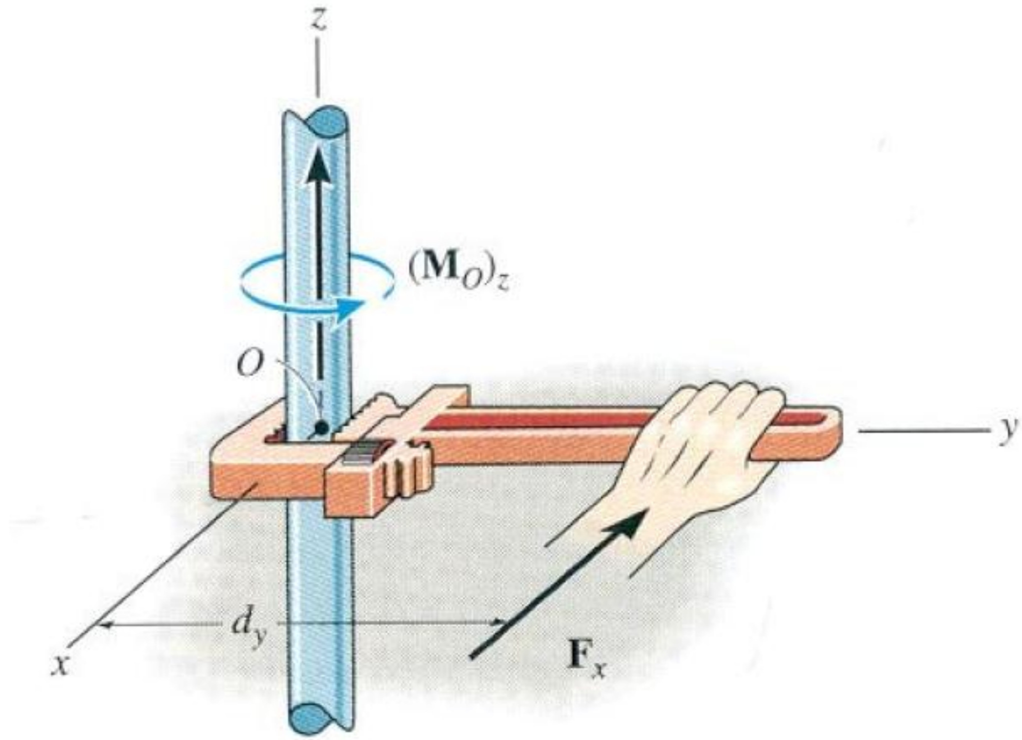
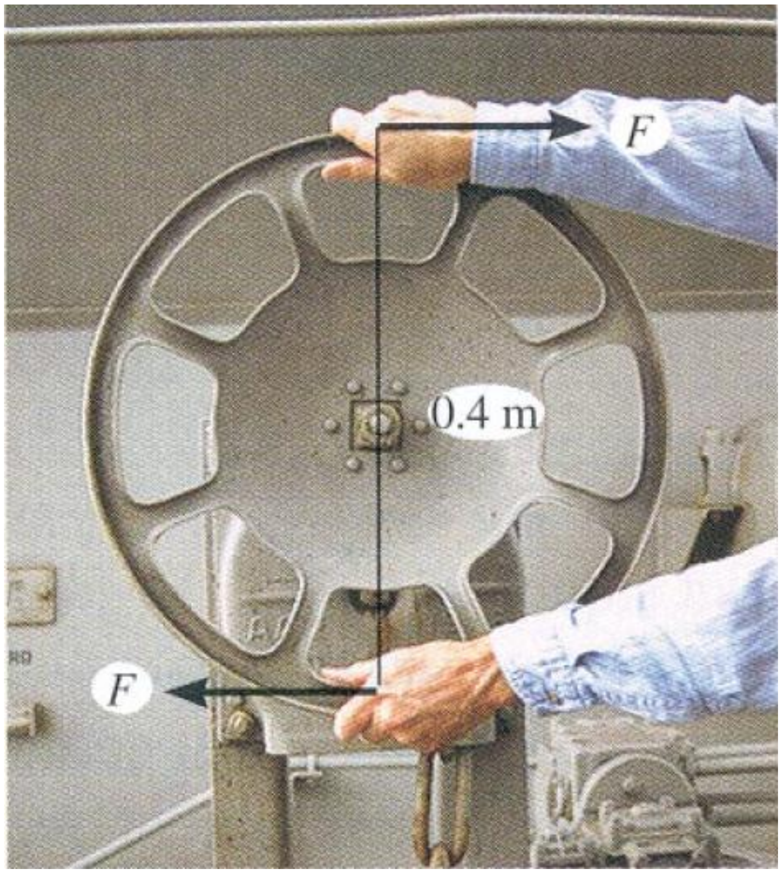
Moment of a force

The *moment of a force* is a measure of the tendency of the force to produce *rotation* of a body about a point or axis.

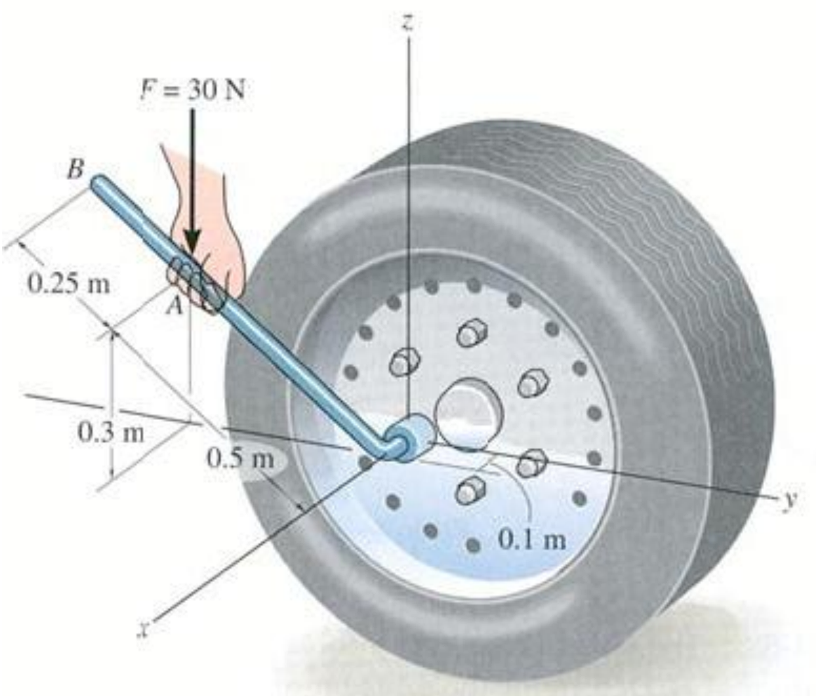


Applications



What is the net effect of the two forces on the wheel?

Applications



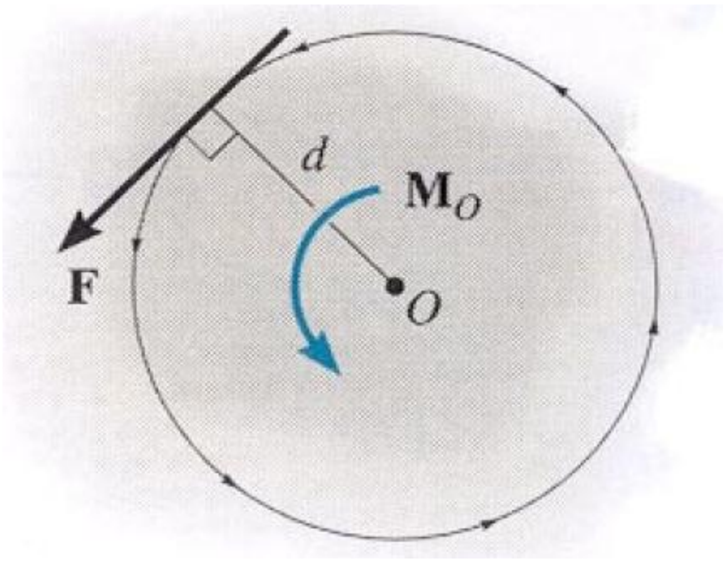
What is the effect of the 30-N force on the lug nut?

Moment of a force ... scalar description

The moment of a force F about a point o is denoted by M_o and has magnitude

$$M_o = F\,d$$

where



F is the magnitude of the force.

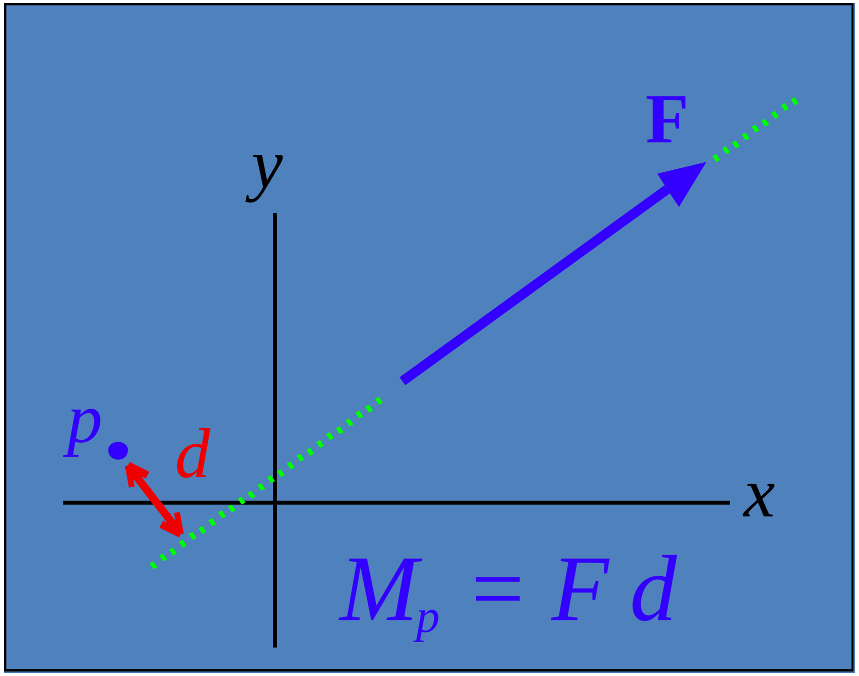
d is the perpendicular distance from O to the line of action of F, and is often called the "moment arm".

