Inclusive Production of the $\rho^\pm(770)$ Meson in Hadronic Decays of the $Z^0$ Boson

Ph.D. Thesis in Engineering Physics University of Gaziantep

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Introduction

High energy collisions of sub-atomic particles can result in events containing a high multiplicity of hadronic particles.

An example was the production of $Z^0$ Bosons at LEP an $e^+e^-$ collider with $E_{CM} = 91.2$ GeV (= $1.5 \times 10^8$ Joules)

\[ e^+e^- \rightarrow Z \rightarrow q\bar{q} \rightarrow \text{hadrons} \]


The Standard Model

Table 2.1: The particles and forces of the Standard Model.

<table>
<thead>
<tr>
<th>Fermions (spin - \frac{1}{2})</th>
<th>Bosons (spin 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarks</td>
<td>Leptons</td>
</tr>
<tr>
<td>u (up)</td>
<td>d (down)</td>
</tr>
<tr>
<td>c (charm)</td>
<td>s (strange)</td>
</tr>
<tr>
<td>t (top)</td>
<td>b (bottom)</td>
</tr>
</tbody>
</table>

This study describes the ALEPH measurement of

- the production rate and
- the differential cross section of the $\rho^\pm(770)$ meson in hadronic decays of the $Z$ boson.

$\rho^\pm$ candidates are reconstructed from the decay channel:

\[ \rho^\pm \rightarrow \pi^0 + \pi^\pm \quad (BR \approx 100\%) \]

The results are compared with:

- JETSET 7.4 (a Monte Carlo event generator software)
- OPAL measurement (unique at LEP)
The Large Electron-Positron Collider (LEP)
- Circumference 27 km (largest e⁻e⁺ collider)
- Located in an underground at a depth about 100 m
- Operated
  - between 1989 – 1995; $E_{CM} = 91.2$ GeV for Z
  - between 1995 – 2000; $E_{CM} = 160-190$ GeV for W+W⁻

for

ALEPH,
DELPHI,
OPAL, and L3
experiments

Different particle types interact differently with matter e.g. photons do not feel a magnetic field

We need different types of detectors to measure different types of particles.

Particle Detectors
Detectors employed in HEP experiments record position, arrival time, momentum, energy and identity of particles.

Charged particles (e⁺, p, K⁺, μ⁺, π⁺) can be detected through their ionisation in tracking chambers
- a measure of the curvature of the track in a magnetic field gives a measure of its momentum
- a measure of the rate of ionisation loss (dE/dx) can be used to determine its type

Neutral particles (γ, n) are detected via calorimeters, where their position and energy are measured.

The ALEPH Detector

Tracking Chambers

<table>
<thead>
<tr>
<th>Detector</th>
<th>Resolution</th>
<th>Photo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex Detector (VDET)</td>
<td>$\sigma(r, \phi) = 12$ μm  $\sigma(z) = 10$ μm</td>
<td></td>
</tr>
<tr>
<td>Inner Tracking Chamber (ITC)</td>
<td>$\sigma(r, \phi) = 20$ μm</td>
<td></td>
</tr>
<tr>
<td>Time Projection Chamber (TPC)</td>
<td>$\sigma(r, \phi) = 180$ μm  $\sigma(z) = 1$ mm  $\sigma(p) = 1.2 \times 10^{-3} p$ GeV/c</td>
<td></td>
</tr>
</tbody>
</table>
**Electromagnetic Calorimeter (ECAL)**

For electrons and photons of high energy, a dramatic result of the combined phenomena of bremsstrahlung and pair production is the occurrence of cascade showers.

A parent electron will radiate photons, which converts to pairs, which radiate and produce fresh pairs in turn, the number of particles increasing exponentially with depth in the medium.

