On the applicability of the linear discrete-time model and system identification by the least squares method

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Although system identification is a widely used method in obtaining a simple and empirical model for system response, material published on its technological aspects is scarce. This paper intends to compile practical aspects which could be useful in application of identification by looking at the time-step response of the system. An analogue computer model of a second order, linear underdamped system is excited by a pseudo-random binary signal generated by a digital computer and the response of the system is recorded digitally in discrete time. The coefficients of a discrete-time model are calculated as to minimize the cummulative error between the position data recorded experimentally and that calculated by the model. Dependence of the model coefficients on the amplitude and discrete-time step of the excitation is clarified on an analytical basis and verified by experiments. Finally, the sensitivity of the model to variations in excitation frequency is searched in amplitude and phase lag, and sample frequency-response curves are presented.

Key words: identification, discrete-time model, least squares method

1. Introduction

Mathematical model or equation of motion of a mechanical system describes the relationship between the actuation forces applied and the resulting motion and uniquely defines the position and effect of each system variable in this relationship. Mathematical model, therefore, contains all the information on how to control the system to follow a required motion pattern, and hence, it is an indispensible tool in designing control strategies. It can also be used to simulate a real test rig on which ideas on control and motion design can be tested. Change of system parameters like masses, mass moments of inertia, locations of mass centers, etc. can easily be done in a simulation program whereas on a real rig, this comes up to be costly and time consuming. Exact mathematical models of mechanical systems are derivable by energy methods like Lagrange, Hamilton, and Newton-Euler formulations. Exact equations of motion are in form of simultaneous linear or non-linear differential

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equations of order at least 2. Minimum number of equations to define the system dynamics is equal to the degrees of freedom and can be obtained by Lagrange's formulation without multipliers [1, 2]. Simulation works modelling the dynamics of mechanical systems like mechanisms, robot manipulators, etc. are numerous [3]. One great difficulty in using equations of motion is in their solution. Realistic equations covering all the related non-linearities like that due to the mechanism kinematics, backlash, Coulomb friction, and magnetic hysteresis have no closed form solutions. Numerical solutions yielding the profile of the motion take too much time and are generally not suitable for real time solution requirements like *model referencing* in control strategies or real time simulators. Analogue computer implementations of the equations are capable of producing real time solutions but they are not too much accurate. Further, analogue computer patching is a tedious task and number of arithmetic making modules on such a computer is always limited [4]. Another difficulty in the utilisation of exact equations of motion is that all the system parameters like masses, mass moments of inertia, stiffnesses, damping coefficients, and physical dimensions are to be explicitly known beforehand. For measurement, generally, system needs be dismantled into its main components where each parameter of the system is lumped. In applications where the difficulties mentioned can not be overcome, *system identification* becomes very useful, generating an empirical mathematical model for the response of the system. The mathematics of the empirical models are so simple that they can be implemented digitally in or near real time for most applications.

In system identification, a mathematical model which may represent the system is defined and an appropriately chosen command signal is applied to the actual system. Response of the system is recorded and a parameter identification of the model that best fits the obtained experimental data is made. If the system is under continuous functional operation, the normal operating data can also be used. A validation test is generally necessary to see the degree of compatibility between the model and the system. Proposed model can be in frequency or time domain, in continuous or discrete time. In time domain modeling, *weighing function, difference equation, and state variable equation* are the choices, among which selection is done due to the identification objective and the types of input-output data [5, 6]. These models are mathematically simpler than the exact equations of motion of the system though presumably not as so accurate. Results of extensive experimentation on system identification have been presented in the work of Kapucu [10] which have been the basis for this paper. This paper aims to bring a clarification to one such model, the *Linear Difference Model* with both mathematical and experimental approaches towards understanding "how dynamic" the model is, and hence, "how safely" it can be used, by means of frequency response.

2. Representation of a dynamic system by a linear difference equation

A linear, time invariant discrete system with one input $u(k)$ and one output $y(k)$ can be characterized by a general n-th order difference equation [7, 8] as:

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_0 u(k) + b_1 u(k-1) + \cdots + b_n u(k-n) \quad (1)$$

or

$$y(k) + \sum_{j=1}^{n} a_j y(k-j) = \sum_{j=0}^{n} b_j u(k-j), \quad (2)$$

where k is an integer index counting the discrete-time steps and a_j and b_j are real, constant coefficients.

