Problems

Binary Number System

1. Convert the following binary numbers to the decimal numbers
   a) 0.11001, b) 00101100, c) 10.01
2. Convert the following decimal numbers to the binary numbers
   a) 0.125, b) 128, c) 15.018

Digital Logic

1. Construct the Truth Table of AND, OR and NOT Gates.
2. Construct NAND and NOR Gates using relevant AND, OR and NOT Gates.
3. Construct Truth Table of the XOR Gates. (For the inputs A and B, XOR gate implements the logical expression A.B’+A’.B)
4. What is the output of the following circuit for the binary inputs?
5. State four basic postulates of the quantum mechanics.

6. For the states given states, determine $a$.
   a) $|\psi\rangle = a|0\rangle + 0.5|1\rangle$
   b) $|\psi\rangle = a|0\rangle + i0.5|1\rangle$
   c) $|\psi\rangle = a|0\rangle - |1\rangle$
   d) $|\psi\rangle = a|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

7. What is the unitary matrix? Write down Pauli X, Y and Z matrices and show that they are unitary.

8. Show that the following matrices are unitary for real $\alpha$.

   $R_x(\alpha) = \exp(-i\alpha X/2) = \begin{pmatrix} \cos(\frac{\alpha}{2}) & -i \sin(\frac{\alpha}{2}) \\ -i \sin(\frac{\alpha}{2}) & \cos(\frac{\alpha}{2}) \end{pmatrix}$

   $R_y(\alpha) = \exp(-i\alpha Y/2) = \begin{pmatrix} \cos(\frac{\alpha}{2}) & -\sin(\frac{\alpha}{2}) \\ \sin(\frac{\alpha}{2}) & \cos(\frac{\alpha}{2}) \end{pmatrix}$

   $R_z(\alpha) = \exp(-i\alpha Z/2) = \begin{pmatrix} e^{-i\alpha/2} & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix}$

   $Ph(\delta) = e^{i\delta} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

9. Determine output of the following quantum circuits
10. Transformation matrix of CNOT gate is given by
\[
S = \begin{pmatrix}
1 & 0 \\
0 & i
\end{pmatrix}
\]
Determine transformation of states.
(Direct sum and Direct Product)

11. Construct transformation matrix for SWAP gate.

FREDKIN gate
The icon for the FREDKIN gate as shown in the figure:

Truth table of the fredkin gate are given by

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x) y z</td>
<td>(x') y' z'</td>
</tr>
<tr>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0 0 1</td>
</tr>
<tr>
<td>0 1 0</td>
<td>0 1 0</td>
</tr>
<tr>
<td>0 1 1</td>
<td>0 1 1</td>
</tr>
</tbody>
</table>
Transformation of the state by using FREDKIN gate is given by:

\[
\begin{array}{c|c}
\text{011} & \text{011} \\
\text{100} & \text{100} \\
\text{101} & \text{110} \\
\text{110} & \text{101} \\
\text{111} & \text{111} \\
\end{array}
\]