ME209 - Numerical Methods

Lecture 1: Introduction

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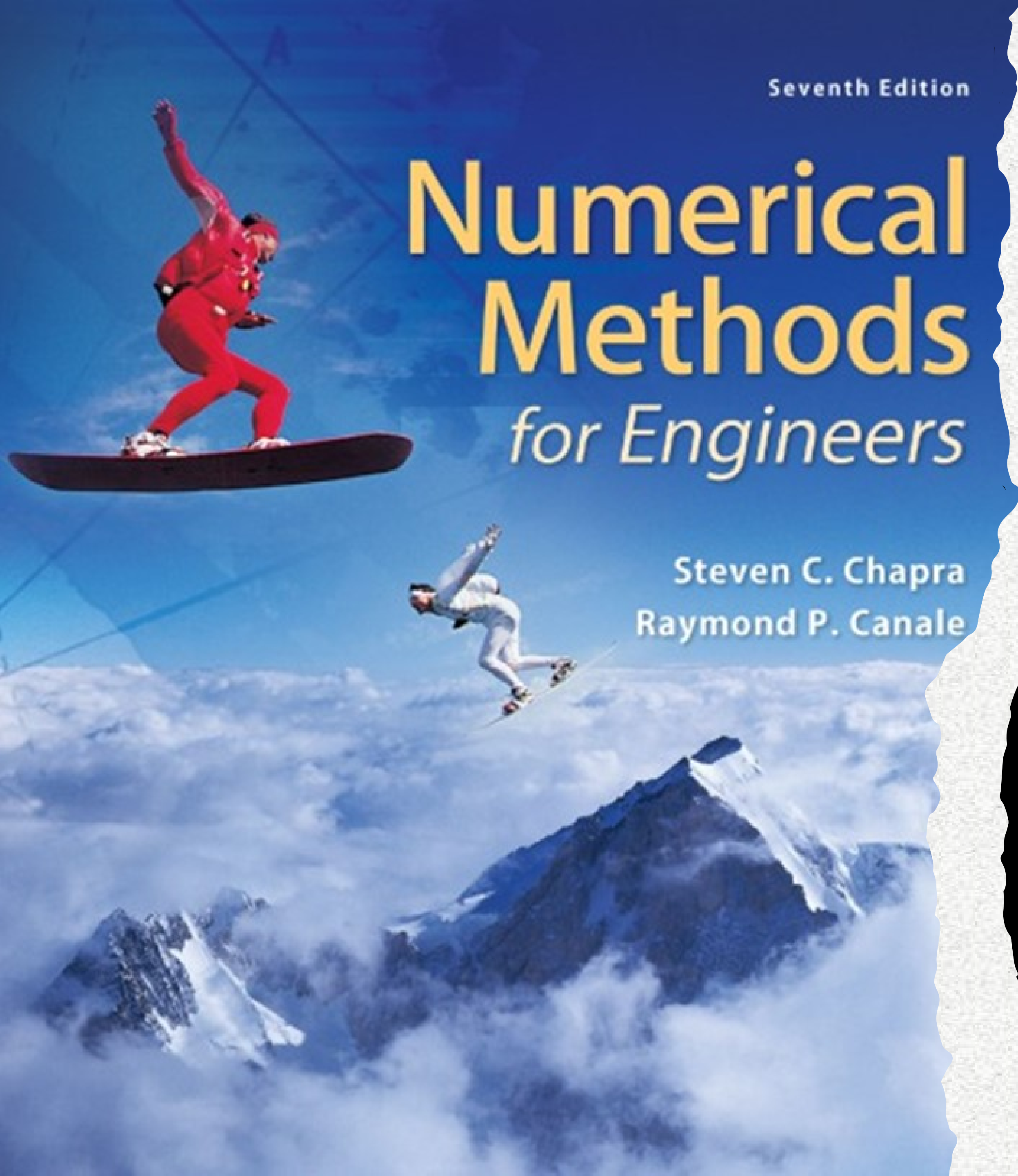


Lecture Book

S.C. Chapra and R.P. Canale, *Numerical Methods for Engineers 7th Edition*, Mc Graw-Hill Education, 2015.

Grading

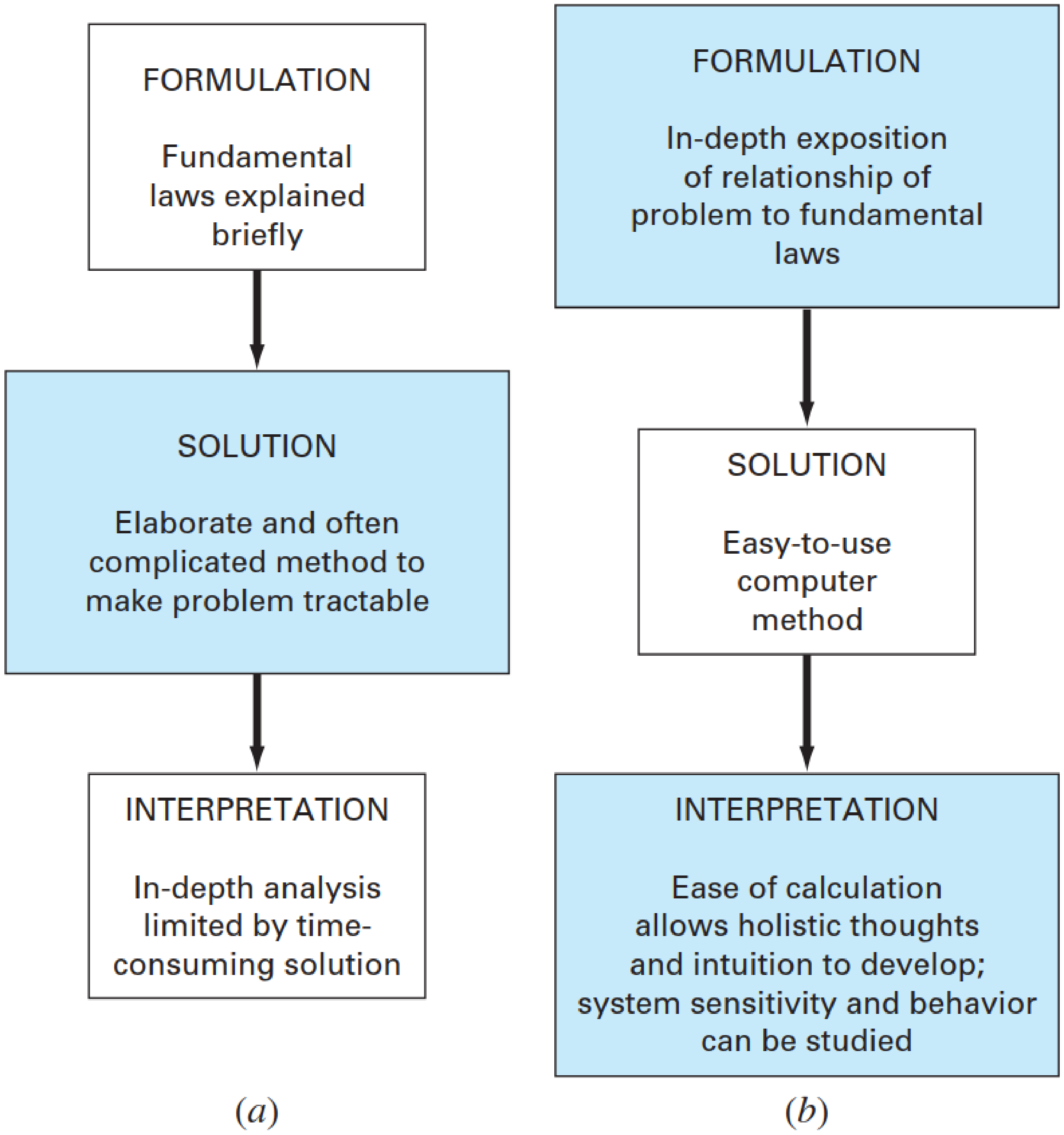
Midterm 1 (30%) +
Midterm 2 (30%) +
Final Exam (40%)

Numerical Methods - Definition

- Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations.
- Numerical methods involve large numbers of tedious arithmetic calculations.
- These methods have gained popularity due to the advances in efficient computational tools such as digital computers and calculators.