Independent variable for Eqs. (1) and (2) is k, standing for discrete time. Real time difference between two successive commands and similarly two response recordings is T. Order n of the equation is arbitrary. Normally, accuracy and hence the reliability of the equation increases with n, but this increases the computation time of model coefficients and $y(k)$. In the work reported here n is taken as 2 in the modeling of second-order linear mechanical systems where input is a force and output a displacement. Eq. (1) for this problem simply indicates that the position y of the lumped mass at the end of k-th discrete-time interval of duration T is in direct proportion with the two previous positions that have occurred 1 and 2 discrete-time intervals ago and with the force values that have been applied 1 and 2 discrete-time intervals ago and the current value of the applied force. Coefficients of Eq. (1) are the proportionality or influence coefficients which certainly are related to the lumped parameters of the mechanical system modelled. Generation of magnitudes of these coefficients is called *the model identification.*

3. Model identification by least squares

Equation (1) contains $(2n+1)$ unknown coefficients and hence they can be calculated by the simultaneous solution of this many equations. Results of such an operation will generally be not so accurate. The usual practice therefore is to collect greater numbers of input and output data and apply the *least squares* technique to minimize the error between the actual discrete data output collected and what the proposed model generates. Real system can be actuated by a variety of inputs. Step input is one type which is widely preferred and hence in this work, the system is actuated by a *pseudo-random binary signal*, abbreviated as PRBS. The *input-output* vector $x(i)$ containing $(2n+1)$ elements is defined as follows:

$$x(i) = [-y(k-1), \ldots, -y(k-n), u(k), \ldots, u(k-n)]^{\mathrm{T}}, \quad (3)$$

where i is a counter enumerating the equations in form of equation (3), i.e. starting from 1 and reaching to the value $(2n+1)$ at minimum. The *parameter vector* $\boldsymbol{\theta}$ containing $(2n+1)$ elements is defined as follows:

$$\boldsymbol{\theta} = [a_1, a_2, \dots a_n, b_0, b_1, \dots b_n]^{\mathrm{T}}. \tag{4}$$

Then, *the least squares equation* becomes

$$y(i) = \boldsymbol{x}^{\mathrm{T}}(i)\boldsymbol{\theta} + e(i), \tag{5}$$

where $e(i)$ is the *error* which is required to be as small as possible. For improving accuracy, a large number N of data for input and output are collected. Substitution of them generates a system of equations, as many as the number of combinations of $(2n+1)$ in N which can be written as follows:

$$\boldsymbol{Y} = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{E}, \tag{6}$$

where

$$\boldsymbol{Y} = [y(n+1), y(n+2), \dots, y(n+N)]^{\mathrm{T}}, \tag{7}$$

$$\boldsymbol{E} = [e(n+1), e(n+2), \dots, e(n+N)]^{\mathrm{T}}, \tag{8}$$

and

$$\mathbf{X} = \begin{bmatrix} -y(n) & \dots & -y(1) & u(n+1) & \dots & u(1) \\ -y(n+1) & \dots & -y(2) & u(n+2) & \dots & u(2) \\ -y(n+2) & \dots & -y(3) & u(n+3) & \dots & u(3) \\ \cdot & \dots & \cdot & \cdot & \dots & \cdot \\ \cdot & \dots & \cdot & \cdot & \dots & \cdot \\ -y(n+N-1) & \dots & -y(N) & u(n+N) & \dots & u(N) \end{bmatrix}. \tag{9}$$

From Eq. (6), $\boldsymbol{\theta}$ can be estimated by means of least squares that minimize the error function $\boldsymbol{J}$,

$$\boldsymbol{J} = \sum_{k=n+1}^{N+n} \boldsymbol{E}^2(k) = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{E} = (\boldsymbol{Y} - \mathbf{X}\boldsymbol{\theta})^{\mathrm{T}}(\boldsymbol{Y} - \mathbf{X}\boldsymbol{\theta}) \tag{10}$$

i.e. the *parameter vector* $\boldsymbol{\theta}$ composed of the coefficients of Eq. (1) is determined from

$$\frac{\delta \boldsymbol{J}}{\delta \boldsymbol{\theta}} = 0. \tag{11}$$

This yields the vector of the optimized values for the unknown model coefficients

$$\overline{\boldsymbol{\theta}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\boldsymbol{Y}. \tag{12}$$

In the experimental work presented here, N is arbitrarily selected as 42.

T in Eqs. (3, 4, 5, 7, 8, 10), and (12) stands for 'transpose' and **not** for the incremental time T.

4. The second-order system

Simplest mechanical system, as seen in Fig. 1(a), can have an inertia element m movable in a single coordinate $y(t)$ under the effect of position and velocity dependent forces and an externally applied arbitrary actuation force $u(t)$. Position dependent forces can be represented by the force of a spring with stiffness k and the velocity dependent forces by the force of a dashpot of damping coefficient c. A great many realistic mechanical systems can effectively be assumed to have this format. Position dependent forces may include the proportional components of the actuation forces generated by PD controllers. Similarly, the velocity dependent forces may include the *Coriolis and centrifugal* forces if any, forces due to viscous damping, and the derivative components of the actuation forces of PD controllers. System may contain more than one inertia element each moving in a non-linear relationship with the input like a 4-bar mechanism, but may still be considered linear if it is functional within a small part of its operation range. Therefore even though the system of Fig. 1(a) is quite simple, it is fundamental and didactic in understanding and appreciation of system identification.

