Residual swing/vibration reduction using a hybrid input shaping method

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Abstract

This paper addresses a hybrid input shaping method to reduce the residual swing of a simply suspended object transported by a robot manipulator or the residual vibration of equivalent dynamic systems. The method is based on the successive use of two input shaping methods. A cycloid is chosen as the trajectory to be preshaped. First of all, a ramp function is superimposed onto the cycloid and then the resulting trajectory is convolved with a sequence of two impulses. The resulting trajectory is preshaped twice. A hydraulically actuated robot manipulator carrying a simply suspended object is employed as the experimental system. Simulation and experimental results for each method constituting the hybrid method are presented and compared to what we call hybrid input shaping method. The results indicate that a swing-free stop is obtainable at the end of a move with a high degree of robustness to uncertainties in the natural frequency of the swinging or vibrating system by employing the hybrid method. The method is simple and easy to implement, and most importantly robust to uncertainties (±15% of \( \omega_n \)) in the natural frequency of the system. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

During pick-and-place operations, some objects may not be grasped by robot manipulators due to the task constraints. Such objects which need to be carried by a hook or a similar device attached to the manipulator endpoint display a motion separate from the point of suspension, [1,8]. Similar applications include the transport of large objects in a factory environment by the use of a bridge crane by which the object is raised, transported and lowered on a target location, [2,12], and a molten-metal filled container carried by a manipulator in a foundry where the molten-metal

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should not splash at the end of the transportation where pouring the metal into the moulds takes place. The transport of such objects generally result in undesired swing at the end of a move.

In order to stop a suspended object in a swing-free state, Starr [1] has suggested a trajectory consisting of an acceleration part, where the suspension point is accelerated stepwise until half the desired transportation velocity is reached and then is further stepwise accelerated to the specified final velocity which is attained at a point roughly at one-fourth of the transportation velocity times the period of natural oscillation of the suspended object, and a deceleration part, where the same process is applied in reverse. Later, Strip [2] has reported on the swing-free transportation of suspended objects that the suspension point has a symmetrical acceleration and deceleration profiles, hence the deceleration starting at a distance from its goal equal to the distance travelled while accelerating. This distance is reported to be one-half the acceleration of the suspension point times the square of the natural period of oscillation of the object. Alici et al. [8] have proposed two methods, (1) adjusting transportation time, thus stopping the manipulator at the instant when the object completes one or more full cycle(s), and (2) adjusting travelling time of each section of a three-piece continuous trajectory provided that a given transportation time is unchanged, to demonstrate the swing-free transportation of suspended objects with robot manipulators.

Input preshaping based on convolving the desired reference input with a sequence of impulses is a method of reducing residual vibrations in computer-controlled machines [3,5,9]. This method requires a perfect knowledge of the system parameters such as natural frequency and damping ratio to reduce residual vibration to zero. The more impulses are used, the more robust the system becomes to system parameter variations but at a penalty of greater response time. Meckl and Seering [10] have developed a set of shaped force profiles to reduce residual vibration of velocity limited dynamic systems. The force profiles are constructed from a versine (1-cosine) function, which has no discontinuity in slope at the beginning and end, and its harmonics with coefficients chosen to minimise the energy of the resulting function near the system natural frequency. This is accomplished by employing an optimization technique. Teo et al. [12] have reported on pulse input sequences to reduce vibration in overhead cranes. The pulse sequences determine robustness to variations in the system parameters. They have presented simulation and experimental results for a four-pulse input sequence without robustness and a six-pulse input sequence with robustness. In a similar work, Cuccio et al. [13] have addressed preshaped input laws based on selecting a limited number of acceleration steps of equal duration in order to reduce the residual vibration of point-to-point moving elastic systems. As the number of steps increases, the preshaped trajectory becomes smoother.