![Image of a calorimeter](image1)

---

**Event Resolution and Spatia Resolution**

![Graphs of energy and spatial resolution](image2)

\[
\frac{\sigma_E}{E} = 0.15 \frac{1}{\sqrt{E}} + 0.009,
\]

\[
\sigma_{\theta_{\text{rel}}} = \left( \frac{2.5}{\sqrt{E}} + 0.2\right) \text{ mrad}
\]

---

**Event Selection**

The ALEPH detector recorded a variety of events:

- $\gamma\gamma$ events
- most of decays of Z boson to its various modes

<table>
<thead>
<tr>
<th>Z Decay Mode</th>
<th>Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow \mu^+\mu^-$</td>
<td>3.366 ± 0.007</td>
</tr>
<tr>
<td>$Z \rightarrow \tau^+\tau^-$</td>
<td>3.379 ± 0.008</td>
</tr>
<tr>
<td>$Z \rightarrow \ell^+\ell^-$ hadrons</td>
<td>69.910 ± 0.060</td>
</tr>
<tr>
<td>$Z \rightarrow \ell^+\ell^-$ invisible</td>
<td>30.000 ± 0.060</td>
</tr>
</tbody>
</table>

In addition, there are small number of background:

- beam-gas interaction
- beam electrons
- cosmic rays

![Event selection diagrams](image3)
To remove events which suffer from a low geometric acceptance, a cut applied on the polar angle, $\theta$, of the event axis as defined by sphericity.

Events are accepted if

$$35^\circ < \theta < 145^\circ \quad (|\cos \theta| < 0.82)$$

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**Hadronic Event Selection**

Hadronic events are selected on the basis of the **total charged multiplicity** and energy within an event. For this, cuts are applied to select ‘good charged tracks’.

**Track Cuts**

removes same badly reconstructed tracks.

A good charged track must have:

- at least 4 TPC hits
- a polar angle $20^\circ < \theta < 160^\circ$
- a transverse impact parameter $|d_0| < 2$ cm
- a longitudinal impact parameter $|z_0| < 5$ cm
- a transverse momentum $p_T > 200$ MeV/c

---

**Event Cuts**

1. a minimum of 5 ‘good’ tracks
2. a minimum of 15 GeV total ‘good’ charged energy

With these cuts a total 3,239,746 hadronic events are selected with

- efficiency: $\epsilon = 95.0\%$
- purity: $P = 99.6\%$

from data recorded by ALEPH from 1991 to 1995 running periods.

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**Track Selection**

$\rho^{\pm}$ candidates are reconstructed from the decay channel:

$$\rho^{\pm} \rightarrow \pi^{0} + \pi^{\pm} \quad (BR \approx 100\%)$$

$\pi^{\pm}$ selection is relatively trivial, while $\pi^{0}$ selection and reconstruction is more complicated.

All selection performances are determined from the Monte Carlo with the aim to maximise both

- purity
- efficiency.
Charged Track Selection

All good tracks originating from IP are considered as pions. The following cuts select $\pi^\pm$ with $\epsilon = 95\%$ and $P = 43\%$

- a transverse impact parameter $b_\perp < 0.5$ cm
- longitudinal impact parameter $b_\parallel < 3.0$ cm
- a transverse momentum $p_\perp > 250$ MeV/c
- if available, $-2 < \gamma(E/dx) < 3$ (with pion hypothesis)

Tight cuts on the impact parameters are introduced to increase the reconstruction purity of $\pi^\pm$ mesons from $\rho^\pm$.

Charged particle identification is performed by the measurement of ionisation energy loss, $dE/dx$.

The deviation from an assumed hypothesis expressed as:

$$\chi^2 = \frac{(dE/dx_{\text{measured}} - (dE/dx)_{\text{expected}})^2}{\sigma^2}$$

Neutral Pion Selection

Neutral pions are built from the decay $\pi^0 \rightarrow \gamma + \gamma$

Invariant mass spectra is built up by the equation:

$$M^2 = 2E_1E_2(1 - \cos \theta_{12})$$

Following cuts are applied:

- Each photon energy $E_{\gamma} > 1$ GeV
- Pion energy $E_{\pi} < 18$ GeV
- Invariant mass of two photon $M(\gamma_1\gamma_2)$, should be within $\pm$2$\sigma$ mass window around pion signal. 
  $\sigma$ is defined as HWHM.