Differential equation of motion of the system shown in Fig. 1(a) is

$$m\ddot{y} + c\dot{y} + ky = u(t). \tag{13}$$

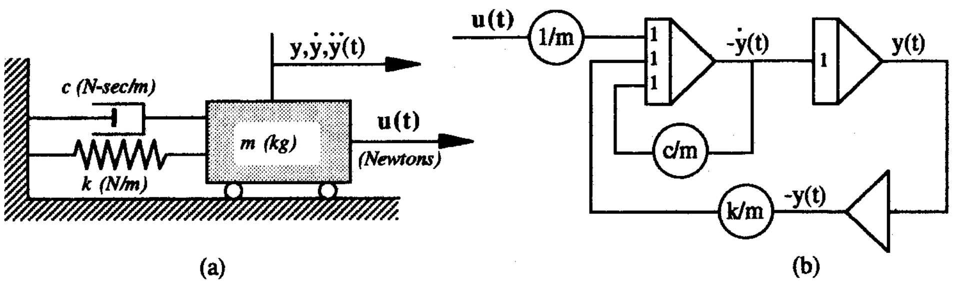

Fig. 1. (a) A simple, single-degree-of-freedom second-order linear mechanical system, (b) analogue simulation circuit of the same system. Time constants of the integrators are 1 s with zero initial conditions.

This equation in time domain can be very complicated for a multi-body system like a mechanism and will probably have no closed-form solution. Representation of the same input/output relationship in frequency domain has been regarded easier to solve and also to understand. Transfer function $G(s)$, derivable from Eq. (13), defined as a ratio of system response to input in frequency domain is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1/m}{s^2 + (c/m)s + k/m}, \tag{14}$$

s is an indication for a function or signal to have different values at different frequencies. Equations (13) and (14) are continuous functions.

The dynamic behaviour of a system can be predicted through its frequency response. For example, the Bode plot shown in Fig. 2 indicates that the system is damped, second-order system with resonant frequency of 0.138 Hz. Its damping ratio is 0.5, and it will follow the command signals without too much distortion up to 0.159 Hz where phase lag becomes 90 degrees. Bode plot shown in Fig. 2 is obtained by direct measurement of the response of the analogue simulation given by Fig. 1, scaled to the following magnitudes: $m = 1$ kg, $k = 1$ N/m, $c = 1$ N s/m. Amplitude of response, X, is in direct proportion to the amplitude of the actuation force U:

$$\frac{X}{U/k} = \frac{1}{\sqrt{(1 - \omega^2/\omega_n^2)^2 + (2\xi\omega/\omega_n)^2}}, \tag{15}$$

where ω is the frequency of the forcing function, ω_n is the system undamped natural frequency, and ξ is the damping ratio. Therefore, the amplitude of the excitation signal used in the determination of frequency response is not important unless the displacements are within the physical limits of the system.

5. Description and purpose of the experimental work

Although system identification is a well known subject, number of publications presenting its know-how is relatively scarce. Purpose of the experimental work presented in this paper has been to display and experimentally verify some practical aspects on the following items:

i) to present an analytical solution for the discrete-time model of an under-damped system using transformations, which yields a mathematical solution for model coefficients;

ii) to do the identification of the system handled in (*i*) experimentally by using the method presented in Section 2;

iii) to repeat the experimentation of step (*ii*) by applying actuation forces of different frequencies and amplitudes to display their effect on model coefficients;

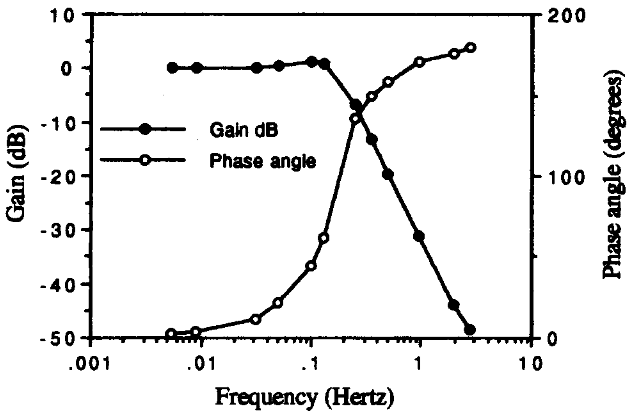

Fig. 2. Experimentally obtained frequency-response characteristics of an analogue simulation representing a second-order system having a 1 kg mass, 1 N/m spring stiffness, and 1 N s/m damping coefficient. Amplitude of excitation is immaterial provided that the amplifiers of the simulation circuit do not saturate.