Noncomputer Methods

- Solutions were derived for some problems using **analytical**, or **exact**, methods.
- **Graphical** solutions used to solve complex problems but the results are not very precise. They are extremely tedious and awkward to implement without aid of computers. graphical techniques are often limited to problems that can be described using three or fewer dimensions.
- **Calculators** and slide rules were used to implement numerical methods manually. Manual calculations are slow and tedious.



The three phases of engineering problem solving in
(a) the precomputer and
(b) the computer era.

Analytical vs. Numerical Methods

- **Examples: Analytical Methods**
  - Differentiation

$$\frac{dy}{dx}\left(x^2 - \sin(x)\right) = 2x - \cos(x)$$

  - Integration

$$\int x^3 + x - e^x = \frac{x^4}{4} + \frac{x^2}{2} - e^x + c$$

  - Root(s) of an Equation

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ax}}{2a}$$

Analytical vs. Numerical Methods

Need for Numerical Methods

- In general, there are few analytical (closed-form) solutions for many practical engineering problems.
- Numerical methods can handle:
  - Large systems of equations
  - Non-linearity
  - Complicated geometries that are common in engineering practice and that are often impossible to solve analytically.

Mathematician and Engineer

The thinking of **engineers** toward mathematics has always been different from that of **mathematicians**.

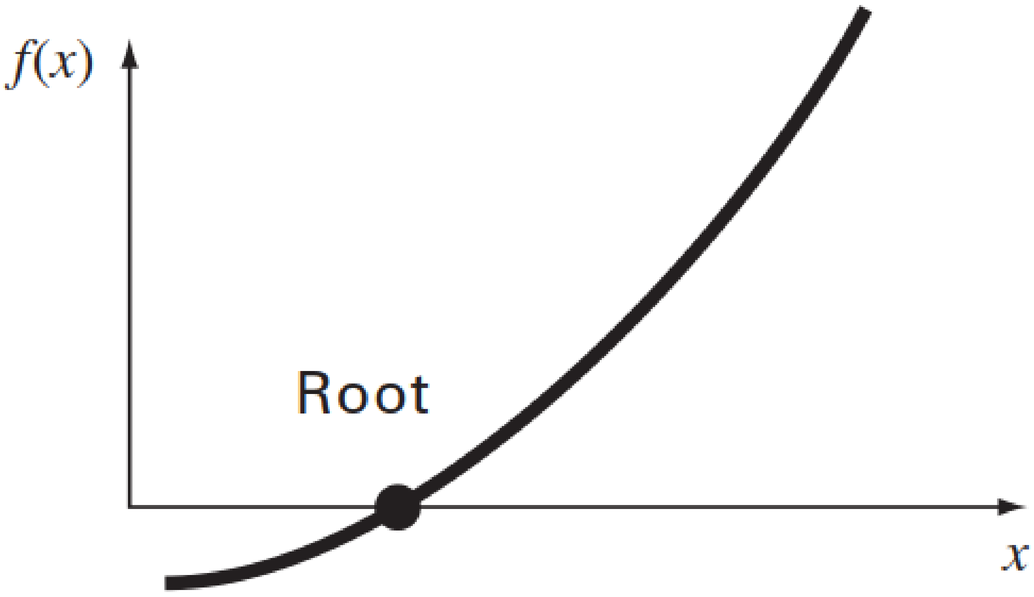
- **Mathematician** may be interested in finding out whether a solution to a differential equation exists.
- An **engineer** simply assumes that the existence of a physical system is proof enough of the existence of a solution and focuses instead on finding it.

Reasons to Study Numerical Methods

- Numerical methods are extremely powerful problem-solving tools. They are capable of handling large systems of equations, nonlinearities, and complicated geometries
- It enables you to intelligently use the commercial software packages as well as designing your own algorithm.
- Numerical Methods are efficient vehicles in learning to use computers
- It reinforce your understanding of mathematics; where it reduces higher mathematics to basic arithmetic operation.

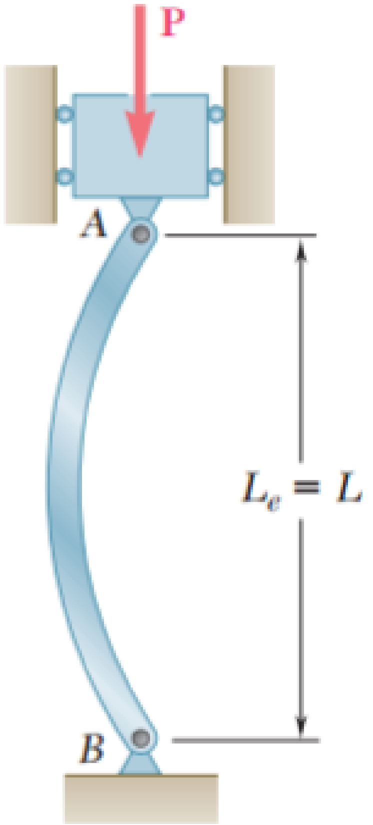
Course Contents

(*a*) *Part 2:* Roots of equations Solve $f(x) = 0$ for x.



Finding roots of nonlinear equations

- Finding the roots of nonlinear equations is a common task in mechanical engineering, especially in the context of structural analysis, control systems, and fluid dynamics.
- **Example: Structural Analysis - Buckling Load Calculation**



The critical buckling load for a slender column can be calculated using Euler's formula, which is a nonlinear equation:

$$P_{\text{critical}} = \frac{\pi^2 \cdot E \cdot I}{K^2 \cdot L^2}$$

Where:

- P_{critical} = Critical buckling load
- E = Young's modulus of the material
- I = Moment of inertia of the column's cross-sectional shape
- K = Effective length factor (depends on boundary conditions)
- L = Length of the column