In this study, we propose a hybrid input shaping method in order to reduce the swing of a simply suspended object at the end of any move with a high degree of robustness to errors in the natural frequency of the swinging or vibrating system. The hybrid input shaping method is based on the two input shaping methods presented in a previous study of the authors of this paper, [11]. They are the method of superimposing a ramp function onto a main desired function, and the method of convolving a sequence of impulses with a desired input to generate a shaped input. Theoretical background, new simulation and experimental results indicating the effectiveness of the two methods in reducing residual swing are presented in this paper in order to compare them to the hybrid method. To demonstrate the feasibility of the methods, we consider the trajectory based on a cycloidal motion which is commonly used as a high-speed cam profile [4], continuous throughout one cycle plus a ramp. The ramp function can be superimposed onto any mathe-
matically described function in order to give an initial velocity to the system, which generates a swing equal in magnitude, but out of phase to the swing imposed by the main trajectory. The amplitude of the ramp and the cycloidal function are determined such that a total distance to be travelled and the transportation time are satisfied. Then the resulting trajectory is convolved with a sequence of two impulses to obtain a twice shaped input. It is believed that this method contributes to the efforts to reduce the swing of the suspended objects with a high degree of robustness to uncertainties in natural frequency of the system.

An object suspended at the end of a robot manipulator by a hook-like device is considered and its equation of motion for the translating link of the manipulator are solved on a digital computer for the desired trajectory of the suspension point. The servo valve and actuator dynamics are considered in the simulations. We assume that the compliance existing in the transmission and structural elements does not cause considerable vibration of the manipulator endpoint during and at the end of the move, and the suspension point follows a pre-determined path with a certain velocity and acceleration. This is verified by the experimental results presented in Section 4, where the translating link of the manipulator tracks a commanded trajectory with a good fit. Simulation results have been verified on a hydraulically actuated robot manipulator whose translational link carrying a compound pendulum. Simulation and experimental results show that the hybrid input shaping method is applicable and easy to implement on a real system.

2. Modelling of system

The experimental system used in this study consists of a single link hydraulically actuated robot manipulator transporting a simply suspended object, which is attached to the manipulator endpoint by a hook or a similar device. A schematic representation of the system is depicted in Fig. 1, where $O$ is the point of suspension, $x$ is the horizontal displacement of point $O$, $\theta$ is the angular displacement of the object with respect to the vertical, $c$ is the coefficient of viscous friction which is one of the predominant characteristics of a hydraulic system, $M$ is the mass of the translating object, $P_{\text{tank}}$, $P_{\text{supply}}$, and $P_{\text{tank}}$ are the input pressures, and $m$, $I_G$, and $G$ are the mass, moment of inertia, and gravity center of the compound pendulum, respectively. The figure shows the forces and displacements involved in the system.

![Fig. 1. Schematic representation of a translating link carrying a simply suspended object via a hook.](image-url)
link, \( m \) is the mass of the object, \( I_G \) is the mass moment of inertia of the suspended object about its center of gravity, \( r \) is the radial distance from the center of gravity to the point of suspension \( O \). We assume that the manipulator endpoint moves and accelerates in one direction, say in \( x \)-direction. The linearized equations of motion describing the system are given as [11]:

\[
(M + m)\ddot{x} + (mr)\ddot{\theta} - (mr\dot{\theta}^2)\theta + c\dot{x} = F_a,
\]

\[
(I_G + mr^2)\ddot{\theta} + (mr)\ddot{x} + mgr\theta = 0,
\]

where \( F_a = P_1A_1 - P_2A_2 \), the force exerted by the actuator. The pressures \( P_1 \) and \( P_2 \) in the actuator are functions of supply pressure, coefficient of leakage flow, flow gain of spool stage, area of the piston, actuator speed and spool displacement, [6], \( A_1 \) and \( A_2 \) are the cross sectional areas of the piston. The swing of the suspended object is described by Eq. (2), where the acceleration \( \ddot{x} \) of the point of suspension \( O \) is the forcing function. If it is possible to describe \( \ddot{x} \) mathematically, the solution for Eq. (2) will be obtained. So, the motion of the suspended object is a simple-periodic motion while the point of suspension \( O \) is accelerating with \( \ddot{x} \) and the natural frequency is

\[
\omega_n = \sqrt{\frac{mgr}{I_O}},
\]

where \( I_O \) is the mass moment of inertia of the swinging object with respect to an axis through \( O \).