iv) to indicate whether the discrete-time model represents definitive system dynamics at frequencies around and beyond resonance via frequency response tests to verify the correctness of the system identification. System tested is an analogue simulation as shown in Fig. 1(b). For control, it is connected to a Macintosh FX II computer via a 12 bit MacADIOS *(Macintosh Analog-Digital Input-Output System)* interface card. Analog circuit is actuated by PRBS *(Pseudo-Random Binary Signal)* of various amplitudes. Unit scaling is such that the voltage of 1.0 V corresponds to a force of 1.0 N on the mass. Mass, spring constant, and damping coefficient are unity hence the coefficient potentiometers on the position and velocity feedback and actuation force lines are set to unity. PRBS is a continuous set of step commands produced at T second intervals. Polarity of the steps is random and the amplitudes arbitrary, but constant. In experiments, amplitudes are set to moderate values which do not cause the amplifiers of the analogue simulation to saturate. The discrete-time interval T is also arbitrary. However, to get reasonable results, the applicable range for T can have some limits.

6. Analytical solution for model coefficients

A damped mechanical system contains inertia, stiffness, and damping elements and its transfer function is given by Eq. (14). Roots of its denominator are:

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\frac{c^2}{4m^2} - \frac{k}{m}}. \tag{16}$$

In underdamped systems, restoring forces are more dominant than damping forces, that is $c^2/4m^2$ is less than k/m and the roots given by Eq. (16) are complex. Free vibration of such a system displays the amplitude reducing in time. The resonant frequency of the system is

$$\omega_d = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}. \tag{17}$$

Substituting the *frequency of damped oscillations* into the expression for the roots,

$$s_{1,2} = b \pm \mathrm{i}\omega_d, \tag{18}$$

where $b = -c/2m$ is also a system parameter. Discrete-time representation of the system response is the transfer function in z-domain and is obtained by z transforming the transfer function in s-domain, operated on by the *zero order hold* as follows:

$$G(z) = z[G_0(s) \cdot G(s)] = z\left[\frac{1 - \mathrm{e}^{-sT}}{s} \frac{1/m}{s(s+b)^2 + \omega_d^2}\right]. \tag{19}$$

The term to be z-transformed in Eq. (19) can be converted into the forms available in tables [7, 8] by separating into partial fractions

$$G(z) = \frac{A}{m} z[1 - \mathrm{e}^{-sT}] \cdot z\left[\frac{A}{s} + \frac{Bs + C}{(s+b)^2 + \omega_d^2}\right], \tag{20}$$

where $A = 1/(b^2 + \omega_d^2)$, $B = -A$ and $C = 2bB$. With further algebra,

$$G(z) = \frac{A}{m}\left[\frac{z-1}{z}\right] \cdot z\left[\frac{1}{s} - \frac{s+b}{(s+b)^2 + \omega_d^2} - \frac{b}{\omega_d} \frac{\omega_d}{(s+b)^2 + \omega_d^2}\right]. \tag{21}$$

Completion of the transformation gives

$$G(z) = \frac{Y(z)}{U(z)} = \frac{A}{m}\left[\frac{z-1}{z}\right]$$
$$\left[\frac{z}{z-1} - \frac{z^2 - z\mathrm{e}^{-bT}\cos\omega_d T}{z^2 - 2z\mathrm{e}^{-bT}\cos\omega_d T + \mathrm{e}^{-2bT}} - \frac{b}{\omega_d} \cdot \frac{z\mathrm{e}^{-bT}\sin\omega_d T}{z^2 - 2z\mathrm{e}^{-bT}\cos\omega_d T + \mathrm{e}^{-2bT}}\right]. \tag{22}$$

Cross multiplication of Eq. (22) yields

$$\begin{aligned} &z^2 Y(z) + [-2\mathrm{e}^{-bT}\cos\omega_d T] z^1 Y(z) + \mathrm{e}^{-2bT} z^0 Y(z) = \\ &= \frac{A}{m}\left[1 - \mathrm{e}^{-bT}(\cos\omega_d T + \frac{b}{\omega_d}\sin\omega_d T)\right] z^1 U(z) + \\ &+ \frac{A}{m}\left[\mathrm{e}^{-bT}(1 - \cos\omega_d T + \frac{b}{w_d}\sin\omega_d T)\right] z^0 U(z). \end{aligned} \tag{23}$$

This equation is of the same structure as Eq. 1, which, for $n = 2$, takes the following form:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_0 u(k) + b_1 u(k-1) + b_2 u(k-2). \qquad (24)$$