Poor purity of the pions are improved by a "Ranking" Method.

The topology of reconstructed $\pi^0$ is found to be important.

Geometric representation of four possible $\pi^0$ topologies are given right.

Photon pairs taken from:

topology 1: one ECAL cluster, within which two subclusters are resolved

topology 2: one ECAL cluster, within which more than two subclusters are resolved

topology 3: two ECAL clusters, within each of which no subclusters are resolved

topology 4: two ECAL clusters, within one or both of which more than one subcluster is resolved

Mass Constraint

Uncertainties in the reconstructed momentum vector of a $\pi^0$ are introduced due to the finite ECAL spatial and energy resolution.

To improve the moment resolution of the pion the reconstructed mass can be constrained to the nominal pion mass of 135 MeV/c$^2$. This is done by modifying the photon energies ($E_1$ and $E_2$) and opening angle ($\theta$) between photons until the required mass is obtained. These parameters can be found by minimising the chi-square form:

$$\chi^2 = \frac{(E_1 - \bar{E}_1)^2}{\sigma_1^2} + \frac{(E_2 - \bar{E}_2)^2}{\sigma_2^2} + \frac{(\cos \theta_{12} - K)^2}{\sigma_{\theta_{12}}^2}$$

with mass constraint: $M_{\pi^0}^2 = 2E_1E_2(1 - K)$.
Neutral Pion Calibration

Selecting $\pi^0$ candidates from a mass window around the $\pi^0$ peak provides initial rejection of most of the combinatorial background.

It is found that: peak mass and width of the $\pi^0$ signal depend on both energy and topology.

So, we need to calibrate two functions $m(E,T)$ and $\sigma(E,T)$

The calibration procedure is as follows:
1. The mass spectra is plotted for the matched pion signal (for each topology in 1 GeV energy intervals)
2. The peak (signal) is fitted with a Gaussian from which positions $a$ and $b$, taken at half height, are measured
3. Half width at half maximum (HWHM) $\sigma = (b-a)/2$ and the center of the mass window $m = (b+a)/2$ are evaluated

The calibration functions give a correct selection of $\pi^0$ peak.

The Ranking Method

Consider $n$ photons taken from the ECAL. We can form $\pi^0$ candidates by building photon pairs as follows:

PHOTONS SELECTED PAIRS (combinations)  
<table>
<thead>
<tr>
<th>$n$</th>
<th>12</th>
<th>23</th>
<th>34</th>
<th>45</th>
<th>...</th>
<th>$(n-1)n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>24</td>
<td>35</td>
<td></td>
<td></td>
<td>13, 24, 35</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>25</td>
<td>36</td>
<td></td>
<td></td>
<td>14, 25, 36</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>...</td>
<td>4n</td>
<td></td>
<td></td>
<td>5, ... , 4n</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>...</td>
<td>3n</td>
<td></td>
<td></td>
<td>3, ... , 3n</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td></td>
<td>2n</td>
<td></td>
<td></td>
<td>2, 2n</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
</tbody>
</table>

Number of combinations: $C(n,2) = \frac{n!}{2(n-2)!} + \frac{n(n-1)}{2}$

we have $n(n-1)/2$ candidates forming $S+B$ but only $n/2$ of them are true forming $S$.

Signal-to-background ratio:

\[ S \frac{n/2}{n(n-1)/2 - n/2} = \frac{1}{n-2} \]

Purity and Efficiency

In this study, we define the neutral pion purity ($P$) and efficiency ($\varepsilon$) as follows:

\[ P = \frac{S}{S+B} \quad \varepsilon = \frac{S}{S+B} \]

\[ \varepsilon \times P = \frac{1}{S+B} \left( \frac{S}{S+B} \right)^2 \]

where:

$S$ is the number of signal, and

$B$ is the number of background for the given mass window

$S_0$ is the total number of signal within $\pm 1\sigma$ mass window.
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The Standard Method