When a ramp function is used as a part of a desired trajectory, it becomes difficult to describe the acceleration of the ramp function at the beginning and end of the trajectory, where it becomes \(+\infty\) and \(-\infty\), respectively. Thus, it becomes problematic to solve Eq. (2) for a ramp input. To overcome this difficulty, we define an equivalent system moving under the desired displacement \( y \). Although the suspended object depicted in Fig. 1, and the equivalent system shown in Fig. 2 are different physically, they are represented by the same mathematical model. Such systems are called equivalent or analogous systems. In this study, we make use of the mathematical model of the equivalent or analogous spring-mass system shown in Fig. 2, composed of an equivalent mass \( m_e \) moving in the coordinate \( x \) and an equivalent stiffness \( k_e \). It is

\[
\ddot{x} + \omega_n^2x = \omega_n^2y, \quad \text{where } \omega_n^2 = \frac{k_e}{m_e}.
\]

The problem here is to plan a desired input that will move the translating link of the manipulator, which carries the suspended object, from one operating point to another in the shortest possible time with zero swing at the end of the move. By preshaping the desired input \( y \), a swing-free move is obtainable. Note that the input is the displacement \( y \), and the output is the angular displacement \( \theta \) of the suspended object. The pressures \( P_1 \) and \( P_2 \) are in quadratic forms and are given in [6]. The natural frequency \( \omega_n \) is the natural frequency of the suspended object given by Eq. (3).

![Fig. 2. An equivalent simple mass-spring system representing the swing of the simply suspended object.](image-url)
3. Theoretical background for input shaping

The translating link of the manipulator is desired to follow the input $y$. The hybrid input shaping technique consists of two methods or stages; (i) superimposing a ramp function onto the specified trajectory, and (ii) the resulting two-piece trajectory is convolved with a sequence of two impulses to generate a shaped input. A cycloid is chosen as the desired trajectory to be preshaped. Note that the resulting trajectory is preshaped twice. The first stage neither limit nor increase the traveling time while the second stage increases the traveling time by a semi-period of the first mode of the system as presented below. Since each stage by itself can be employed to obtain a preshaped reference input [11] it is of use to present theoretical background for them.

3.1. Superimposing a ramp onto a specified function

The reference input for the translating link consists of two functions. The total distance to be covered from the beginning to the end of a move within a specified move time is the sum of the distances to be travelled by each of the two functions within the same duration. By adjusting the excursion distance of each function, the swing of the suspended object can be eliminated provided that the specified move time and the total distance are unchanged. Each component of the reference input for the translating link creates a swing of equal amplitude, out of phase such that they cancel each other and no swing results.

A cycloid plus ramp motion is expressed as

$$y = \frac{Y_1}{2\pi} [2Rt - \sin (2Rt)] + \frac{Y_2}{\tau} t,$$

where $Y_1$ is the cycloidal motion distance to be travelled, $t$ is time into motion, $\tau$ is the travelling time, $R = \pi/\tau$, and $Y_2$ is the ramp distance (lift) to be travelled at the end of the travelling time $\tau$.

For zero initial conditions, the equation of motion for the analogous mass-spring system given by Eq. (4) is solved for the ramp and cycloid inputs defined by Eq. (5). In order to have no swing after $t > \tau$, the distances $Y_1$ and $Y_2$ must be

$$Y_1 = Y \left[ 1 - \left( \frac{\tau_n}{\tau} \right)^2 \right], \quad Y_2 = Y \left( \frac{\tau_n}{\tau} \right)^2,$$

where $\tau_n$ is the period of natural oscillation or swing and $Y$ is the total distance to be travelled. Note that if the travelling time is the period of natural swing, the shaped trajectory consists only of a ramp.

Eqs. (1) and (2) are solved on a digital computer for the cycloidal input plus the ramp input with the distances determined from Eq. (6), using Runge–Kutta numerical integration algorithms. The period of oscillation of the compound pendulum considered is taken as 1.95 s or the natural frequency 3.22 rad/s which is that of the experimental system described in the next section. The distance $Y$ to be covered and the travelling time $\tau$ are arbitrarily taken as 0.2 m and 1.95 s, respectively, in the simulation. The same data is used in Section 4 for experimental verification. The simulation result is depicted in Fig. 3 where the shaped input $y$, the response of the link $x$, and the resulting swing $\theta$ of the suspended object are given together. Note that there is no swing when $t > \tau$, and that there is a significant time delay in the simulated response of the manipulator and
therefore in the simulated swing of the suspended object. This time delay could be due to the servo
valve and actuator dynamics considered in the simulations. The same time delay can also be seen
in the experimental results given in Section 4.