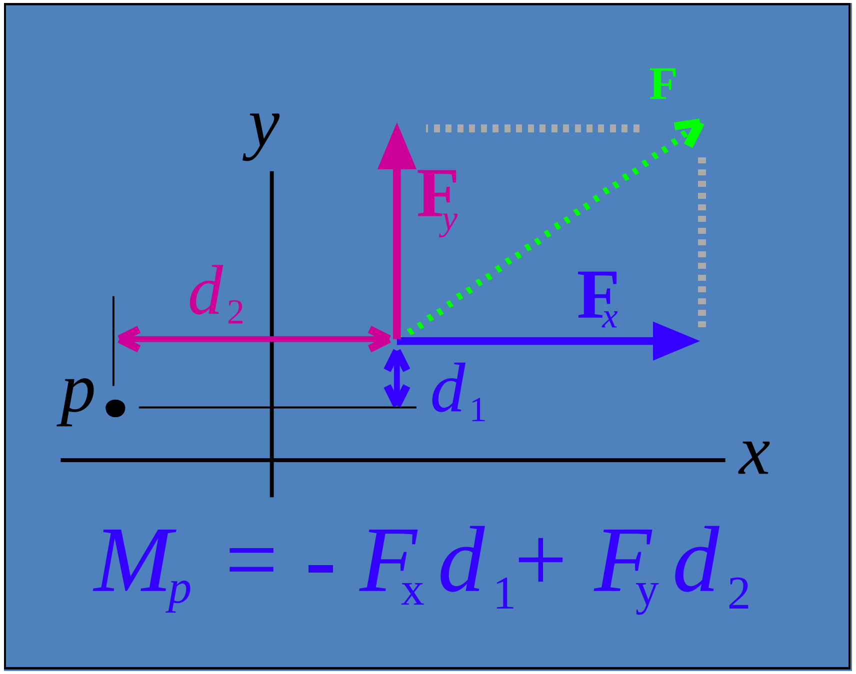
M_o is the tendency for F to "twist" the axis through point o.

- In 2-D, the direction of MO is either clockwise or counter-clockwise depending on the tendency for rotation.
- M_o has units of (force-length); e.g. ft-lb, N-m.
- "Moment" and "torque" are used synonymously.

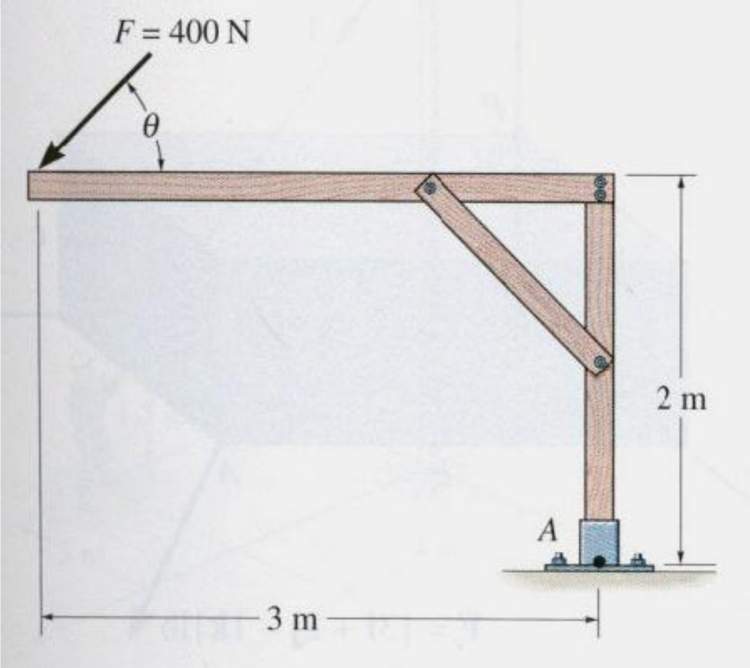
Principle of moments: Consider a force F. The moment of F about some point p can be computed by finding the perpendicular distance from p to the line of action of F *(this is the definition of moment)*. Or ... the force F can be resolved into components, and the moments from each component about point p can be computed and summed to obtain the same result.

illustration:





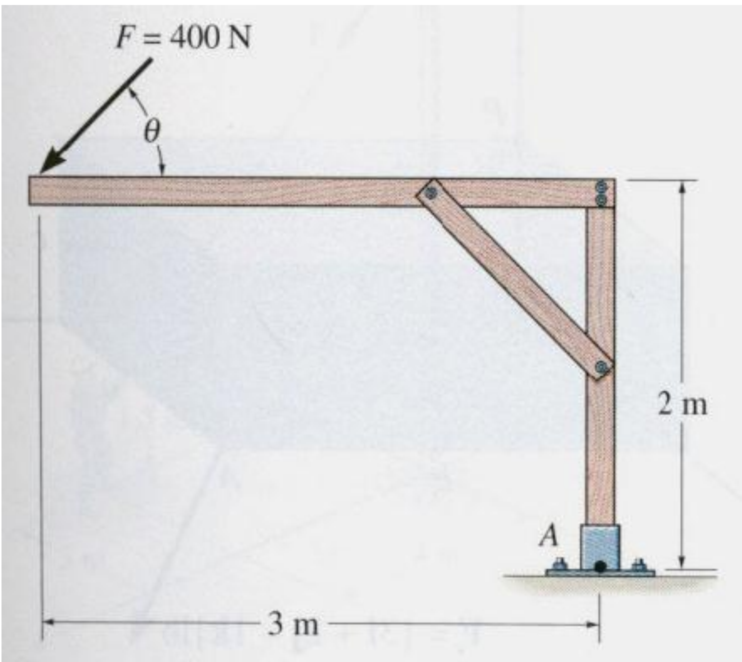
Example 1



Given: A 400 N force is applied to the frame and θ = 20°.

Find: The moment of the force at A.

Solution



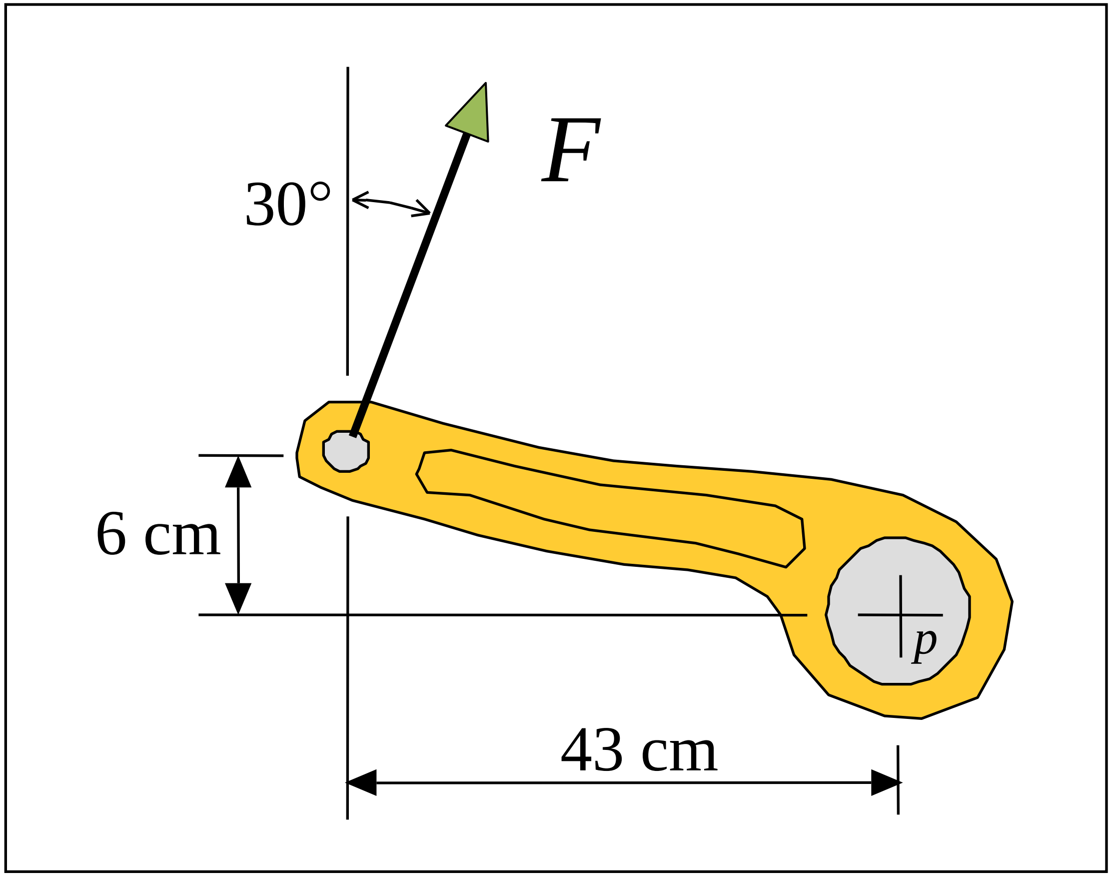
$+\uparrow F_y = -400 \sin 20^\circ$ N

$+\rightarrow F_x = -400 \cos 20^\circ$ N

$+\circlearrowleft M_A = \{(400 \cos 20^\circ)(2) + (400 \sin 20^\circ)(3)\}$ N·m