Equating the coefficients of their corresponding terms, the coefficients of the discrete-time model of the underdamped system come up as:

$$\begin{aligned} a_1 &= -2\mathrm{e}^{-bT}\cos\omega_d T, \quad a_2 = \mathrm{e}^{-2bT}, \quad b_0 = 0, \\ b_1 &= \frac{1}{k}\left[1 - \mathrm{e}^{-bT}(\cos\omega_d T + \frac{b}{\omega_d}\sin\omega_d T)\right], \\ b_2 &= \frac{1}{k}\left[\mathrm{e}^{-bT}(1 - \cos\omega_d T + \frac{b}{\omega_d}\sin\omega_d T)\right]. \end{aligned} \qquad (25)$$

Powers of z imply the relative positions of the associated terms on the *time axis*. For the discrete-time model of $n = 2$, z^2 terms correspond to the current time, z^1 and z^0 terms correspond to the time of T and $2T$ seconds before the current time, respectively. For the system under consideration, $m = 1$ kg, $c = 1$ N s/m, $k = 1$ N/m, and once the incremental time T is arbitrarily defined, numerical values of the coefficients can be calculated. The profiles of the coefficients through T, as defined by Eq. (25), are shown in Fig. 3. Coefficients are continuous for all values of T, but

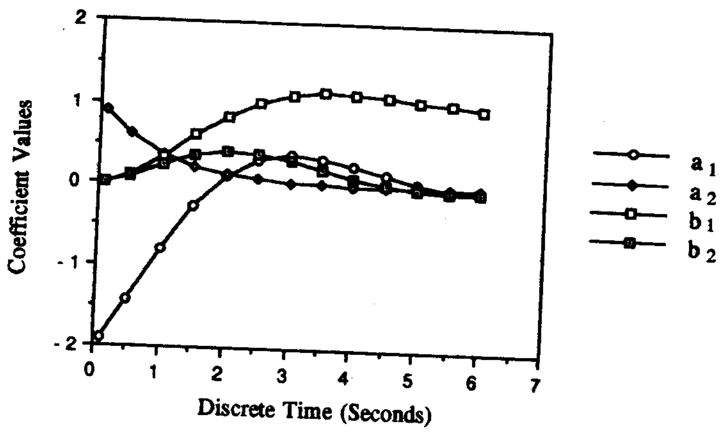

Fig. 3. Coefficients of a second-order discrete-time model for a second-order linear underdamped system as a function of discrete time T, resulting from a theoretical analysis. Mass of the system is 1 kg, spring stiffness is 1 N/m, and damping coefficient of a dashpot parallel to the spring is 1 N s/m.

solution for $T = 0$ is physically trivial. Eq. (25) indicates that profile of a_1 starts from the value of -2, displays an oscillation with frequency ω_d, and approaches to zero in T. a_2 starts from 1 and exponentially reduces to zero in T. b_1 and b_2 both start from zero and display oscillations with frequency ω_d and decay to $1/k$ and zero, respectively. A lightly damped system will respond to a command swiftly and attain the commanded position after a few transient oscillations. Therefore, as the incremental time gets longer, the effect of previous commands and position recordings become negligible on the current position. At the end of a long T, the mass displacement simply becomes equal to the static deflection of the spring and it comes to rest.

7. Dependence of model coefficients on the amplitude and discrete time of the PRBS excitation signal

Analytical solution for model coefficients given by Eq. (25) indicate that they are dependent on the discrete time of the PRBS excitation but not on its amplitude. They are functions of the lumped parameters of the system. Once these parameters are known, coefficients can be calculated for any excitation frequency. But generally, the problem emerges in the opposite sense, system parameters are not known and model coefficients are required to be experimentally determined. At this point, assuming that PRBS will be used to excite the system, the problem of deciding on the discrete time and the amplitude of the excitation signal comes up. To see the picture, the model coefficients of the analogue computer simulation are experimentally determined with various discrete-time durations at three different amplitudes of the exciting PRBS. Results are shown in Fig. 4. PRBS is nothing than a series of step signals with random polarity. The steady-state response of a second-order system to a step input is a step displacement equal to the static deflection of the spring. Similarly to frequency response, one intuitively expects that the amplitude of the PRBS should not effect the coefficient values. Fig. 4 (a), (b) and (c) show the coefficient values obtained with 0.25, 0.5, and 1 volt amplitudes, respectively. They all show the same profile up to about $T = 3$ seconds and further, these profiles show a good fit to the analytically obtained profiles of Fig. 3. However, experimental and analytical coefficient values differ from each other at higher discrete-time values, corresponding to lower excitation frequencies. Experimental curves show a fall in the profiles of a_1, a_2 and b_2 at $T = \pi/\omega_d$ and then try to recover towards the analytical profiles. It is clearly seen that the recovery is faster if the system is driven by a higher amplitude excitation. Using excitation signals as high as possible may appear a rule of thumb unless any part of the system, mechanical or electrical, does not saturate and induce extra non--linearities. This value, i.e. $T = \pi/\omega_d$, is equal to a half of the natural vibration