As given in Eq. (6), \( Y_1 \) and \( Y_2 \) are the function of the natural frequency which is the identity of a
dynamic system. Simulations are, therefore, carried out in order to demonstrate the robustness of
this method to uncertainties in the natural frequency of the system. The maximum amplitude of
the residual swing (as a percentage of the maximum amplitude of step response of the system) for
\( Y = 0.2 \) m and a range of travelling times resulting in \( 0.2 \leq \omega'_n / \omega_n \leq 2.0 \) is plotted against the ratio
of \( \omega'_n / \omega_n \) in Fig. 4, where \( \omega'_n = 3.22 \) rad/s is the actual natural frequency of the system. Note that
this method can move the system without causing swing when the system natural frequency is
exact and with very little swing when the natural frequency is not known exactly. If the amplitude
of the residual swing for a simple swinging system is less than 5\%, it is widely accepted that the
system is in a swing-free state after following a trajectory.

3.2. Convolving a specified function with a sequence of two impulses

This method, which involves convolving a sequence of impulses with the main desired traject-
ory in order to produce a shaped reference input, has been proposed by Singer and Seering. A full
account of this method is given in [5,9]. It is based on the transient response of a second order
system with the natural frequency of \( \omega_n \), and the expected damping ratio of \( \zeta \) to an impulse input.
The transient response \( c(t) \) is expressed as

\[
c(t) = \left[ A - \frac{\omega_n}{\sqrt{1 - \zeta^2}} \exp \left( -\zeta \omega_n (t - t_0) \right) \right] \sin \left( \omega_n \sqrt{1 - \zeta^2} (t - t_0) \right),
\]

(7)
where $A$ is the amplitude of the position impulse command, $t$ is time, $t_0$ is the time of the impulse input. Since any arbitrary function can be formed from a sequence of impulses, the impulse sequence can be used to reduce the vibration of dynamic systems or the swing of suspended objects under arbitrary trajectories. This superposition is accomplished by convolving any desired trajectory with a sequence of impulses in order to yield the shortest actual system input. This operation, therefore, becomes a prefilter for any input to be given to the system. The time penalty resulting from prefiltering the input equals to the length of the impulse sequence.

The amplitudes and the durations of the impulses are determined such that the system moves without swing after the motion has ended. When a sequence of two impulses is used to reduce the residual vibration of a dynamic system, the following impulse durations and amplitudes are found [5]:

$$t_1 = 0, \quad t_2 = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}},$$

$$A_1 = \frac{1}{1 + K}, \quad A_2 = \frac{K}{1 + K},$$

where $K = \exp \left(-\zeta \pi / \sqrt{1 - \zeta^2}\right)$. Note that the amplitudes of the two impulses are normalised so that they sum to unity. These are the shortest time-duration sequences that eliminate residual vibrations, assuming that only positive amplitudes are used. It has been reported, [9], that the response is extremely sensitive to uncertainties in the natural frequency and damping ratio of the system. Some additional constraints can be added to increase robustness against the uncertainties. This necessitates the use of more than two impulses at a penalty of increasing the move time, [9], which is not very desirable.

Recalling that the system under consideration has no damping, that is, $\zeta = 0.0$. The durations and amplitudes of the impulses are found as

![Fig. 4. Sensitivity of the system to uncertainties in the natural frequency when the first method is used.](image-url)
\[ t_1 = 0, \quad t_2 = \frac{\pi}{\omega_n}, \]
\[ A_1 = \frac{1}{2}, \quad A_2 = \frac{1}{2}, \]

where \( K = 1 \). Note that the time \( t_2 \) of the second impulse is a half of the period of natural oscillation of the system and the duration of the motion is lengthened by \( t_2 \). This sequence of two impulses is convolved with any desired trajectory. In this study, a cycloidal function which has continuous first and second derivatives at the beginning and at the end of a move is chosen as the desired trajectory. It is important to note that the preshaped trajectory does not contain impulses once the convolution is performed. The convolution of any desired trajectory with some impulse sequence results in a trajectory that has the same vibration or swing reducing effects as the impulse sequence.

