AE 306 AERODYNAMICS II

HOMEWORK-4

Consider the velocity potential for 2D compressible flow field for air is given:

$$\phi = \frac{\sqrt{1-M_\infty}}{4\pi}\ln(x^2+y^2) - V_\infty y$$

$V_\infty = 150$ m/s $P_\infty = 98$ kPa $T_\infty = 20\ C^\circ = 293.15$ K $(x,y) = (0.02, 0.03)$

a) Velocity components u and v:

$$M_\infty = V_\infty / a_\infty = V_\infty / \sqrt{\gamma R T_\infty} = 150/\sqrt{1.4 * 287 * 293.15} = 0,4371$$

$$u = \frac{\partial\phi}{\partial x} = \Phi_x = \frac{\sqrt{1-M_\infty}}{4\pi}\frac{2x}{x^2+y^2} = \frac{\sqrt{1-0,4371}}{4\pi}\left(\frac{2*0,02}{0,02^2+0,03^2}\right) = 1,8371 \text{ m/s}$$

$$v = \frac{\partial\phi}{\partial y} = \Phi_y = \frac{\sqrt{1-M_\infty}}{4\pi}\frac{2y}{x^2+y^2} - V_\infty = \frac{\sqrt{1-0,4371}}{4\pi}\left(\frac{2*0,03}{0,02^2+0,03^2}\right) - 150 = -147,244 \text{ m/s}$$

b) Speed of sound a:

$$\frac{T_0}{T_\infty} = 1 + \frac{(\gamma-1)}{2}M_\infty^2 = 1 + \frac{(1.4-1)}{2}(0.4371^2) = 1,0382$$

$$T_0 = 1,0382 * T_\infty = 1.0382 \times 293.15 = 304,35 \text{ K}$$

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 * 287 * 304.35} = 349,7 \text{ m/s}$$

From the Equation 8,18 in Modern Compressible Book:

$$a^2 = a_0^2 - \frac{\gamma-1}{2}(\Phi_x^2 + \Phi_y^2 + \Phi_z^2) \quad \text{where } \Phi_z = 0 \text{ in 2D flow}$$

$$a^2 = 349,7^2 - \frac{(1,4-1)}{2}(1,8371^2 + 147,244^2) = 117951,65$$

$a = 343,44$ m/s

c) Mach number M :

$$V = \sqrt{u^2 + v^2} = \sqrt{1,8371^2 + 147,244^2} = 147,255 \text{ m/s}$$

$$M = \frac{V}{a} = \frac{147,255}{343,44} = 0,429$$

d) Pressure P;

$$\frac{P_0}{P_\infty} = \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \left(\frac{1.4-1}{2}\right) 0,43712^2\right]^{\frac{1.4}{1-1.4}} = 1,14$$

$$P_0 = 1,14 * P_\infty = 1,14 * 98 * 10^3 = 111,74 \text{ kPa}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{(1.4-1)}{2} 0,429^2\right]^{\frac{1.4}{1-1.4}} = 1,135$$

$$P = \frac{P_0}{1,135} = \frac{111,74 \text{ kPa}}{1,135} = 98,461 \text{ kPa}$$

e) Temperature T;

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 = 1 + \left(\frac{1.4-1}{2}\right) 0,429^2 = 1,037$$

$$T = \frac{T_0}{1,037} = \frac{304,35}{1,037} = 293,545 \text{ K}$$

f) Density ρ;

$$\rho = \frac{P}{RT} = \frac{98,461 * 10^3}{287 * 293.545} = 1,169 \text{ kg/m}^3$$