![Diagram](image)

![Diagram](image)

![Diagram](image)

![Diagram](image)
**π^0 Estimators**

An estimator, based on kinematics of \( \pi^0 \), can be used to discriminate signal and background. Following distributions define our estimators:

- Chi-square (\( \chi^2 \)) values from mass constraint
- Photon pair opening angles, \( \theta_{12} \)
- 2D scatter distribution of \( \chi^2 \) vs \( \theta_{12} \)

Solid curve has the form:

\[
(\frac{\chi^2}{A})^2 + (\frac{\theta_{12}}{B})^2 = 1
\]

---

**Ranking Method**

After the initial mass window selection, additional improvement can be achieved by applying the estimator indirectly with a ‘Ranking’ method.

The algorithm is as follows:

1. Pion estimator values are calculated for each pion in an event
2. Pions candidates are ranked according to their estimator values
3. A scan is then made through the list for pairs of pions which share photons. When such a pair exist, one or both candidates must be false: the candidate with largest estimator value is removed

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**Example of applying Ranking method:**

**SELECTED CANDIDATES BEFORE RANKING**

<table>
<thead>
<tr>
<th>Pion</th>
<th>Photon</th>
<th>Truth</th>
<th>#</th>
<th>est.</th>
<th>1</th>
<th>2</th>
<th>info.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.05</td>
<td>A1</td>
<td>1</td>
<td>TRUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.12</td>
<td>B1</td>
<td>1</td>
<td>TRUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>0.13</td>
<td>X1</td>
<td>1</td>
<td>FALSE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.28</td>
<td>C1</td>
<td>1</td>
<td>C2</td>
<td></td>
<td></td>
<td>TRUE</td>
</tr>
<tr>
<td>Y</td>
<td>0.45</td>
<td>Y2</td>
<td>1</td>
<td>TRUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0.87</td>
<td>Z1</td>
<td>1</td>
<td>Z2</td>
<td></td>
<td></td>
<td>FALSE</td>
</tr>
</tbody>
</table>

**SELECTED CANDIDATES AFTER RANKING**

<table>
<thead>
<tr>
<th>Pion</th>
<th>Photon</th>
<th>Truth</th>
<th>#</th>
<th>est.</th>
<th>1</th>
<th>2</th>
<th>info.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.05</td>
<td>A1</td>
<td>1</td>
<td>TRUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.12</td>
<td>B1</td>
<td>1</td>
<td>TRUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>0.19</td>
<td>X1</td>
<td>1</td>
<td>X2</td>
<td></td>
<td></td>
<td>FALSE</td>
</tr>
<tr>
<td>C</td>
<td>0.28</td>
<td>C1</td>
<td>1</td>
<td>C2</td>
<td></td>
<td></td>
<td>TRUE</td>
</tr>
</tbody>
</table>

To investigate performance of the Ranking method, the mass window, and pion energy is varied.

The results are compared with the standard method.

Performance is measured in terms of the product: \( \varepsilon \times \mathcal{P} \)

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**Performance of the Ranking Method**

**Example Applications**

- **Mass window**
- **Energy**

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- **University of Gaziantep**
- **Department of Engineering Physics**

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**Extraction of the $\rho^\pm(770)$ Signal**

The rate and cross section of the $\rho^\pm$ meson are extracted from the invariant mass distribution of its daughter pions, 

$$\rho^\pm \rightarrow \pi^0 + \pi^\pm \quad (BR \approx 100\%)$$

by fitting the invariant mass to a sum of signal and background functions.

