AE305 AERODYNAMICS I
PROBLEM HOUR 2

Q6. The measured lift slope for the NACA 23012 airfoil is 0.1080 degree^{-1}, and $\alpha_{L=0} = -1.3°$. Consider a finite wing using this airfoil, with AR = 8 and taper ratio = 0.8. Assume that δ = τ. Calculate the lift and induced drag coefficients for this wing at a geometric angle of attack = 7°.

$$a = \frac{a_o}{1 + \frac{a_o}{\pi AR}(1+\tau)}, \qquad \text{where } a_o = 0.1080 \text{ per degree} = 6.188 \text{ per radian}$$

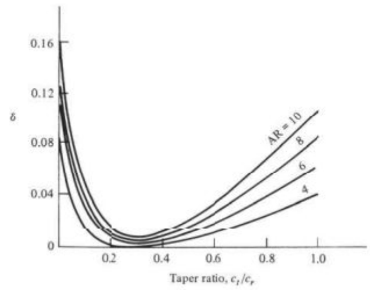
From Fig. 5.18: δ = τ = 0.054.

$$a = \frac{6.188}{1 + \frac{6.188}{\pi(8)}(1+0.054)} = 4.91 \text{ per rad.}$$

$$= 0.0857 \text{ per degree}$$

$$C_L = a\,(\alpha - \alpha_{L=0}) = 0.0857\,[7 - (-1.3) = \boxed{0.712}$$

$$C_{D_i} = \frac{C_L^2}{\pi aR}\,(1+\delta) = \frac{(0.712)^2}{\pi(8)}\,(1.054) = \boxed{0.0212}$$



τ:Lift efficiency factor
δ: Induced drag factor

Q7. Consider an elliptic wing with the distribution of circulation given by:

$$\Gamma(y) = \Gamma_0 \left[1 - \left(\frac{2y}{b}\right)^2\right]^{1/2}$$

From this expression, show that the downwash behind the wing is constant.

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

$$L'(y) = \rho U_\infty \Gamma(y)$$

$$\Rightarrow L'(y) = \rho U_\infty \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

Downwash:

$$\frac{d\Gamma}{dy} = -\frac{4\Gamma_0}{b^2} \frac{y}{\left(1 - \frac{4y^2}{b^2}\right)^{\frac{1}{2}}}$$

$$\Rightarrow w(y_0) = \frac{\Gamma_0}{\pi b^2} \int_{-b/2}^{b/2} \frac{y}{\left(1 - \frac{4y^2}{b^2}\right)^{\frac{1}{2}} (y_0 - y)} dy$$

insert $y = \frac{b}{2} \cos\theta$
$dy = -\frac{b}{2} \sin\theta \, d\theta$

$$\Rightarrow w(\theta_0) = -\frac{\Gamma_0}{2\pi b} \int_0^\pi \frac{\cos\theta}{\cos\theta - \cos\theta_0} d\theta$$

$$\int_0^\pi \frac{\cos(n\theta)}{\cos\theta - \cos\theta_0} = \frac{\pi \sin n\theta_0}{\sin\theta_0}$$

$$\Rightarrow \boxed{w(\theta_0) = -\frac{\Gamma_0}{2b}}$$ downwash const.

Q8.

The Cessna Citation Jet aircraft (figure 3) has a tapered wing that uses the NASA NLF-0213 airfoil from root to tip with $\alpha_{L=0} = -4°$. Use a collocation method with 10 stations to determine the lift coefficient of the whole wing in the following situations.

1. When flying at zero angle of attack.
2. When flying at $\alpha = 5°$.
3. When flying at $\alpha = 5°$ with the flaps deployed ($\alpha_{L=0} = -10°$ for those spanwise sections of the wing where the flaps are located).

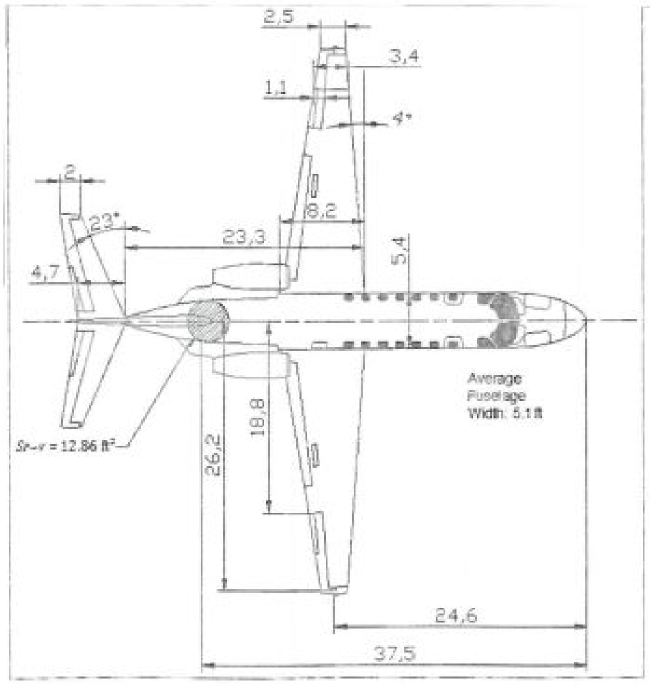


Figure 3: Top CAD view of a Cessna Citation CJ13 Aircraft. Units are in feet.

The equation for the chords of the wing is:

$* \; c(y) = c_r + (c_t - c_r) \cdot \frac{|y|}{b/2}$

Making the change of variable: $y = \frac{b}{2} \cos\theta$:

$* \; c(\theta) = c_r + (c_t - c_r) \cdot |\cos\theta|$

where y is measured from the wing chord.

