1-(15 points) Consider a system whose output $y(n)$ is related to the input $x(n)$ by

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) x(n + k)$$

Determine whether or not the system is
a) memoryless, b) causal, c) linear, d) shift-invariant, e) stable.

2-(10 points) The response of an LTI system to a step input, $x(t) = u(t)$, is $y(t) = (1 - e^{-2t}) u(t)$.

What is the response of the system to an input of $x(t) = 4u(t) - 4u(t - 1)$?

3-(15 points) A linear time invariant system has the following impulse response:

$$h(t) = 2e^{-t} u(t)$$

Use convolution integral to find the response $y(t)$ to the following input:

$$x(t) = u(t) - u(t - 4)$$

4-(20 points) Consider the following finite-length sequences:

$$x(n) = 0.5n[u(n) - u(n - 6)]$$

$$h(n) = 2\sin\left(\frac{n\pi}{2}\right)[u(n + 3) - u(n - 4)]$$

a) (10 points) Make a sketch and label each of $x(n)$ and $h(n)$.

b) (10 points) Find the convolution of $x(n)$ and $h(n)$.

5-(15 points) Consider a continuous-time system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

with $x(t) = 2u(t)$ and $y(0) = 4$

Express $y(t)$ in terms of the zero-input and zero-state responses.

6-(15 points) An LTI system has the impulse response

$$h(t) = u(t + 1) - u(t - 3)$$

a) (5 points) Determine whether or not this system is causal.

b) (5 points) Determine whether or not this system is stable.

c) (5 points) Find the system response to the input

$$x(t) = \delta(t - 1) - 2\delta(t + 1)$$

7-(10 points) Obtain the state-space representation of the following circuit with $L=0.5\text{H}$, $R=6\text{ohm}$, $C=0.25\text{F}$ and the input current $i(t)$ and considering $y(t)$ as the output.

NOTE: YOU SHOULD SHOW/EXPLAIN YOUR WORK TO RECEIVE FULL CREDIT. THE CORRECT ANSWER WITH NO SUPPORTING WORK MAY RESULT IN NO CREDIT.