Course Contents

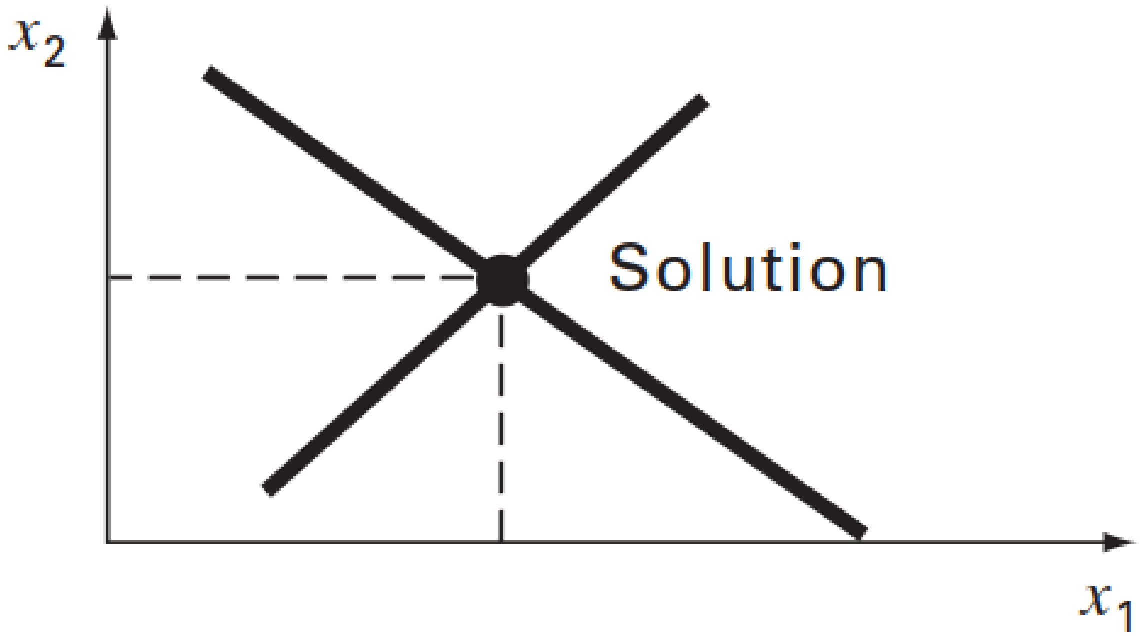
(*b*) *Part 3:* Linear algebraic equations

Given the a's and the c's, solve

$$a_{11}x_1 + a_{12}x_2 = c_1$$

$$a_{21}x_1 + a_{22}x_2 = c_2$$

for the x's.



Solving linear algebraic system of equations

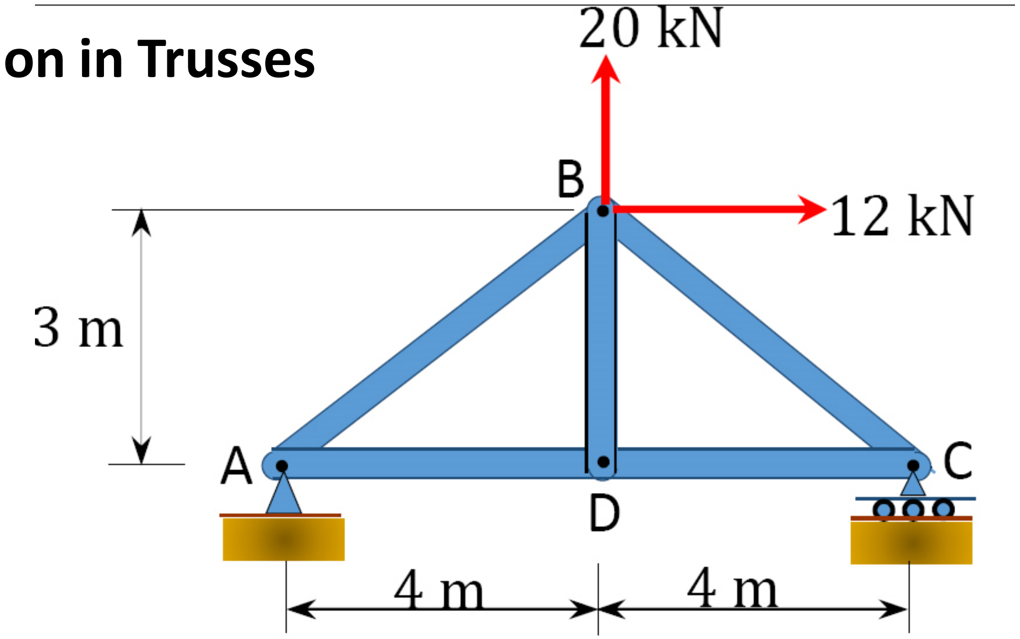
- Solving linear algebraic systems of equations is a fundamental task in mechanical engineering, as it is often used in various engineering simulations and analyses.
- **Example: Structural Analysis - Internal Load Calculation in Trusses**

The truss structure consists of interconnected bars and nodes, and the engineer wants to determine the internal forces (tensions and compressions) in each bar when subjected to external loads.

To solve this problem, the engineer can create a set of linear equations based on the principles of static equilibrium for each node in the truss. The equations relate the forces acting on each node to zero in both the horizontal and vertical directions.

The equations can be written in matrix form as follows:

$$Ax = b$$

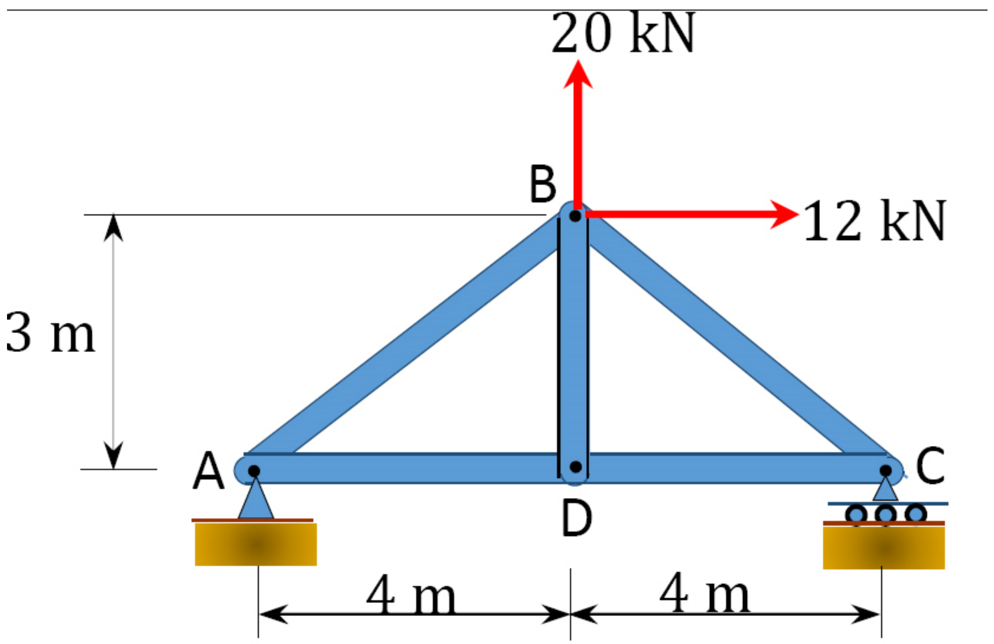


Solving linear algebraic system of equations

- **Example: Structural Analysis - Internal Load Calculation in Trusses**

$$Ax = b$$

Where:
•*A* is the coefficient matrix representing the connectivity of the truss members.
•*x* is the vector of unknown forces in each truss member.
•*b* is the vector of external loads applied to the truss.