**Signal Reconstruction**

The data is analysed in:
- six intervals of scaled momentum: $x_p = p_p/p_{beam}$
- nine intervals of scaled energy: $x_E = E_p/E_{beam}$

Here $p_{beam} \approx E_{beam}$ (about 45.6 GeV) is the LEP momentum.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$x_p$ range</th>
<th>$x_E$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05 &lt; $x_p$ &lt; 0.10</td>
<td>0.05 &lt; $x_E$ &lt; 0.10</td>
</tr>
<tr>
<td>2</td>
<td>0.10 &lt; $x_p$ &lt; 0.20</td>
<td>0.10 &lt; $x_E$ &lt; 0.20</td>
</tr>
<tr>
<td>3</td>
<td>0.20 &lt; $x_p$ &lt; 0.30</td>
<td>0.20 &lt; $x_E$ &lt; 0.30</td>
</tr>
<tr>
<td>4</td>
<td>0.30 &lt; $x_p$ &lt; 0.40</td>
<td>0.30 &lt; $x_E$ &lt; 0.40</td>
</tr>
<tr>
<td>5</td>
<td>0.40 &lt; $x_p$ &lt; 0.50</td>
<td>0.40 &lt; $x_E$ &lt; 0.50</td>
</tr>
<tr>
<td>6</td>
<td>0.50 &lt; $x_p$ &lt; 1.00</td>
<td>0.50 &lt; $x_E$ &lt; 1.00</td>
</tr>
<tr>
<td>7</td>
<td>1.00 &lt; $x_p$ &lt; 1.00</td>
<td>1.00 &lt; $x_E$ &lt; 1.00</td>
</tr>
</tbody>
</table>

**Signal Extraction and Fitting Procedure**

**Signal Shape**

Basic line shape for the $\rho^\pm$ signal is a relativistic p-wave Breit-Wigner:

$$BW(m) = \frac{1}{\Gamma(m)} \left( \frac{m - m_0}{\sqrt{(m - m_0)^2 + \Gamma^2}} \right)^3$$

with

$$\Gamma(m) = \Gamma_m \left( \frac{q}{q_0} \right)^3 \sqrt{1 + \frac{q^2}{q_0^2}}$$

$m$ is the two-pion invariant mass

$m_0$ is the resonance peak mass

$\Gamma_m$ is the mass dependent width

$\Gamma_{FWHM}$ is the nominal width (FWHM)

$q$ is the momentum of the decay products in the rest frame of parent

$q_0$ is the momentum when $m = m_0$

Monte Carlo ($JETSET$) uses a non-relativistic Breit-Wigner:

$$BW(m) = \frac{(\Gamma_m/2)^3}{(m - m_0)^2 + (\Gamma_m/2)^2}$$

nominal values are:

$m_0 = 775.8 \pm 0.5 \text{ MeV}/c^2$

$\Gamma_0 = 151.5 \pm 1.2 \text{ MeV}/c^2$

[PDG 2006]
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Partially Reconstructed Signal

An important consideration is the effect of partially reconstructed $\rho^{\pm}$ mesons where a $\pi^0$ is reconstructed from originating from $\rho^{\pm}$ signal and one that is not.

Such a combination contains most of the kinematics of the $\rho^{\pm}$ signal for this reason partial signal has similar, but wider shape.

Summary for the Signal function:

- In the fits, nominal values ($\Gamma_{12}$, $m_{12}$) are replaced by $\Gamma_{12}^T$ and $m_{12}^T$.
- The signal function is parameterised as a function of fully reconstructed signal height ($H_2$), since they are correlated. The ratio $r = H_2/H_1$ is taken from the Monte Carlo predictions.
- Signal function is parameterised as a sum of two RBW functions:
  \[ f_{rbw}(m) = p_0 RBW_1(m) + RBW_2(m)/r \]
  \[ p_0 \] is the normalisation constant.

Note that RBW is replaced by BW in the Monte Carlo fits.

Appropriate functions representing each reflection are selected and fitted to the Monte Carlo.

\[ f(m) = f_{rbw}(m) \times \text{reflection function} \]

Bose-Einstein Correlations

Bose-Einstein Correlations (BECs) are an apparent attraction in phase-space between identical bosons.

Features of BEC:

- BEC is relevant if and only if identical bosons (such as pions) are close to each other in phase-space.
- Most of the particles generated in hadronic events are pion triplets obeying Bose statistics. As a result, BECs affect the dynamics of the pions in the final state.
- BECs are quantum mechanical effect that must appear during the fragmentation state. Hence, measurement of BECs can help the understanding of QCD studies.
- BECs are not implemented in MC programs effectively.