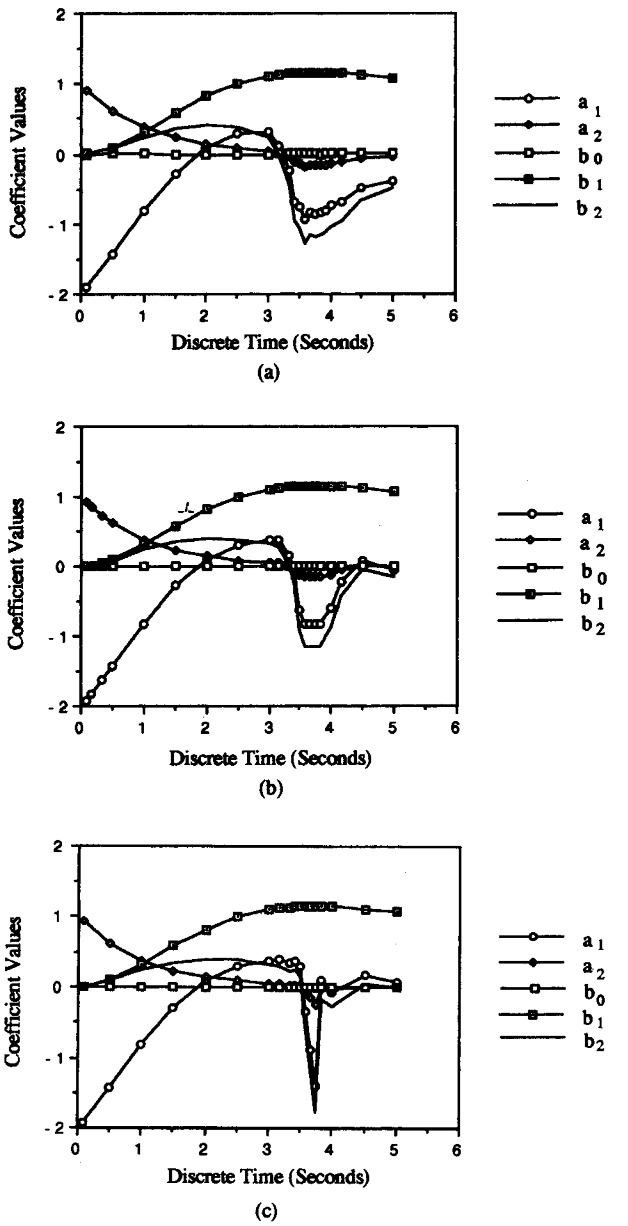

Fig. 4. Profiles of the experimentally obtained model coefficients for the system referred to by Fig. 3. PRBS amplitudes used are 0.25 V in (a), 0.5 V in (b) and 1 V in (c).

period of the damped system. An alternating actuation with the period twice this value brings the system to resonance. This point is where aliasing problems dominate. Normally, a mechanical system should not be driven at frequencies near or beyond resonance. Shannon indicates that sampling must be done at a rate five to ten times the highest frequency, thought to be present if aliasing problems are to be avoided [9]. Minimum allowable sampling frequency is defined in [8] as follows:

$$\omega_s = 2\omega_d \tag{26}$$

for an underdamped second-order system. For excitation by PRBS, sampling should be done at $\pi/(2\omega_d)$ s intervals. According to Eq. (26), the parts of the curves in Fig. 4 after $T = 1.814$ s are not reliable. The drop in the coefficient profiles is within this unreliable region. Experiments show that drop in the profiles is due to the damping and no such a drop is observed in the identification of undamped systems. This point needs further study and explanation. In the characteristic system identification problem, system parameters and hence ω_d are unknown. The procedure should be to keep the discrete time as low as the instrumentation allows, and to check whether the discrete time used is within the $\pi/(2\omega_d)$ limit, use Eq. (25) to get an estimation for system parameters and hence ω_d. Repeating the identification test at various discrete-time values to get a profile similar to that shown in Fig. 4 would be the best.

An instant problem is to know whether the discrete-time model and the experimentally obtained coefficients are fully definitive of the dynamics of the system or not. The uncertainty here can be clarified by looking at the frequency response of the discrete-time model. The model is acted upon by a sinusoidal force and the amplitude of the output motion and its phase lag from the excitation are directly measured from the plot of the output motion. To see the influence of the discrete-time values, three different sets of coefficients are used, taken at 0.083, 0.5 and 1 s, all of which are within the reliable region. Coefficients are experimentally identified as: $a_1 = -1.914204$, $a_2 = 0.920750$, $b_0 = -0.000410$, $b_1 = 0.002587$, $b_2 = 0.003890$ for $T = 0.083$ s, $a_1 = -1.427500$, $a_2 = 0.612530$, $b_0 = -0.000080$, $b_1 = 0.096777$, $b_2 = 0.087309$, for $T = 0.5$ s, and $a_1 = -0.813160$, $a_2 = 0.375270$, $b_0 = -0.000300$, $b_1 = 0.322237$, $b_2 = 0.237742$ for $T = 1$ s with PRBS amplitude of 0.5 V. Fig. 4 implies that same results would be obtained with any other amplitude. Frequency response of the models are given by the Bode plot shown in Fig. 5. The frequency response of the models behave independently of the amplitude of the excitation sinusoid, as expected. The amplitude used in the tests that produced Fig. 5 is 0.5 V. Repetition of the same test with a 1 V amplitude sinusoid produced exactly the same gain and phase lag profiles. Plot of gain in Fig. 5 shows a good fit to that of Fig. 2. Further, models with coefficients identified at different time steps show the same trend in gain.