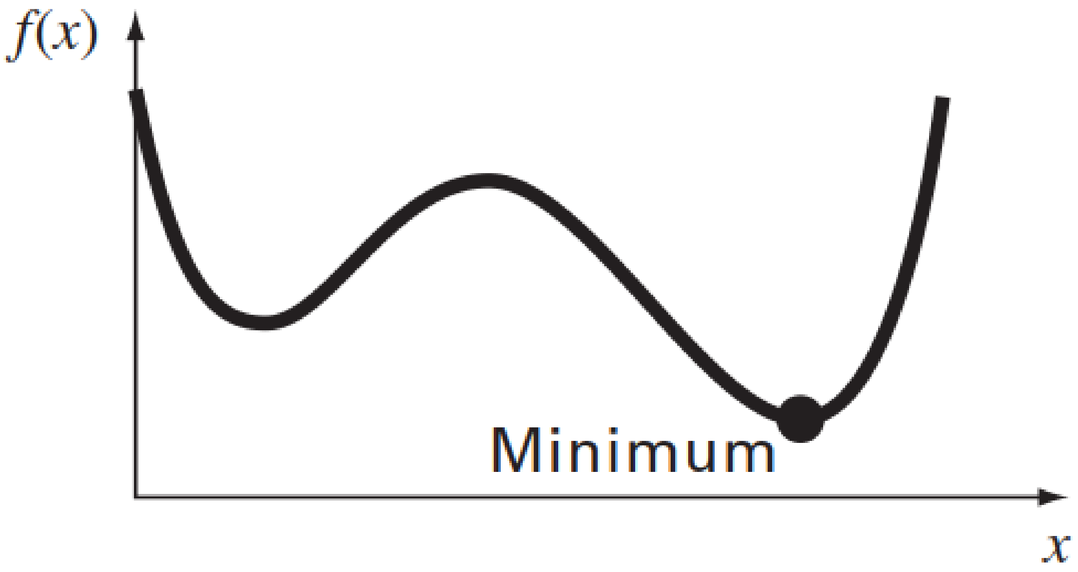
Once the internal forces are known, the engineer can assess whether any member is in tension or compression and check if they meet the design criteria for safety. This analysis is crucial for ensuring the truss structure's stability and integrity under the applied loads.

Similar linear algebraic techniques are applied in various other mechanical engineering applications, such as finite element analysis, heat transfer calculations, and control systems design

Course Contents

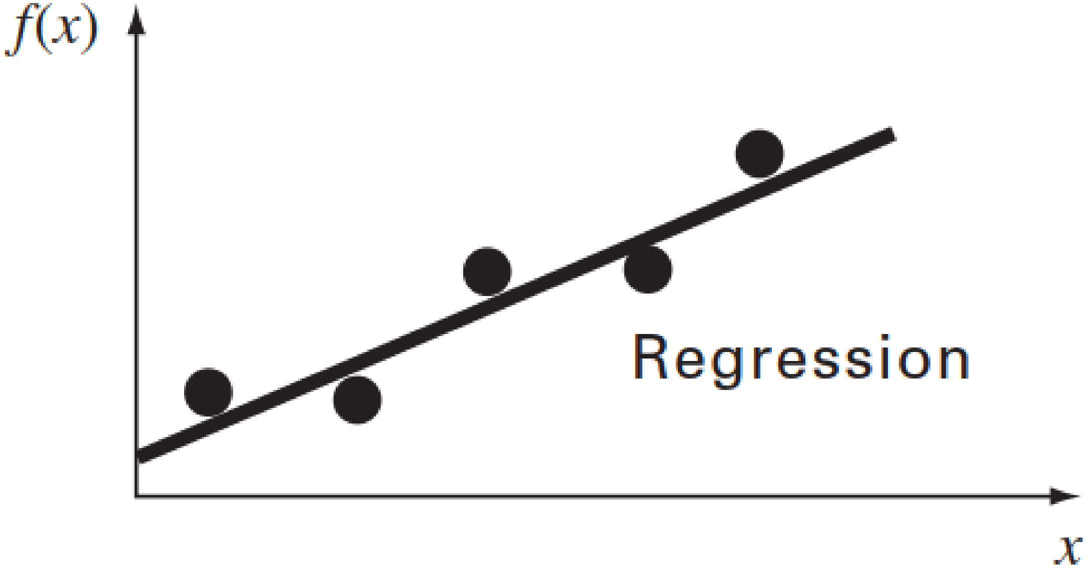
(*c*) *Part 4:* Optimization

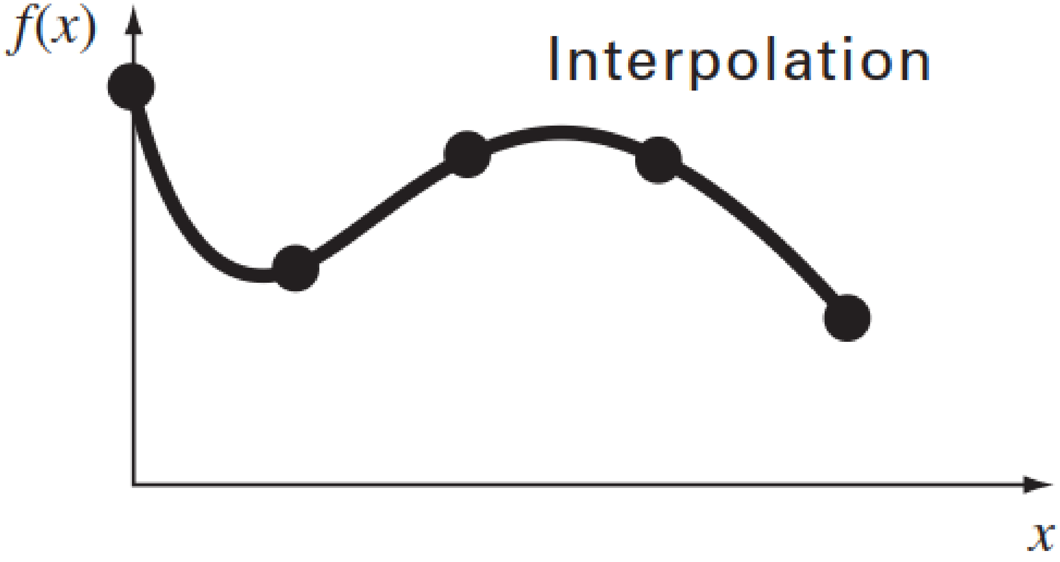
Determine x that gives optimum $f(x)$.



Course Contents

(d) *Part 5:* Curve fitting



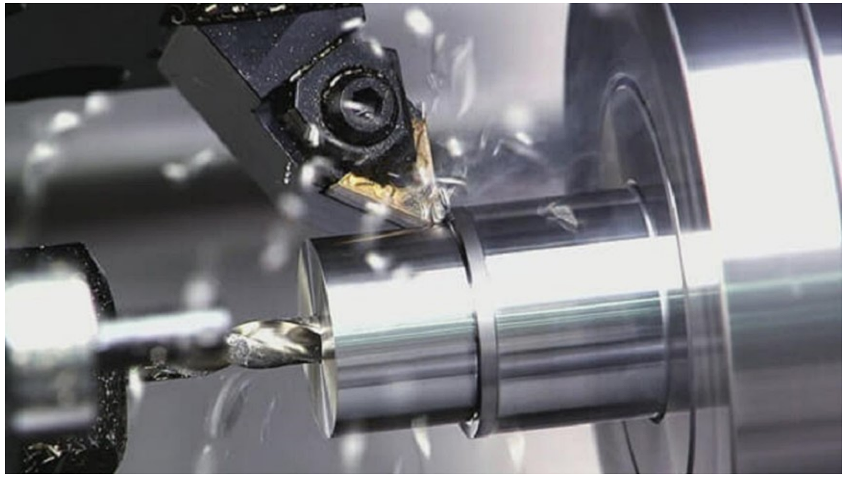


Curve Fitting

- Curve fitting is a valuable technique in mechanical engineering for modeling and analyzing experimental data, as well as for designing and optimizing mechanical systems.

