time: T_{pr} = 4.205 min
Optimum number of passes: N = 2
Below are the resulting optimal values of cutting conditions:
According to the 2nd alternative:

Pass	a (mm)	T_{pr} (min)	V (m/min)	f_z (mm/tooth)
1st	3	2.195	26.4	0.338
2nd	2	2.01	25.16	0.57

According to the 4th alternative:

Pass	a (mm)	T_{pr} (min)	V (m/min)	f_z (mm/tooth)
1st	2	2.01	25.16	0.57
2nd	3	2.195	26.4	0.338

10. Discussion and conclusion

In this study, a mathematical model has been developed for the constrained optimization of cutting parameters that are used in the multipass plain and face milling operations. It is based on the “maximum production rate” criterion. For automating the procedures given in the previous sections, a program is implemented on an IBM compatible PC by using C language. The developed optimization strategy mainly utilizes the two main mathematical techniques: dynamic programming and geometric programming. One of the important steps in executing the program is to select a proper number of sections for the problems, since higher precision, i.e. selecting a higher number for the number of sections, will increase the execution time dramatically, although more effective times are calculated for the objective function. This value should be selected always according to the total depth. The values obtained by the program have been tested for various sizes of material stock and values of constraints to see the effectiveness of the proposed optimization strategy over the ones proposed by single-pass based optimization systems reported in the literature and handbook recommendations. The results have shown that the multipass approach to the optimization of cutting conditions is superior to the single-pass production rates and handbook values. It has been also recognized that in the multipass operations it is always better to use unequal cutting conditions in each pass instead of using equal depths of cut for all passes which is often used as “canned cycles” in CNC machine tool controllers. With the developed program, it would be possible to have an increase in the productivity which is always considered as a main goal of process planning applications.

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