1-(20p) You are given a system with impulse response \( h(t) = e^t u(t) \).

a)(5p) Is the system denoted by \( h(t) \) bounded-input bounded-output (BIBO) stable?

b)(10p) You now hook the system up into a feedback system, as shown in figure below. Find the new transfer function, \( H_{\text{new}}(s) \), from the input \( x(t) \) to the output \( y(t) \).

c) (5p) Find the range of parameter \( A \) such that the new feedback system is bounded-input bounded-output (BIBO) stable?

\[ x(t) \xrightarrow{\Sigma} w(t) \xrightarrow{h(t)} y(t) \]

2-(20p) You are given a transfer function:

\[ H(s) = \frac{1}{(s + a)(s + b)} \]

where \( H(s) \) is the Laplace transform of \( h(t) \), an impulse response of a system, and "\( a \)" and "\( b \)" are real constants and \( a > b \).

a)(10p) If \( h(t) \) were **causal**, over what range of values of "\( a \)" and "\( b \)" would the system be BIBO stable?

Determine also the Region of Convergence (ROC) for both causal and BIBO stable system.

b)(10p) If \( h(t) \) were **noncausal**, over what range of values of "\( a \)" and "\( b \)" would the system be BIBO stable?

Determine also the Region of Convergence (ROC) for both noncausal and BIBO stable system.

3-(20p) For the system given below and assuming \( f(t) \) as the input and \( y(t) \) as the output;

a)(6p) Find the system transition matrix.

b)(8p) Find the system state variables.

c)(6p) Find the system output response \( (y(t)) \) due to the system input \( f(t) = u(t) \).

\[
\begin{bmatrix}
\frac{dx_1(t)}{dt} \\
\frac{dx_2(t)}{dt}
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + 
\begin{bmatrix}
0 \\
1
\end{bmatrix} f(t)
\]

\[ x_1(0^-) = [0] \quad x_2(0^-) = [1] \quad y(t) = [4 \quad 5] x_1(t) x_2(t) \]

4-(20p) Find the following inverse z-transforms and z-transforms (with corresponding ROCs):

a)(5p) \( X(z) = 4z^2 + 2 + 3z^{-1} \), \( 0 < |z| < \infty \).

b)(5p) \( H(z) = \frac{2z^2 - 5z}{z^2 - \frac{5}{2}z + 1} \), \( |z| > 2 \).

c)(5p) \( x(n) = u(n) - u(n - 10) \).

d)(5p) \( h(n) = (\frac{1}{3})^n u(-n) \).

5-(20p) Consider an LTI system for which the input \( x(n) \) and the output \( y(n) \) satisfy the linear constant-coefficient difference equation

\[ y(n) - \frac{1}{2} y(n - 1) = x(n) + \frac{1}{3} x(n - 1) \]

a)(10p) Determine the transfer function \( H(z) \).

b)(10p) Find the impulse response \( h(n) \) of the system assuming that the system is **causal**.
### Laplace Transform:

<table>
<thead>
<tr>
<th>$X(s)$</th>
<th>$x(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$\delta(t)$</td>
</tr>
<tr>
<td>$\frac{1}{s}$</td>
<td>$u(t)$</td>
</tr>
<tr>
<td>$\frac{1}{s^2}$</td>
<td>$tu(t)$</td>
</tr>
<tr>
<td>$\frac{k!}{s^{k+1}}$</td>
<td>$t^ku(t)$</td>
</tr>
<tr>
<td>$\frac{1}{s + a}$</td>
<td>$e^{-at}u(t)$</td>
</tr>
<tr>
<td>$\frac{1}{(s + a)^2}$</td>
<td>$te^{-at}u(t)$</td>
</tr>
<tr>
<td>$\frac{s}{s^2 + \omega^2}$</td>
<td>$\cos(\omega t)u(t)$</td>
</tr>
<tr>
<td>$\frac{\omega}{s^2 + \omega^2}$</td>
<td>$\sin(\omega t)u(t)$</td>
</tr>
</tbody>
</table>

### Z-Transform:

<table>
<thead>
<tr>
<th>$X(z)$</th>
<th>$x(n)$</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$\delta(n)$</td>
<td>All $z$</td>
</tr>
<tr>
<td>$\frac{z}{z - 1}$</td>
<td>$u(n)$</td>
<td>$</td>
</tr>
<tr>
<td>$\frac{z}{z - 1}$</td>
<td>$-u(-n - 1)$</td>
<td>$</td>
</tr>
<tr>
<td>$z^{-m}$</td>
<td>$\delta(n - m)$</td>
<td>All $z$ except zero if $m &gt; 0$ or $\infty$ if $m &lt; 0$</td>
</tr>
<tr>
<td>$\frac{z}{z - a}$</td>
<td>$a^n u(n)$</td>
<td>$</td>
</tr>
<tr>
<td>$\frac{z}{z - a}$</td>
<td>$-a^n u(-n - 1)$</td>
<td>$</td>
</tr>
<tr>
<td>$\frac{az}{(z - a)^2}$</td>
<td>$na^n u(n)$</td>
<td>$</td>
</tr>
<tr>
<td>$\frac{az}{(z - a)^2}$</td>
<td>$-na^n u(-n - 1)$</td>
<td>$</td>
</tr>
</tbody>
</table>

Laplace Transform:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} \, dt$$

Z-Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

For a given state equations and output equations ($x$ is state vector, $u$ is the input and $y$ is the output vector):

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

$$x(t_0) = x_0$$

$$x = \phi(t - t_0)x_0 + \int_{t_0}^{t} \phi(t - \tau)Bu(\tau)\,d\tau$$

$$y = C\phi(t - t_0)x_0 + \int_{t_0}^{t} C\phi(t - \tau)Bu(\tau)\,d\tau + Du$$

**NOTE:** YOU SHOULD SHOW/EXPLAIN YOUR WORK TO RECEIVE FULL CREDIT.

**THE CORRECT ANSWER WITH NO SUPPORTING WORK MAY RESULT IN NO CREDIT.**