Chapter 3

Properties in Tension and Compression (Part I)
In daily life, engineering materials in service may be subjected to different types of loadings such as:

- axial (tension & compression)
- moment (bending)
- shear (direct & torsional)
- cyclic (fatigue)
- time-dependent (creep)

To make sure that there is no failure/fracture of materials under such loads, we have to know their load carrying limits/capacities.

For this purpose, we need to test these materials in laboratories to their upmost limits before they are put in service. Such tests must be performed under controllable conditions and comply to some standards.

However, actual service conditions are different from test conditions. Thus, results of laboratory tests will not be directly applicable to actual conditions. They have to be somehow modified before used in actual conditions.
Properties of materials under tensile and compressive loads are defined by uniaxial type of tension and compression tests. These tests:

- are the easiest type of tests to evaluate material properties.
- represent the condition of principal stresses that are reasons of failures.
- give results to be utilized for combined stress situations.

Tests are conducted on tensile test machines using specimens of standard size and shape.
The procedure is as follows:

- Take a standard test specimen
- Make necessary measurements on specimen before test
- Place specimen on testing machine
- Apply load on specimen starting from zero and increasing gradually
- Make a note of load and elongation at different times of test
- Proceed until specimen fractures
- Make necessary measurements on fractured specimen
- Convert load-elongation graph into stress-strain diagram
Stress-Strain Curves for Various Materials

Plain Carbon Steel
- Stress: 855
- Strain: 0.4004

Tempered Steel Alloy
- Stress: 1626.4
- Strain: 0.1484

Gray Cast Iron
- Stress: 655
- Strain: 0.0054

2024-T351 Aluminum Alloy
- Stress: 324
- Strain: 0.2194

6Al-4V Titanium Alloy
- Stress: 654.1
- Strain: 0.1201

Rubber
- Stress: 159
- Strain: 4.9453

Nylon
- Stress: 44
- Strain: 0.445

High-density Polyethylene
- Stress: 12.2
- Strain: 6.3567

Phenol Formaldehyde (Bakelite)
- Stress: 52
- Strain: 0.0129
For all types of materials, there are two modes of behaviour under loading:

- **Elastic behaviour:** This is the initial mechanical behaviour during which the specimen returns to its original dimensions upon release of the load. The termination of elastic behavior is known as “elastic limit” of material.

- **Plastic behaviour:** When the load is increased beyond elastic limit, a part of deformation on the specimen is permanent and does not disappear upon release of the load. Such deformation is called “plastic deformation”.

![Diagram showing different modes of behavior](image)
For most engineering materials, elastic behaviour of material obeys “Hooke’s Law” (i.e. there is a linear relationship between stress and strain, as given in Fig. 1a). Such materials are called “linearly elastic”.

On the other hand, some materials (e.g. rubber) are not linearly elastic. They exhibit a nonlinear stress-strain curve as in Fig. 1b.

Upon unloading, both types of materials will follow the loading curves in the reversed direction.

“Elastic limit” is defined as the greatest stress that can be applied without resulting in any permanent strain upon release of load. It is an important material property in design applications since allowable stress values in design work are based on the elastic limit.
The chief design principle concerning elasticity is that the allowable stress must lie within the elastic range. This is ensured by either thicker cross sections or materials of greater “elastic strength” that resist loads without being deformed plastically (“yielding”).

Elastic (yield) strength of various materials is shown in Fig. 2.

In most cases, high strength materials are chosen for purpose of weight saving though they can be costly for specific applications.
Elastic behaviour of a metal is not necessarily linear up to elastic limit. In Fig. 3, the point marking the end of linear relationship is "proportional limit". For practical purposes, linear relationship is assumed to be valid until elastic limit without introducing serious error.

As in Fig. 4, the transition from elastic to plastic behaviour may be sudden (as in annealed and hot rolled steels). This plastic behaviour is "yielding", and "yield point" refers to the elastic limit. The maximum elastic stress that could be carried by a metal is its "yield strength".

A metal behaves elastically when it obeys Hooke’s law and stress (σ) - strain (ε) response is simultaneous. Time dependence of elastic stress-strain relationship is called "anelasticity".
“Stiffness” is the ratio of incremental normal stress to corresponding direct strain for tensile/compressive stress below proportional limit of material.

Mathematically, it is the slope of stress-strain diagram at any point within elastic region (i.e. $d\sigma/d\varepsilon$ in Fig. 5). For linearly elastic materials, this slope is constant, expressed by “Elastic Modulus (E)” or “Young’s Modulus”. Magnitude of stress to produce a given strain increases with the value of E (i.e. greater is the slope, stiffer is the material).

When proportional limit is so low and a constant ratio cannot be obtained (e.g. for cases of brittle materials), there are other definitions of stiffness (explained in the next slide).
(a) **Initial Tangent Modulus:** the slope of stress-strain curve at the origin (i.e. the slope of OM in Fig. 6a).

(b) **Tangent Modulus:** the slope of stress-strain curve at any given stress (i.e. the slope of TPM in Fig. 6b).