We use the $N=10$ points:

$* \; \theta_j = \frac{j}{N+1} \cdot \pi = \frac{j}{11} \cdot \pi$

$* \; j = 1, \dots, 10$

The wing doesn't have twist, so $\alpha(\theta) = $ constant, and $\alpha_{l=0}(\theta) = -4^\circ = -\frac{\pi}{45}$ rad. We have to solve the system of equations:

$$* \; \alpha - \alpha_{l=0} = \frac{2b}{\pi \cdot [c_r + (c_t - c_r) \cdot |\cos\theta_j|]} \cdot \sum_{n=1}^{10} A_n \sin(n\theta_j) + \sum_{n=1}^{10} n A_n \frac{\sin(n\theta_j)}{\sin\theta_j} =$$

$$= \sum_{n=1}^{10} A_n \cdot \sin(n\theta_j) \cdot \left\{ \frac{2b}{\pi[c_r + (c_t - c_r) \cdot |\cos\theta_j|]} + \frac{n}{\sin\theta_j} \right\}$$

Using:

$* \; \alpha_{l=0} = -\frac{\pi}{45}$ rad

$* \; c_r = 8.2 \; ft = 2.5 m$

$* \; c_t = 2.5 \; ft = 0.762 \; m$

$* \; \frac{b}{2} = (26.2 - \frac{5.4}{2}) \; ft = 23.5 \; ft = 7.163 \; m$

the equations to solve are:

$$
* \; B = \begin{bmatrix}
4.1 & 9.8 & 16.3 & 22.9 & 28.4 & 31.9 & 32.6 & 29.7 & 23.2 & 13.1 \\
5.8 & 11.4 & 14.2 & 12.2 & 5.1 & -5.6 & -16.4 & -23.3 & -23.1 & -14.8 \\
6.1 & 9.2 & 5.8 & -3.4 & -12.1 & -13.3 & -4.5 & 9.3 & 18.4 & 15.1 \\
5.7 & 5.5 & -2.4 & -9.4 & -5.7 & 6.3 & 12.7 & 3.9 & -11.4 & -14.7 \\
5.0 & 1.7 & -6.4 & -4.4 & 6.9 & 7.6 & -6.0 & -11.0 & 3.7 & 14.0 \\
5.0 & -1.7 & -6.4 & 4.4 & 6.9 & -7.6 & -6.0 & 11.0 & 3.7 & -14.0 \\
5.7 & -5.5 & 2.4 & 9.4 & -5.7 & -6.3 & 12.7 & -3.9 & -11.4 & 14.7 \\
6.1 & -9.2 & 5.8 & 3.4 & -12.1 & 13.3 & -4.5 & -9.3 & 18.4 & -15.1 \\
5.8 & -11.4 & 14.2 & -12.2 & 5.1 & 5.6 & -16.4 & 23.3 & -23.1 & 14.8 \\
4.1 & -9.8 & 16.3 & -22.9 & 28.4 & -31.9 & 32.6 & -29.7 & 23.2 & -13.1
\end{bmatrix}
$$

$$
* \; [B] \cdot \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{Bmatrix} = \left(\alpha + \frac{\pi}{45}\right) \cdot \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}
$$

1) When $\alpha = 0°$, the solution of the equations is:

$$
* \; \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{Bmatrix} = \begin{Bmatrix} 0.0128 \\ 0 \\ -0.0003 \\ 0 \\ 0.0006 \\ 0 \\ 0.0001 \\ 0 \\ 0.0001 \\ 0 \end{Bmatrix}
$$

The aspect ratio of the wing is calculated as:

$$
* \; \Lambda = \frac{b^2}{S}
$$

The surface of the wing is:

$$* \; S = \cancel{2} \times \frac{(C_r + C_t)}{\cancel{2}} \cdot \frac{b}{2}$$

So:

$$* \; \Lambda = \frac{b^2}{S} = \frac{2 \cdot b^{\cancel{2}}}{(C_r + C_t) \cdot \cancel{b}} = \frac{940}{107}$$

The lift coefficient is calculated as:

$$* \; \boxed{C_L = \pi \Lambda A_1 = 0.35}$$

2)

When $\alpha = 5° = \frac{\pi}{36}$ rad:

$* \; A_1 = 0.0288$

$* \; \boxed{C_L = 0.80}$

3)

The left flap is located between the sections:

$* \; y = -13.53$ ft $\Rightarrow \theta = 2.18$ rad

and:

$* \; y = -1.42$ ft $\Rightarrow \theta = 1,63$ rad

The right flap is located between the sections:

$* \; y = 1.42$ ft $\Rightarrow \theta = 1.51$ rad

and:

$* \; y = 13.53$ ft $\Rightarrow \theta = 0.96$ rad

The equations to solve are, then:

$$* \; [B] \cdot \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{Bmatrix} = \begin{Bmatrix} \alpha + \pi/45 \\ \alpha + \pi/45 \\ \alpha + \pi/45 \\ \alpha + \pi/18 \\ \alpha + \pi/18 \\ \alpha + \pi/18 \\ \alpha + \pi/18 \\ \alpha + \pi/45 \\ \alpha + \pi/45 \\ \alpha + \pi/45 \end{Bmatrix}$$

For $\alpha = 5°$ we get:

$$* \; \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{Bmatrix} = \begin{Bmatrix} 0.0417 \\ 0 \\ -0.0069 \\ 0 \\ 0.0028 \\ 0 \\ 0.0012 \\ 0 \\ -0.0009 \\ 0 \end{Bmatrix}$$

$$* \; \boxed{C_L = \pi \Lambda A_1 = 1.15}$$