A full cycloid is chosen as the desired trajectory to be convolved with a sequence of two impulses in order to demonstrate the effectiveness of this method in reducing the residual swing. The delay due to the impulse sequence is \( t_2 = 0.975 \) s. For \( \tau = 0.975 \) s, the total travelling time including \( t_2 \) thus becomes 1.95 s. By using the preshaped trajectory consisting of a full cycloid convolved with a sequence of two impulses, the differential equation of motion given by Eqs. (1) and (2) are solved on a digital computer for \( Y = 0.2 \) m and the total travelling time of 1.95 s. The solution obtained is depicted in Fig. 5, where it is clearly seen that there is no swing when \( t > \tau \). The maximum amplitude of the residual swing (as a percentage of the maximum amplitude of step response of the system) for \( Y = 0.2 \) m and a range of travelling times resulting in \( 0.2 \leq \omega'_n/\omega_n \leq 2.0 \) is plotted against \( \omega'_n/\omega_n \) in Fig. 6 which indicates that this method is also robust to uncertainties in the natural frequency of the system.

Fig. 5. Simulated swing-free motion of the suspended object when the second method is used. The total travelling time is 1.95 s.
3.3. Hybrid input shaping

This method is based on the two methods just presented above. The distances $Y_1$ and $Y_2$ for a cycloid and a ramp function are calculated from Eq. (6), and subsequently the resulting trajectory is convolved with the two impulse sequence defined by Eq. (9). For the specified travelling time $\tau = 0.975$ s, $Y = 0.2$ m, $\omega_n = 3.22$ rad/s, $t_2$, $Y_1$ and $Y_2$ are calculated as $-0.975$ s, $-0.6$ and $0.8$ m, respectively. Note that the total travelling time becomes $\tau + t_2 = 1.95$ s which is the same travelling time for the two methods. By using the resulting preshaped trajectory, the differential equation of motion given by Eqs. (1) and (2) are solved on a digital computer. The solution obtained is depicted in Fig. 7, where it is clearly seen that there is no swing when $t > \tau$. The maximum amplitude of the residual swing (as a percentage of the maximum amplitude of step response of the system) for $Y = 0.2$ m and a range of travelling times resulting in $0.2 \leq \omega'_n/\omega_n \leq 2.0$ is plotted against $\omega'_n/\omega_n$ in Fig. 8 which indicates that this method is also robust to uncertainties in the natural frequency of the system.

4. Experimental evaluation of input shaping

The experimental set-up shown in Fig. 9 consists of a hydraulically actuated manipulator of Stanford type, a compound pendulum attached to the tip of the translating link comprising the dynamic load, a conductive plastic servo potentiometer to measure the swing of the suspended object, and hardware to control and command the manipulator, and read the potentiometer output. The manipulator links are controlled by Bosch regulator valves of 0811 404 028 type [7] in a closed loop fashion with rotary potentiometer feedback to obtain the desired position of the translating link. The block diagram of the control strategy is shown in Fig. 10.

Fig. 6. Sensitivity of the system to uncertainties in the natural frequency when the second method is used.
The object of the experimental work has been to illustrate the effectiveness and robustness of the methods in preventing the swing of suspended objects. The translational link of the manipulator was kept parallel to the ground and was given the preshaped trajectories described in Section 3.

Fig. 7. Simulated swing-free motion of the suspended object when the hybrid input shaping method is used. The total travelling time is 1.95 s.