(c) **Secant Modulus:** the slope of secant drawn from origin to any specified point on the stress-strain curve (i.e. the slope of OP in Fig. 6c).

(d) **Chord Modulus:** the slope of chord between any two specified points on the stress-strain curve (i.e. the slope of PQ in Fig. 6d).
Stiffness of a material should not be confused with the overall “rigidity” of a machine element that depends upon the dimensions as well. Rigidity is the design terminology when the functional requirements demand that deformations must be small.

For a material that obeys Hooke’s law, the extension (under tension) or the contraction (under compression) is defined by $\delta$ as follows:

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{A} \frac{\delta}{L} \quad \Rightarrow \quad \delta = \frac{F \cdot L}{A \cdot E}$$

$E$ : Young's Modulus (kg/mm$^2$)

$F$ : applied force (kg)

$A$ : cross sectional area (mm$^2$)

$L$ : the strained length (mm)

$E$ : Young’s Modulus (kg/mm$^2$)

When the imposed conditions demand the deformations to be very small, instead of changing the material type, desired rigidity can be obtained by adjusting the geometrical parameters only (i.e. $L$ and $A$ in above equation) without disturbing the other functions.
The designer must reduce all expressions to independent variables first, and then group them as “given”, “fixed” (those that were determined previously from other design equations), “material” or “geometrical” parameters.

This example shows systematic approach for material selection based on rigidity:

For a cylindrical bar: \( F = 500 \text{ kg}, \ E = 7 \times 10^3 \text{ kg/mm}^2 \) (for an Al alloy), \( L = 1000 \text{ mm}, \ r = 7 \text{ mm} \) (here; “radius” is facilitated as an independent parameter).

1. Let’s calculate elongation: 
   \[
   \delta_{Al} = \frac{F \cdot L}{A \cdot E} = \frac{F \cdot L}{\pi r^2 \cdot E} = \frac{500 \times 1000}{\pi \times 7^2 \times 7 \times 10^3} = 0.464 \text{ mm}
   \]

2. If we use steel \( (E = 2 \times 10^4 \text{ kg/mm}^2) \), \( \delta_{St} = 0.1624 \text{ mm} \), which is about 2.85 times smaller. This shows that contribution of material to rigidity can be very significant.

3. Let’s see what increase in radius of Al bar is required to have the same rigidity:
   \[
   r_{Al} = \sqrt{\frac{F \cdot L}{\pi \delta_{St} \cdot E_{Al}}} = 11.83 \text{ mm} \Rightarrow \Delta r = 11.83 - 7 = 4.83 \text{ mm}
   \]
4. How about comparing weights \( W = A \times L \times \rho \):

\[
W_{Al} = \left( \pi 11.83^2 \times 1000 \right) \times \left( 2.66 \times 10^{-6} \right) = 1.169 \text{ kg}
\]

\[
W_{St} = \left( \pi 7^2 \times 1000 \right) \times \left( 7.65 \times 10^{-6} \right) = 1.177 \text{ kg}
\]

This proves if low weight is one of requirements, the designer can easily be deluded by low density of aluminum.

5. In relation with above case, suppose that bar length is fixed as 350 mm and radius as 7 mm, and elongation of up to 0.464 mm is permissible. The respective weights will be \( W_{Al} = 0.143 \text{ kg} \) and \( W_{St} = 0.412 \text{ kg} \). Such difference in results of case 4 & 5 states that a design problem relies completely upon the conditions imposed.

6. It was possible to arrive at these results immediately by making a careful definition of “measures of value”. The first case was comparing the rigidity against weight. Using \( E \) as the primary index, our measure of value is to be high \( E/\rho \) ratio:

\[
\left( \frac{E}{\rho} \right)_{Al} = 2.63 \times 10^{10} \text{ mm} \quad \text{&} \quad \left( \frac{E}{\rho} \right)_{St} = 2.61 \times 10^{10} \text{ mm}
\]

This result indicates little difference in favour of aluminum \( 2.61/2.63 = 0.9924 \), which was also proved in case 4 \( W_{Al} / W_{St} = 1.169/1.177 = 0.9924 \). On the other hand, if our measure of value was based on the minimum weight (decided by density), then \( \rho_{Al} / \rho_{St} = 2.87 \) which would totally change our design philosophy.
Elastic Behaviour - Elastic Modulus

- **Elastic modulus (E)** of a material is determined by binding forces between atoms. It is a structure-insensitive property (such forces cannot be changed without changing the basic nature of material). Elastic moduli of various material groups are given in Fig. 7.

- **Cast irons** do not obey Hooke’s law. Thus, unlike steels, they do not have specific elastic modulus.

- Due to very low elastic moduli of polymer group, achieving rigidity with plastics requires even more experience than designing with metals or other materials.