Example: Constructing a Tool Life Equation

The following results were obtained from experiments done while milling AISI-4140 steel using fixed values for feed rate and depth of cut.



Cutting speed , V (m/min)	160	180	200	220	240
Tool life, T (min)	7.0	5.5	5.0	3.5	2.0

To determine the tool life equaation method of least-square can be used. By using this equation, tool life of the turning machine can be predicted.

$$VT^a = b$$

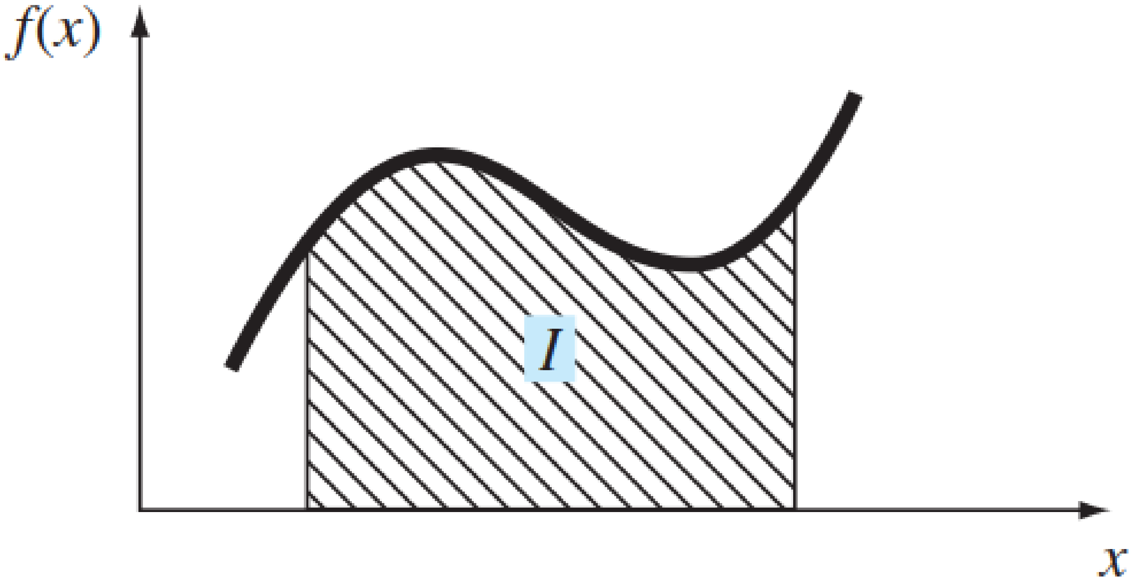
Where *a* and *b* are constants.

Course Contents

(*e*) *Part 6:* Integration

$I = \int_a^b f(x)\, dx$

Find the area under the curve.

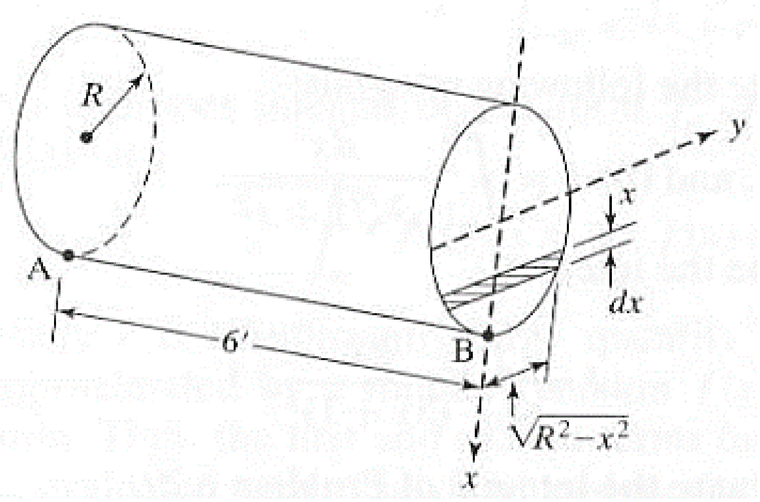


Numerical Integration

Numerical integration is widely used in mechanical engineering for various applications.

Example: The force exerted by the fluid on the circular side

A closed cylindrical barrel, of radius *R* and length *L*, is half full of a fluid with a density ρ and lies on the ground on the edge AB as shown.



The force exerted by the fluid on the circular side is given by

$$F = \int_0^R 2\rho(\sqrt{R^2 - x^2}).x.dx$$

Course Contents

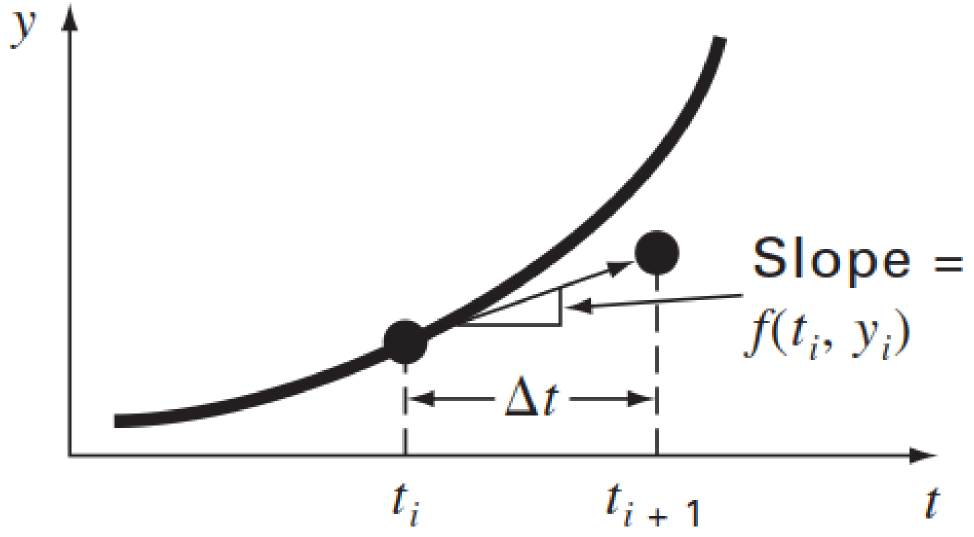
(*f*) *Part 7:* Ordinary differential equations

Given

$$\frac{dy}{dt} \simeq \frac{\Delta y}{\Delta t} = f(t, y)$$

solve for y as a function of t.

