# EP220 First Midterm Exam (Duration: $\frac{5}{3}$ hour, 12.04.2013)

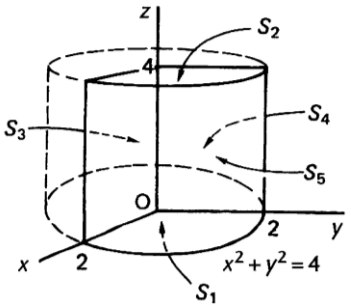
**1 (60).** If the potential φ of a particle is given as $\varphi = x^2y + y^2z + z^2x$ at the point *P(1,-1,2)*. Find

**a)** the directional derivative of φ in the direction of the vector $\boldsymbol{A} = 4\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ at P(1,3,2),
**b)** the unit normal to the surface φ at that point. c) div (grad φ) and d) curl (grad φ).

***Ans: a)$\sqrt{5}/15$, b) (2i-3j+5k)/ $\sqrt{38}$, c)4, d)0***

**2 (50).** A particle moves in 3 dimensions under the force $\boldsymbol{F} = x^2y^2\,\boldsymbol{i} + y^3 z\,\boldsymbol{j} + z^2\,\boldsymbol{k}$. Evaluate *the work done* on this particle along the curve $x = 2u^2$, $y = 3u$ and $z = u^3$ between *A(2,-3,-1)* and *B(2,3,1)*. ***Ans:23.8 Joule***

**3 (50).** A vector field of $\boldsymbol{F} = x\,\boldsymbol{i} + 2\,\boldsymbol{j} + z^2\,\boldsymbol{k}$ taken over the region bounded the planes $z = 0$, $z = 4$, $x = 0$, $y = 0$ and the surface $x^2 + y^2 = 4$ in the first octant. Evaluate the *volume integral* of div **F**. ***Ans: 20π***




**4 (70).** Solve the following problems.

**a)** **(20)** Show that *A = {1,0,2,-2}* and *B = {-2,1,1,0}* vectors are *orthogonal*. Then, find an *orthonormal* set of vectors forming from these vectors.
**b)** **(20)** Find the rank of matrix

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

**c)** **(20)** Determine two numbers *s* and *t* such that the following matrix is *symmetric*.

$$A = \begin{bmatrix} 2 & s & t \\ 2s & 0 & s+t \\ 3 & 3 & t \end{bmatrix}$$

**d)** **(10)** Find $5\,Tr(A^T)$ of the matrix forming from vectors *A = {1,2,3}*, *B = {2,4,3}* and *C = {1,-5,11}*.

***Ans: a) {1/3 X, 1/$\sqrt{6}$ Y}, b) 2, c) t=3, s=0, d) 80.***

**5 (60).** Find *the inverse of* matrix with any methods.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}, \quad \textbf{Ans:} \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

**6 (60).** Solve the system of current equations of an electric circuit by using *Cramer's rule*.

$$-2i_1 + 3i_2 - i_3 = 1; \qquad i_1 + 2i_2 - i_3 = 4; \qquad -2i_1 - 6i_2 + i_3 = -3.$$

***Ans: $i_1$=11/13 , $i_2$=-6/13, $i_3$=-53/13.***

**Hint:** Cylindrical Polar coordinates: $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = z$, $dV = \rho d\rho d\varphi dz$, $ds = \rho d\varphi dz$. Spherical coordinates: $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, $dV = r^2\sin\theta dr d\theta d\varphi$, $ds = r^2\sin\theta d\theta d\varphi$.