Fig. 8. Sensitivity of the system to uncertainties in the natural frequency when the hybrid input shaping method is used.
4.1. Experimental results

Three sets of experiments have been conducted. The first set is for the method of superimposing a ramp function onto a full cycloid and the second is for that of convolving a sequence of two impulses with a full cycloid and the third is for the hybrid input shaping method of convolving sequence of two impulses with the trajectory obtained using the first method. In all the results presented in this section, the shaped input $y$, the resulting movement $x$ of the point of suspension $O$, and the resulting experimentally measured and simulated swing $\theta$ of the suspended object are shown in the same figure for comparison of simulation and experimental results. Fig. 11 shows the experimental result when the travelling time is $\tau = 1.95$ s and the distance travelled is 0.2 m. This experimental result corresponds to the simulation result presented in Fig. 3. Fig. 12 shows the comparison of experimental and simulated sensitivity of the system to uncertainties in the natural frequency when the first method is used. Note that there is a close correspondence between the experimental and simulation results.

The experimental result shown in Fig. 13 is for the second method when the total travelling time is adjusted according to the theory presented in Section 3.2. The data used for this result is the same data for the simulation result depicted in Fig. 5. Fig. 14 shows the comparison of
experimental and simulated sensitivity of the system to uncertainties in the natural frequency when the second method is used. Note that the total travelling time and the total distance travelled are unchanged and there is a close correspondence between experimental and simulation results.

The experimental result shown in Fig. 15 is for the third method - hybrid input shaping method, initially a cycloid and ramp function distances $Y_1$ and $Y_2$ are calculated and then the resulting trajectory is convolved with two impulses as explained in Section 3.3. The data used for this result is the same data for the simulation result depicted in Fig. 7, i.e., $t_2 = 0.975$ s, the
distance travelled is 0.2 m, the total travelling time is 1.95 s. Fig. 16 shows the comparison of experimental and simulated sensitivity of the system to uncertainties in the natural frequency when the hybrid input shaping method is used.

Fig. 13. Experimental and simulated swing-free motion of the suspended object when the second method is used.

Fig. 14. Comparison of experimental and simulated sensitivity of the system to uncertainties in the natural frequency when the second method is used.

distance travelled is 0.2 m, the total travelling time is 1.95 s. Fig. 16 shows the comparison of experimental and simulated sensitivity of the system to uncertainties in the natural frequency when the hybrid input shaping method is used.
Fig. 15. Experimental and simulated swing-free motion of the suspended object when the hybrid input shaping method is used.

Fig. 16. Comparison of experimental and simulated sensitivity of the system to uncertainties in the natural frequency when the hybrid input shaping method is used.
It is clear from Figs. 11, 13 and 15 that the three methods perform as well on a real system as illustrated by simulation. Comparing Figs. 12, 14 and 16, it is obvious that the hybrid input shaping method is more robust than the method of superimposing a ramp function onto another function, and the method of convolving a sequence of two impulses with a desired reference. Note that even when there is an error of $\pm 15\%$ in the natural frequency $\omega_n$ of the system, the amplitude of the residual swing is negligibly small. Please bear in mind that the robustness of the first method is limited and that of the second method depends on the number of impulses. As the number of impulses increases, the response time is lengthened by the total time of impulses. So, the hybrid input shaping method we propose is robust to natural frequency uncertainties and effective in reducing residual swing.

5. Conclusions

A hybrid input shaping method based on the successive use of two input shaping methods has been worked out to reduce the residual swing of suspended objects in transportation or the residual vibration of equivalent dynamic systems. All together the three methods for the same purpose has been presented. The simulation and experimental results demonstrate that it is possible to obtain a swing-free motion at the end of a move by employing (i) the method of superimposing a ramp function onto another function, (ii) the method of convolving a desired input with a sequence of two impulses, and (iii) the method of hybrid input shaping, which is based on using the first and the second methods successively to generate a shaped input. While the first method neither limit nor increase the move time to reduce swing at the end of a move and to be robust to uncertainties in the natural frequency of the system, the second lengthens the travelling time and necessitates more impulses to reduce swing and to be robust. But, hybrid shaping method has the characteristics of the first and second methods such as travelling time can be adjusted arbitrarily, can be applied to multi-mode system, and is more robust than the two methods. Simulations and experimental results indicate that a swing-free stop is obtainable at the end of a move with a high degree of robustness to uncertainties in the natural frequency of the swinging or vibrating system by employing the hybrid input shaping method we propose. The method is simple and easy to implement, and most importantly robust to uncertainties ($\pm 15\%$ of $\omega_n$) in the natural frequency of the system.

References