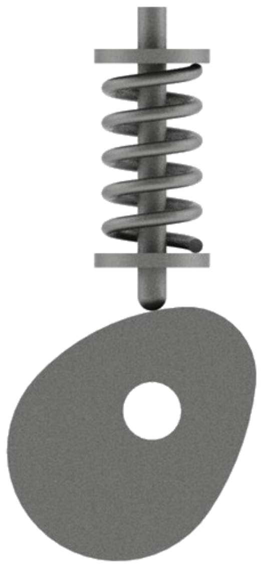
$$y_{i+1} = y_i + f(t_i, y_i)\,\Delta t$$



Numerical Differentiation

- Numerical differentiation plays a crucial role in mechanical engineering for various applications, particularly when dealing with experimental data or numerical simulations.

Example: Velocity and Acceleration Analysis in Mechanism Design



Consider a mechanical engineer working on the design of a cam-follower mechanism. In this mechanism, a cam rotates, and a follower connected to it moves up and down.
The engineer wants to determine the velocity and acceleration of the follower as it moves over a complete cycle of cam rotation.

To analyze the motion, the engineer collects position data for the follower as a function of time. This data is typically obtained experimentally or through simulation.

Numerical Differentiation

Example: Velocity and Acceleration Analysis in Mechanism Design

Numerical Differentiation for Velocity: To find the velocity of the follower at each time point, the engineer can use numerical differentiation techniques, such as finite differences. The first-order finite difference formula for velocity is:

$$V(t) = \frac{y(t + \Delta t) - y(t)}{\Delta t}$$

Numerical Differentiation for Acceleration: Similarly, to find the acceleration of the follower at each time point, the engineer can apply numerical differentiation once more. The second-order finite difference formula for acceleration is:

$$a(t) = \frac{V(t + \Delta t) - V(t)}{\Delta t}$$

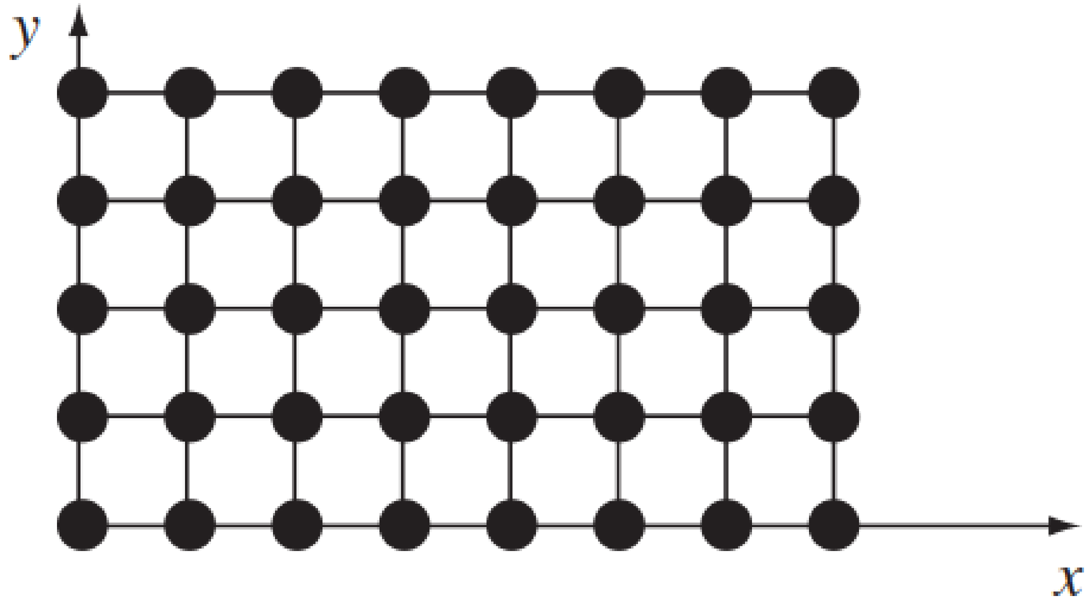
Course Contents

(g) *Part 8:* Partial differential equations

Given

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

solve for u as a function of x and y



03.10.2023	Basic definitions in numerical analysis	ch1
10.10.2023	Numerical error analysis - Taylor series	CH3-4
17.10.2023	Finding the roots of equations: Bisection method	ch5-6
24.10.2023	Finding the roots of equations: Newton-Raphson - Secant method	ch6-7
31.10.2023	Solutions of Linear Equation Systems	ch9
07.11.2023	Solutions of Linear Equation Systems	ch10-11
14.11.2023	Midterm 1	
21.11.2023	One-Dimensional Unconstrained Optimization	ch13
28.11.2023	Least Squares Regression	ch17
05.12.2023	Polynomial Interpolation	ch18
12.12.2023	Numerical integration: Trapezoid Method - Simpson Method	ch21
19.12.2023	Integration of equations	ch22
26.12.2023	Midterm 2	
02.01.2024	Numerical differentiation	ch23

Chapter 1
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Mathematical Modeling

A Simple Mathematical Model

- A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms.
- Generally, it can be represented as a functional relationship of the form

$$\begin{matrix}\text{Dependent} \\ \text{variable}\end{matrix} = f\left(\begin{matrix}\text{independent} \\ \text{variables}\end{matrix}, \text{parameters}, \begin{matrix}\text{forcing} \\ \text{functions}\end{matrix}\right)$$

A Simple Mathematical Model

$$\begin{matrix}\text{Dependent} \\ \text{variable}\end{matrix} = f\left(\begin{matrix}\text{independent} \\ \text{variables}\end{matrix}, \text{parameters}, \begin{matrix}\text{forcing} \\ \text{functions}\end{matrix}\right)$$

- Dependent variable is a characteristic that usually reflects the behavior or state of the system;
- Independent variables are usually dimensions, such as time and space, along which the system's behavior is being determined;
- Parameters are reflective of the system's properties or composition;
- Forcing functions are external influences acting upon the system.

A Simple Mathematical Model

- **Example: Newton's Second Law of Motion**
  - the time rate of change of momentum of a body is equal to the resultant force acting on it

$$F = ma \qquad a = \frac{F}{m}$$

- F = net **force** acting on the body (N, or kg m/s2), **Forcing function**
- m = mass of the object (kg), **The parameter** representing a property of the system
- a = its acceleration (m/s2). **The dependent variable**

Typical of mathematical models of the physical world:

- It describes a natural process or system in mathematical terms.
- It represents an idealization and simplification of reality.
- It yields reproducible results and can be used for predictive purposes.

Because of its simple algebraic form, the solution of Eq. (1.2) can be obtained easily. However, other mathematical models of physical phenomena may be much more complex, and either cannot be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solution.

Complex Mathematical Model

- **Example: Newton's Second Law of Motion**

A model for this case can be derived by expressing the acceleration as the time rate of change of the velocity (dydt) and substituting it into Eq. (1.3) to yield

$$\frac{dv}{dt} = \frac{F}{m}$$

where y is velocity (m/s) and *t* is time (s).

- If the net force is positive, the object will accelerate.
- If it is negative, the object will decelerate.
- If the net force is zero, the object's velocity will remain at a constant level.

Schematic diagram of the forces acting on a falling parachutist. FD is the downward force due to gravity. FU is the upward force due to air resistance.

