EP220 Final Exam

(Duration: $\frac{5}{3}$ hour)

1 (70). Show that the vector field $\boldsymbol{F} = 2x\cos(2y)\,\boldsymbol{i} - (2x^2\sin(2y) + 4\,y^2)\boldsymbol{j}$ is conservative and then find the corresponding potential function V, which obeys, $\boldsymbol{F} = Grad\ V$.

$$\varphi(x, y) = x^2\cos(2y) - \frac{4}{3}y^3.$$

2 (80). Consruct an orthonormal set of eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

3 (100). **a)** Find the matrix A given that $(I_2 + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$

$\{a_{11} \to -\frac{9}{13}, a_{12} \to \frac{1}{13}, a_{22} \to -\frac{6}{13}, a_{21} \to \frac{2}{13}\}.$

b) Solve the linear system of equations by using any method.

$$\begin{aligned} x + y &= 5 - z, \\ x - 3z &= -1 + 2y \\ 2x + y &= 3 + z \end{aligned}$$

Thus the unique solution of the system is $x = N_x/D = 4,\ y = N_y/D = -2,\ z = N_z/D = 3$, that is, the vector $u = (2, -1, 0)$.

4 (100). For the given the matrix B as following;

$$B = \begin{bmatrix} \frac{1}{2}i & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2}i \end{bmatrix}$$

Find;

- **a)** det(B),
- **b)** the inverse of B by using adj(B) expression, and
- **c)** B^*
- **d)** $B^\dagger$
- **e)** State the type of matrix if it is Hermitian, anti-Hermition or unitary matrix.

Ans) Det[B]=-1,b) $\{\{-\frac{i}{2}, \frac{\sqrt{3}}{2}\}, \{\frac{\sqrt{3}}{2}, -\frac{i}{2}\}\}$ c) $\left\{\left\{-\frac{i}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{\frac{\sqrt{3}}{2}, -\frac{i}{2}\right\}\right\}$, d)$\{\{-\frac{i}{2}, \frac{\sqrt{3}}{2}\}, \{\frac{\sqrt{3}}{2}, -\frac{i}{2}\}\}$,

e) B is a unitary matrix.

5 (60). Find the integral of $\int \frac{1}{(z+4)(z-1)^3}dz$ using ***residues.***

6 (90). Answer the following integral by using special functions

- **a)** $\int_0^\infty x^4\ e^{-3x}\text{dx}$
- **b)** $\int_0^\infty \frac{x^3}{\sqrt{3-x}}dx$
- **c)** Using the Rodrigues formula, $H_n(x) = (-1)^2 e^{x^2}(\frac{d}{dx})^n e^{-x^2}$, find $H_4(x)$.