Measurement and Mathematics

- Measurement constitutes the first steps towards mathematics.
- In other words, associating numbers with physical objects...
- Thus, compare the objects by comparing the associated numbers...

1 FOOT FOR LENGTH
SEEDS AND BEANS FOR WEIGHT
Brief History of Measurement

Egyptians (around 3000 BC)
- Earliest known measurement was **Egyptian cubit**.

Babylonians (around 1700 BC)
- Their weights and measures had a wider influence.
- Their system was approximately same with Egyptians.
- However, there were small differences about lengths.

Harappans (between 2500 BC and 1700 BC)
- They had flourished in Punjab (in India).
- They adopted a uniform system of weights and measures.
- Several scales for the measurement of length were also discovered during excavations.

In summary:
- European system based on Roman measures
- Roman system based on Greek measures
- Greek system based on breadth of finger

Current measurement systems have been derived from 3000 BC.
Anthropic Measurements and Units

Digit: Egyptians - breadth of forefinger
Inch: breadth of thumb
Palm: breadth of four fingers
Span: tip of thumb to tip of little finger (hand spread)
Cubit: Egyptians - elbow to tip of middle finger
Foot: length of man's foot
Yard: King Henry I of England - tip of his nose to end of thumb
Mile: 5000 Roman feet
Pace: Romans - a full stride

and many others...
Early Measurements

But there was a problem!

There were a lot of different lengths which change from human to human.

Solution was to develop a standard measurement: a black granite rod!!

There were 28 digits in a cubit, 4 digits in a palm, 5 digits in a hand, 3 palms (so 12 digits) in a small span, 14 digits (or a half cubit) in a large span, 24 digits in a small cubit, and so on...

But a problem again: How could they measure a measurement smaller than 1 digit?

For this, the Egyptians used measures composed of unit fractions...
**SI System**

- *Le systeme International d’unites* or *International System of Units*
- Also called *Metric System* (i.e. all measurements based on m, kg, s).
- Seven **base** units (m, kg, s, A, °K, mol, cd) and other **derived** units.
- Used throughout the world except USA and some part of UK.

**English (British or Imperial) System**

- Various measurements based on **different units**:
  - Length: inches / feet / yards / miles / ...
  - Mass: ounces / pounds / tones / ...
  - Volume: fluid ounce / cups / quarts / gallons / ...

**Why do we prefer SI system?**

- SI units **work in 10s**, so it is **very easy to convert** big units to small units, or vice versa (e.g. 1 m = 100 cm). However, English units are **not easy to convert**. Can you easily convert mile to inches?
- In SI system, a **unique unit** is used for a quantity (e.g. the unit of distance is meter). However, **various units** may be used in English system (e.g. for distance: inch, foot, yard, mile, and so on.)
1670  **Gabriel Moulton**, a French mathematician, proposes a measurement system based on a physical quantity of nature and not on human anatomy.

1790  **The French Academy of Science** recommends the adoption of a system with a unit of length equal to one ten-millionth of the distance on a meridian between the Earth’s North Pole and equator.

1870  A French conference is set up to work out standards for a unified metric system.

1875  **The Treaty of Meter** is signed by 17 nations (including United States). This has established a permanent body with the authority to set standards.

1893  The United States officially adopts the metric system standards as bases for weights and measures (but continues to use British units).

1975  **The Metric Conversion Act** is enacted by Congress. It states “The policy of the United States shall be to coordinate and plan the increasing use of the metric system in the United States and to establish a United States Metric Board to coordinate the voluntary conversion to the metric system.” (This means that no mandatory requirements are made.)
SI Base Units - Meter

**Definition**

- **Meter (m)** is the unit of length.
- It was originally defined as a unit equal to one ten-millionth part of a quadrant of Earth's meridian as measured from North Pole to Equator.
- It is now precisely equal to the length of path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.

**International Prototype**

- **International Prototype Metre bar** was the standard from 1889 to 1960.
- It is kept at *Bureau International des Poids et Measures* (International Bureau of Weights and Measures) near Paris.

Made of 90% platinum & 10% iridium with "Tresca section" (named after French engineer Henri Tresca) to minimise the effects of torsional strain.
SI Base Units - Kilogram

Definition

- **Kilogram (kg)** is the unit of mass.
- Originally, it was defined as the mass of a volume of pure water equal to a cube of one tenth meter at the temperature of melting ice.
- It is now equal to the mass of international prototype of kilogram.

International Prototype

- There are currently two prototypes: the one kept at *Bureau International des Poids et Measures (BIPM)* in Paris, and the other one at *National Institute of Standards and Technology (NIST)* in Washington D.C.
- **The prototype at BIPM** is made of a platinum alloy known as “Pt-10Ir”, which is 90% platinum and 10% iridium (by mass).
- It was machined into a right-circular cylinder with equal height and diameter (39.17 mm) to minimize its surface area.
- **The prototype at NIST** is actually 0.999 999 961 kg of the one at BIPM.
Mass vs Weight

- **Mass** is **stuff** (a quantity of matter) regardless of what it is made of, any number of other conditions (including shape, temperature, color), where it is, and how it moves (disregarding relativistic effects).