The width ($\Gamma_{12}$) and peak mass ($m_{12}$) are different from the nominal values due to resolution effects, which is dominated by $\pi^0$ component.

The mass resolution ($\Gamma_{mass}$) and peak mass ($m_{mass}$) of the $\rho^{\pm}$ meson are determined from the Monte Carlo.

\[ \Gamma_{mass} = m_{mass} - m_{mass}\text{MC} - m_{mass}\text{MC} \]

The free parameters $p_1 - p_6$ are adjusted by the fitting procedure.

Reflections

\[ \pi^0\pi^\pm\text{ mass spectra contain reflections from the decays:} \]

\[ \omega(782) \rightarrow \pi^0\pi^+\pi^- \]
\[ \eta(548) \rightarrow \pi^0\pi^+\pi^- \]
\[ K^{\pm\pm}(892) \rightarrow \pi^0K^{\pm} \]
Residual Bose-Einstein Correlations

Experimental studies reveal that BECs affect the distribution of effective masses of $\pi^+\pi^-$ pairs originating from the IP. The life-time ($10^{-22}$s) and therefore decay length ($1\text{fm}$) of the $\rho^\pm$ meson is sufficiently short that pions from the decay $\rho^\pm \rightarrow \pi^\pm + \pi^0$ can be considered as coming from the IP.

Hence a ‘Residual’ BECs must affect pions of the $\rho^\pm$ meson. Most apparent sign of residual BEC is a distortion in the mass spectra.

The interference term is a model to describe the distortion affecting both signal and background. This can be shown by removing the interference term from the real data mass distribution.

Volatility:

- $C = 0.00 \rightarrow$ original distribution
- $C = 0.30 \rightarrow$ close to its fitted value of 0.28

Rates & Cross Sections

Production Rate:

$$R = \frac{S}{N} \frac{1}{\varepsilon} \frac{d\sigma}{dx}$$

Differential Cross Section:

$$\frac{d\sigma}{dx} = \frac{R}{\frac{1}{\varepsilon} \frac{S}{N}}$$

$N$: number of selected hadronic events
$S$: number of fitted signal
$\delta x$: width of the energy interval
$\varepsilon$: reconstruction efficiency defined as:

$$\varepsilon = \frac{n_{rec}}{n_{gen}} = \frac{\text{number of reconstructed rho-mesons in MC}}{\text{number of generated rho-mesons in MC}}$$

Systematic Error Analysis

Statistical errors originate from counting (statistical) uncertainties that result in measured values being randomly high or low.

Systematic errors originate from detector effects, uncertainties in models, measurement procedures resulting in measured values being systematically high or low.

Possible source of systematic errors:

- track selection cuts
- fitting procedure
- reflection models
- signal function
- efficiency correction
- uncertainty in the extrapolation to full $x_1$ and $x_2$ ranges
Table 3.1: Measured multiplicities and differential cross-sections for the $p^+$ in $x_y$ intervals. The result of removing the measured $x_y$ intervals is also given, including extrapolation to full $x_y$ range with an additional error due to the uncertainty in the extrapolation.