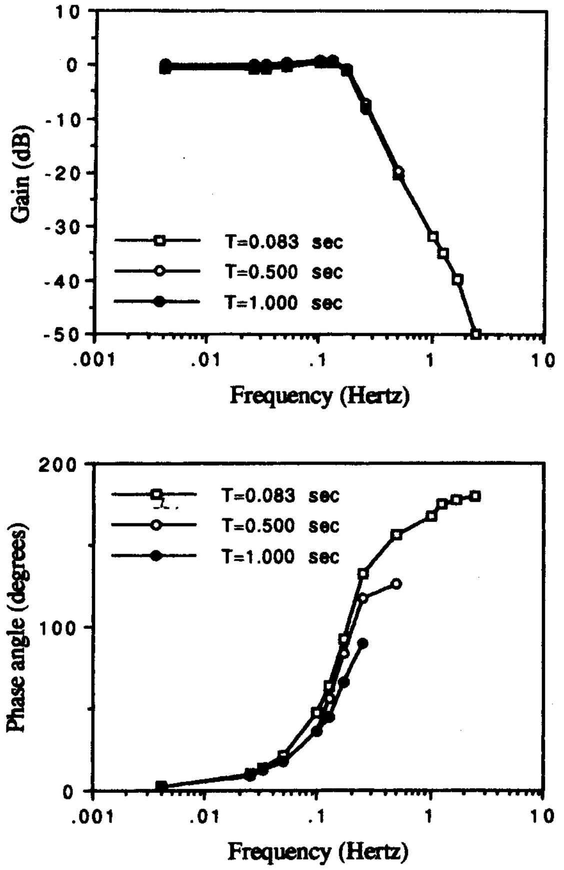

Fig. 5. Bode plot showing the frequency response of the discrete-time model for the second-order underdamped system referred to in Fig. 2, with experimentally obtained coefficients. To show the effect of the discrete time, coefficients obtained with 3 different discrete-time values are subjected to frequency response tests. Amplitude of the sinusoidal excitation is 0.5 N.

However, it must be noted that due to aliasing problems, higher frequency excitations have been impossible for longer time step models. Fig. 5 implies that models obtained by different time steps show a variation in phase shift. It must be noted that discrepancies start to dominate at higher frequency ranges where neither a mechanical system nor its model should be operating. The reason for the apparent phase-shift discrepancies is that the position and forcing on the system 1 and 2 time steps before the initiation of the solution are not known and so assumed equal to zero. This error can be compensated by assigning better initial values. Contribution of this error on the phase shift becomes more effective as the frequency of the excitation increases. The lower frequency region of this plot is presumably more accurate and displays a good fit to that of Fig. 2. One conclusion that can be extracted from Fig. 5 is that the time step used in system identification should be as small as possible for the model to cover a larger frequency range.

8. Conclusion

In this paper it is put forth that most second-order mechanical systems can be assumed equivalent to a simple linear spring-mass-damper system. Response of such systems can be represented in form of a discrete-time model, whose coefficients can be extracted from experimentally obtained position data. A procedure for experimental identification of the system is presented with the necessary formulations. Then, it is shown that the discrete-time model can be obtained by z-transforming the continuous model, operated on by the zero-order hold. This analysis yielded an analytical definition of model coefficients in terms of system parameters and discrete time. Experimental work and theoretical analyses have brought the following conclusions:

i) A second-order linear mechanical system can be represented by a second-order discrete-time model. If higher-order models are used, experimentation will yield that the influence of the terms of the order greater than 2 are negligible.

ii) System identification is generally used when the parameters of the system are unknown. Knowing the analytical expressions for the coefficients, system parameters can be extracted from the experimentally obtained coefficient values.

iii) Second-order discrete-time models provide an estimation for the dynamic behaviour of a system very proximate to that of the real system in terms of gain and time lag in response.

iv) Mathematics involved in the calculation of a position using the discrete-time model is so little that such a model can effectively be used to provide *near-real time* solutions in simulation or control applications to replace analogue models.

v) In identification, discrete-time values up to π/ω_d yield a good model. Normally, discrete-time steps should be as small as possible. Amplitude of the excitation PRBS can be of any suitable value that does not cause saturation.