![Figure 7](image-url)

- **Figures and Data**
  - Ceramics
  - Porous ceramics
  - Glasses
  - Metals and alloys
  - Composites
  - Woods and wood products
  - Polymers
  - Rubbers
  - Polymer foams

- **Graph Details**
  - Y-axis: Young's Modulus (Stiffness) (GPa)
  - X-axis: Flexible to Stiff

- **Value Ranges**
  - Ceramics: > 1000
  - Porous ceramics: 8
  - Glasses: 50
  - Metals and alloys: 13
  - Composites: 8
  - Woods and wood products: 25
  - Polymers: 0.08
  - Rubbers: 0.1
  - Polymer foams: 0.5

- **Graph Legend**
  - Flexible
  - YOUNG'S MODULUS (STIFFNESS) (GPa)
  - Stiff
Elastic Behaviour - Elastic Modulus

- Elastic modulus of a material is almost a constant value, slightly affected by: 
  *alloying condition*, 
  *heat treatment*, 
  *cold working*.

- Above room temperatures, elastic modulus of metals decreases.

- Elastic modulus also depends on the directionality ("anisotropy"). This is important in rolling (e.g. cold-rolled iron has modulus of 23.06, 20.6 and 27.49 kg/mm\(^2\) at 0°, 45° and 90° respectively.)

- As mentioned before, "specific stiffness" (modulus/density ratio) is also an important consideration in material selection (see Fig. 8).

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Modulus (x 10^3 kg/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 °C</td>
</tr>
<tr>
<td>Carbon Steel</td>
<td>21.0</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>19.7</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>11.6</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Figure 8

As mentioned before, "specific stiffness" (modulus/density ratio) is also an important consideration in material selection (see Fig. 8).
Resilience \((U)\) is the capacity of a material for returning to its original dimensions after elastic deformation.

Consider a bar subjected to an axial load \((F)\) causing an elastic deformation \((\delta)\). The work done by this force is: \(U = (F \times \delta) / 2\)

Assuming that the material obeys Hooke’s law, this work is converted into an elastic potential energy \((U)\) of material \((A\) is the area over which \(\sigma\) acts uniformly, and uniform straining is produced along the bar length \(L)\):

\[
U = \frac{F \times \delta}{2} = \frac{(\sigma \times A)(\varepsilon \times L)}{2} = \frac{(\sigma \times A)((\sigma/E) \times L)}{2} = \frac{1}{2} \left( \frac{\sigma^2 \times A \times L}{E} \right)
\]

The maximum elastic energy is reached when the bar is strained to its proportional limit \((the\ elastic\ limit\ can\ also\ be\ used\ in\ equation)\).

Total elastic energy also depends upon volume of material \((as\ it\ is\ indicated\ with\ the\ term\ A \times L\ in\ the\ equation)\).
In a broad sense, resilience is the area under stress-strain curve until elastic limit, as illustrated in Fig. 9. For linearly elastic materials:

\[
U = \frac{1}{2} \cdot \frac{S_y^2}{E}
\]

- \(U\) : modulus of resilience (kg·mm/mm³)
- \(S_y\) : yield strength (kg/mm²)
- \(E\) : Young’s Modulus (kg/mm²)

Its unit is kg·mm/mm³. Hence, total elastic energy to be absorbed by an element depends also upon the volume.

An ideally resilient material has high elastic limit and low elastic modulus. This states that not all metals have high modulus of resilience (e.g. rubber is more resilient than carbon steel).

Resilience is an important property in design where energy absorption is required. Some examples are springs, parts subjected to impact loading, vibrating components, etc.

<table>
<thead>
<tr>
<th>Material</th>
<th>(U) (kg·mm/mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tool Steel (S1)</td>
<td>11000 \cdot 10^{-4}</td>
</tr>
<tr>
<td>Carbon Steel (1040)</td>
<td>306 \cdot 10^{-4}</td>
</tr>
<tr>
<td>Al. Annealed (1100)</td>
<td>8.75 \cdot 10^{-4}</td>
</tr>
<tr>
<td>Brass</td>
<td>147 \cdot 10^{-4}</td>
</tr>
<tr>
<td>Rubber</td>
<td>2100 \cdot 10^{-4}</td>
</tr>
<tr>
<td>Acrylic</td>
<td>28 \cdot 10^{-4}</td>
</tr>
</tbody>
</table>
Special Definitions of Elastic Limit

- Special definitions of elastic limit are introduced for engineering materials without a clear yield point (e.g. cold work steels, nonferrous metals, etc.):

1. **Offset Yield Strength (Proof Stress):** An offset strain (OA), 0.2% for St & Al and 0.5% for Cu and its alloys, is measured from the origin and a line parallel to linear portion of \( \sigma - \varepsilon \) curve is drawn. The intersection of this line with the curve (point P) is offset yield strength (Fig. 10). This method cannot be applied for metals undergoing more than 0.5% elastic strain (equivalent to stress of Sy”).

2. **Johnson’s Apparent Elastic Limit:** A line (OA) with a slope of 50% of initial slope is drawn, and line (xy) is drawn tangent to \( \sigma - \varepsilon \) curve and parallel to OA. Point of tangency (P) gives the elastic limit (Fig. 11). This method usually is not preferred due to greater possibility of inaccuracy as compared with above method.