<table>
<thead>
<tr>
<th>$x_y$ range</th>
<th>Multiplicity</th>
<th>$d^2N/dx dy$</th>
<th>$d^2N_{extr}/dx dy$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05-0.10</td>
<td>0.562 ± 0.012 ± 0.003</td>
<td>11.214 ± 0.287 ± 0.200</td>
<td>11.214 ± 0.287 ± 0.200</td>
</tr>
<tr>
<td>0.10-0.15</td>
<td>0.532 ± 0.009 ± 0.013</td>
<td>6.320 ± 0.134 ± 0.089</td>
<td>6.320 ± 0.134 ± 0.089</td>
</tr>
<tr>
<td>0.15-0.20</td>
<td>0.501 ± 0.007 ± 0.014</td>
<td>4.975 ± 0.076 ± 0.057</td>
<td>4.975 ± 0.076 ± 0.057</td>
</tr>
<tr>
<td>0.20-0.30</td>
<td>0.235 ± 0.002 ± 0.010</td>
<td>2.334 ± 0.026 ± 0.018</td>
<td>2.334 ± 0.026 ± 0.018</td>
</tr>
<tr>
<td>0.30-0.50</td>
<td>0.163 ± 0.003 ± 0.009</td>
<td>0.326 ± 0.007 ± 0.004</td>
<td>0.326 ± 0.007 ± 0.004</td>
</tr>
<tr>
<td>0.50-1.00</td>
<td>0.042 ± 0.000 ± 0.005</td>
<td>0.036 ± 0.001 ± 0.000</td>
<td>0.036 ± 0.001 ± 0.000</td>
</tr>
<tr>
<td>0.05-1.00</td>
<td>1.502 ± 0.004 ± 0.008</td>
<td>2.567 ± 0.021 ± 0.019 ± 0.0428</td>
<td>2.567 ± 0.021 ± 0.019 ± 0.0428</td>
</tr>
</tbody>
</table>
### Table 11.3: Comparisons of the total multiplicity of the \( \rho^0 \), \( N(\rho^0) \) and the ratio \( 2N(\rho^0)/N(\mu^+) \) as measured by ALEPH to OPAL and Monte Carlo predictions.

<table>
<thead>
<tr>
<th>Data set</th>
<th>( N(\rho^0) )</th>
<th>( 2N(\rho^0)/N(\mu^+) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH data</td>
<td>2.79 ( \pm 0.03 \pm 0.15 )</td>
<td>1.12 ( \pm 0.05 \pm 0.17 )</td>
</tr>
<tr>
<td>OPAL data</td>
<td>2.40 ( \pm 0.06 \pm 0.43 )</td>
<td>1.08 ( \pm 0.04 \pm 0.20 )</td>
</tr>
<tr>
<td>JETSET 7.4</td>
<td>2.77</td>
<td>1.06</td>
</tr>
<tr>
<td>PYTHIA 6.4</td>
<td>2.85</td>
<td>1.07</td>
</tr>
<tr>
<td>HERWIG 6.5</td>
<td>1.93</td>
<td>1.04</td>
</tr>
</tbody>
</table>

### Conclusion

- Inclusive production of the \( \rho^0 \) mesons in hadronic Z decays has been observed with the ALEPH detector.
- Measured rate and differential cross-section are in good agreement with OPAL measurements within the error bars.
- Monte Carlo rates obtained from JETSET and PYTHIA are consistent with the real data measurements of two independent experiments.
- The model for BEC used in our study is same as the OPAL. This model successfully describes distortion in two-pion invariant mass.

### Future Work

Experiences gained in this study can be applied to future work in a number of possible areas:

- Particle production involving \( \rho^0 \) decay products
- A measurement of \( \rho^0(980) \rightarrow \eta\pi^\pm \)
- Much work needs to be done to implement BEC's in the MC models, since BEC's are not implemented in MC correctly. This is important for LEP and LHC.
- Work on the ATLAS Experiment.

### Publications

- A. Beddall, A. Beddall, A. Bingil, Y. Darmaz
- A. Beddall, A. Beddall, A. Bingil, Y. Darmaz
- A. Beddall, A. Beddall, A. Bingil
- A. Beddall, A. Beddall, A. Bingil

* A Ranking Method for Neutral Pion and Eta Selection in Hadronic Events, TFO 22 (2004)
* Inclusive Production of rho-- mesons in Hadronic Z Decays, AIP 899 (2007)

### Questions

- What are the results of the ALEPH and OPAL data in terms of \( N(\rho^0) \) and the ratio \( 2N(\rho^0)/N(\mu^+) \)?
- How do the Monte Carlo rates from JETSET, PYTHIA, and HERWIG compare to the real data?
- What are the implications of the findings for future work in the field of particle production and decay?