Complex Mathematical Model

- the net force $F = F_D + F_U$

- the force due to gravity $F_D = mg$
  - g = gravitational constant, 9.81 (m/s^2)

- Air resistance $F_U = -cv$
  - c = drag coefficient (kg/s)

Complex Mathematical Model

$$\frac{dv}{dt} = \frac{F}{m} \longrightarrow \frac{dv}{dt} = \frac{mg - cv}{m} \longrightarrow \frac{dv}{dt} = g - \frac{c}{m}v$$

More advanced techniques, such as those of calculus, must be applied to obtain an exact or analytical solution.

For example, if the parachutist is initially at rest ($v = 0$ at $t = 0$), calculus can be used to solve equation

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

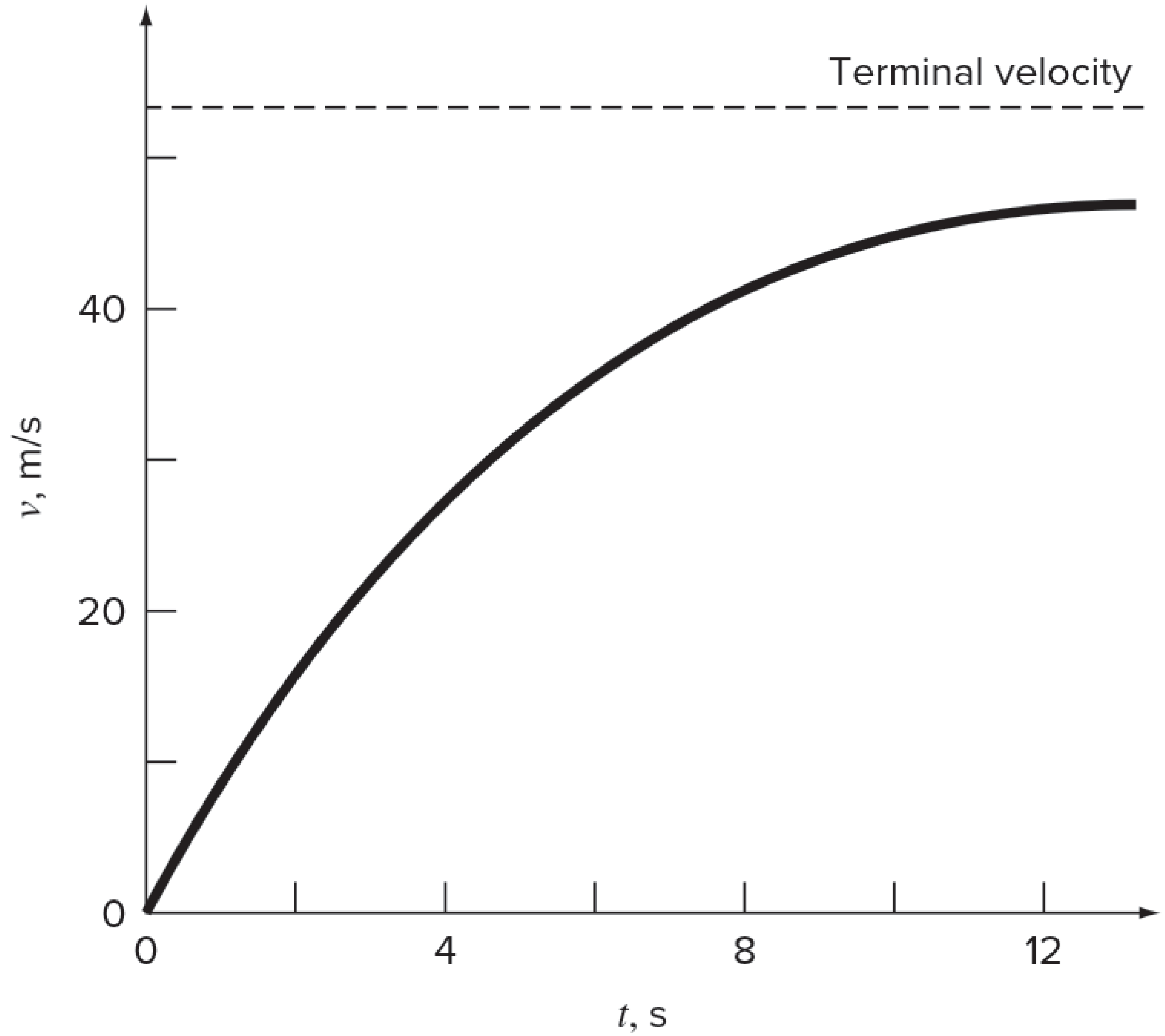
where $v(t)$ = the dependent variable, t = the independent variable, c and m = parameters, and g = the forcing function.

Analytical Solution to the Falling Parachutist Problem

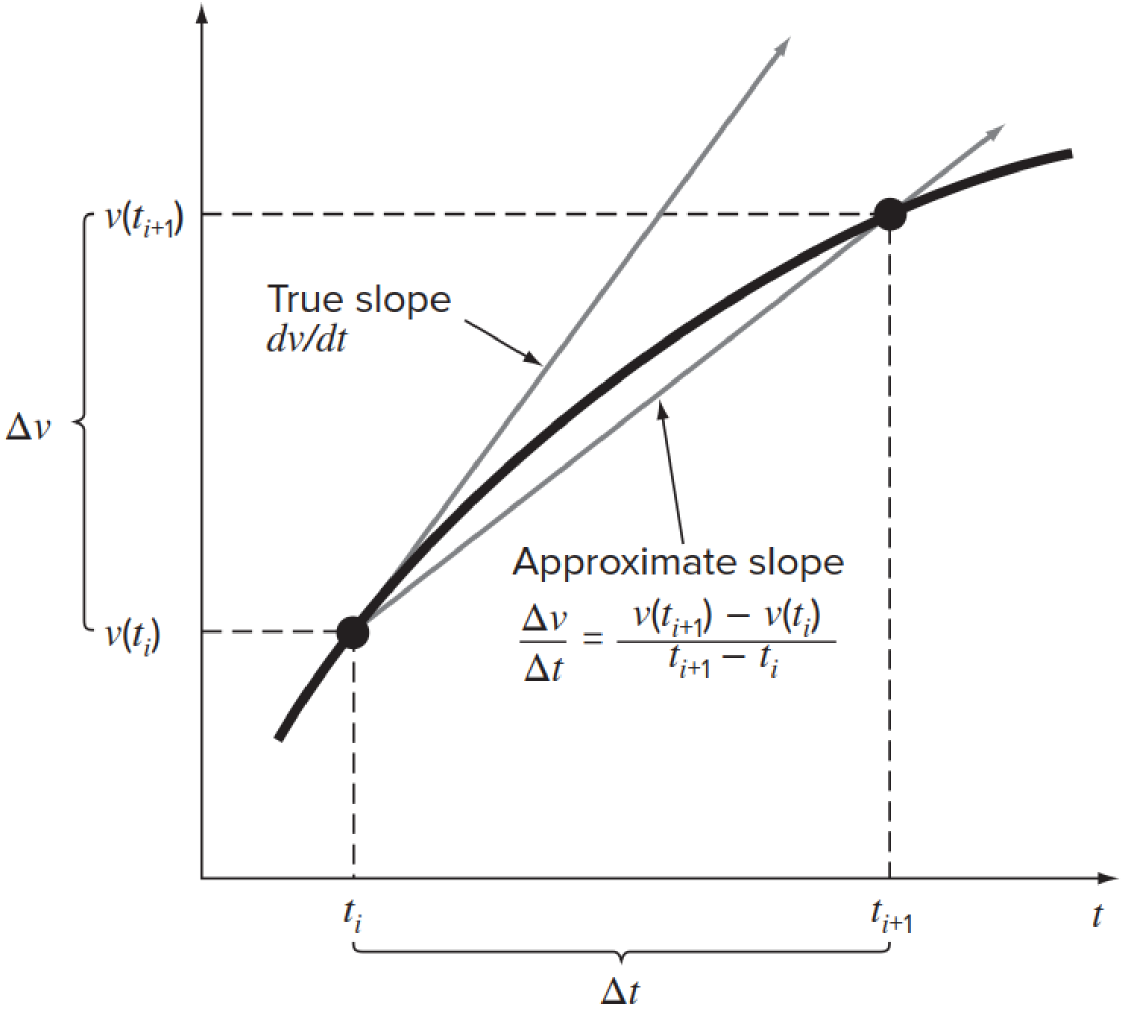
- **Problem Statement.** A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. Use Eq. (1.10) to compute velocity prior to opening the chute. The drag coefficient is equal to 12.5 kg/s.
- **Solution.**