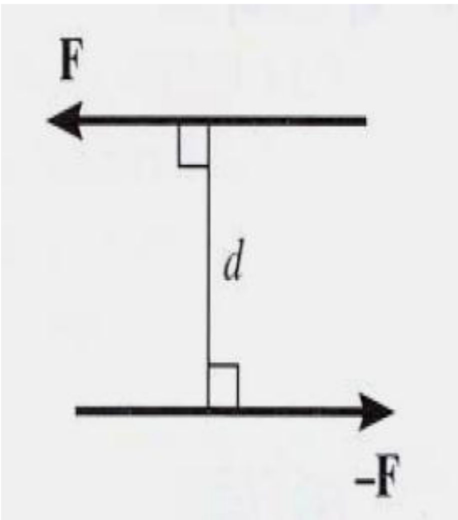
$= 1160$ N·m

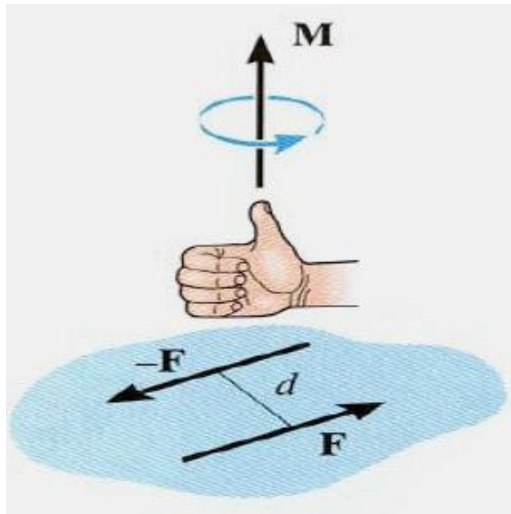
Example 2: The wrench shown is used to turn drilling pipe. If a torque (moment) of 800 N.m about point p is needed to turn the pipe, determine the required force F.



A:
$F = 239$ N

Moment of a Couple





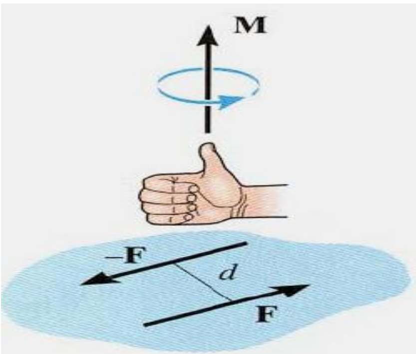
A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

The moment of a couple is defined as
$M_O = F\ d$ (using a scalar analysis) or as

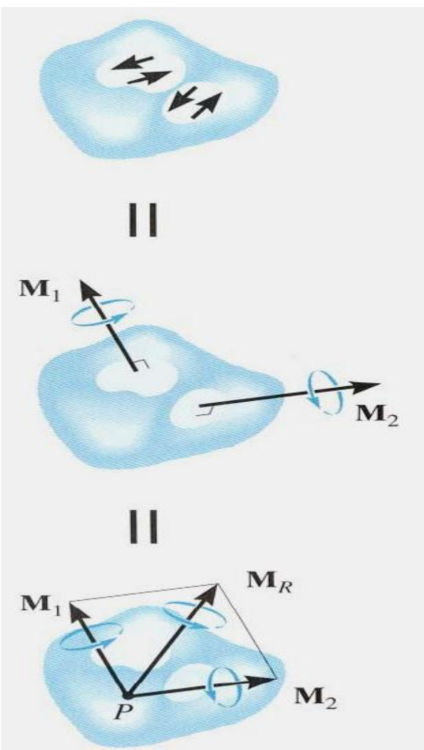
$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here ***r*** is any position vector from the line of action of ***–F*** to the line of action of ***F***.

Moment of a Couple



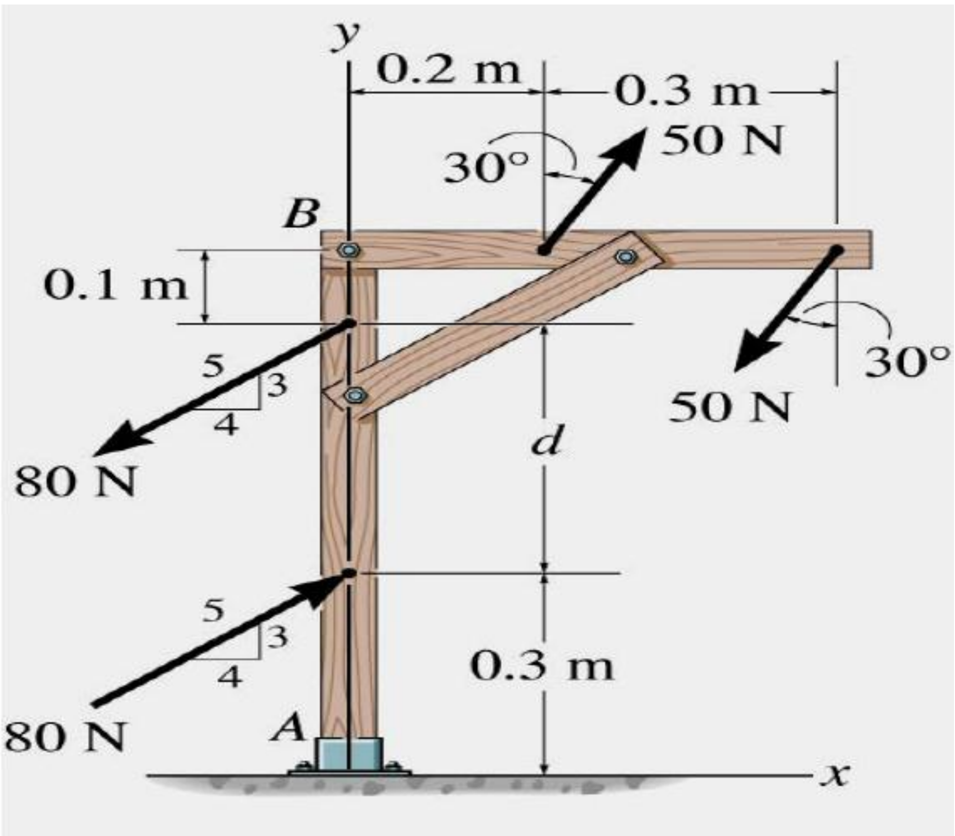
The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals (F d)



Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a **free vector**. It can be moved anywhere on the body and have the same external effect on the body.

Moments due to couples can be added using the same rules as adding any vectors.

Example 1



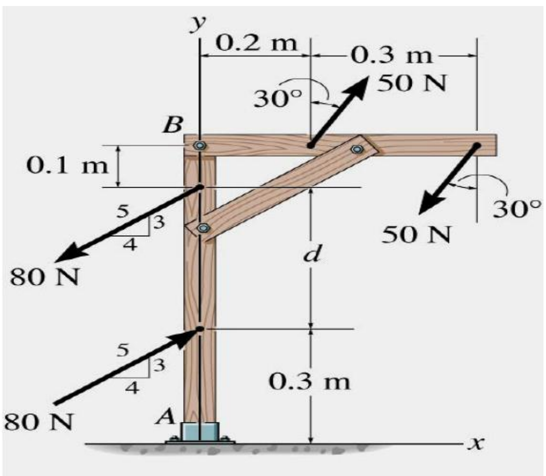
Given: Two couples act on the beam and d equals 0.4 m.

Find: The resultant couple

Plan:

1) Resolve the forces in x and y directions so they can be treated as couples.
2) Determine the net moment due to the two couples.

Solution



The x and y components of the top 50 N force are:

(50 N) (cos 30°) = 43.3 N up

(50 N) (sin 30°) = 25 N to the right

Similarly for the top 80 N force:

(80 N) (3/5) down

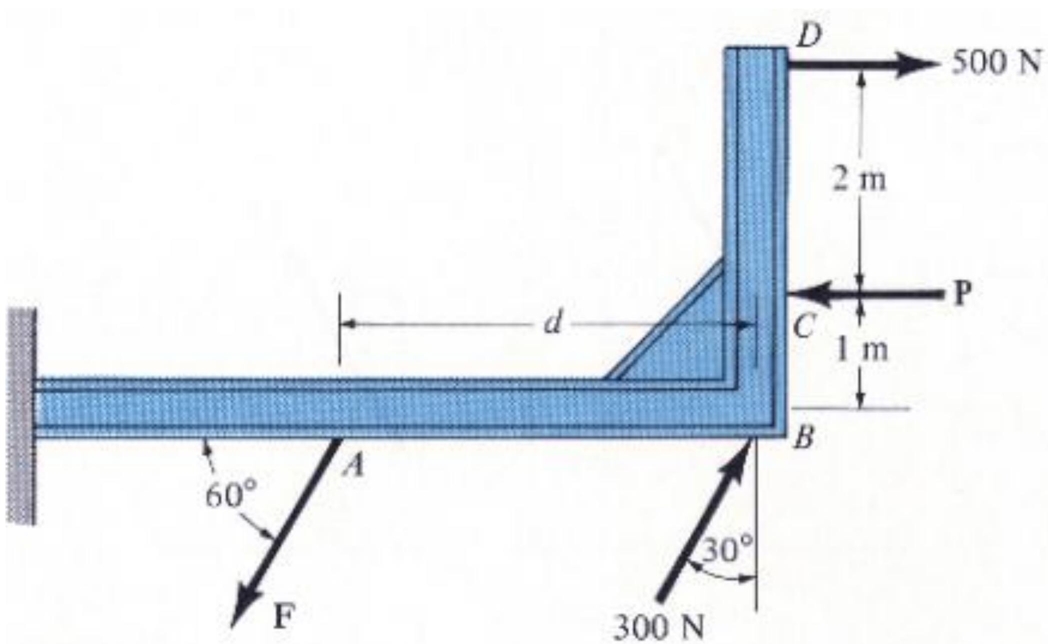
+ (80 N)(4/5)(0.4 m)

(80 N) (4/5) to the left

12.61 N m

The net moment equals to

Example 2



Two couples act on the beam. One couple is formed by the forces at A and B, and other by the forces at C and D. If the resultant couple is zero, determine the magnitudes of **P** and **F**, and the distance *d* between A and B.