- **Weight** is **force** (specifically gravitational force) acting on the stuff in a special way. It is the force necessary to cause the stuff to accelerate at gravitational acceleration of 9.81 m/s² (32.174 ft/s²).

mass\:(kg\ or\ \textit{lb})\ \text{remains constant}

but

weight\:(kgf\ or\ \textit{lbf})\ \text{varies}

1 kg of cotton and 1 kg of iron are **EQUAL** in mass

(NOT EQUAL in weight due to effects of gravity and geometry)
**Definition**

- **Second (s)** is the unit of **time**.
- It was originally defined as $1/86\,400$ of a solar day.
- Today, it is equal to the duration of $9\,192\,631\,770$ periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of cesium-133 atom.
- The frequency of a specific transition between two of cesium's energy levels can be accurately monitored, and thus it is used to define the second.
Ampere

- **Ampere (A)** is the unit of *electric current*.
- Being maintained in two straight parallel conductors of infinite length with negligible circular cross section and placed 1 m apart in vacuum, it is equal to that constant current which would produce a force (between these conductors) equal to $2 \times 10^{-7}$ Newton per meter of length.

Kelvin

- **Kelvin (°K)** is the unit of *thermodynamic temperature*.
- It is equal to the fraction of thermodynamic temperature of triple point of water (i.e. the point where water exists in gaseous, liquid and solid phases simultaneously) at 273.16 °K (0.01 °C, 32.02 °F).

Mole

- **Mole (mol)** is the unit of *the amount of substance of a system*.
- It is equal to the amount of substance of a system which contains as many elementary entities (i.e. atoms, molecules, ions, electrons) as there are atoms in 0.012 kilogram of carbon-12.

Candela

- **Candela (cd)** is the unit of *luminous intensity*.
- It is equal to luminous intensity of a source in a given direction that emits monochromatic radiation of frequency (540 x 1012 hertz) and that has a radiant intensity of $1/683$ watt per steradian.
### SI Coherent Derived Units

<table>
<thead>
<tr>
<th><strong>SI Base Units</strong></th>
<th><strong>SI Coherent Derived Units without Special Names</strong></th>
<th><strong>SI Coherent Derived Units with Special Names and Symbols</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>kilogram (kg)</td>
<td>mass (kg)</td>
<td>force (N)</td>
</tr>
<tr>
<td>meter (m)</td>
<td>length (m)</td>
<td>pressure (Pa)</td>
</tr>
<tr>
<td>second (s)</td>
<td>time (s)</td>
<td>heat flow rate (W)</td>
</tr>
<tr>
<td>mole (mol)</td>
<td>amount of substance (mol)</td>
<td>activity (Bq)</td>
</tr>
<tr>
<td>ampere (A)</td>
<td>electric current (A)</td>
<td>inductance (H)</td>
</tr>
<tr>
<td>kelvin (K)</td>
<td>thermodynamic temperature (K)</td>
<td>magnetic flux density (T)</td>
</tr>
<tr>
<td>candela (cd)</td>
<td>luminous intensity (cd)</td>
<td>voltage (V)</td>
</tr>
</tbody>
</table>

Mathematical symbols and units are connected by lines indicating multiplication (solid) or division (broken lines). Special names and symbols are denoted in parentheses.

- **N**: newton (kg·m/s²)
- **Pa**: pascal (N/m²)
- **Gy**: gray (J/kg)
- **Sv**: sievert (J/kg)
- **Hz**: hertz (1/s)
- **Bq**: becquerel (1/s)
- **Wb**: weber (V·s)
- **H**: henry (Wb/A)
- **T**: tesla (Wb/m²)
- **C**: coulomb (A·s)
- **V**: volt (W/A)
- **F**: farad (C/V)
- **Ω**: ohm (V/A)
- **S**: siemens (1/Ω)
- **lx**: lux (lm/m²)
- **lm**: lumen (cd·sr)
- **sr**: steradian (m²/m² = 1)
- **rad**: radian (m/m = 1)

*Image credit: National Institute of Standards and Technology (NIST)*
### SI Prefixes (Multipliers)

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>$10^n$</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>yotta</td>
<td>Y</td>
<td>$10^{24}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>zetta</td>
<td>Z</td>
<td>$10^{21}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>exa</td>
<td>E</td>
<td>$10^{18}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>peta</td>
<td>P</td>
<td>$10^{15}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^{9}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^{6}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>kilo</td>
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<tr>
<td>deca</td>
<td>da</td>
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</tr>
<tr>
<td>-</td>
<td>-</td>
<td>$10^{0}$</td>
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<tr>
<td>deci</td>
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</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>$10^{-2}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>micro</td>
<td>u</td>
<td>$10^{-6}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
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<td>pico</td>
<td>p</td>
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<tr>
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<td>a</td>
<td>$10^{-18}$</td>
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</tr>
<tr>
<td>zepto</td>
<td>z</td>
<td>$10^{-21}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
<tr>
<td>yocto</td>
<td>y</td>
<td>$10^{-24}$</td>
<td>1 000 000 000 000 000 000 000 000 000</td>
</tr>
</tbody>
</table>