$$v(t) = \frac{9.81(68.1)}{12.5}\left(1 - e^{-(12.5/68.1)t}\right) = 53.44\left(1 - e^{-0.18355t}\right)$$

t, s	*v*, m/s
0	0.00
2	16.42
4	27.80
6	35.68
8	41.14
10	44.92
12	47.54
∞	53.44



Numerical Solution



$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

$v(t_i)$ = velocity at an initial time t_i,

$v(t_{i+1})$ = velocity at some later time t_{i+1}

from calculus $$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} v(t_i)$$

equation can then be rearranged to yield

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m} v(t_i) \right] (t_{i+1} - t_i)$$

If you are given an initial value for velocity at some time t_i, you can easily compute velocity at a later time t_{i+1}. This new value of velocity at t_{i+1} can in turn be employed to extend the computation to velocity at t_{i+2} and so on. Thus, at any time along the way,

New value = old value + slope × step size

Note that this approach is formally called *Euler's method.*

Numerical Solution to the Falling Parachutist Problem

- Problem Statement. Perform the same computation as in Example 1.1 but use Eq. (1.12) to compute the velocity. Employ a step size of 2 s for the calculation.
- Solution. At the start of the computation ($t_i = 0$), the velocity of the parachutist is zero. Using this information and the parameter values from Example 1.1, Eq. (1.12) can be used to compute velocity at $t_{i+1} = 2$ s:

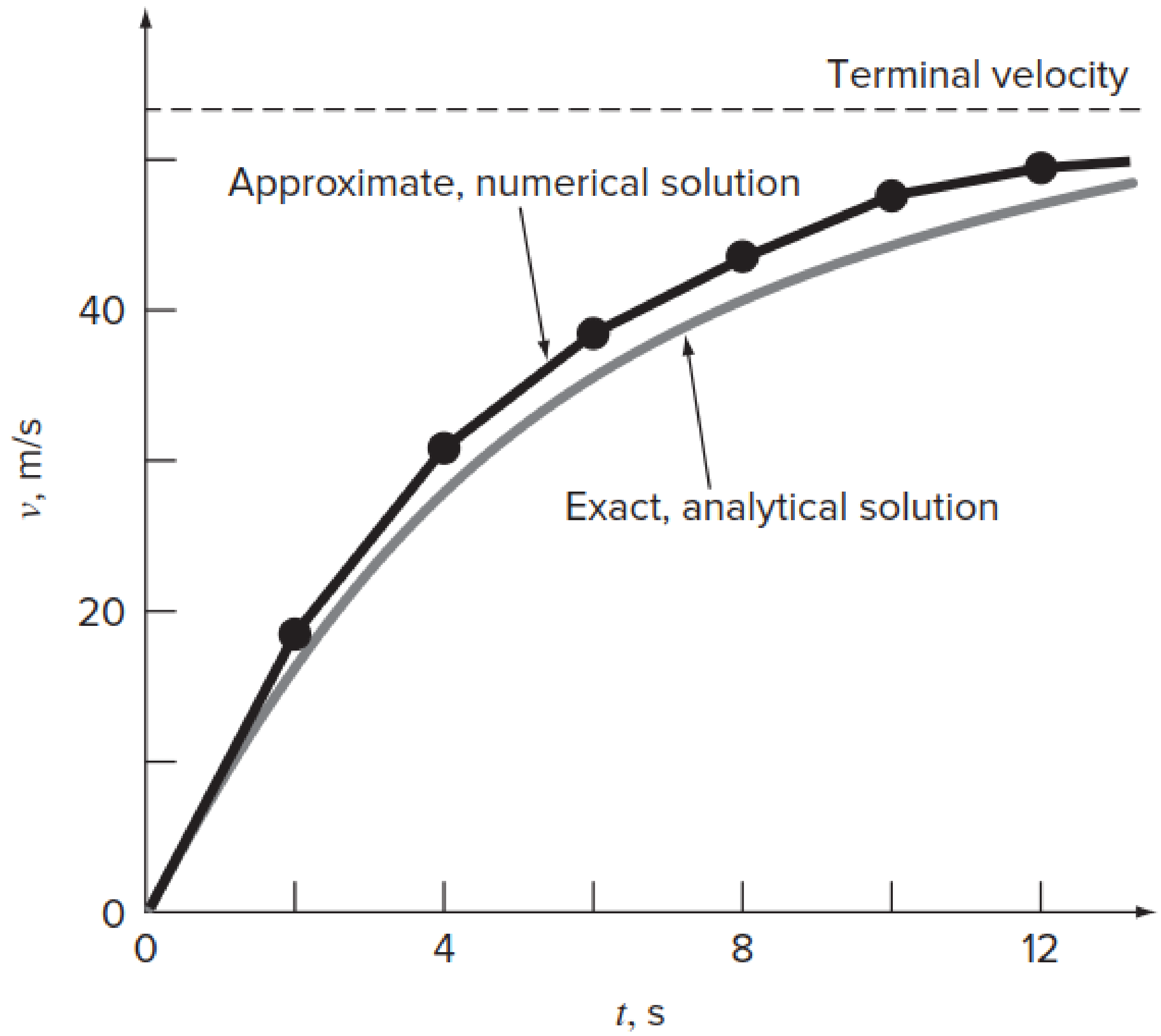
$$v = 0 + \left[9.81 - \frac{12.5}{68.1}(0)\right]2 = 19.62 \text{ m/s}$$

For the next interval (from t = 2 to 4 s)

$$v = 19.62 + \left[9.81 - \frac{12.5}{68.1}(19.62)\right]2 = 32.04 \text{ m/s}$$

The calculation is continued in a similar fashion to obtain additional values:

t, s	*v*, m/s
0	0.00
2	19.62
4	32.04
6	39.90
8	44.87
10	48.02
12	50.01
∞	53.44



Conservation Laws And Engineering

- Aside from Newton's second law, there are other major organizing principles in engineering.
- Among the most important of these are the conservation laws.
- Change = 0 = increases − decreases

or

- Increases = decreases