Solution

Since these are couples we must have:

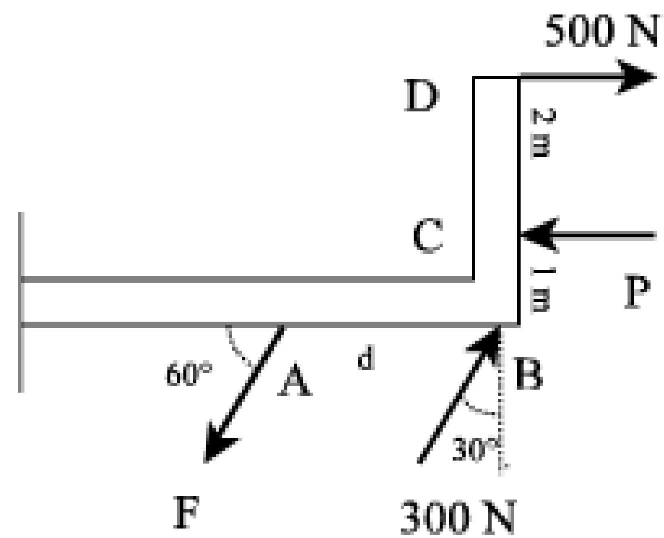
F = 300 N

P = 500 N

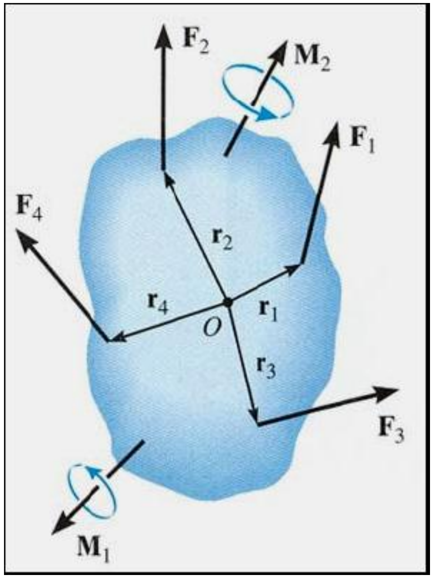
The resultant coupe is:

$M = - 500 * 2 + 300 * d \cos 30^0 = 0$

Thus d = 3.85 m



Equivalent force systems



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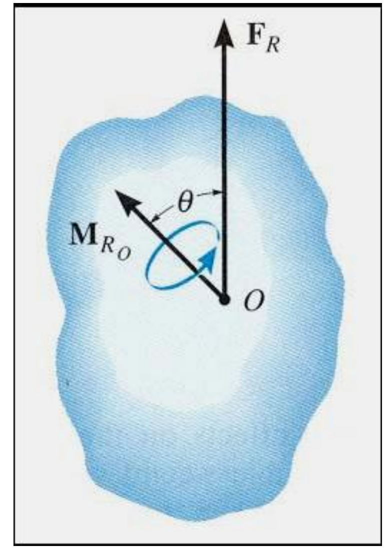
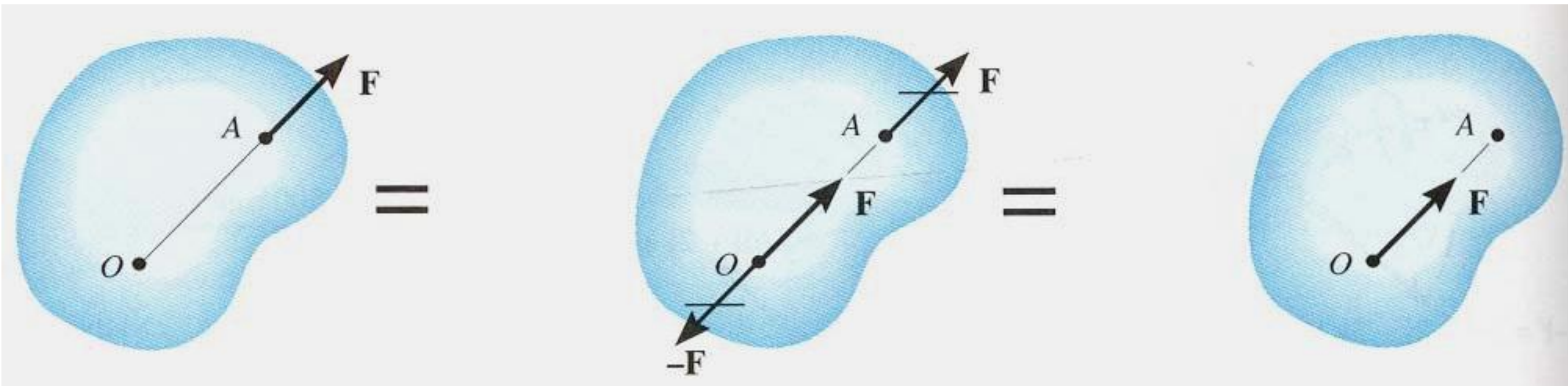


When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

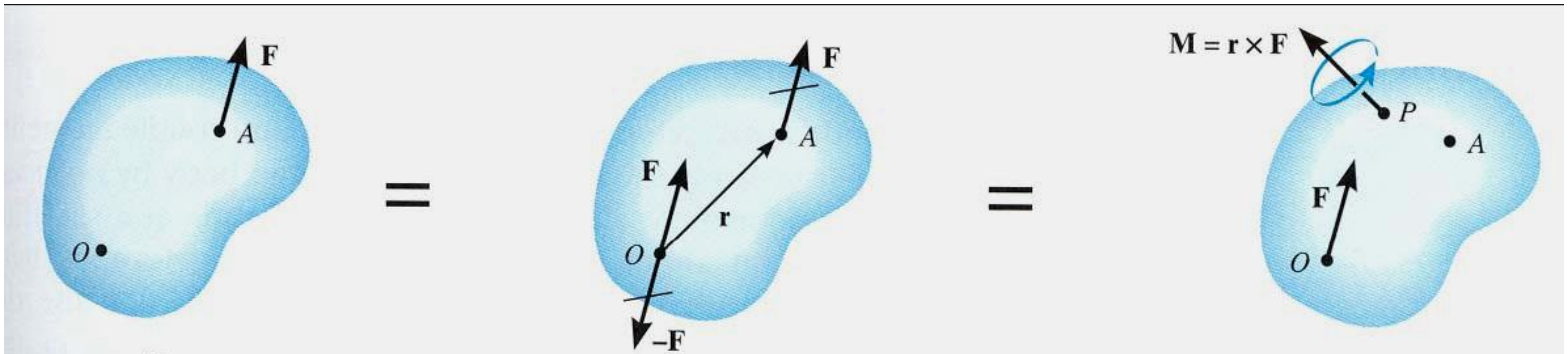
*The two force and couple systems are called **equivalent systems** since they have the same **external** effect on the body.*

Moving a Force on its Line of Action



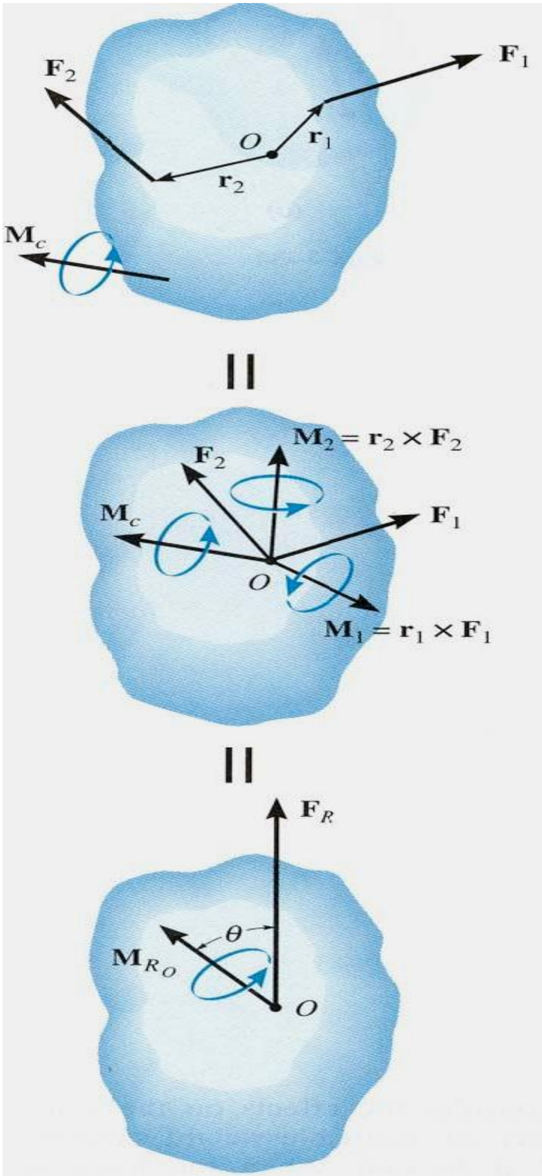
Moving a force from A to O, when both points are on the vectors' line of action, does not change the ***external effect****. Hence, a force vector is called a* ***sliding vector****. (But the internal effect of the force on the body does depend on where the force is applied).*