Note: The multipliers are given in scientific notation, where each digit represents a power of 10.
## Conversion of Units

<table>
<thead>
<tr>
<th>ENGLISH UNITS</th>
<th>ENGLISH EQUIVALENTS</th>
<th>METRIC EQUIVALENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 INCH</td>
<td>12 INCHES</td>
<td>= about 2-1/2 CENTIMETERS</td>
</tr>
<tr>
<td>1 FOOT</td>
<td>12 INCHES = 12 INCHES</td>
<td>= about 30 CENTIMETERS</td>
</tr>
<tr>
<td>1 YARD</td>
<td>3 FEET = 36 INCHES</td>
<td>= about 1 METER</td>
</tr>
<tr>
<td>1 HAND</td>
<td>about 4 INCHES</td>
<td>= about 10 CENTIMETERS</td>
</tr>
<tr>
<td>1 CUBIT</td>
<td>about 1/2 YARD</td>
<td>= about 46 CENTIMETERS</td>
</tr>
<tr>
<td>1 BRACCIO</td>
<td>15 to 39 INCHES</td>
<td>= about 1/2 to 1 METER</td>
</tr>
<tr>
<td>1 FATHOM</td>
<td>6 FEET</td>
<td>= about 2 METERS</td>
</tr>
<tr>
<td>1 MILE</td>
<td>5,280 FEET</td>
<td>= about 1-1/2 KILOMETERS</td>
</tr>
<tr>
<td>1 OUNCE</td>
<td>16 OUNCES</td>
<td>= about 28 GRAMS</td>
</tr>
<tr>
<td>1 POUND</td>
<td>16 OUNCES</td>
<td>= about 1/2 KILOGRAM</td>
</tr>
<tr>
<td>1 TEASPOON</td>
<td></td>
<td>= about 5 MILLILITERS</td>
</tr>
<tr>
<td>1 TABLESPOON</td>
<td>3 TEASPOONS</td>
<td>= about 15 MILLILITERS</td>
</tr>
<tr>
<td>1 CUP</td>
<td>16 TABLESPOONS</td>
<td>= about 250 MILLILITERS</td>
</tr>
<tr>
<td>1 QUART</td>
<td>4 CUPS</td>
<td>= about 1 LITER</td>
</tr>
<tr>
<td>1 GALLON</td>
<td>4 QUARTS</td>
<td>= about 4 LITERS</td>
</tr>
</tbody>
</table>


Almost all developed and developing countries have a **National Metrology Institute (NMI)**, having relations with **BIPM**.

The main objectives of NMIs are **to build and maintain national standards for all measurements carried out within that country and to calibrate the measurement standards and devices of lower level laboratories**.

This chain extends to production, quality control and to all scientific, commercial and military devices which are used for measurements.

This way, all measurements are traceable to the national standards. National standards are linked to the standards in other countries or those of BIPM by a process of international comparisons. This is called **traceability**.

**Ulusal Metroloji Enstitüsü (UME)** of Turkey was established in **1992** as part of TÜBİTAK (Scientific and Technical Research Council of Turkey).

**List of NMIs in the world can be found at**: [www.ume.tubitak.gov.tr/menu_links.php?f=301](http://www.ume.tubitak.gov.tr/menu_links.php?f=301)
The studies related to international measurement system had started on **25 May 1875** when Miralay Husnu Bey had signed the **Meter Convention** on behalf of the Ottoman Empire.

Though the Ottoman Empire was one of the 20 states who were the founders of Meter Convention, there were no significant development in this field until **the law of Weights and Measures (law no 1781)** has been put in act on **26 March 1931**.

After the second world war, need for an integrated system of metrology was felt strongly in Turkey. On the other hand, the volume of the market for calibrations was not large enough to justify a major investment in metrology until 1980.

The Prime Ministry of Turkey has asked TÜBİTAK to establish the national measurement system in the early eighties. Initial studies began in 1982, the feasibility study was accepted by all the relevant parties in Turkey, and **UME was founded in 1992** as part of TÜBİTAK.
Accredited by **Turkish Accreditation Agency (TÜRKAK)** according to **TS EN ISO/IEC 17025**.

Being a signatory of **CIPM Mutual Recognition Arrangement**, 87 institutes and 3 international organizations recognize the calibration certificates and measurement reports issued by UME.

Member of **EURAMET, MENAMET, EURACHEM, and IMEKO**.
Useful Links

- Bureau International des Poids et Measures (BIPM):
  www.bipm.org

- International Organization for Standardization (ISO):
  www.iso.org

- European Association of National Metrology Institutes (EURAMET):
  www.euramet.org

- Ulusal Metroloji Enstitüsü (UME):
  www.ume.tubitak.gov.tr

- Türk Akreditasyon Kurumu (TÜRKAK):
  www.turkak.org.tr

- Online unit converter:
  www.digitaldutch.com/unitconverter

- Scale of universe:
  http://scaleofuniverse.com