EP220 Second Midterm Exam

(Duration: $\frac{5}{3}$ hour)

1 (60). Find the eigenvalues and the eigenvectors of matrix A,

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}$$

$$\lambda_1 = 2,\ \mathbf{x_1} = \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix};\ \lambda_2 = 3,\ \mathbf{x_2} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix};\ \lambda_3 = -5,\ \mathbf{x_3} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

2 (60). Find a matrix P that diagonalizes

$$A = \begin{bmatrix} -6 & 5 \\ 4 & 2 \end{bmatrix}.$$

$$\begin{pmatrix} 1 & 5 \\ 2 & -2 \end{pmatrix}$$

3 (50). Consider the complex function is defined as $F(z) = z^2 + i^2\ Re[z] - i\ 2\ Im[z] - Conjugate[z]$.

By considering $z = 2- i$, write the function in both

- **a)** Cartesian and
- **b)** Polar form for corresponding z.

4 (60).). Consdier the real function $u(x, y) = x^2 - y^2 + x - y$.

- **a)** state if $u(x,y)$ is a harmonic function or not,
- **b)** if $u(x,y)$ is an harmonic function, then find the value of analytic function $f(z)$ by using Cauchy-Riemann equations at the point $z = 1+i$.

$$f(z) = z^2 + (1+i)z + ia.$$

5 (60). Verify Cauchy's theorem by evaluating the integral of $f(z) = \bar{z}^2$ for the triangle path forming through points $z = 0$, $z = 2$, and $z = 2 + 2i$.

6 (60). Find the integral of $\int \frac{\sinh(z)\exp(-z)}{(z^2-2z-4)(z^2+\left(\frac{\pi}{2}\right)^2)} dz$.

Hints: Cylindrical Polar coordinates: $x = \rho cos\varphi$, $y = \rho sin\varphi$, $z = z$, $dV = \rho d\rho d\varphi dz$, $ds = \rho d\varphi dz$.

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \cosh(x) = \frac{e^x + e^{-x}}{2}, \int \frac{f(z)}{z - z_0} dz = 2\pi i f(z_0),\ \ a_{-1} = \lim_{z\to z_0}\left[\frac{1}{(n-1)!}\frac{d^{n-1}}{dz^{n-1}}((z-z_0)^n f(z))\right]$$