Moving a Force off its Line of Action



*Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a **"free" vector**, it can be applied at any point P on the body.*

Finding the Resultant of a Force and Couple System



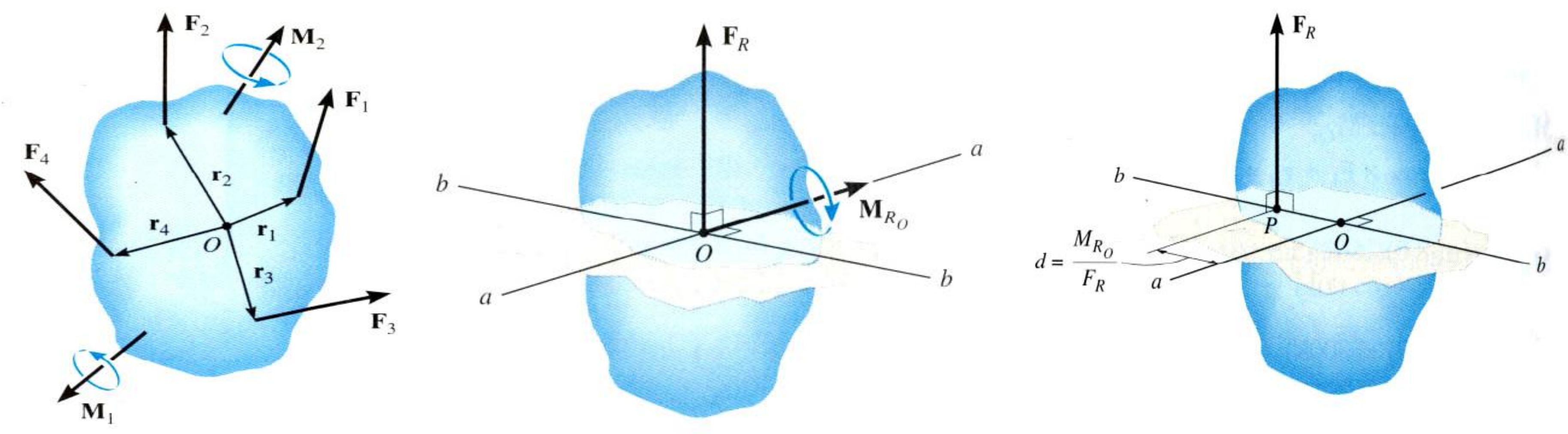
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

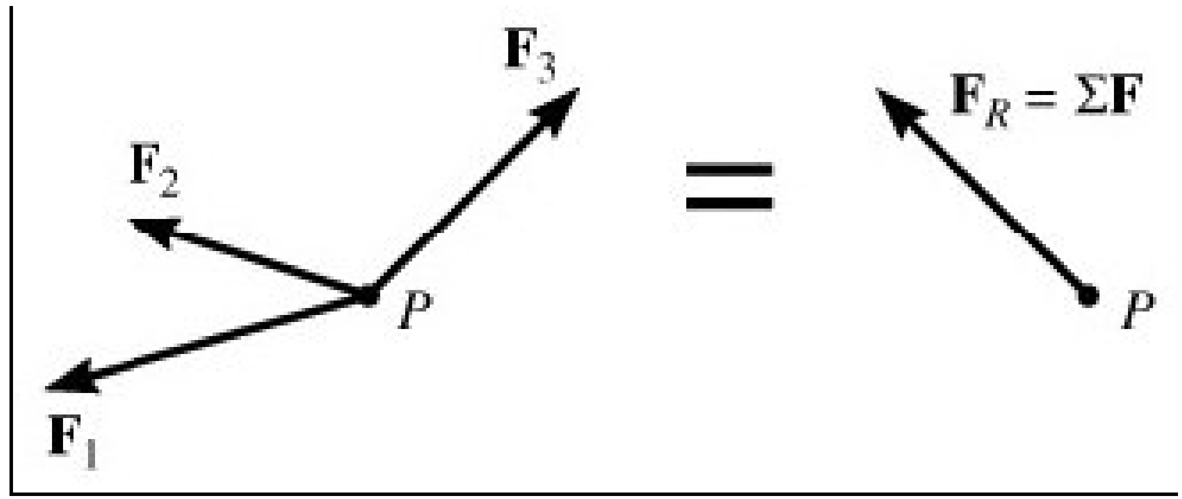
Reducing a Force Moment to a Single Force



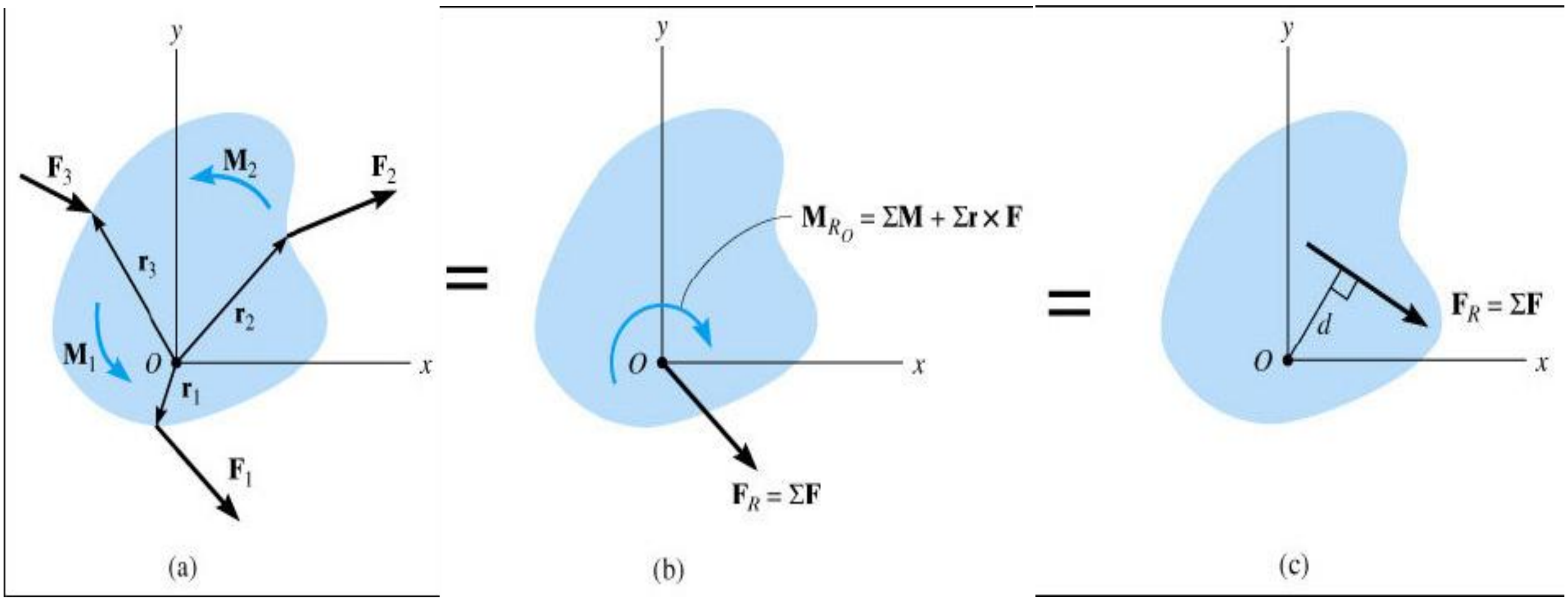
If $\boldsymbol{F_R}$ and $\boldsymbol{M_{RO}}$ are perpendicular to each other, then the system can be further reduced to a single force, $\boldsymbol{F_R}$, by simply moving $\boldsymbol{F_R}$ from O to P (at distance d).

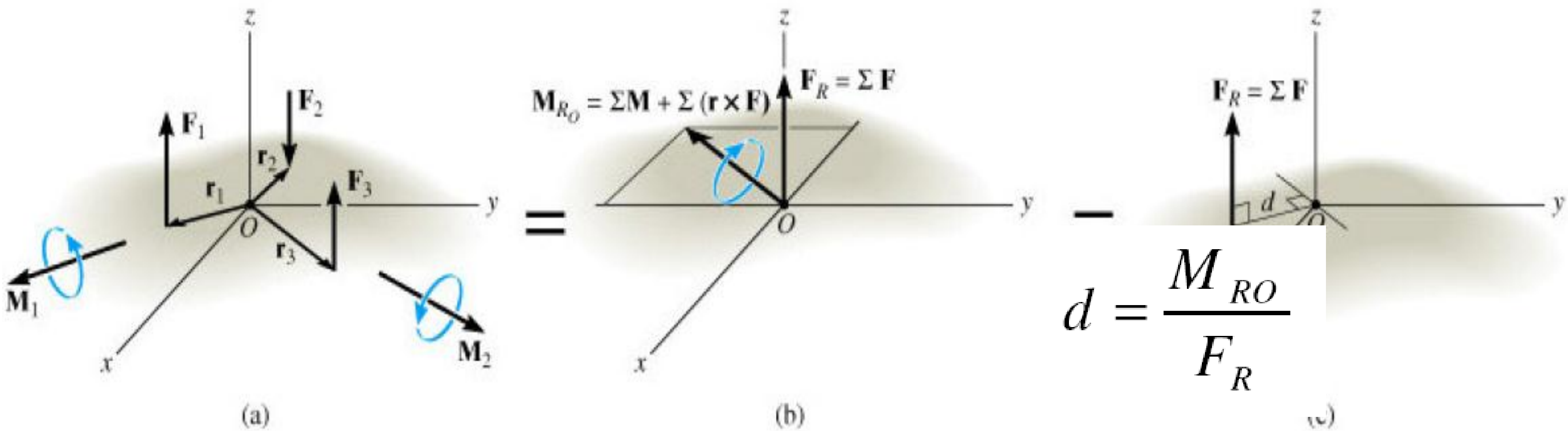
This will be true in three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