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99-1767
Pickrel, C.R. Examples of variation in measured modal parameters of a single test specimen. Proceedings of the NVH (Noise and Vibration Harshness): Source, Path and Receiver Modeling Symposium, Novi, MI: 108-113, Oct. 26-27, 1998 (sem@sem1.com).

KEYWORDS: aircraft, frequency response function, multi-degree-of-freedom systems, noise, parameter identification techniques, random excitation

Some examples of estimating variance in measured modal parameters are discussed. The effect of random noise on the variance of mode frequency and damping of a synthesized ten-degree-of-freedom system is assessed using a "jackknife" approach. Multiple, redundant pole estimates from different solution sets are used to assess variance in the modal parameter estimation process. Examples of variance in mode frequency and damping are shown from ground and flight tests of a transport airplane. Variation is shown to be dominated by nonlinearity.

Reliability Analysis

99-1768
Red-Horse, J.R., and T.L. Paez. Uncertainty evaluation in dynamic system response. Proceedings of the 16th International Modal Analysis Conference, Santa Barbara, CA: 1206-1212, Feb. 2-5, 1998.

KEYWORDS: computer programs, dynamic response, Monte Carlo method, probabilistic methods

The advanced mean value (AMV) method for probabilistic system analysis is a technique for the estimation of the cumulative distribution function of a random variable. The random variable is a deterministic, often implicitly defined function of a vector of random variables. The technique uses a number of simplifying approximations in an iterative scheme. The approximations in AMV allow the procedure to be used with only a limited knowledge of the physics of the underlying problem and to enhance the computational feasibility through an associated reduction in required results. This article presents an AMV code and an advanced AMV code for the probabilistic analysis of system behavior.

Sensitivity Analysis

99-1769
Balmès, E. Efficient sensitivity analysis based on finite element model reduction. Proceedings of the 16th International Modal Analysis Conference, Santa Barbara, CA: 1118-1124, Feb. 2-5, 1998 (balmes@mss.ecp.fr).

KEYWORDS: algorithms, design, engines, mode shapes, multi-degree-of-freedom systems, sensitivity analysis

Iterative methods are used widely for finite element model updating and structural optimization. Most of these approaches use partial derivatives, called sensitivities, of properties with respect to physical parameters of the full-order model. Accurate and inexpensive evaluations of sensitivities are thus important. This study presents a general categorization of approximation methods, along with suggestions for new approaches to obtain low-cost predictions of both mode shapes and their sensitivities.

Stability Analysis

99-1770
Kondou, T., A. Sueoka, and T. Inoue. Forced vibration analysis of a nonlinear structure connected in series (stability analysis based on the argument principle). *JSME International Journal, Mechanical Systems, Machine Elements and Manufacturing* (Japan) 41(3):583-591, Sept. 1998.

KEYWORDS: forced vibration, multi-degree-of-freedom systems, stability analysis, vibration analysis

It is very difficult to determine the stability of the periodic steady-state vibrations generated in a large-sized nonlinear system with multiple degrees of freedom. To overcome this, a new practical method is presented. The method, called the vector locus method, is based on the argument principle. It is applicable to the stability analysis of general multi-degree-of-freedom systems with parametric excitation.

System Identification Techniques

99-1771
Kapucu, S., and S. Baysec. On the applicability of the linear discrete-time model and system identification by the least squares method. *Strojnícky Casopis* (Slovakia) 50(2):104-118, 1999 (in Slovak).

KEYWORDS: least squares method, mathematical models, system identification techniques

Although system identification is used widely to obtain a simple and empirical model for system response, material on its technological aspects is limited. This article, therefore, compiles practical aspects that can be used in identification by considering the time-step response of the system. An analogue computer model of a second-order, linear underdamped system is excited by a pseudo-random binary signal generated by a digital computer, and the response is recorded digitally in discrete time. The coefficients of the model are calculated to minimize the cumulative error between the position data recorded experimentally and that calculated by the model.

99-1772
Van der Auweraer, H., and L. Hermans. Applications of structural model identification during normal operating conditions: an overview of the Eureka project SINOPSYS. Proceedings of the 17th International Modal Analysis Conference, Kissimmee, FL: 27-34, Feb. 8-11, 1999.

KEYWORDS: automobiles, case histories, damping, design, modal tests, natural frequencies, rockets

Experimental techniques such as modal testing and modal analysis are used widely. However, these techniques are limited to dedicated, controlled laboratory tests where a low-level excitation is applied, and the corresponding system response is measured. During actual operation, however, the loading conditions may be substantially different from the ones used in the modal tests. This article presents an in-operation modal analysis technique, which is shown