Concurrent Force Systems



Coplanar Force Systems





$$d = \frac{M_{RO}}{F_R}$$

Summary

An equivalent force system at a point o of a structure or body consists of a resultant force, $\mathbf{F}_R$*, and a resultant moment,* $\mathbf{M}_{Ro}$ *where:*

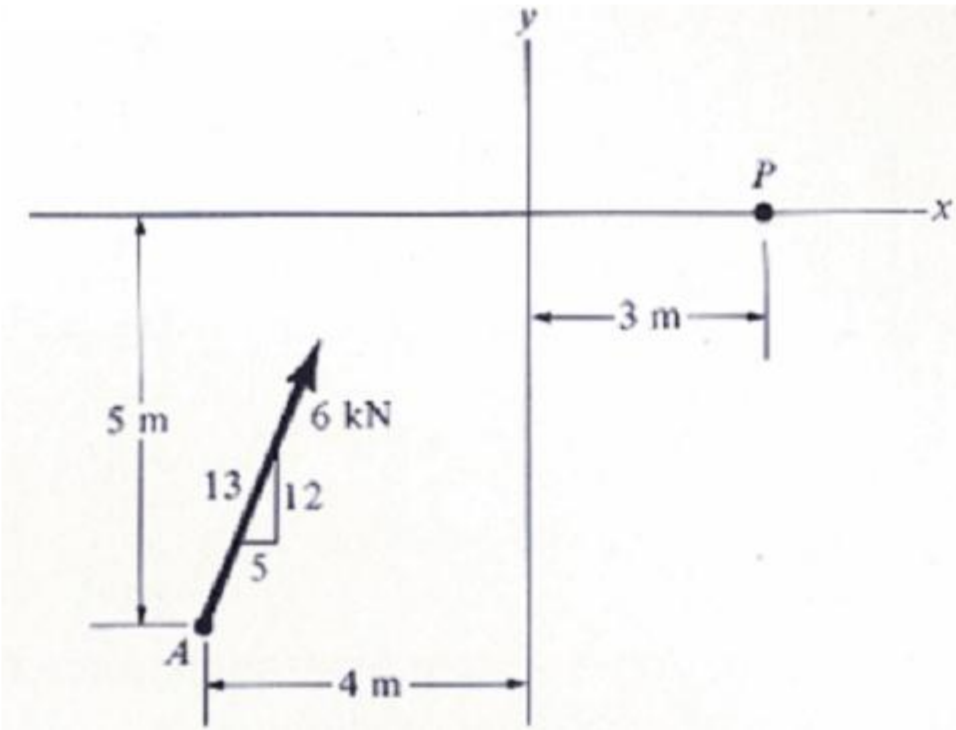
$$\mathbf{F}_R = \sum \mathbf{F} \quad \textbf{and} \quad \mathbf{M}_{Ro} = \sum \mathbf{M}_o$$

The above summations are over all external forces.

Two force systems are equivalent if:

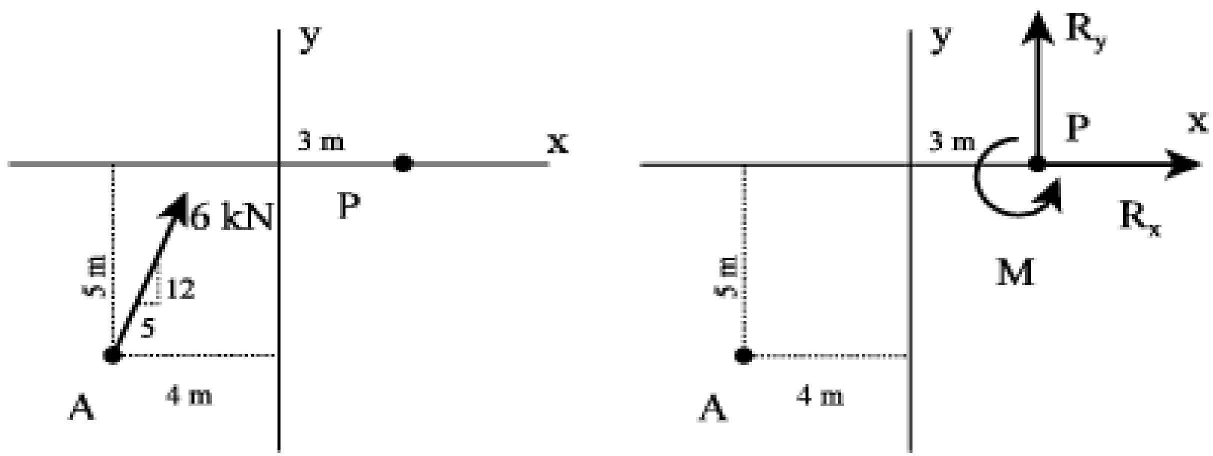
1) $\Sigma\mathbf{F}$ *is the same, and*
2) $\Sigma\mathbf{M}$ *about an arbitrary point is the same.*

Example 1



Replace the force at A by an equiavelent force and couple moment at point P.

Solution



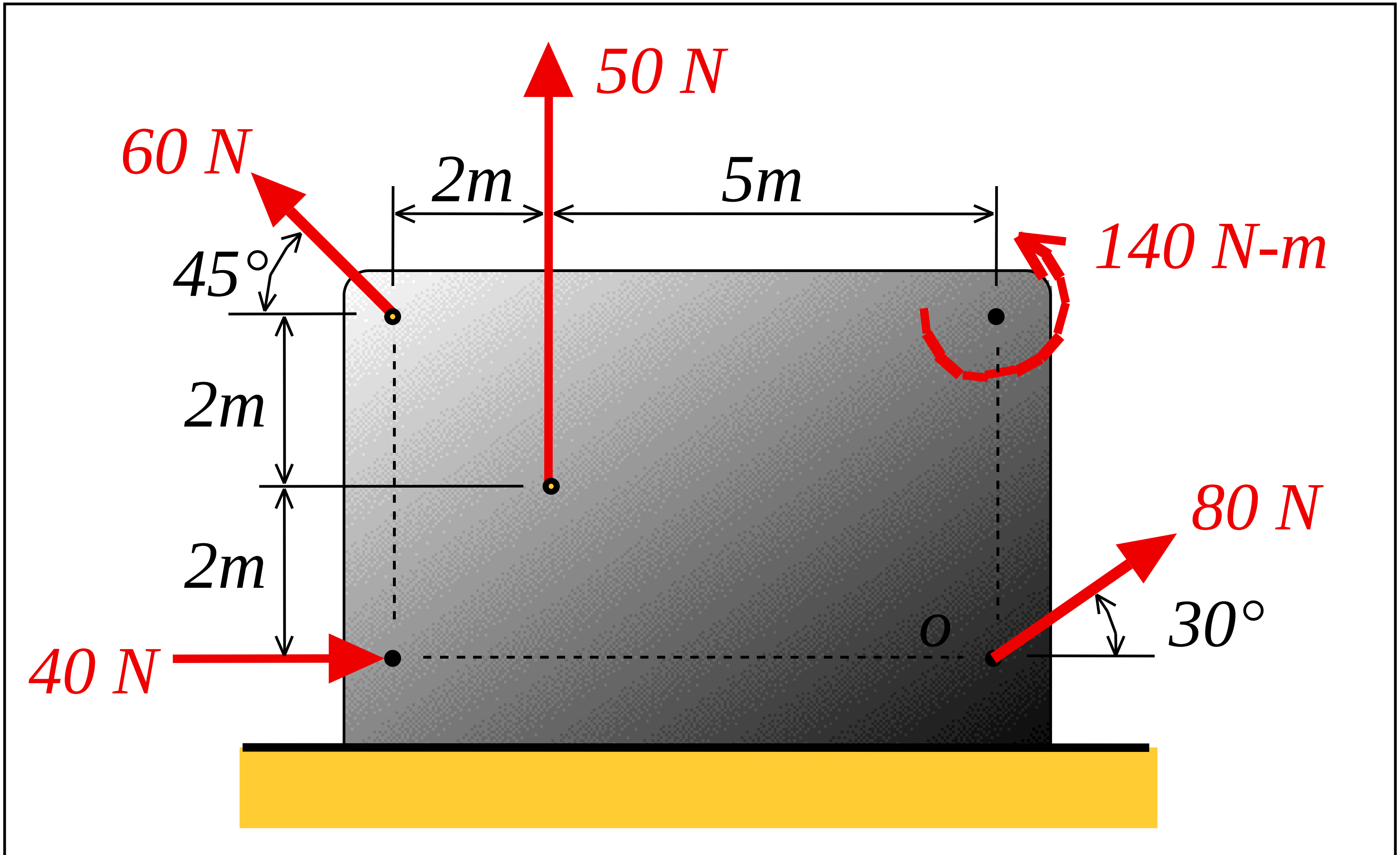
Our job is to make the two pictures represent equivalent force-couple systems. If the systems are equivalent then they must have the same net force and the same moment about any point. Thus we have to satisfy the following equations:

$$\sum F_x: \tfrac{5}{13} 6\,kN = Rx \qquad\qquad R_x = 2.31\,N$$
$$\sum F_y: \tfrac{12}{13} 6\,kN = Ry \qquad \Rightarrow \qquad R_y = 5.54\,N$$
$$\sum M_P: \left(\tfrac{5}{13} 6\,kN\right)(5m) - \left(\tfrac{12}{13} 6\,kN\right)(7\,m) = M \qquad\qquad M = -27.2\,Nm$$

Thus the force-couple system at P is:

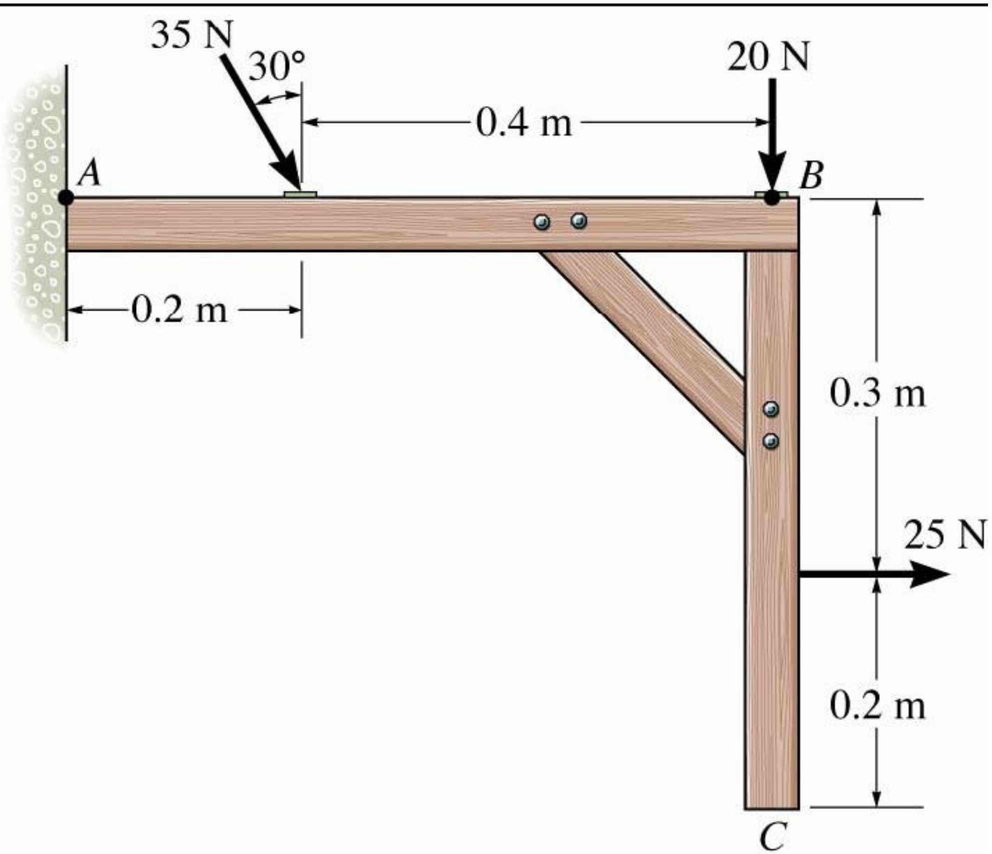
$$\mathbf{R} = \{2.31\mathbf{i} + 5.54\,\mathbf{j}\}\,N; \quad M = 27.2\,Nm \quad CW$$

Example 2: *Determine an equivalent force system that acts at point O.*



A: $F_{Rx} = 66.9$ N, $F_{Ry} = 132$ N, $M_{RO} = 237$ N-m cw.

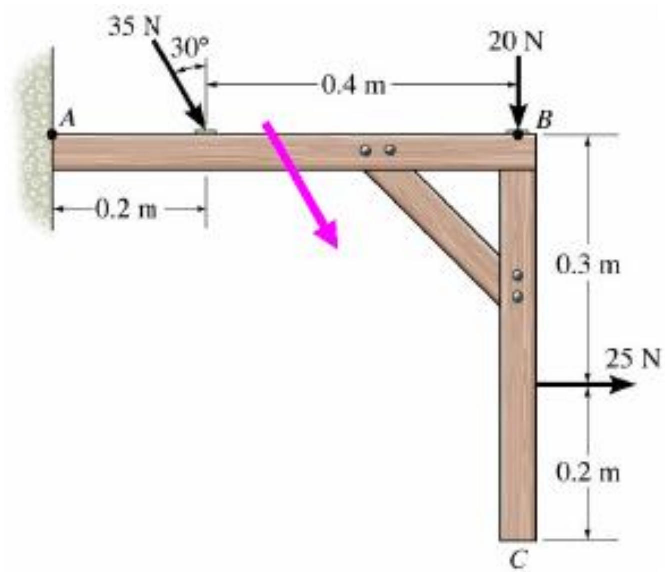
Example 3



Given: *A 2-D force and couple system as shown.*

Find: *The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.*

Solution



$$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5\ N$$

$$+\downarrow \Sigma F_{Ry} = 20 + 35 \cos 30° = 50.31\ N$$

$$+\circlearrowleft M_{RA} = 35 \cos 30°(0.2) + 20(0.6) - 25(0.3)$$

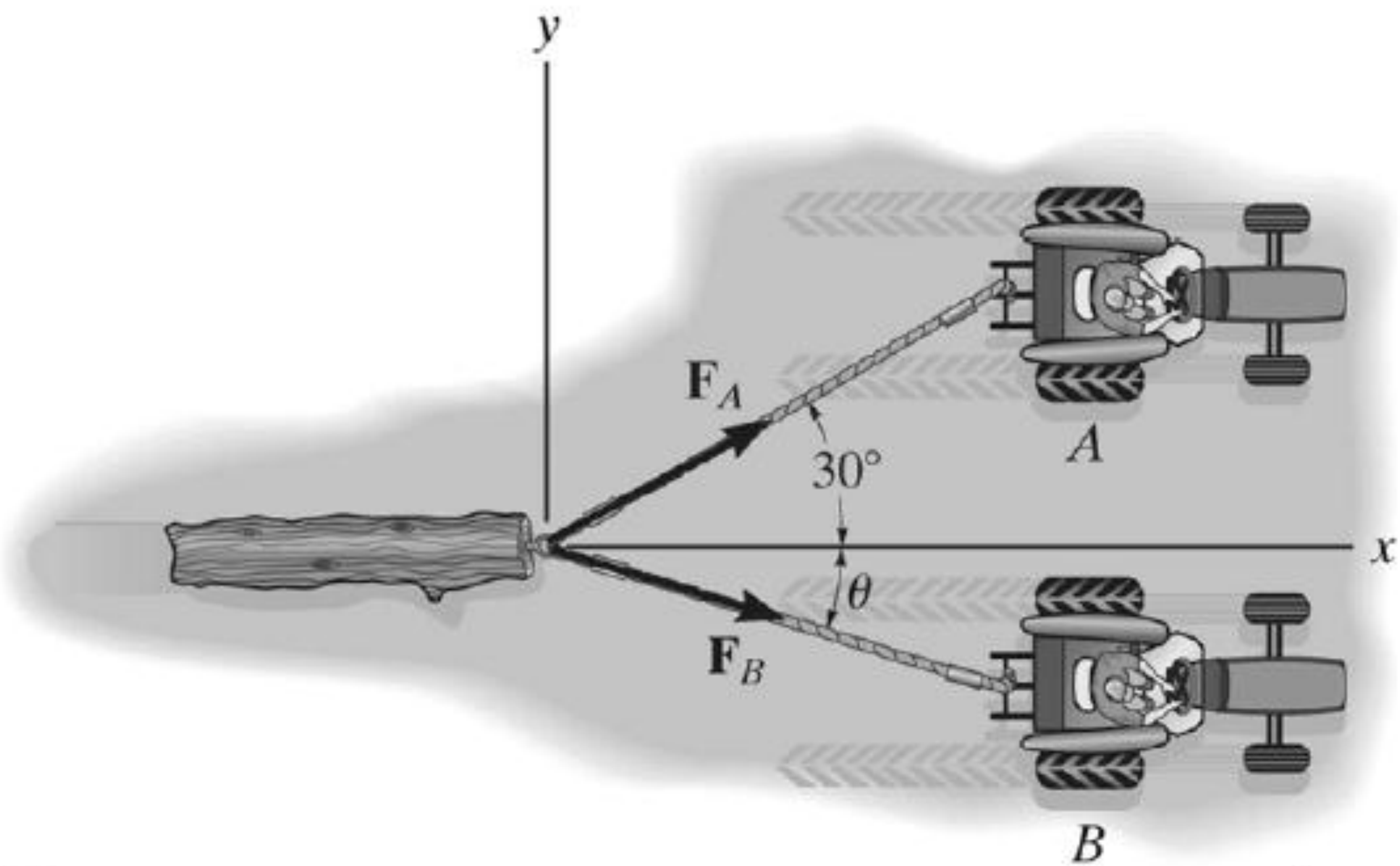
$$= 10.56\ N.m$$

$$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9\ N$$

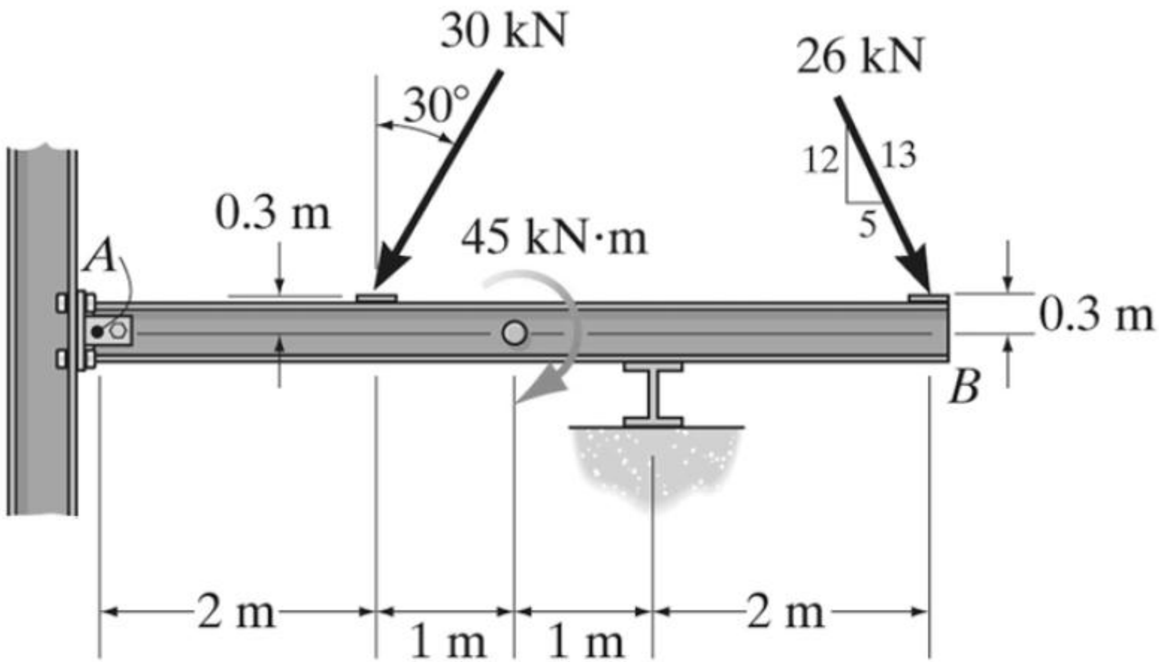
$$\searrow\ \theta = \tan^{-1}(50.31/42.5) = 49.8°$$

The equivalent single force F_R can be located on the beam AB at a distance d measured from A.

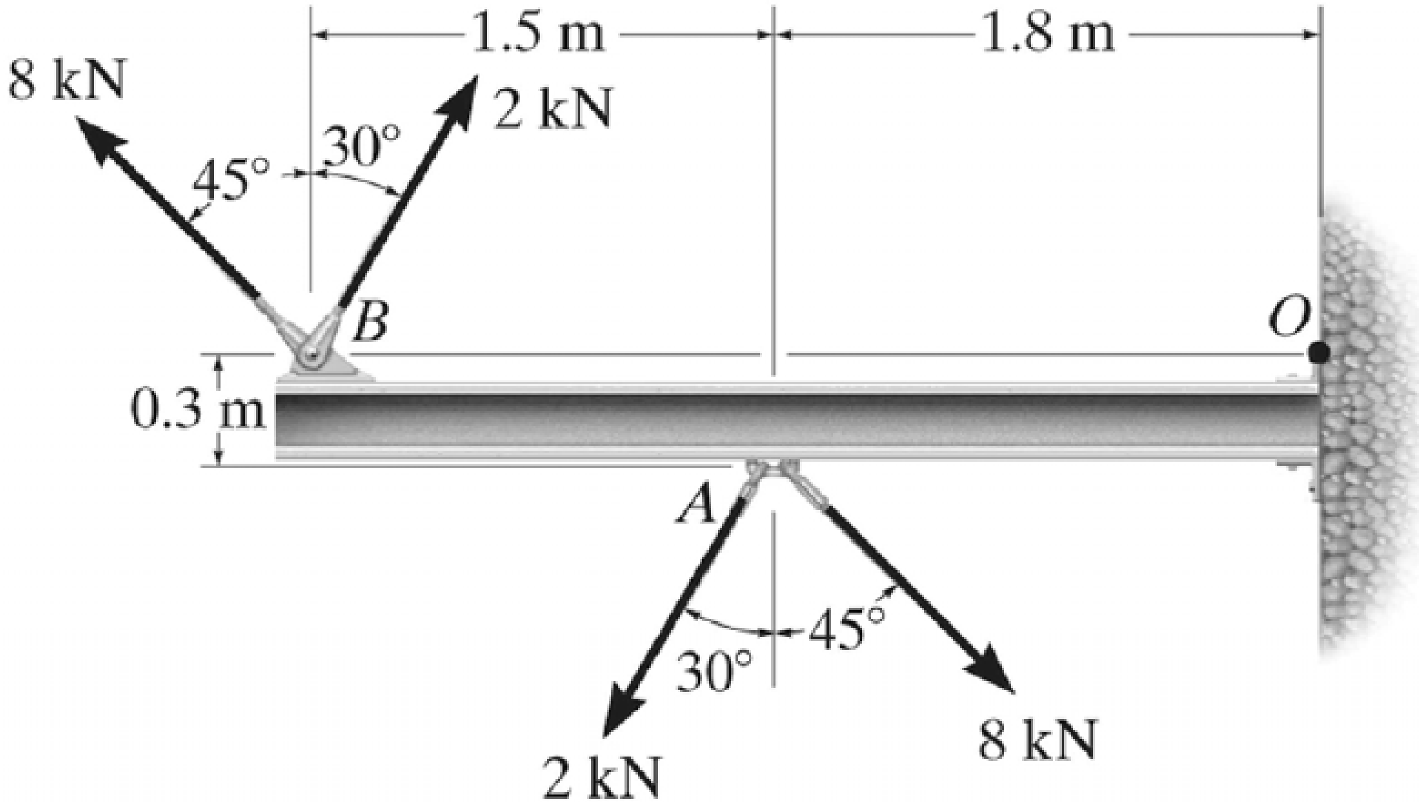
$$d = M_{RA}/F_{Ry} = 10.56/50.31 = 0.21\ m.$$



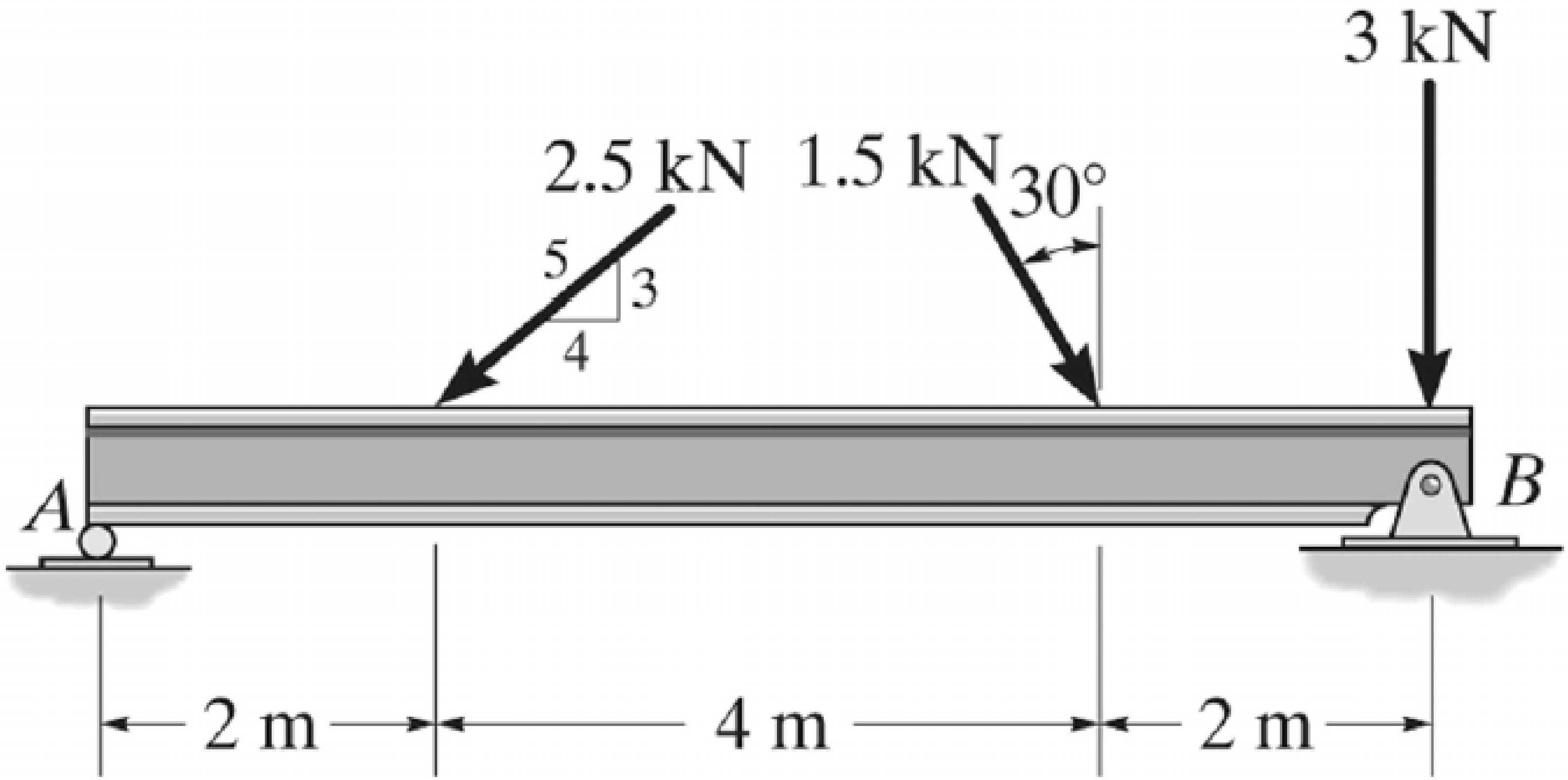
The log is being towed by two tractors A and B. Determine the magnitudes of the two towing forces $\mathbf{F}_A$ and $\mathbf{F}_B$ if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the x axis. Set $\theta = 15°$.



Replace the force and couple moment system acting on the overhang beam by a resultant force, and specify its location along AB measured from point A.



Determine the resultant couple moment acting on the beam. Solve the problem two ways: (a) sum moments about point O; and (b) sum moments about point A.



Replace the force system acting on the beam by an equivalent force and couple moment at point A.

Replace the force system acting on the beam by an equivalent force and couple moment at point B.